Multi-criteria Decision making in Components
Importance Analysis applied to a Complex Marine System

Summary
This paper presents a possible application of the Analytic Hierarchy Process (AHP) in a multi-criteria analysis of components importance to a system reliability structure illustrated by a stern tube sealing system installed on deep sea ships. We used the AHP to determine the relevance of the importance ranks and made an importance analysis for the criteria of reliability, safety and repair costs. We compared our results with the measures analysed and presented in literature before. We also presented some conclusions on using the AHP for multi-criteria analysis of components importance in Complex Technical Systems (CTS). We finished with suggestions for further research on quantitative and qualitative methods of importance analysis.

Sažetak
U radu je predstavljena moguća primjena AHP (analitičkog hijerarhijskog procesuiranja) u multikriterijskoj analizi značajnosti komponenti za strukturu pouzdanosti sustava, koja je prikazana na primjeru sustava brtvljenja statvene cijevi ugrađene na prekooceanske brodove. Koristio se AHP kako bi se odredila relevantnost razine značajnosti te je analizirana značajnost za kriterije pouzdanosti, sigurnosti i troškove popravaka. Rezultati su uspoređeni s vrijednostima mjerenja koje su analizirane i prikazane u ranijoj literaturi. Također su izneseni zaključci o upotrebi AHP-a za multikriterijsku analizu značajnosti komponenti u kompleksnim tehničkim sustavima (CTS). Na kraju rada dane su implikacije za daljnja kvantitativna i kvalitativna istraživanja analize značajnosti.

1. INTRODUCTION / Uvod
Reliability theory concentrates on the operation of systems, both in terms of statistics and physics of failure, and is effective when it comes to determining measures for reliability, availability and safety. With regard to a system as a whole, basic dependability measures are important information as for intact system operation but as far as system components go, these measures give very general information on their vulnerability and, except for a series reliability structure, are unable to describe the impact of a component on the whole system. The system tolerance for its components failure depends on their reliability and the structure of the system where a particular component is located [2].

Very often while analysing failure consequences for technical system components, the analyst has no detailed statistical knowledge concerning the vulnerability of given components, repair costs and quantitative indices describing failure consequences [23]. In such situations it seems useful to use expert knowledge applied to a qualitative analysis of systems components importance, e.g. by means of the method of comparison of alternatives for selected components. The AHP method (Analytic Hierarchy Process) gives such a possibility and allows the opportunity to obtain quantitative results on the basis of qualitative information. The method has its limitations though which have been indicated by its critics since publishing it [1, 10, 21] where multiplication of alternatives was presented as disrupting the decision process. In the published literature many applications and modifications removing the limitations of the AHP method were described [4, 8] but they will not be discussed here in detail because they are outside the scope of this paper.

The AHP method has been used in many areas [3, 12, 16, 20], such as: management, political science, transport, sociology or...
manufacturing engineering. It is impossible to characterize all its applications, for instance in [20] the number of 291 papers describing its different applications was presented. In particular the attention could be paid to papers discussing CTS risk analysis. In the published literature we meet two approaches of risk management:

- making decisions under risk: in a situation when we know the scenario and we can objectively estimate its probability,
- making decisions under uncertainty: in a situation when we know the scenario however the objective estimation of its probability is not possible.

Because of lack of field data, reliability analysis very often belongs to the latter and that is how the AHP might be used. The decision making process under uncertainty might take place using many criteria, e.g. Laplace ("When there is no information on the probability of the analysed scenarios, it must be assumed that they are equally probable" – the approach was followed in this paper). Equally, both Hurwicz criterion (using subjective coefficient of optimism) and Savage criterion (mineralization of the maximum regret) might be used for further research.

It has also been used in the theory of decision making to select a given product from among a wide assortment according to selected criteria of product importance evaluation. As proposed in [6], the method may well be used to indicate the most important components \( E_i \) in a system, i.e. components belonging to many assortment groups \([5, 11, 14, 15, 23]\). Such an attitude allows us to indicate the most important components according to selected importance criteria \( k_i \) (Fig. 1).

The AHP method can be applied in the components importance analysis for the system reliability structure by using it in two stages:

1. determination of components importance criteria and their relations in terms of their ability to be quantified. This allows to build a multi-criteria model of system components importance with calculated weight coefficients for the criteria of reliability, safety, repair costs of the component which failed and for other components,
2. determination of mutual importance relation between system components according to all analysed criteria which allows to obtain the final importance ranking of system components.

By means of grades aggregation in the model hierarchical structure, an aggregated values matrix is created and is the basis for the creation of a reduced grades matrix giving final grades determined for the main criteria, main alternatives and grading alternatives [9].

2. THE OBJECT OF ANALYSIS / Predmet analize

The components importance analysis in a complex technical system was conducted on the example of one of the subsystems of a marine power plant: stern tube seals lubricating oil system of a container vessel [24]. The system diagram is shown in Fig. 2.

The reliability structure of the system was modelled, using reliability block diagrams, as shown in Fig. 3. The structure of the assumed level of decomposition refers to the main components of the system specified in terms of their function in the system and selected as a separate machine or device.

<table>
<thead>
<tr>
<th>Importance ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
</tr>
</tbody>
</table>

\( k \) is the criterion

\( E_1 \) \( E_2 \) \( E_3 \) \( E_4 \) \( E_5 \) \( E_6 \) is the component

The basic reliability data of the system components have been summarized in Table 1. It was assumed that all the components are renewable objects.

The distribution of time to failure is exponential with \( \lambda \) parameter [failure/h]. The repair process is described by mean time to repair \( T_R \) [h]. The accepted failure rate and the average duration of restoration have been taken from [6]. The filter - pump system is duplicated, the analysis has assumed an average value of failure and restoration process parameters due to the periodic replacement of these devices between the operating and stand-by systems.

It was assumed that both sub-branches (pumping systems) break down in the same way. A similar assumption relating to the gravity oil tanks has been made.
3. Multi-Criteria Analysis of Components Importance by Means of AHP / Multikriterijska analiza značajnosti komponenti s pomoću AHP-a

For the object analysis (lubricating oil system for the propeller shaft stern tube seal), 3 components importance criteria have been taken into account: reliability, safety and costs effectiveness.

The criteria have been selected in a way that allowed us to unify their mutual evaluation. Their character was matched for the evaluation process to consider their maximising:

1. reliability - makes the system certain to operate despite failure of a given component – greater reliability is connected with longer periods in between the planned maintenance work,
2. safety – understood as an inverse proportion of negative consequences for the system operation, connected with component failure – greater safety means lesser hazard for the staff, environment and the system itself,
3. cost effectiveness – understood as a characteristic inversely proportional to system repair costs (spare parts, manpower and system operation interruption costs) connected with failure of a given component. Greater cost effectiveness means smaller restoration costs.

The mutual verbal evaluation of relations between criteria has been created on the basis of the opinions given by specialists of technical systems operation. The Saaty scale has been used to create a quantitative evaluations matrix for mutual relations. Following, a mutual relevance matrix for the analysed criteria has been created which is presented in Table 2.

After normalizing the matrix, criteria relevance coefficients were obtained. They are shown in Table 3.

Table 3 Importance criteria relevance [7]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Sum</th>
<th>Weight</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability</td>
<td>9.0000</td>
<td>0.1061</td>
<td>0.9554</td>
</tr>
<tr>
<td>Safety</td>
<td>1.5333</td>
<td>0.6333</td>
<td>0.9711</td>
</tr>
<tr>
<td>Cost effectiveness</td>
<td>4.3333</td>
<td>0.2604</td>
<td>1.1288</td>
</tr>
</tbody>
</table>

We can notice that, relying on expert opinions, it has to be taken into account that parameter safety equals over 63% of relevance, cost effectiveness 26%, while reliability less than 11%.

Consistency ratio CR for the matrix equals 0.0532 and it allows to assume that the matrix is consistent (value CR<0.1 is required [19]). The research can be continued.

The obtained results correlate with common sense interpretation of reality where safety is the most significant and reliability can be considered less important if the repair/component exchange costs are not high.

3.1. Components importance and their failure influence on the system reliability / Značajnost komponenti i utjecaj kvara na pouzdanost sustava

In the next part of the analysis, the relevance of given system components has been evaluated with regard to their influence on the reliable system operation. After normalising the matrix of a relative mutual relations matrix of particular components according to the criterion reliability, relevance parameters of given components were obtained according to the criterion reliability as shown in Table 4.
We can notice that according to the criterion reliability, E6 (with over 27% relevance) is the most significant system component and also E4, E5 and E6 (almost 12% relevance). The influence of other components on the system operational safety is relatively little – below 5.2% and less. Consistency ratio CR for the input data equals 0.0358 and it allows to assume that the matrix is consistent.

### 3.3. Components importance and their failure influence on the operational costs / Značajnost komponenti i utjecaj kvara na operativne troškove

After normalizing the matrix of particular components according to the cost effectiveness, relevance parameters of given components were obtained according to the criterion cost effectiveness as shown in Table 6.

### 3.4. Aggregated system components importance / Ukupna značajnost komponenti sustava

The last part of the analysis is to indicate an aggregated measure describing the system components relevance considering all the criteria simultaneously. Table 7 shows the multi-criteria ranking of components importance.

The aggregated relevance evaluation shows that E1, whose relevance is almost 37%, is undoubtedly the most significant system component considering all the criteria. The relevance of E6 and E4 is over 11%. The aggregated system components importance is presented in Fig. 3.
The results prove that the propeller shaft stern tube seal with bearings is the most significant component when it comes to the aggregated relevance evaluation. Next, about 3 times smaller component importance value is for the pipeline and equipment as well as oil cooler. Other ranking positions are for the lube oil sump tank, pumps, gravity tanks and filters.

4. COMPARATIVE RESULTS ANALYSIS / Komparativna analiza rezultata

Major in [6] an identical technical system was considered where selected importance criteria and multi-criteria analysis of components importance were used. The author assumed the importance value according to the criterion safety which was obtained taking into account the opinion of 30 experts (marine officer engineers with at least 5 years of professional experience on see vessels which were selected at random out of 70 experts). The assumed number is eligible to be qualified as representative and which is:

$$I_{SAFETY}(i) = \frac{K(i)}{\max_{k=1...n} K(k)}$$  \hspace{1cm} (4.1)$$

where:

$K$ is the average value of measure describing failure consequences determined by experts, 0 - no impact of component failure on the emergency state of the system safety, 2 - low severity, 5 - medium severity, accident of an operator possible, 8 – high severity for the service and the environment, fatal accident risk 10 – very high severity, loss of life by the crew possible, risk of sinking of the ship, environmental disaster possible. $n$ - number of system components.

The value of the importance measure $I_{SAFETY}(i)$ for the analysed system is shown in Fig. 4.

R. E. Barlow and F. Proschan proposed an importance measure independent of lifetime. The Barlow-Proschan’s measure is equal to probability that the cause of the system failure lies in the i-th component changing its state to failed. This measure can be treated as the average Birnbaum’s measure with regard to the component.

$$I_{B-P}(i) = \int_{0}^{\infty} f_i(t) \cdot I^B (i \mid t) \cdot dt$$  \hspace{1cm} (4.2)$$

Figure 3 Aggregated system components importance in percentage considering all criteria (own analysis)
Slika 3. Ukupna značajnost komponenti sustava u postocima, uzimajući u obzir sve kriterije (izradili autori)

Figure 4 The component importance measure considering failure consequences in operational safety (own analysis)
Slika 4. Izmjerene vrijednosti značajnosti komponente, uzimajući u obzir posljedice kvara na operativnu sigurnost (izradili autori)
where:
\( f_i \) – probability density function of time to failure distribution for i-th component;
\( \frac{\partial R(t)}{\partial t} \) – Birnbaum’s reliability importance measure;
\( R(t) = [r_1(t), r_2(t), ..., r_n(t)] \) – system component reliability vector at the moment \( t \);
\( R(t) \) – system reliability.

In [6] a two-criteria components importance \( I_{SAFETY-BP} \) was determined as:

\[
I_{SAFETY-BP}(i) = I^{B-P}(i) \cdot I^{SAFETY}(i) \quad (4.3)
\]

The two-criteria components importance of the analysed system so defined was shown in Fig. 5.

The results of the two-criteria importance analysis for reliability and safety show that, in this case, pipelines with equipment are the most important component. Then, there is propeller shaft stern tube seal with bearings and oil cooler. Lower in the ranking there are pumps, lube oil sump tank, filters to end with gravity tanks.

Table 8 presents importance measures for system components considering the mentioned criteria and weighted sum of criterion \( I^{SAFETY} \) and \( I^{B-P} \) for weights equal to \( w_1=0.8 \) and \( w_2=0.2 \) respectively:

\[
I^{SAFETY-BP}(i) = \sum_{i=1}^{n} w_i I_i = \frac{n}{\sum_{i=1}^{n} w_i} \quad (4.4)
\]

where: \( I_i=I^{SAFETY} \), \( I_i=I^{B-P} \).

To determine the correlation of results series obtained by means of the AHP method demonstrated in Table 10 and measure values shown in Table 8, Pearson linear correlation coefficient values \( r_{XY} \) for particular series were calculated:

\[
r_{XY} = \frac{\text{cov}(I_{AHP}, Y)}{\sigma_{I_{AHP}} \sigma_Y} \quad (4.5)
\]

where:
\( \text{cov}(I_{AHP}, Y) \) - co-variation of random variables \( I_{AHP} \) and \( Y \),
\( \sigma_{I_{AHP}} \) - standard deviation of random variable \( I_{AHP} \),
\( \sigma_Y \) - standard deviation of random variable \( Y \).

Table 9 shows the correlation coefficients results.

Table 9 Pearson linear correlation coefficient values for determining correlation between \( I_{AHP} \) with other measures [7]

<table>
<thead>
<tr>
<th>Random variable Y</th>
<th>( r_{XY} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I^{SAFETY} )</td>
<td>0.95</td>
</tr>
<tr>
<td>( I^{B-P} )</td>
<td>0.13</td>
</tr>
<tr>
<td>( I^{SAFETY-BP})</td>
<td>0.57</td>
</tr>
<tr>
<td>( I^{S-B-P} )</td>
<td>0.95</td>
</tr>
</tbody>
</table>

According to the criteria relevance shown in Table 3, safety was ranked highest – 63%. Hence, the aggregated system components importance has the values which are most dependent on the importance measure considering operational safety. The results for the aggregated system components values obtained by means of the AHP method, prove that there is a

Table 8 Measure values of component importance [7]

<table>
<thead>
<tr>
<th>Component</th>
<th>( E_1 )</th>
<th>( E_2 )</th>
<th>( E_3 )</th>
<th>( E_4 )</th>
<th>( E_5 )</th>
<th>( E_6 )</th>
<th>( E_7 )</th>
<th>( E_8 )</th>
<th>( E_9 )</th>
<th>( E_{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I^{SAFETY} )</td>
<td>1.0000</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.5000</td>
<td>0.5000</td>
<td>0.5000</td>
<td>0.2000</td>
<td>0.2000</td>
<td>0.2000</td>
<td>0.2000</td>
</tr>
<tr>
<td>( I^{B-P} )</td>
<td>0.0800</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.1700</td>
<td>0.0350</td>
<td>0.2400</td>
<td>0.1950</td>
<td>0.1950</td>
<td>0.0500</td>
<td>0.0500</td>
</tr>
<tr>
<td>( I^{SAFETY-BP})</td>
<td>0.0800</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0850</td>
<td>0.0175</td>
<td>0.1200</td>
<td>0.0390</td>
<td>0.0390</td>
<td>0.0100</td>
<td>0.0100</td>
</tr>
<tr>
<td>( I^{S-B-P} )</td>
<td>0.8160</td>
<td>0.2402</td>
<td>0.2402</td>
<td>0.4340</td>
<td>0.4070</td>
<td>0.4480</td>
<td>0.1990</td>
<td>0.1990</td>
<td>0.1700</td>
<td>0.1700</td>
</tr>
</tbody>
</table>
big correspondence with the importance measures obtained for the criterion safety by means of the method presented in ref. 13 based on the expert opinions and proposed importance measure (relation 4.1). The comparison of the two-criteria components importance shows a difference (Pearson linear correlation coefficient equal to 0,57). It is caused by assuming in [6, 7] that the multi-criteria measure of components importance is the product of Barlow-Proshan’s reliability measure and safety measure. Using the product results in a sharp decrease of value where one of the product factors is close to 0 (like for components 2, 3, 5).

For the weighted mean value presented in this paper (relation 4.2), the two-criteria components importance for weights equal to: 0.8 – criterion safety and 0.2 – criterion reliability, the correlation between the results obtained by means of the AHP method shows a similarity in the results (Pearson linear correlation coefficient equal to 0,95). It proves the usefulness of the AHP method for the multi-criteria analysis of CTS components importance.

5. CONCLUSIONS / Zaključci

The fact that the AHP method gives a possibility to obtain importance measure values useful in practice and allowing to analyse the importance of CTS components, and that the method is simple, it can be widely applied in dependability theory. Its particular advantage is the possibility it gives to quantify qualitative measures, such as the importance measures according to the criterion safety shown in the paper, which often are difficult to be described by means of numbers.

It must be underlined that using the AHP method requires to calculate the consistency ratio CR every time which makes it possible to simultaneously evaluate the mutual relations matrix obtained on the basis of expert opinions.

Further research in the topic area shown in this paper will be conducted with the use of other methods [13, 17, 18, 22] allowing to combine qualitative and quantitative measures for the purpose of the multi-criteria analysis of components importance for CTS in their reliability structure. Additionally, other analyses will be carried out but for more criteria including reliability, safety, spare parts availability in stock, repair costs effectiveness, ergonomics and maintainability (available spare parts and manpower).

What is more, the problems connected with the group decision area are minimized because for calculating consistency ratio CR, average expert opinions are considered. When CR is too high, obviously there are some conflicts in the analysed data which can be corrected at the very start. Owing to that, analyses carried out by means of other than the AHP methods, can be made on the basis of correct, reliable data.

Acknowledgment / Zahvala

The author’s research presented in this article was carried out under the Grant NCN 2011/01/D/ST8/07827: “Importance analysis of components in reliability structure of complex technical systems illustrated by a marine power plant” and under research grant of the Ministry of Science and Higher Education of Poland 4/S/ITESO/14: “Diagnostics methods and efficient operation of complex technical systems in terms of failure prevention and environmental protection”. REFERENCES / Literatura