VENDOR SELECTION AND SUPPLY QUOTAS DETERMINATION BY USING THE ANALYTIC HIERARCHY PROCESS AND A NEW MULTI-OBJECTIVE PROGRAMMING METHOD

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ABSTRACT

In this article, we propose a new methodology for solving the vendor selection and the supply quotas determination problem.

The proposed methodology combines the Analytic Hierarchy Process (AHP) for determining the coefficients of the objective functions and a new multiple objective programming method based on the cooperative game theory for vendor selection and supply quotas determination.

The proposed methodology is tested on the problem of flour purchase by a company that manufactures bakery products.

For vendor selection and supply quotas determination we use three complex criteria: (1) purchasing costs, (2) product quality, and (3) vendor reliability.

KEY WORDS

analytic hierarchy process, multi-objective programming methods, vendor selection

CLASSIFICATION

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INTRODUCTION

The problem of vendor selection and supply quotas determination is increasingly becoming the key element in achieving competitive advantages in manufacturing companies. The manufacturing companies that purchase products for their production process must decide which vendors they should contract with and they must determine the appropriate order quantity for each vendor selected.

In this article, we discuss the vendor selection problem of companies which purchase flour used for producing bakery products. Before the start of the selection process, decision makers (DMs) should define (1) the minimum and the maximum number of vendors from which they plan to purchase the flour, and (2) the maximum quantity purchased from an individual vendor. Here we propose a new methodology that combines two operational research methods: Analytic Hierarchy Process (AHP) [1], and a new multiple objective linear programming method [2]. The AHP method is used to reduce the complex criteria functions to simple ones determining the coefficients of complex criteria functions (quality and reliability), which present the coefficients of the objective functions in the multiple objective integer linear programming model providing the final vendor selection and the quantity supplied from a particular vendor. We use the multiple objective linear integer programming model with three objective functions: (1) purchasing costs, (2) vendor quality and (3) vendor reliability, and three constraints: (1) the total demand, (2) the minimum and maximum number of vendors and (3) the limitations of vendor capacities. In order to solve the multi-objective integer linear programming (MOILP) model we use a new multi-objective linear programming method based on the idea of the cooperative game theory [2] and thus help DMs in the process of the MOILP model solving and finding the preferred solution.

Generally speaking vendor selection and supply quotas determination is an important practical and scientific problem dealt with by numerous researchers. Great efforts have been made to define appropriate models for vendor selection and determination of supply quotas from the selected vendors and to apply the appropriate methods to solve such models.

The literature dealing with vendor selection uses various methods. The AHP method in combination with linear and multi-objective linear programming methods has been used in many articles. For instance Ghodsypour and O’Brien [3] use the AHP method in combination with linear programming, Wang, Huang and Dismukes [4] use the AHP and goal programming. Kumar Shankar and Yadav [5] use the AHP method and fuzzy linear programming, while Kumar, Vrat and Shankar [6, 7] use only fuzzy goal programming with this aim. Perić, Babić and Veža [8] use the AHP and fuzzy linear programming to solve the vendor selection and the supplied quotas determination problem in a bakery. A smaller number of articles combine the revised weighting method and multi-objective linear programming methods. Perić and Babić [9] solve the vendor selection and the supply quotas determination problem by using the revised weighting method and fuzzy multi-criteria programming while Perić [10] uses the revised weighting method and a new multiple objective programming method to solve the vendor selection and the supply quotas determination problem in a bakery. In this article we use the AHP method and the new multi-objective linear programming method based on the cooperative game theory to solve this problem.

The main idea of the study is to create a new methodology for vendor selection and supply quotas determination to solve a specific problem, which would be more objective and easier to use than previously used methodologies. There are criteria which by their nature can be complex and have a hierarchical structure. It has been shown that those criteria can be
simplified by the application of the AHP method or the revised weighting method [10-12]. In our case, two complex criteria functions referring to quality and reliability are simplified by using the AHP method. In this way three objective functions (cost, quality and reliability) are formed. After that a multi-objective integer linear programming model is formed. When the number of objective functions is greater than two, it is not easy for the decision maker to choose the preferred solution. The problem becomes more complex when the number of decision makers (DMs) is greater than one. In the situations when several decision makers participate in the problem solving process, we suggest developing the process of obtaining the preferred solution according to the idea of cooperative games. To solve the problem of vendor selection and supply quotas determination we form the MOLIP model. In order to solve the MOLIP model we use the new MOLP method developed by Matejaš and Perić [2].

The rest of the article is organized as follows: First we present the methodology of vendor selection and supply quotas determination by using the AHP method, and a new multiple objective linear programming method based on the cooperative game theory. Then we test the proposed methodology on the real vendor selection problem of a bakery. In the conclusion we point to the advantages of using the proposed methodology in comparison to the use of similar methodologies.

**METHODOLOGY OF VENDOR SELECTION AND SUPPLY QUOTAS DETERMINATION**

In order to solve the vendor selection and the supply quotas determination problem we use the AHP method, and a new multiple objective linear integer programming method based on the cooperative game theory. The AHP method is used to simplify the complex criteria functions. The proposed methodology consists of the following steps:

I) Selecting criteria for vendor selection,

II) Applying the AHP method to simplify complex criteria functions,

III) Building and solving the MOLIP model to determine marginal solutions,

IV) Solving the MOLIP model by applying the new multiple objective linear programming method based on the cooperative game theory to reach the preferred solution of the problem.

**SELECTING CRITERIA FOR VENDOR SELECTION**

The selection of criteria for vendor selection presents the first step in the proposed methodology. Numerous criteria are listed in the literature and their selection depends on each particular problem [12]. The total purchasing costs in a particular period, product quality offered by particular vendors, and vendor reliability are presented as the most important criteria for vendor selection. Each of these criteria is presented through a number of sub-criteria, which can further be expressed through a number of sub-sub-criteria, etc. This reveals the hierarchical structure of criteria for vendor selection, which enables the application of the AHP method aimed at solving the problem of complex criteria functions.

**ANALYTIC HIERARCHY PROCESS**

The Analytic Hierarchy Process (AHP) is one of the most outstanding multi-criteria decision-making approaches. The AHP method [1] has a great importance in problem structuring and decision making. Its application allows an interactive creation of the problem hierarchy that serves as a preparation for the decision-making scenario. The next step is a pairwise comparison of the hierarchy elements (goals, criteria and alternatives) and eventually all the mutual comparisons are synthesized and weight coefficients for each element are determined.
The sum of weight elements on each hierarchy level is equal to 1 and allows the decision
maker to rank all the hierarchy elements in terms of importance.

In the AHP, multiple paired comparisons are based on a standardised evaluation scheme
(1 = equally important; 3 = slightly more important; 5 = much more important; 7 = very much
more important; 9 = absolutely more important). The result of the pairwise comparisons of n
elements can be summarised in an nxn evaluation matrix A in which every element \( a_{ij} \) is an
estimate of the ratios of the relative weights, e.g. \( a_{ij} = \frac{w_i}{w_j} \), where small errors in the
consistency of judgements are acceptable.

In the next step the largest eigenvalue \( \lambda_{\text{max}} \) of the evaluation matrix has to be determined. It
can easily be seen that \( \lambda_{\text{max}} \geq n \). The difference \( \lambda_{\text{max}} - n \) can therefore be used as a measure
of consistency and consistency index is \( CI = (\lambda_{\text{max}} - n) / (n - 1) \). The consistency ratio (CR)
is given by \( CR = CI / RI \), where \( RI \) is the random index (the average of 500 randomly filled
matrices). If this ratio is significantly small (about 10% or less), we accept the estimates
obtained by this process. Otherwise, we attempt to improve inconsistency.

Namely, the value is adopted as the consistency index (CI). This value is compared to the
same index obtained as an average over a large number of reciprocal matrices \( (\lambda_{\text{max}} - n) / (n - 1) \) of the same order, whose entries are random.

The estimates of relative weights of elements on the corresponding level of a hierarchical
structure will allow the calculation of local priorities (weights) of criteria, sub-criteria and
alternatives, which are then synthesized into global priorities of alternatives. The global
priority of an alternative is calculated by addition of its local priorities weighted with the
weights of elements on the higher level.

**MULTIPLE OBJECTIVE LINEAR PROGRAMMING (MOLP) MODEL**

The general form of MOLP can be presented in the following way:

If \( f_k(x) = c_k^T x \), \( x \in \mathbb{R}^n \), \( c_k \in \mathbb{R}^n \), then

\[
\max_{x \in S} f_k(x), \quad k = 1, 2, ..., K, \tag{1}
\]

where \( S = \{ x \in \mathbb{R}^n : x \geq 0, Ax \leftrightarrow b, A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m \} \).

Thus, the model (1) contains \( K \) linear functions and \( m \) constraints, with the variables which
must be nonnegative. The variables of the model can be continuous, integer and binary or
their combination.

By solving the model (1) in such a way that each of the objective functions is separately
maximized on the set \( S \), we obtain marginal solutions of this model. Since the objective
functions in MOLP models are mutually conflicting, the values of objective functions will be
significantly different for marginal solutions.

DMs almost certainly will not choose any of the obtained marginal solutions, but will look
for a compromise solution which will satisfy their preferences towards objective function
values.

In order to find the preferred efficient solution, we can use a number of standard multi-
objective programming (MOP) methods (see [13]). However, those methods have different
efficiency and give different solutions, so the problem of choosing the appropriate method occurs almost always.

A NEW ITERATIVE METHOD FOR SOLVING MOILP MODELS

A new iterative method for solving multi-objective linear programming problems with one or more decision makers has been proposed by Matejaš and Perić in [2]. This method is based on the idea of the cooperative game theory [14; 15, pp.239-270] and helps decision makers significantly in the process of obtaining the preferred efficient solution.

If several decision makers (players) optimize their utilities at the same time and on the same constraint set (budget), they can achieve their aspirations at different optimal points. If only one point should be chosen in those circumstances we have a multiple objective programming problem (MOPP), which is a problem frequently encountered in practice. It can be stated in the form,

$$
\max_{x \in S} f_k(x), \quad k = 1, 2, \ldots, K,
$$

where $f_k(x), \ x \in \mathbb{R}^n, \ k = 1, 2, \ldots, K$ is the given objective function for a decision maker (player) $k$ ($P_k$) and $S \subset \mathbb{R}^n$ is the given set (budget). If the objective functions are linear and the budget is a convex polyhedron (which means that it is defined as the intersection of linear constraints) then we have a multi-objective linear programming problem (MOLPP). In [2] an efficient method (MP-method) for solving such problems is presented. We shall briefly explain this method.

It is natural that each player $P_k$ has his aspiration $d_k$ which he wants to achieve ($f_k(x) \geq d_k$). The MP-method fully respects these aspirations under the frame of given possibilities (budget $S$). For this purpose the desired budget $D$ is defined,

$$
D = \{x \in \mathbb{R}^n : x \geq 0, \ f_k(x) \geq d_k, \ k = 1, 2, \ldots, K\}.
$$

The players are aware that they will probably fail to realize their aspirations fully. For that reason the shifted desired budget $D_\lambda$ is also defined,

$$
D_\lambda = \{x \in \mathbb{R}^n : x \geq 0, \ f_k(x) \geq \lambda d_k, \ k = 1, 2, \ldots, K\}, \quad \lambda \geq 0.
$$

Now, the method can be stated in a very simple form. We are looking for the largest $\lambda$ such that $D_\lambda \cap S \neq \emptyset$ (geometric form) or equivalently,

$$
\max_{(x,\lambda) \in G} \lambda, \quad \text{where} \quad G = \{(x,\lambda) \in \mathbb{R}^{n+1} : x \in S, \ \lambda \geq 0, \ f_k(x) \geq \lambda d_k, \ k = 1, 2, \ldots, K\},
$$

which is a standard linear programming problem (LPP). The optimal solution $\lambda^*$ shows to which (minimum) extent all the players can realize their aspirations. If $x^*$ is the optimal point then the indicator

$$
\lambda_k = \frac{f_k(x^*)}{d_k}, \quad k = 1, 2, \ldots, K
$$

shows to which extent the player $P_k$ can realize his own aspiration. If $\lambda_k = \lambda^*$ then the realization is minimal. It is equal to the optimal one and the corresponding constraint is
active, \( f_k(x^*) = \lambda^* d_k \). For \( \lambda_k > \lambda^* \) the realization is better than the optimal one and the constraint is passive, \( f_k(x^*) > \lambda^* d_k \). Thus, the indicators measure the reality of players’ aspirations and they can be used to improve the solution if it is unsatisfactory. Namely, the change of an aspiration in a passive constraint up to \( d'_k \leq (\lambda_k / \lambda^*) \cdot d_k \) will not change the optimal solution while any change of an aspiration in an active constraint will immediately change it. It means that the aspirations of players with active constraints are set too high in comparison to others which prevent the solution from being better (larger). The optimal solution \( \lambda^* \) can be improved, which means increased, by decreasing aspirations in active constraints. In this way the problem (3) represents one step of the method intended for solving the initial problem (2). If players are satisfied with the obtained solution, then the problem is solved. If not, then they need to redefine their aspirations and perform the next step of the method. The indicators \( \lambda_k \) help players make the right redefinitions ensuring the desired improvements.

We see that the MP-method is an iterative method which is based on the principles of the game theory (cooperation among the players). It includes the basic step which may be repeated until the satisfactory equilibrium is attained. At each stage of the process the players can understand why they have obtained such a solution and what should be done to drive the solution in the desired direction. As a result, the players can adjust their aspirations until they reach the state of equilibrium which is satisfactory to everyone or until they detect that such a state does not exist.

It is also important to note that the MP-method can be applied to the general MOPP (not only to the linear one). All conclusions concerning optimal points, indicators and possible iterations remain the same. But since (3) is not LPP any more, the numerical application would require further research on computational methods.

**CASE STUDY**

**CRITERIA FOR VENDOR SELECTION**

Vendor selection and determination of supply quotas per each selected vendor is a multi-criteria problem. Literature offers a large number of criteria that can be used in vendor selection. Which criteria will be chosen by a decision maker depends on the kind of problem to be solved. In this study we consider criteria that can be used by producers of bakery products when selecting flour vendors. More about the criteria can be read in [8].

**DATA REQUIRED FOR VENDOR SELECTION AND SUPPLY QUOTAS DETERMINATION**

The following text presents an example of vendor selection for a bakery. It is to be noted that in the production of bread and bakery products, the purchase of flour is contracted for the period of one year, from harvest to harvest, which usually does not correspond to the calendar year. After the harvest, flour producers have the information on the available wheat quantity, price and quality which allows them to define the price, quality and quantity of flour they can supply in the subsequent one-year period.

In the one-year period the bakery plans to consume 6 000 tons of flour Type 550. The company contacts 6 potential flour suppliers and defines the upper limit of flour supplied by a single vendor in the amount of 4 000 tons. The management have decided to sign supply contracts with at least two suppliers. Besides, they decided that the number of suppliers may not exceed four.
Table 1. Purchasing costs for flour Type 550. Source: Supplier offers.

<table>
<thead>
<tr>
<th>Vendor</th>
<th>Purchasing price in euros/ton (B1)</th>
<th>Transportation cost in euros/ton (B2)</th>
<th>Total purchasing costs in euros/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>240</td>
<td>20</td>
<td>260</td>
</tr>
<tr>
<td>2</td>
<td>215</td>
<td>25</td>
<td>240</td>
</tr>
<tr>
<td>3</td>
<td>230</td>
<td>20</td>
<td>250</td>
</tr>
<tr>
<td>4</td>
<td>275</td>
<td>15</td>
<td>290</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>10</td>
<td>210</td>
</tr>
<tr>
<td>6</td>
<td>260</td>
<td>35</td>
<td>295</td>
</tr>
</tbody>
</table>

Table 2. Quality indicators for flour Type 550. Source: Supplier analysis.

<table>
<thead>
<tr>
<th>Quality indicators</th>
<th>Criteria weights</th>
<th>Vendor 1</th>
<th>Vendor 2</th>
<th>Vendor 3</th>
<th>Vendor 4</th>
<th>Vendor 5</th>
<th>Vendor 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>General characteristics of flour (A1)</td>
<td>(0,20)</td>
<td>14.2</td>
<td>14.56</td>
<td>13.6</td>
<td>14.1</td>
<td>13.09</td>
<td>14.85</td>
</tr>
<tr>
<td>Moisture in % (B3)</td>
<td>min (0,30)</td>
<td>0.56</td>
<td>0.55</td>
<td>0.59</td>
<td>0.51</td>
<td>0.54</td>
<td>0.48</td>
</tr>
<tr>
<td>Ash in % (B4)</td>
<td>min (0.20)</td>
<td>1.8</td>
<td>1.8</td>
<td>1.6</td>
<td>1.8</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Acidity level in ml/100 grams (B5)</td>
<td>min (0,10)</td>
<td>26.5</td>
<td>26.8</td>
<td>29.4</td>
<td>24.6</td>
<td>24.7</td>
<td>28.7</td>
</tr>
<tr>
<td>Wet gluten in % (B6)</td>
<td>max (0,40)</td>
<td>60.2</td>
<td>56.3</td>
<td>57</td>
<td>56</td>
<td>57.8</td>
<td>55.8</td>
</tr>
<tr>
<td>Farinograph (A2)</td>
<td>(0,30)</td>
<td>110</td>
<td>102.1</td>
<td>128</td>
<td>104.3</td>
<td>98</td>
<td>133</td>
</tr>
<tr>
<td>Water absorption in % (B7)</td>
<td>min (0,40)</td>
<td>163</td>
<td>146</td>
<td>167</td>
<td>161</td>
<td>175</td>
<td>165</td>
</tr>
<tr>
<td>Extensograph (A3)</td>
<td>(0,30)</td>
<td>380</td>
<td>400</td>
<td>605</td>
<td>390</td>
<td>330</td>
<td>395</td>
</tr>
<tr>
<td>Energy in cm² (B9)</td>
<td>max (0,40)</td>
<td>55</td>
<td>30</td>
<td>33</td>
<td>40</td>
<td>80</td>
<td>50</td>
</tr>
<tr>
<td>Elasticity in mm (B10)</td>
<td>min (0,30)</td>
<td>1110</td>
<td>1015</td>
<td>1255</td>
<td>1610</td>
<td>1126</td>
<td>1460</td>
</tr>
<tr>
<td>Resistance (B11)</td>
<td>min</td>
<td>1</td>
<td>0.95</td>
<td>1</td>
<td>0.45</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>Amylograph (A4)</td>
<td>max (0,20)</td>
<td>1</td>
<td>0.87</td>
<td>0.80</td>
<td>0.40</td>
<td>0.45</td>
<td>0.60</td>
</tr>
<tr>
<td>Peak viscosity in BU (B12)</td>
<td>max</td>
<td>95</td>
<td>111</td>
<td>92</td>
<td>69</td>
<td>80</td>
<td>87</td>
</tr>
</tbody>
</table>

Table 3. Vendor reliability indicators. Source: Croatian Financial Agency.

<table>
<thead>
<tr>
<th>Reliability indicators</th>
<th>Criterion</th>
<th>Vendor 1</th>
<th>Vendor 2</th>
<th>Vendor 3</th>
<th>Vendor 4</th>
<th>Vendor 5</th>
<th>Vendor 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial stability, indebtedness and liquidity (A5)</td>
<td>(0,60)</td>
<td>max (0,20)</td>
<td>1.15</td>
<td>0.90</td>
<td>0.85</td>
<td>0.80</td>
<td>0.99</td>
</tr>
<tr>
<td>Coverage of fixed assets and stocks by capital and long term resources, (B13)</td>
<td>max (0,20)</td>
<td>51.46</td>
<td>20.7</td>
<td>41.0</td>
<td>55.77</td>
<td>40.2</td>
<td>37.6</td>
</tr>
<tr>
<td>Share of capital in source of funds in %, (B14)</td>
<td>max (0,10)</td>
<td>8</td>
<td>20</td>
<td>15</td>
<td>16</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Indebtedness factor, number of years (B15)</td>
<td>Min (0,10)</td>
<td>0.68</td>
<td>0.50</td>
<td>0.55</td>
<td>0.40</td>
<td>0.45</td>
<td>0.60</td>
</tr>
<tr>
<td>Total assets turnover coefficient (B16)</td>
<td>max (0,10)</td>
<td>7.25</td>
<td>1.2</td>
<td>1.15</td>
<td>0.85</td>
<td>3.13</td>
<td>1.70</td>
</tr>
<tr>
<td>General liquidity coefficient (B17)</td>
<td>max (0,30)</td>
<td>0.95</td>
<td>1.55</td>
<td>2.34</td>
<td>1.12</td>
<td>2.10</td>
<td>1.80</td>
</tr>
<tr>
<td>Short term receivables collection period, in days (B18)</td>
<td>min (0,20)</td>
<td>95</td>
<td>111</td>
<td>92</td>
<td>69</td>
<td>80</td>
<td>87</td>
</tr>
<tr>
<td>Performance indicators (A6)</td>
<td>(0,40)</td>
<td>max (0,20)</td>
<td>1.07</td>
<td>1.04</td>
<td>1.03</td>
<td>1.01</td>
<td>1.02</td>
</tr>
<tr>
<td>Coefficient of total revenue and expenditure ratio (B19)</td>
<td>max (0,20)</td>
<td>3.95</td>
<td>1.55</td>
<td>2.34</td>
<td>1.12</td>
<td>2.10</td>
<td>1.80</td>
</tr>
<tr>
<td>Share of profit in total income in % (B20)</td>
<td>max (0,20)</td>
<td>3.28</td>
<td>0.99</td>
<td>1.45</td>
<td>1.05</td>
<td>1.30</td>
<td>1.05</td>
</tr>
<tr>
<td>Share of profit in assets in % (B21)</td>
<td>max (0,20)</td>
<td>7405</td>
<td>2560</td>
<td>1383</td>
<td>1705</td>
<td>2260</td>
<td>4206</td>
</tr>
</tbody>
</table>
The proposed prices of flour and transportation costs (Criterion C1) are shown in Table 1. The potential vendors should supply data on flour quality which they have to maintain throughout the contract period (Criterion C2). The vendors should also supply data on their reliability in an appropriate form (Criterion C3). Tables 2 and 3 indicate flour quality and vendor reliability. The weights expressing the relative importance of criteria and sub-criteria are given in brackets, and are determined by the decision makers where the sum of weights in every group of sub-criteria is 1 [10].

APPLICATION OF ANALYTIC HIERARCHY PROCESS

Taking into consideration the data from Tables 1, 2 and 3, a hierarchical structure of goals and criteria for vendor selection is formed. The hierarchical structure is shown in Figure 1.

Figure 1. Hierarchical Structure of the Supplier Selection Problem [10].
Vendor selection and supply quotas determination by using the analytic hierarchy process ...

Figure 2. Weights of criteria and sub-criteria for the criterion quality.

Figure 3. Weights of criteria and sub-criteria for the criterion quality.

The calculated weights of suppliers for quality and reliability criteria are presented in Figures 4 and 5. The obtained weights present the coefficients of the quality and reliability objective functions respectively.

Figure 4. Weights of suppliers for the criterion quality.
Taking into consideration the data on the normalized coefficient weights with the cost variable, weights of suppliers for quality and reliability criteria functions (obtained from the AHP method application), the total demand for flour in the given period, limited quantities supplied by single vendors and the constraint of the minimum and the maximum number of vendors, the following MOLIP model is formed:

\[
\max_{x \in S} \left[ f_1(x), f_2(x), f_3(x) \right]
\]

where

\[
\begin{align*}
  f_1(x) &= 0.168285x_1 + 0.15534x_2 + 0.161812x_3 + 0.187702x_4 + 0.135922x_5 + 0.190939x_6, \\
  f_2(x) &= 0.131x_1 + 0.160x_2 + 0.231x_3 + 0.142x_4 + 0.139x_5 + 0.197x_6, \\
  f_3(x) &= 0.365x_1 + 0.074x_2 + 0.093x_3 + 0.116x_4 + 0.181x_5 + 0.170x_6,
\end{align*}
\]

and

\[
S = \left\{ (x, y) : \sum_{j=1}^{6} x_j = 6000; \ x_j \leq 4000, \ x_j \leq M \cdot y_j, \ -x_j + M \cdot y_j \leq M - x_j^{\min}, \ 2 \leq \sum_{j=1}^{6} y_j \leq 4, \ x_j \geq 0; \ y_j \in \{0,1\}; \ j = 1,2,...,6 \right\}
\]

\(y_j\) are artificial binary variables and they show us whether a supplier \(j\) has been chosen. These variables are related to variables \(x_j\), in such a way that if the problem solution contains the variable \(x_j\), then the variable \(y_j\) must equal 1, and if in the problem solution the variable \(x_j\) is zero then \(y_j\) must also be zero, and vice versa. \(M\) is a very big number, and \(x_j^{\min}\) \((j = 1, 2, ..., 6)\) is the minimal value which the variable \(x_j\) can have if the variable \(y_j\) is included in the solution.

The model (7) is first solved by the linear integer programming method using the Microsoft Excel Solver optimizing separately each of the three objective functions on the given set of constraints. The results are given in Table 5:

**Table 5.** Payoff values.

<table>
<thead>
<tr>
<th>Solution</th>
<th>(f_1(x))</th>
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<th>(f_3(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>max (f_1(x))</td>
<td>1159.58</td>
<td>876</td>
<td>872</td>
</tr>
<tr>
<td>max (f_2(x))</td>
<td>964.714</td>
<td>1318</td>
<td>712</td>
</tr>
<tr>
<td>max (f_3(x))</td>
<td>1054.818</td>
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Table 5 shows that the objective functions have different values when we separately maximize objective functions on the given set of constraints. So the function \(f_1(x)\) has values

---

**Synthesis with respect to: Reliability (C3)**

[Goal: TVP > Reliability (C3) (6:1.000)]

Overall inconsistency = .02

**Figure 5.** Weights of suppliers for the criterion reliability.

**MOLIP MODEL BUILDING AND SOLVING**

Taking into consideration the data on the normalized coefficient weights with the cost variable, weights of suppliers for quality and reliability criteria functions (obtained from the AHP method application), the total demand for flour in the given period, limited quantities supplied by single vendors and the constraint of the minimum and the maximum number of vendors, the following MOLIP model is formed:

\[
\max_{x \in S} \left[ f_1(x), f_2(x), f_3(x) \right]
\]

where

\[
\begin{align*}
  f_1(x) &= 0.168285x_1 + 0.15534x_2 + 0.161812x_3 + 0.187702x_4 + 0.135922x_5 + 0.190939x_6, \\
  f_2(x) &= 0.131x_1 + 0.160x_2 + 0.231x_3 + 0.142x_4 + 0.139x_5 + 0.197x_6, \\
  f_3(x) &= 0.365x_1 + 0.074x_2 + 0.093x_3 + 0.116x_4 + 0.181x_5 + 0.170x_6,
\end{align*}
\]

and

\[
S = \left\{ (x, y) : \sum_{j=1}^{6} x_j = 6000; \ x_j \leq 4000, \ x_j \leq M \cdot y_j, \ -x_j + M \cdot y_j \leq M - x_j^{\min}, \ 2 \leq \sum_{j=1}^{6} y_j \leq 4, \ x_j \geq 0; \ y_j \in \{0,1\}; \ j = 1,2,...,6 \right\}
\]

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</table>

Table 5 shows that the objective functions have different values when we separately maximize objective functions on the given set of constraints. So the function \(f_1(x)\) has values
Vendor selection and supply quotas determination by using the analytic hierarchy process ...

between 964,714 and 1159,58, the function $f_2(x)$ between 802 and 1318, and the function $f_3(x)$ between 712 and 1822. Therefore, the decision makers have three conflicting goals and they have to choose a compromise solution.

The proposed methodology for vendor selection and supply quotas determination uses the MP method which is based on the idea of the cooperative game theory.

The process of the vendor selection and solving the supply quotas determination problem by applying the MP method starts by informing the DMs (players) $P_k (k=1,2,...,K)$ on the maximal and minimal values of the objective functions.

$$
964,714 \leq f_1 \leq 1159,58 \quad (8)
$$

$$
802 \leq f_2 \leq 1318 \quad (9)
$$

$$
712 \leq f_3 \leq 1822 \quad (10)
$$

After informing the decision makers on the highest and the lowest value of their objective function, the decision makers determine the initial acceptable value of the objective functions. In the first stage the decision makers have determined the following acceptable values for their objective functions: $f_1 = 1150$, $f_2 = 1300$, $f_3 = 1600$. It is normal that each decision maker aims to achieve the maximum value of its objective function. The DMs know that they can hardly reach the determined acceptable level from the first step. The final acceptable level of the objective function values should be reached after several steps of the method application, which requires an active participation of the DMs and negotiations in the process of problem solving.

In the second stage of the method application the following integer linear programming model is solved:

$$
\max_{(x,y,\lambda)} \lambda
$$

where

$$
G = \{(x,y,\lambda): \sum_{j=1}^{6} x_j = 6000, \ x_j \leq 4000, \ x_j \leq M \cdot y_j, \ -x_j + M \cdot y_j \leq M - x_j^{\min}, \ 2 \leq \sum_{j=4}^{6} y_j \leq 4, \ f_1 \geq 1150 \lambda, \ f_2 \geq 1300 \lambda, \ f_3 \geq 1600 \lambda, \ x_j \geq 0, \ y_j \in \{0,1\}, \ j = 1,2,...,6 \}
$$

The following solution has been obtained:

Table 6. The solution of stage 2, step 1.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Variable values</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$x_1 = 2905.374$, $x_3 = 3094.626$, $\lambda = 0.842664$, $x_2 = x_4 = x_5 = x_6 = 0$,</td>
<td>1026,852</td>
<td>985,383</td>
<td>1120,204</td>
<td>0.867</td>
<td>0.843</td>
<td>0.843</td>
</tr>
</tbody>
</table>

Note: The indicators $\lambda_k$ ($k = 1, 2, 3$) are calculated using (4).

The DMs were not satisfied with the obtained solution.

In the second step of the method the decision makers determine new reduced aspiration levels (Values $\lambda = 0.843$ suggest decreasing the aspiration level value of the decision makers 2 and 3). They agreed to determine: $d_1 = 1150$, $d_2 = 1200$, $d_3 = 1400$. 

99
After solving the model (11) with the changed constraints, \( f_2 \geq 1200 \lambda \) instead of \( f_2 \geq 1300 \lambda \), \( f_3 \geq 1400 \lambda \) instead of \( f_3 \geq 1600 \lambda \), the following solution has been obtained:

**Table 7.** The solution of stage 2, step 2.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Variable values</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
</tr>
</thead>
</table>
| II       | \( x_1 = 2115.396 \\
\( x_3 = 3884.604, \lambda = 0.95556, \) \\
x_2 = x_4 = x_5 = x_6 = 0 | 1002.612 | 1174.46 | 1133.388 | 0.872     | 0.979     | 0.872     |

After the second step, only the DM1 was not satisfied with the obtained solution. The decision makers agreed that the DM2’s acceptance level should not be less than 1174.46, and that the increase of the value of the function \( f_1 \) should be achieved by decreasing the acceptance level of the function \( f_3 \).

**Step 3.** In this step the DMs have agreed to reduce the value of the function \( f_3 \) to 1200.

After solving the model (11) with the changed constraints \( f_2 \geq 1174.46 \) instead of \( f_2 \geq 1300 \lambda \) and \( f_3 \geq 1200 \lambda \) instead of \( f_3 \geq 1600 \lambda \), the following solution has been obtained:

**Table 8.** The solution of stage 2, step 3.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Variable values</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
</tr>
</thead>
</table>
| III      | \( x_1 = 2000, x_3 = 4000, \lambda = 0.8725, \) \\
x_2 = x_4 = x_5 = x_6 = 0 | 1003.364 | 1186    | 1102    | 0.873     | 1.01      | 0.918      |

The DM 1 was not satisfied with the obtained value of the function \( f_1 \). The DM 3 has decreased the acceptance level of the function \( f_3 \) to 1000.

**Step 4.** After solving the model (11) with the changed constraints \( f_2 \geq 1174.46 \) instead of \( f_2 \geq 1300 \lambda \) and \( f_3 \geq 1000 \lambda \) instead of \( f_3 \geq 1600 \lambda \), the following solution has been obtained:

**Table 9.** The solution of stage 2, step 4.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Variable values</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
</tr>
</thead>
</table>
| IV       | \( x_1 = 2000, x_3 = 4000, \lambda = 0.8725, \) \\
x_2 = x_4 = x_5 = x_6 = 0 | 1003.364 | 1186    | 1102    | 0.873     | 1.01      | 0.918      |

However, decreasing the acceptance level of the function \( f_3 \) did not change the previous solution. The DMs agreed to continue decreasing the acceptance level of the function \( f_3 \) to 800.

**Step 4.** After solving the model (11) with the changed constraints \( f_2 \geq 1174.46 \) instead of \( f_2 \geq 1300 \lambda \) and \( f_3 \geq 880 \lambda \) instead of \( f_3 \geq 1600 \lambda \), the following solution has been obtained:
The DMs have accepted the obtained solution. Therefore after only five steps the preferred solution has been obtained. Thus the solution process has been completed.

It should be noted that the method presupposes that the DMs know or can determine the acceptable level of their objective functions. The solution process ensures obtaining the preferred efficient solution in the minimal number of steps.

To show that the proposed methodology is better than the application of standard MOILP methods in solving this problem, the model (7) has been solved by the $\varepsilon$–Constraints MOILP method [13, pp.250-252]. First, function $f_1$ has been maximized, while functions $f_2$ and $f_3$ are put into the constraints set, gradually reducing the value of their objective functions in the constraint set, then function $f_2$ has been maximized with functions $f_1$ and $f_3$ in the constraints set, gradually reducing their goals, and at the end function $f_3$ has been maximized, while the functions $f_1$ and $f_2$ are included into the constraint set, gradually reducing their goals. In this way a large number of efficient solutions have been obtained. The set of efficient solutions has been presented to the DMs. However, for the DMs it was very difficult to choose the preferred solution.

**CONCLUSION**

This article presents a new methodology for vendor selection and supply quotas determination by means of the AHP method and a new multi-objective linear programming method based on the cooperative game theory. The results of the research carried out in a bakery business indicate a high performance of the proposed methodology in vendor selection and supply quotas determination.

The methodology has a number of advantages compared to the previously applied techniques for solving the same problem. Namely, the AHP method can be efficiently applied for reducing complex and hierarchical criteria quality and reliability to simpler forms. Applying the new MOLP method decision makers can actively participate in the whole process of the supply quotas determination. This method guaranties obtaining the preferred solution in a finite number of steps and it can be used when there is a large number of decision makers.

The efficiency of the proposed methodology is tested on a practical case of one year flour purchasing for a company that produces bakery products. The simplicity of the proposed methodology and decision makers’ high level confidence of the obtained results were the main advantages of the applied methodology. When solving the MOIFP model by using the new method, decision makers are actively included in the process of finding the preferred solution. If some of them are not satisfied with the obtained solution they know where the ‘problem’ is, i.e. who should reduce the aspiration level so as to enable the unsatisfied ones to improve their objective function value. Here decision makers choose the preferred solution in the process of negotiations among themselves and by solving an integer linear programming problem through a finite number of steps.

Further improvements of the proposed methodology of vendor selection and the supply quotas determination problem in terms of a dynamic process and a simultaneous application
of quantity discounts as well as a discount of quantity value in a particular period will be the subject of the future research.

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REFERENCES


