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Control for Formation of Multi-Agent Systems with Time-varying Delays and Uncertainties based on LMI

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Original scientific paper

In this paper, robust stability of vehicles formation system with structural uncertainties and time-varying delays are analyzed. The vehicles are modeled as general linear systems and the feedback control is based only on relative information about vehicle states in an undirected communication topology graph. Sufficient conditions are proposed for asymptotical stability of vehicles formations system. Both system model without uncertainties and system model with uncertainties are considered. Stability and robust stability criteria of vehicles formations system are obtained in terms of linear inequality matrix and free-weighting matrix method. Numerical examples are given to illustrate the effectiveness of the results.

Key words: Formation stability, Decentralized control, Time-varying delays, Linear matrix inequality, Robust stability

Upravljanje formacijom multiagentskog sustava s vremenski promjenjivim kašnjenem i nesigurnostima zasnovano na linearnim matričnim nejednakostima. U ovome članku analizira se robusnost formacije vozila sa strukturnim nesigurnostima i vremenski promjenjivim kašnjenjem. Vozila su modelirana kao općeniti linearni sustav, a upravljanje po povratnoj vezi je zasnovano na informaciji o relativnom stanju sustava u neusmjerenom komunikacijskom grafu. Predloženi su dovoljni uvjeti za asimptotsku stabilnost formacije vozila te je razmotren model sustava sa i bez nesigurnosti. Kriteriji stabilnosti i robusnosti formacije vozila dobiveni su u obliku linearnih matričnih nejednakosti. Na kraju članka, dani su numerički primjeri kako bi se ilustrirala učinkovitost rezultata.

Ključne riječi: stabilnost formacije, decentralizirano upravljanje, vremenski promjenjivo kašnjenje, linearne matrične nejednakosti, robusnost

1 INTRODUCTION

Recent years have seen the emergence of formation of swarm vehicles as a topic of significant interest to the control community. Multi-agent systems have appeared widely in many applications including mobile vehicles, formation flight of unmanned air vehicles (UAVs), clusters of satellites, automated highway systems. The coordinated motion of multiple autonomous vehicles has received increasing attention recently [1–4].

The research on vehicles formation is motivated by the motion of aggregates of individuals in nature [5–7]. Some simulations and explanations are given by researchers from different areas [1, 8–10].

Since formations is concerned with the agent dynamics, it is very natural for researchers to study formation of agents with different dynamics. The motions of vehicles modelled as double integrators are investigated [11–13]. Multi-agent systems with general linear dynamics is studied [14]. A distributed adaptive control problem is analyzed for formation of agents where the dynamics of the agents are nonlinear, nonidentical [15]. An excellent survey of formation control of multi-agent systems is found in [16].

The current studies on formation control mainly are: 1)formation control protocol design for various multi-agent systems [17]; 2) conditions for formation over time-varying topology [18]; 3) communication topology design to optimize the system's performance [19]. Most of the above researches are not concerned with the dynamics of the agent with model uncertainties and time-varying delays, which is very important in practical engineering applications.

In this paper, we aim to design the robust formation controller in terms of linear matrix inequality. Furthermore, the free-weighting matrix method [20] is introduced into formation control of multi-agent systems to overcome the conservativeness of LMI caused.

This paper is organized as follows. In Section 2 some results and notations are introduced. In Section 3 stability of vehicles formation is studied. Both stabilities of vehicles formation without model uncertainties and with model uncertainties are analyzed in terms of linear matrix inequalities (LMI) in Section 4. Several theorems are concluded. In Section 5, illustrative examples and some simulations are given to show validity of the conclusions. Finally, concluding remarks are made in Section 6.

2 PRELIMINARIES

Some results from algebraic graph theory, matrix theory are introduced. The models of vehicles are also described.

2.1 Algebraic graph theory

In this section, basic concepts and notations in graph theory are stated [21, 22].

Let $\mathbb{G} = (\mathbb{V}, \mathbb{E}, \mathbb{A})$ be a weighted graph of order n with the set of vehicles $\{v_1, v_2, \cdots, v_n\}$, set of edges $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$, and a weighted adjacency matrix $\mathbb{A} = [w_{ij}]$ with nonnegative adjacency elements w_{ij}. The vehicle indices belong to a finite index set $\mathbb{I} = \{1, 2, \cdots, n\}$. An edge of $\mathbb{G}$ is denoted by $e_{ij} = (v_i, v_j)$. The adjacency elements associated with the edges of the graph are positive, i.e. $e_{ij} \in \mathbb{E} \Leftrightarrow w_{ij} > 0$. Moreover, we assume $w_{ii} = 0$ for all $i \in \mathbb{I}$. The set of neighbors of vehicle v_i is denoted by $\mathbb{N}_i = \{v_j \in \mathbb{V} : (v_i, v_j) \in \mathbb{E}\}$. The in-degree and out-degree of vehicle are, respectively, defined as follows

$$\deg_{in}(v_i) = \sum_{j=1}^{n} w_{ji}, \deg_{out}(v_i) = \sum_{j=1}^{n} w_{ij}.$$

For a graph with 0-1 adjacency elements, $\deg_{out}(v_i) = |\mathbb{N}_i|$. The degree matrix of the digraph $\mathbb{G}$ is a diagonal matrix $D = [d_{ij}]$ where $d_{ij} = 0$ for all $i \neq j$ and $d_{ii} = \deg_{out}(v_i)$. The graph Laplacian associated with the digraph $\mathbb{G}$ is defined as $L(\mathbb{G}) = D - A$. For undirected graph, the adjacency matrix is symmetric, i.e., $w_{ij} = w_{ji}$. Its in-degree and out-degree are equal, i.e, $\deg_{in}(v_i) = \deg_{out}(v_i)$. Then the Laplacian matrix is symmetric and defined by

$$l_{ij} = \begin{cases} \sum\limits_{k=1, k\neq i}^{n} w_{jk}, j = i \\ -w_{ij}, j \neq i \end{cases}. \tag{1}$$

We note that zero is an eigenvalue of L and the associated eigenvector is $1_{n\times 1}$. If graph $\mathbb{G}$ is strongly connected, 0 is an isolated eigenvalue of L and all other eigenvalues of $\mathbb{G}$ are real-valued and are strictly positive.

2.2 Matrix theory

Some results that are useful for analysis in later sections are introduced [23].

Definition 1 The Kronecker product of $P = [p_{ij}]$ and $Q = [Q_{ij}]$ is denoted by $P \otimes Q$ and is defined to be the $P \otimes Q = [p_{ij} Q]$.

Lemma 1 Given $A \in \mathbb{R}^{N\times N}$ with eigenvalues $\lambda_1, \cdots, \lambda_N$ in any prescribed order, there is a unitary matrix $T \in \mathbb{R}^{N\times N}$ such that $T^{-1}AT = U = [u_{ij}]$ is upper triangular, with diagonal entries $u_{ii} = \lambda_i, i = 1, \cdots, N$.

Lemma 2 If A and $B \in \mathbb{R}^{N\times N}$ are nonsingular, then so is $A \otimes B$, and $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$.

Lemma 3 Let $X \in \mathbb{R}^{r\times s}$ and $Y \in \mathbb{R}^{N\times N}$, then $(I_N \otimes X)(Y \otimes I_s) = (Y \otimes I_r)(I_N \otimes X)$.

Lemma 4 If there is a unitary matrix $T \in \mathbb{R}^{N\times N}$ such that $T^{-1}AT = U = [u_{ij}]$ is upper triangular, then $(T \otimes I_n)^{-1}(A \otimes I_n)(T \otimes I_n) = U \otimes I_n$.

Lemma 5 The LMI

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} < 0$$

where $Q(x) = Q^T(x), R(x) = R^T(x)$, and $S(x)$ depend affinely on x, is equivalent to

$$R(x) < 0, Q(x) - S(x)R(x)^{-1}S^T(x) < 0.$$

Lemma 6 Let Y, D and E be arbitrary matrices with appropriate dimensions, Y is symmetric, then for all matrices F satisfying $F^T F \leq I$, the following inequality holds

$$Y + DFE + E^T F^T D^T < 0$$

if and only if there exists a positive constant $\varepsilon > 0$ such that

$$Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0.$$

2.3 Model description

We assume given vehicles with the same dynamics

$$\dot{x}_i = A_{veh} x_i + B_{veh} u_i, i = 1, \cdots, N, x_i \in \mathbb{R}^{2n} \tag{2}$$

where the entries of x_i represent $2n$ configuration variables for vehicle i, and u_i represents control inputs. The matrices A_{veh} and B_{veh} have the form

$$A_{veh} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & a_{24} & 0 & a_{26} & \cdots & a_{2(2n)} \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & a_{42} & 0 & a_{44} & 0 & a_{46} & \cdots & a_{4(2n)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix},$$

$$B_{veh} = I_n \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The form of A_{veh} is determined by the fact that the odd-numbered entries of x_i represent the position variables and the even-numbered entries represent velocity variables, and that the control input affects the acceleration. It is assumed that each vehicle is allowed to see only some of its neighbors and apply the same linear feedback as the others, that is, u_i is determined by relative information and is a linear feedback control law. As an example, consider vehicles in $\mathbb{R}^{2\times 2}$ so that vehicle i has position(x_{ipx}, x_{ipy}), velocity (x_{ivx}, x_{ivy}). Assume vehicle i see only vehicles k and j. With the linear feedback the equations of motion for vehicle become

$$\begin{aligned}
\dot{x}_{ipx} &= x_{ivx} \\
\dot{x}_{ivx} &= a_{22}x_{ivx} + a_{24}x_{ivy} + a_{26}x_{ivz} + u_{ix} \\
\dot{x}_{ipy} &= x_{ivy} \\
\dot{x}_{ivy} &= a_{42}x_{ivx} + a_{44}x_{ivy} + a_{46}x_{ivz} + u_{iy}
\end{aligned}$$

$$\begin{aligned}
u_{ix} =& f \times (l_{ii}x_{ipx} - l_{ij}x_{jpx} - l_{ik}x_{kpx}) \\
&+ g \times (l_{ii}x_{ivx} - l_{ij}x_{jvx} - l_{ik}x_{kvx}) \\
u_{iy} =& f \times (l_{ii}x_{ipy} - l_{ij}x_{jpy} - l_{ik}x_{kpy}) \\
&+ g \times (l_{ii}x_{ivy} - l_{ij}x_{jvy} - l_{ik}x_{kvy})
\end{aligned}$$

where, f and g are feedback coefficients, l_{ij} is defined in (1). So the feedback matrix is considered as

$$F_{veh} = \begin{pmatrix} f & g & 0 & 0 & \cdots \\ 0 & 0 & f & g & \cdots \\ \cdots & \cdots & \cdots & \cdots & \ddots \end{pmatrix}.$$

We can rewrite (2) as

$$\dot{x}_i = A_{veh}x_i + \sum_{j=1}^{N} l_{ij}B_{veh}F_{veh}x_j, i = 1, \cdots, N \quad (3)$$

Definition 2 [24] A formation is a vector $h = h_p \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in \mathbb{R}^{2n\times N}$. The N vehicles are in formation at time if there are vectors $q, \omega \in \mathbb{R}^n$ such that $x_{ip}(t) - (h_{ip}) = q$ and $x_{iv}(t) = \omega$, for $i = 1, \cdots, N$. The vehicles converge to formation h if there exist $\mathbb{R}^n$-valued functions $q(\cdot), \omega(\cdot)$ such that $x_{ip}(t) - (h_{ip}) - q(t) \to 0$ and $x_{iv}(t) - \omega(t) \to 0$, as $t \to \infty$, for $i = 1, \cdots, N$.

In reality, there usually are some time delays in communication network due to finite speeds of transmission and traffic congestions. Assume the time delays are time-varying and same in the network. We introduce the following dynamical model.

$$\dot{x}_i(t) = A_{veh}x_i(t) + \sum_{j=1}^{N} l_{ij}B_{veh}F_{veh}x_j(t - \tau(t)), \quad (4)$$

where, $i = 1, \cdots, N$. The time delay $\tau(t)$ is a time-varying and satisfies $0 \leq \tau(t) \leq d$ and $\dot{\tau}(t) \leq \mu$, where $d > 0$ and μ are constants. Now let us consider vehicles formation.

Fig.1 shows the interpretation of the vectors in the definition. h_j is a formation vector defined in Definition 2. Thus, single vehicle model with formation in h is written as

$$\dot{x}_i(t) = A_{veh}x_i(t) + \sum_{j=1}^{N} l_{ij}B_{veh}F_{veh}(x_j(t - \tau(t)) - h_j). \quad (5)$$

Now all the vehicles model with vehicles formation could be written in compact form as

$$\dot{x} = I_N \otimes A_{veh}x + (L_N \otimes (B_{veh}F_{veh}))(x(t - \tau(t)) - h). \quad (6)$$

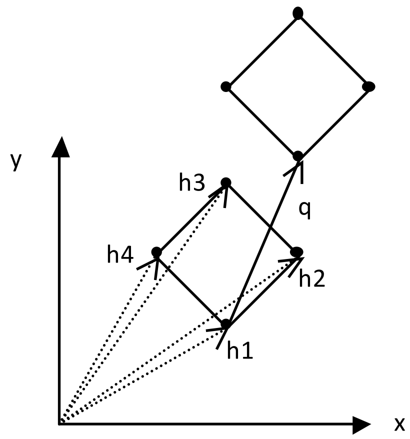

Fig. 1. Vehicles in formation

3 STABILITY OF VEHICLES FORMATION

In this section, we show that the stabilities of N vehicles formation can be equivalent to stabilities of $N - 1$ systems. Two cases are considered.

3.1 Stability of system without model uncertainties

According to Definition 2, vehicles achieve in formation h if $x_1(t) - h_1 = x_2(t) - h_2 = \cdots = x_N(t) - h_N = s(t)$ as $t \to \infty$. We note the fact that $x_i(t) - h_i = s(t)$. Using the structural properties of the matrices of A_{veh} and

Laplacian matrix L, we have

$$A_{veh}h_i = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & a_{24} & 0 & a_{26} & \cdots & a_{2(2n)} \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & a_{42} & 0 & a_{44} & 0 & a_{46} & \cdots & a_{4(2n)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} h_{ipx} \\ 0 \\ h_{ipy} \\ 0 \\ \cdots \end{pmatrix}$$

It can be seen that the odd-numbered columns of A_{veh} and even-numbered rows of h are zero vector, we can obtain $A_{veh}h_i = 0$. And

$$\begin{aligned} &\sum_{j=1}^{N} l_{ij}B_{veh}F_{veh}s(t-\tau(t)) = \\ &l_{i1}B_{veh}F_{veh}s(t-\tau(t)) + l_{i2}B_{veh}F_{veh}s(t-\tau(t)) + \cdots \\ &l_{ij}B_{veh}F_{veh}s(t-\tau(t)) + \cdots + l_{iN}B_{veh}F_{veh}s(t-\tau(t)) \\ &= B_{veh}F_{veh}(l_{i1} + l_{i2} + \cdots + l_{iN})s(t-\tau(t)) \end{aligned}$$

According to (1),
$l_{i1} + l_{i2} + \cdots + l_{iN} = 0$.
Thus, $\sum\limits_{j=1}^{N} l_{ij}B_{veh}F_{veh}s(t-\tau(t)) = 0$. Noticing (5), then

$$\begin{aligned} \dot{s}(t) &= A_{veh}(s(t) + h_i) + \sum_{j=1}^{N} l_{ij}B_{veh}F_{veh}s(t-\tau(t)) \\ &= A_{veh}s(t). \end{aligned} \tag{7}$$

Theorem 1 is used to prove that the formation stability of (6) is equivalent to the asymptotically stability of (9).

Theorem 1. Consider a network of agents with equal communication time-varying delay $\tau(t) > 0$ in all links. Assume the network topology is fixed, undirected, and connected. Eigenvalues of graph Laplacian can be ordered sequentially in an ascending order as $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$. If the following $N-1$ linear time-varying delayed differential equations described as (9) are asymptotically stable about their zero solutions, then (6) is formation stable, that is, the vehicles converge to formation h.
Proof.
Let $x_i(t) - h_i = s(t) + e_i(t)$, we rewrite (5) as $\dot{s}(t) + \dot{e}_i(t) + \dot{h}_i = A_{veh}(s(t) + e_i(t) + h_i) + \sum\limits_{j=1}^{N} l_{ij}B_{veh}F_{veh}(s(t-\tau(t)) + e_j(t-\tau(t)))$. Note that $\dot{h}_i = 0$ and (7), the equation can be

$$\dot{e}_i(t) = A_{veh}e_i(t) + \sum_{j=1}^{N} l_{ij}B_{veh}F_{veh}e_j(t-\tau(t)).$$

Collecting the equations for all $e_i(t)$ into one system we get

$$\dot{e}(t) = I_N \otimes A_{veh}e(t) + L \otimes B_{veh}F_{veh}e(t-\tau(t)). \tag{8}$$

Let U be a matrix such that $\tilde{L} = U^{-1}LU$ is upper triangular. Then let $\tilde{e}(t) = U \otimes I_{2n}e(t)$, note that Lemma 1, Lemma 2, Lemma 3, and Lemma 4, (8) can be equivalent to

$$\dot{\tilde{e}}(t) = I_N \otimes A_{veh}\tilde{e}(t) + \tilde{L} \otimes B_{veh}F_{veh}\tilde{e}(t-\tau(t)).$$

Because $\tilde{L}$ is upper triangular, its diagonal blocks are of the form

$$\dot{\tilde{e}}_i(t) = A_{veh}\tilde{e}_i(t) + \lambda_i B_{veh}F_{veh}\tilde{e}_i(t-\tau(t)).$$

Let $v(t) = \tilde{e}_i(t)$, this equation can be rewritten as

$$\dot{v}(t) = A_{veh}v(t) + \lambda_i B_{veh}F_{veh}v(t-\tau(t)). \tag{9}$$

Thus, we have transformed the stability of N subsystems described as (6) to the stability of N linear delayed differential systems (9) when $i = 1, 2, \cdots, N$. We note that when $\lambda_1 = 0$ (9) become identical with (7). Then if the following $N-1$ equations

$$\dot{v}(t) = A_{veh}v(t) + \lambda_i B_{veh}F_{veh}v(t-\tau(t)), \quad i = 2, \cdots, N$$

are asymptotically stable, (6) is formation stable, namely, the vehicles converge to formation h. Thus, Theorem 1 is proved.

3.2 Stability of system with model uncertainties

In this section, we will analyze the vehicles with model uncertainties. Since most engineering systems designs are based on mathematical model, models and the reality they represent are always different. To be practical, model uncertainties must be considered. Consider these we give a modified model of (5) as follows

$$\begin{aligned} \dot{x}_i(t) =& (A_{veh} + \Delta A_{veh}(t))x_i(t) \\ &+ \sum_{j=1}^{N} l_{ij}(B + \Delta B(t))x_j(t-\tau(t)) - h_j). \end{aligned}$$

It can be rewritten in compact form as

$$\begin{aligned} \dot{x} =& I_N \otimes (A_{veh} + \Delta A_{veh}(t))x \\ &+ (L_N \otimes (B + \Delta B(t))(x(t-\tau(t)) - h), \end{aligned} \tag{10}$$

where, $B = B_{veh}F_{veh}$. The uncertainties matrices $\Delta A_{veh}(t)$, $\Delta B(t)$ are time-varying and have the same structural property as A_{veh} and B, which are defined as

$$\begin{aligned} \Delta A_{veh}(t) &= DF(t)E_1 \\ \Delta B(t) &= DF(t)E_2 \end{aligned}$$

where D, E_1, E_2 are known as real constant matrices with appropriate dimensions, $F(t)$ are unknown real time-varying matrices bounded by

$$F^T(t)F(t) \leq I.$$

Theorem 2 is given to show that the formation stability of (10) is equivalent to the asymptotically stability of (11).

Theorem 2. Consider a network of agents with equal communication time-delay $\tau(t) > 0$ in all links. Assume the network topology $\mathbb{G}$ is fixed, undirected, and connected. Eigenvalues of graph Laplacian can be ordered sequentially in an ascending order as

$$0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N.$$

If the following $N-1$ linear time-varying delayed differential equations are asymptotically stable about their zero solutions

$$\dot{v} = (A_{veh} + \Delta A_{veh}(t))v(t) + \lambda_i(B + \Delta B(t))v(t - \tau(t)), \tag{11}$$

where $i = 2, 3, \cdots, N$. Then (10) is formation robust stable, the vehicles converge to formation h.
Proof.
The proofs of Theorem 2 are almost the same as those of Theorem 1, so are omitted here.

4 STABILITY ANALYSIS OF VEHICLES FORMATIONS

Some sufficient conditions for stability of vehicles formation are given based on free-weighting matrix method [20] and LMI. Two theorems are concluded in this section.
Theorem 3 provides the sufficient conditions of the asymptotically stability of (9).

Theorem 3 Consider a network of agents with equal communication time-delay $\tau(t) > 0$ in all links. Assume the network topology $\mathbb{G}$ is fixed, undirected, and connected. Eigenvalues of graph Laplacian can be ordered sequentially in an ascending order as

$$0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$$

Given that $d > 0$ and $\mu > 0$, if there exists matrix $P = P^T > 0, Q = Q^T \geq 0, R = R^T \geq 0, Z_1 = Z_1{}^T > 0, Z_2 = Z_2{}^T > 0, M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}, N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}, S = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}$ such that

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & dN_1 & dS_1 & dM_1 & \Psi_{17} \\ \Psi_{21} & \Psi_{22} & \Psi_{23} & dN_2 & dS_2 & dM_2 & \Psi_{27} \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & dN_3 & dS_3 & dM_3 & 0 \\ dN_1^T & dN_2^T & dN_3^T & -dZ_1 & 0 & 0 & 0 \\ dS_1^T & dS_2^T & dS_3^T & 0 & -dZ_1 & 0 & 0 \\ dM_1^T & dM_2^T & dM_3^T & 0 & 0 & -dZ_2 & 0 \\ \Psi_{71} & \Psi_{72} & 0 & 0 & 0 & 0 & \Psi_{77} \end{bmatrix} < 0 \tag{12}$$

for $i = 2, \cdots, N$, then (9) is asymptotically stable, (6) is formation stable, that is , vehicles converge to formation h. Here,

$$\begin{aligned}
&\Psi_{11} = PA_{veh} + A_{veh}^T P + Q + R + N_1^T + N_1 + M_1 + M_1^T \\
&\Psi_{21}^T = \Psi_{12} = \lambda_i P B_{veh} F_{veh} - N_1 + S_1 + M_2^T + N_2^T \\
&\Psi_{31}^T = \Psi_{13} = -M_1 - S_1 + M_3^T + N_3^T \\
&\Psi_{22} = -(1-\mu)Q - N_2 + S_2 - N_2^T + S_2^T \\
&\Psi_{32}^T = \Psi_{23} = -M_2 - S_2 - N_3^T + S_3^T \\
&\Psi_{33} = -R - M_3 - S_3 - M_3^T - S_3^T \\
&\Psi_{71}^T = \Psi_{17} = dA_{veh}^T(Z_1 + Z_2) \\
&\Psi_{72}^T = \Psi_{27} = d\lambda_i (B_{veh}F_{veh})^T (Z_1 + Z_2) \\
&\Psi_{77} = -d(Z_1 + Z_2).
\end{aligned}$$

Proof.
We select the following Lyapunov-Krasovskii function

$$\begin{aligned}
V(v(t)) =& v^T(t)Pv(t) + \int_{t-\tau(t)}^{t} v^T(\alpha)Qv(\alpha)d\alpha \\
&+ \int_{t-d}^{t} v^T(\alpha)Rv(\alpha)d\alpha \\
&+ \int_{-d}^{0}\int_{t+\beta}^{t} \dot{v}^T(\alpha)(Z_1 + Z_2)\dot{v}(\alpha)d\alpha d\beta
\end{aligned}$$

By Newton-Leibniz Formula, for matrix M, N and S, we can obtain

$$2\zeta_1^T(t)N\left[v(t) - v(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{v}(\alpha)d\alpha\right] = 0$$

$$2\zeta_1^T(t)S\left[v(t-\tau(t)) - v(t-d) - \int_{t-d}^{t-\tau(t)} \dot{v}(\alpha)d\alpha\right] = 0$$

$$2\zeta_1^T(t)M\left[v(t) - v(t-d) - \int_{t-d}^{t} \dot{v}(\alpha)d\alpha\right] = 0$$

where, $\zeta_1(t) = \begin{bmatrix} v^T(t) & v^T(t-\tau(t)) & v^T(t-d) \end{bmatrix}^T$. The derivative of V along (9) is

$$
\begin{aligned}
&\dot V(v(t)) = \\
&\dot v^T(t)Pv(t) + v^T(t)P\dot v(t) + v^T(t)Qv(t) \\
&- (1-\dot\tau(t))v^T(t-\tau(t))Qv(t-\tau(t)) \\
&+ v^T(t)Rv(t) - v^T(t-d)Rv(t-d) \\
&+ d\dot v^T(t)(Z_1+Z_2)\dot v(t) \\
&- \int_{t-d}^{t} \dot v^T(\alpha)(Z_1+Z_2)\dot v(\alpha)d\alpha \\
&\leq \\
&\dot v^T(t)Pv(t) + v^T(t)P\dot v(t) + v^T(t)(Q+R)v(t) \\
&- (1-\mu)v^T(t-\tau(t))Qv(t-\tau(t)) \\
&- v^T(t-d)Rv(t-d) + d\dot v^T(t)(Z_1+Z_2)\dot v(t) \\
&- \int_{t-\tau(t)}^{t} \dot v^T(\alpha)Z_1\dot v(\alpha)d\alpha - \int_{t-d}^{t-\tau(t)} \dot v^T(\alpha)Z_1\dot v(\alpha)d\alpha \\
&- \int_{t-d}^{t} \dot v^T(\alpha)Z_2\dot v(\alpha)d\alpha \\
&+ 2\zeta_1^T(t)N\left[v(t) - v(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot v(\alpha)d\alpha\right] \\
&+ 2\zeta_1^T(t)S\left[v(t-\tau(t)) - v(t-d) - \int_{t-d}^{t-\tau(t)}\dot v(\alpha)d\alpha\right] \\
&+ 2\zeta_1^T(t)M\left[v(t) - v(t-d) - \int_{t-d}^{t}\dot v(\alpha)d\alpha\right] \\
&= \\
&(v^T(t)A_{veh}^T + v^T(t-\tau(t))\lambda_i(B_{veh}F_{veh})^T)Pv(t) \\
&+ v^T(t)P(A_{veh}v(t) + \lambda_i B_{veh}F_{veh}v(t-\tau(t))) \\
&+ v^T(t)(Q+R)v(t) - (1-\mu)v^T(t-\tau(t))Qv(t-\tau(t)) \\
&- v^T(t-d)Rv(t-d) + dv^T(t)A_{veh}^T(Z_1+Z_2)A_{veh}v(t) \\
&+ dv^T(t)A_{veh}^T(Z_1+Z_2)\lambda_i B_{veh}F_{veh}v(t-\tau(t)) \\
&+ dv^T(t-\tau(t))\lambda_i(B_{veh}F_{veh})^T)(Z_1+Z_2)A_{veh}v(t) \\
&+ dv^T(t-\tau(t))\lambda_i(B_{veh}F_{veh})^T)Z_1\lambda_i B_{veh}F_{veh}v(t-\tau(t)) \\
&+ dv^T(t-\tau(t))\lambda_i(B_{veh}F_{veh})^T)Z_2\lambda_i B_{veh}F_{veh}v(t-\tau(t)) \\
&- \int_{t-\tau(t)}^{t} \dot v^T(\alpha)Z_1\dot v(\alpha)d\alpha - \int_{t-d}^{t-\tau(t)} \dot v^T(\alpha)Z_1\dot v(\alpha)d\alpha \\
&- \int_{t-d}^{t} \dot v^T(\alpha)Z_2\dot v(\alpha)d\alpha \\
&+ 2\zeta_1^T(t)N\left[v(t) - v(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot v(\alpha)d\alpha\right] \\
&+ 2\zeta_1^T(t)S\left[v(t-\tau(t)) - v(t-d) - \int_{t-d}^{t-\tau(t)}\dot v(\alpha)d\alpha\right] \\
&+ 2\zeta_1^T(t)M\left[v(t) - v(t-d) - \int_{t-d}^{t}\dot v(\alpha)d\alpha\right] \qquad (13) \\
&= \\
&(v^T(t)A_{veh}^T + v^T(t-\tau(t))\lambda_i(B_{veh}F_{veh})^T)Pv(t) \\
&+ v^T(t)P(A_{veh}v(t) + \lambda_i B_{veh}F_{veh}v(t-\tau(t))) \\
&+ v^T(t)(Q+R)v(t) - (1-\mu)v^T(t-\tau(t))Qv(t-\tau(t)) \\
&- v^T(t-d)Rv(t-d) + dv^T(t)A_{veh}^T(Z_1+Z_2)A_{veh}v(t) \\
&+ dv^T(t)A_{veh}^T(Z_1+Z_2)\lambda_i B_{veh}F_{veh}v(t-\tau(t)) \\
&+ dv^T(t-\tau(t))\lambda_i(B_{veh}F_{veh})^T)(Z_1+Z_2)A_{veh}v(t) \\
&+ dv^T(t-\tau(t))\lambda_i(B_{veh}F_{veh})^T) \\
&(Z_1+Z_2)\lambda_i B_{veh}F_{veh}v(t-\tau(t)) \\
&- \int_{t-\tau(t)}^{t} \dot v^T(\alpha)Z_1\dot v(\alpha)d\alpha - \int_{t-d}^{t-\tau(t)} \dot v^T(\alpha)Z_1\dot v(\alpha)d\alpha \\
&- \int_{t-d}^{t} \dot v^T(\alpha)Z_2\dot v(\alpha)d\alpha \\
&+ 2\zeta_1^T(t)N\left[v(t) - v(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot v(\alpha)d\alpha\right] \\
&+ 2\zeta_1^T(t)S\left[v(t-\tau(t)) - v(t-d) - \int_{t-d}^{t-\tau(t)}\dot v(\alpha)d\alpha\right] \\
&+ 2\zeta_1^T(t)M\left[v(t) - v(t-d) - \int_{t-d}^{t}\dot v(\alpha)d\alpha\right] \\
&- \int_{t-\tau(t)}^{t} \zeta_1^T(t)NZ_1^{-1}N^T\zeta_1(t)d\alpha \\
&+ \int_{t-\tau(t)}^{t} \zeta_1^T(t)NZ_1^{-1}N^T\zeta_1(t)d\alpha \\
&- \int_{t-d}^{t-\tau(t)} \zeta_1^T(t)SZ_1^{-1}S^T\zeta_1(t)d\alpha \\
&+ \int_{t-d}^{t-\tau(t)} \zeta_1^T(t)SZ_1^{-1}S^T\zeta_1(t)d\alpha \\
&- \int_{t-d}^{t} \zeta_1^T(t)MZ_2^{-1}M^T\zeta_1(t)d\alpha \\
&+ \int_{t-d}^{t} \zeta_1^T(t)MZ_2^{-1}M^T\zeta_1(t)d\alpha \\
&\leq \\
&(v^T(t)A_{veh}^T + v^T(t-\tau(t))\lambda_i(B_{veh}F_{veh})^T)Pv(t) \\
&+ v^T(t)P(A_{veh}v(t) + \lambda_i B_{veh}F_{veh}v(t-\tau(t))) \\
&+ v^T(t)(Q+R)v(t) - (1-\mu)v^T(t-\tau(t))Qv(t-\tau(t)) \\
&- v^T(t-d)Rv(t-d) + dv^T(t)A_{veh}^T(Z_1+Z_2)A_{veh}v(t) \\
&+ dv^T(t)A_{veh}^T(Z_1+Z_2)\lambda_i B_{veh}F_{veh}v(t-\tau(t)) \\
&+ dv^T(t-\tau(t))\lambda_i(B_{veh}F_{veh})^T)(Z_1+Z_2)A_{veh}v(t) \\
&+ dv^T(t-\tau(t))\lambda_i(B_{veh}F_{veh})^T) \\
&(Z_1+Z_2)\lambda_i B_{veh}F_{veh}v(t-\tau(t)) \\
&- \int_{t-\tau(t)}^{t} \dot v^T(\alpha)Z_1\dot v(\alpha)d\alpha - \int_{t-d}^{t-\tau(t)} \dot v^T(\alpha)Z_1\dot v(\alpha)d\alpha
\end{aligned}
$$

$$
\begin{aligned}
&-\int_{t-d}^{t}\dot{v}^T(\alpha)Z_2\dot{v}(\alpha)d\alpha \\
&+2\zeta_1^T(t)N\left[v(t)-v(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{v}(\alpha)d\alpha\right] \\
&+2\zeta_1^T(t)S\left[v(t-\tau(t))-v(t-d)-\int_{t-d}^{t-\tau(t)}\dot{v}(\alpha)d\alpha\right] \\
&+2\zeta_1^T(t)M\left[v(t)-v(t-d)-\int_{t-d}^{t}\dot{v}(\alpha)d\alpha\right] \\
&-\int_{t-\tau(t)}^{t}\zeta_1^T(t)NZ_1^{-1}N^T\zeta_1(t)d\alpha+d\zeta_1^T(t)NZ_1^{-1}N^T\zeta_1(t) \\
&-\int_{t-d}^{t-\tau(t)}\zeta_1^T(t)SZ_1^{-1}S^T\zeta_1(t)d\alpha+d\zeta_1^T(t)SZ_1^{-1}S^T\zeta_1(t) \\
&-\int_{t-d}^{t}\zeta_1^T(t)MZ_2^{-1}M^T\zeta_1(t)d\alpha+d\zeta_1^T(t)MZ_2^{-1}M^T\zeta_1(t)
\end{aligned}
$$

Noting that

$$
\begin{aligned}
&-\int_{t-\tau(t)}^{t}\dot{v}^T(\alpha)Z_1\dot{v}(\alpha)d\alpha-2\zeta_1^T(t)N\int_{t-\tau(t)}^{t}\dot{v}(\alpha)d\alpha \\
&-\int_{t-\tau(t)}^{t}\zeta_1^T(t)NZ_1^{-1}N^T\zeta_1(t)d\alpha \\
&=-\int_{t-\tau(t)}^{t}[\zeta_1^T(t)N+\dot{v}^T(\alpha)Z_1]Z_1^{-1}[N^T\zeta_1(t) \\
&+Z_1\dot{v}(\alpha)]d\alpha
\end{aligned}
$$

Similarly, the other integral items can be combined. Then we obtain

$$
\begin{aligned}
&\dot{V}(v(t))\leq \\
&\begin{bmatrix} v(t) \\ v(t-\tau(t)) \\ v(t-d)\end{bmatrix}^T\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} \\ \Xi_{21} & \Xi_{22} & \Xi_{23} \\ \Xi_{31} & \Xi_{32} & \Xi_{33}\end{bmatrix}\begin{bmatrix} v(t) \\ v(t-\tau(t)) \\ v(t-d)\end{bmatrix} \\
&-\int_{t-\tau(t)}^{t}[\zeta_1^T(t)N+\dot{v}^T(\alpha)Z_1]Z_1^{-1}[N^T\zeta_1(t)+Z_1\dot{v}(\alpha)]d\alpha \\
&-\int_{t-d}^{t-\tau(t)}[\zeta_1^T(t)S+\dot{v}^T(\alpha)Z_1]Z_1^{-1}[S^T\zeta_1(t)+Z_1\dot{v}(\alpha)]d\alpha \\
&-\int_{t-d}^{t}[\zeta_1^T(t)M+\dot{v}^T(\alpha)Z_2]Z_2^{-1}[M^T\zeta_1(t)+Z_2\dot{v}(\alpha)]d\alpha
\end{aligned}
$$

where

$$
\begin{aligned}
\Xi_{11}=&PA_{veh}+A_{veh}^TP+Q+R+N_1^T+N_1+M_1+M_1^T \\
&+d(A_{veh}^T(Z_1+Z_2)A_{veh}+N_1Z_1^{-1}N_1^T+S_1Z_1^{-1}S_1^T \\
&+M_1Z_2^{-1}M_1^T) \\
\Xi_{21}^T=&\Xi_{12}=\lambda_iPB_{veh}F_{veh}-N_1+S_1+M_2^T+N_2^T \\
&+d(\lambda_iA_{veh}^T(Z_1+Z_2)B_{veh}F_{veh}+N_1Z_1^{-1}N_2^T \\
&+S_1Z_1^{-1}S_2^T+M_1Z_2^{-1}M_2^T) \\
\Xi_{31}^T=&\Xi_{13}=-M_1-S_1+M_3^T+N_3^T \\
&+d(N_1Z_1^{-1}N_3^T+S_1Z_1^{-1}S_3^T+M_1Z_2^{-1}M_3^T) \\
\Xi_{22}=&-(1-\mu)Q-N_2+S_2-N_2^T+S_2^T \\
&+d\lambda_i^2(B_{veh}F_{veh})^T(Z_1+Z_2)B_{veh}F_{veh} \\
&+d(N_2Z_1^{-1}N_2^T+S_2Z_1^{-1}S_2^T+M_2Z_2^{-1}M_2^T) \\
\Xi_{32}^T=&\Xi_{23}=-M_2-S_2-N_3^T+S_3^T \\
&+d(N_2Z_1^{-1}N_3^T+S_2Z_1^{-1}S_3^T+M_2Z_2^{-1}M_3^T) \\
\Xi_{33}=&-R-M_3-S_3-M_3^T-S_3^T \\
&+d(N_3Z_1^{-1}N_3^T+S_3Z_1^{-1}S_3^T+M_3Z_2^{-1}M_3^T).
\end{aligned}
$$

Since $Z_1>0$,$Z_2>0$, then the last three parts of (13) are all less than 0. So if

$$
\Xi=\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} \\ \Xi_{21} & \Xi_{22} & \Xi_{23} \\ \Xi_{31} & \Xi_{32} & \Xi_{33}\end{bmatrix}<0 \tag{14}
$$

then $\dot{V}(v(t))<0$, (6) is formation stable, that is, vehicles converge to formation h. According to Lemma 5, (14) is equivalent to (12). Thus, the proof of Theorem 3 is completed.

Theorem 4 gives the sufficient conditions that (11) asymptotically converges to its zero point.

Theorem 4 Consider a network of agents with equal communication time-delay $\tau>0$ in all links. Assume the network topology $\mathbb{G}$ is fixed, undirected, and connected. Eigenvalues of graph Laplacian can be ordered sequentially in an ascending order as $0=\lambda_1\leq\lambda_2\leq\cdots\leq\lambda_N$. Given that $d>0$ and μ, if there exists matrix $P=P^T>0$, $Q=Q^T\geq 0$, $R=R^T\geq 0$, $Z_1={Z_1}^T>0$, $Z_2={Z_2}^T>0$, $M=\begin{bmatrix}M_1 \\ M_2 \\ M_3\end{bmatrix}$, $N=\begin{bmatrix}N_1 \\ N_2 \\ N_3\end{bmatrix}$, $S=\begin{bmatrix}S_1 \\ S_2 \\ S_3\end{bmatrix}$ such that

$$
\begin{bmatrix}
\Psi_{11} & \Psi_{12} & \Psi_{13} & dN_1 & dS_1 & dM_1 & \Psi_{17} & PD & E_1^T \\
\Psi_{21} & \Psi_{22} & \Psi_{23} & dN_2 & dS_2 & dM_2 & \Psi_{27} & 0 & \lambda_iE_2^T \\
\Psi_{31} & \Psi_{32} & \Psi_{33} & dN_3 & dS_3 & dM_3 & 0 & 0 & 0 \\
dN_1^T & dN_2^T & dN_3^T & -dZ_1 & 0 & 0 & 0 & 0 & 0 \\
dS_1^T & dS_2^T & dS_3^T & 0 & -dZ_1 & 0 & 0 & 0 & 0 \\
dM_1^T & dM_2^T & dM_3^T & 0 & 0 & -dZ_2 & 0 & 0 & 0 \\
\Psi_{71} & \Psi_{72} & 0 & 0 & 0 & 0 & \Psi_{77} & -\Psi_{77}D & 0 \\
D^TP & 0 & 0 & 0 & 0 & 0 & -D^T\Psi_{77} & -I & 0 \\
E_1 & \lambda_iE_2 & 0 & 0 & 0 & 0 & 0 & 0 & -I
\end{bmatrix}<0 \tag{15}
$$

for $i=2,\cdots,N$, then (11) is asymptotically stable, (10) is formation robust stable, that is, vehicles converge to formation h.

Proof.
According to Theorem 3, substituting matrices $A_{veh} + \Delta A_{veh}$ and $B + \Delta B$ for A_{veh} and B in Theorem 3,(11) is asymptotically stable if there exist symmetric matrices $\bar{P} = \bar{P}^T > 0$,$\bar{Q} = \bar{Q}^T \geq 0$,$\bar{R} = \bar{R}^T \geq 0$,$\bar{Z}_1 = \bar{Z}_1^T > 0$, $\bar{Z}_2 = \bar{Z}_2^T > 0$,$\bar{M} = \begin{bmatrix}\bar{M}_1\\ \bar{M}_2\\ \bar{M}_3\end{bmatrix}$,$\bar{N} = \begin{bmatrix}\bar{N}_1\\ \bar{N}_2\\ \bar{N}_3\end{bmatrix}$,$\bar{S} = \begin{bmatrix}\bar{S}_1\\ \bar{S}_2\\ \bar{S}_3\end{bmatrix}$ satisfying

$$\Psi' = \begin{bmatrix} \Psi'_{11} & \Psi'_{12} & \bar{\Psi}_{13} & d\bar{N}_1 & d\bar{S}_1 & d\bar{M}_1 & \Psi'_{17} \\ \Psi'_{21} & \bar{\Psi}_{22} & \bar{\Psi}_{23} & d\bar{N}_2 & d\bar{S}_2 & d\bar{M}_2 & \Psi'_{27} \\ \bar{\Psi}_{31} & \bar{\Psi}_{32} & \bar{\Psi}_{33} & d\bar{N}_3 & d\bar{S}_3 & d\bar{M}_3 & 0 \\ d\bar{N}_1^T & d\bar{N}_2^T & d\bar{N}_3^T & -d\bar{Z}_1 & 0 & 0 & 0 \\ d\bar{S}_1^T & d\bar{S}_2^T & d\bar{S}_3^T & 0 & -d\bar{Z}_1 & 0 & 0 \\ d\bar{M}_1^T & d\bar{M}_2^T & d\bar{M}_3^T & 0 & 0 & -d\bar{Z}_2 & 0 \\ \Psi'_{71} & \Psi'_{72} & 0 & 0 & 0 & 0 & \bar{\Psi}_{77} \end{bmatrix} < 0$$

Here,

$$\begin{aligned} \Psi'_{11} &= \bar{\Psi}_{11} + \bar{P}\Delta A_{veh} + \Delta A_{veh}^T \bar{P} \\ \Psi'^T_{21} &= \Psi'_{12} = \bar{\Psi}_{12} + \lambda_i \bar{P}\Delta B \\ \Psi'^T_{71} &= \Psi'_{17} = \bar{\Psi}_{17} + d\Delta A_{veh}^T(\bar{Z}_1 + \bar{Z}_2) \\ \Psi'^T_{72} &= \Psi'_{27} = \bar{\Psi}_{27} + d\lambda_i(\Delta B)^T(\bar{Z}_1 + \bar{Z}_2) \end{aligned}$$

Then

$$\Psi' = \bar{\Psi} + \begin{bmatrix} \bar{P}D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ d(Z_1+Z_2)D \end{bmatrix} F \begin{bmatrix} E_1 & \lambda_i E_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} E_1 & \lambda_i E_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T F^T \begin{bmatrix} \bar{P}D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ d(Z_1+Z_2)D \end{bmatrix}^T < 0$$

Applying Lemma 6, this inequality is equivalent to

$$\bar{\Psi} + \varepsilon \begin{bmatrix} \bar{P}D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ d(Z_1+Z_2)D \end{bmatrix} \begin{bmatrix} \bar{P}D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ d(Z_1+Z_2)D \end{bmatrix}^T + \varepsilon^{-1} \begin{bmatrix} E_1 & \lambda_i E_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} E_1 & \lambda_i E_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0$$

where ε is a positive constant. We multiply ε on both side of this inequality, then we have

$$\varepsilon\bar{\Psi} + \begin{bmatrix} \varepsilon\bar{P}D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ d(Z_1+Z_2)D \end{bmatrix} \begin{bmatrix} \varepsilon\bar{P}D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ d(Z_1+Z_2)D \end{bmatrix}^T + \begin{bmatrix} E_1 & \lambda_i E_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} E_1 & \lambda_i E_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0.$$

We substitute $\varepsilon\bar{P}$, $\varepsilon\bar{Q}$, $\varepsilon\bar{R}$, $\varepsilon\bar{Z}_1$, $\varepsilon\bar{Z}_2$, $\varepsilon\bar{M}$, $\varepsilon\bar{N}$, $\varepsilon\bar{S}$ respectively by P, Q, R, Z_1, Z_2, M, N, S. Then the inequality can be rewritten as

$$\begin{aligned} &\Psi + \begin{bmatrix} PD \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ d(Z_1+Z_2)D \end{bmatrix} \begin{bmatrix} PD \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ d(Z_1+Z_2)D \end{bmatrix}^T + \begin{bmatrix} E_1 & \lambda_i E_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} E_1 & \lambda_i E_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &= \Psi - \begin{bmatrix} PD & E_1^T \\ 0 & \lambda_i E_2^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\Psi_{77}D & 0 \end{bmatrix} \begin{bmatrix} -I & 0 \\ 0 & -I \end{bmatrix}^{-1} \begin{bmatrix} PD & E_1^T \\ 0 & \lambda_i E_2^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\Psi_{77}D & 0 \end{bmatrix}^T < 0. \end{aligned}$$

Applying Lemma 5, this inequality is equivalent to (15). Thus, Theorem 4 is proved.

5 NUMERICAL SIMULATIONS

Consider a network composed of four vehicles each has the same dynamics as (2) depicts. The network topology

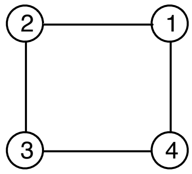

Fig. 2. Topology structure composed of four vehicles

structure is given in Fig.2. The Laplacian of the graph with the adjacency elements is

$$L = \begin{bmatrix} 1 & -0.5 & 0 & -0.5 \\ -0.5 & 1 & -0.5 & 0 \\ 0 & -0.5 & 1 & -0.5 \\ -0.5 & 0 & -0.5 & 1 \end{bmatrix}$$

The eigenvalues of L are $\lambda_i = 0, 1, 1, 2$. The feedback matrix is

$$F_{veh} = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}.$$

When $x_i \in \mathbb{R}^{2n}$ and $n = 2$, it is assumed that

$$A_{veh} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_{veh} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix}, F(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos t & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos t \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix},$$

$$\tau(t) = 0.15 + 0.15\sin(t), d = 0.3, \dot{\tau}(t) \leq \mu = 0.15.$$

Two formations are considered here as depicted in Fig3. In the Diamond formation, the formation vector

$$h_i = \begin{pmatrix} h_{ipx} \\ h_{ipy} \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} h_{ipx} \\ 0 \\ h_{ipy} \\ 0 \end{pmatrix}.$$

Specifically, one group of choices for Diamond formation vectors could be

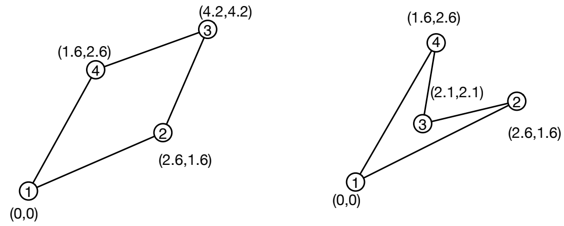

Fig. 3. Two formations: Diamond and Forktail

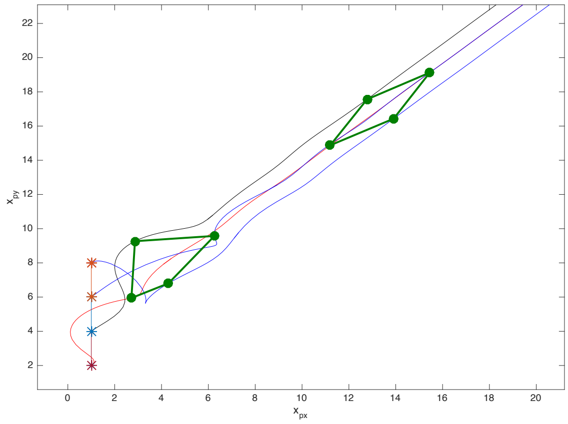

Fig. 4. Trajectories of vehicles ($\tau(t) = 0.15 + 0.15\sin(t)$, $D = 0$)

$$h_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, h_2 = \begin{pmatrix} 2.6 \\ 0 \\ 1.6 \\ 0 \end{pmatrix}, h_3 = \begin{pmatrix} 4.2 \\ 0 \\ 4.2 \\ 0 \end{pmatrix}, h_4 = \begin{pmatrix} 1.6 \\ 0 \\ 2.6 \\ 0 \end{pmatrix}.$$

In fact, the vectors h_i meeting that the coordinate positions of vehicles can make a Diamond formation are all feasible. The whole formation vector are composed of h_i, that is, $h = \begin{pmatrix} h_1^T & h_2^T & h_3^T & h_4^T \end{pmatrix}^T$.
Similarly, in the Forktail formation, the formation vectors also could be determined.

In both cases of Diamond and Forktail formations, Theorem 3 and Theorem 4 are used to determine the stability of system (9) and (11).Theorem 3 is used to test if (9) is stable with given time-varying delays. Theorem 4 is used to test if (11) is stable with given time-varying delays and model uncertainties. According to results, the feasible solutions can be obtained by MATLAB LMI Toolbox in both cases, so formation is stable. Fig.4 and Fig.5 shows that four vehicles with initial positions in a line marked by asterisks eventually converge to a diamond formation marked by circle dots. And Fig.6 and Fig.7 shows the vehicles converge

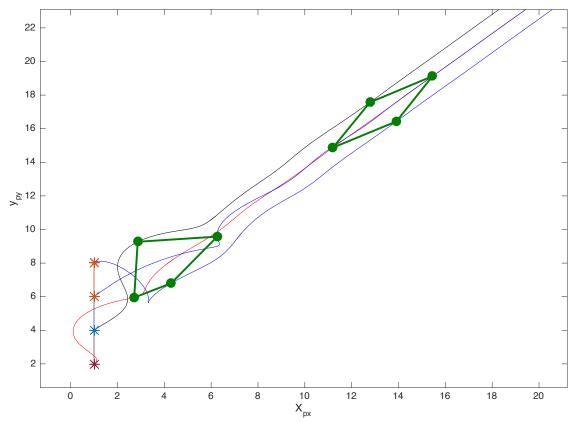

Fig. 5. Trajectories of vehicles($\tau(t) = 0.15 + 0.15\sin(t)$, $D = 0.2I$)

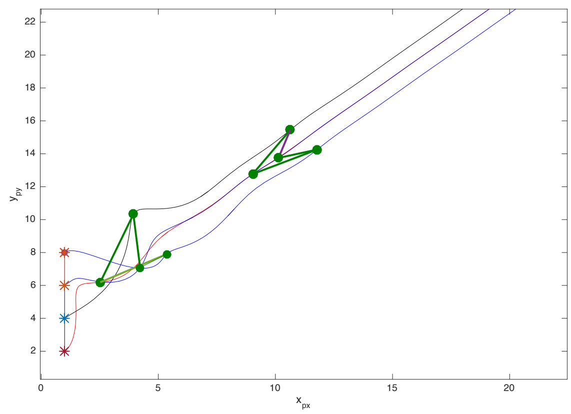

Fig. 6. Trajectories of vehicles ($\tau(t) = 0.15 + 0.15\sin(t)$, $D = 0$)

to a Forktail formation finally.

6 CONCLUSIONS

This paper provides a theoretical analysis for stability of continuous-time vehicles formation with time-varying delays in undirected graphs. The model of vehicle is considered as general linear dynamics. Also the feedback law is linear. It is proved that stability of N Vehicles formations is equivalent to the stability of $N-1$ subsystems which are related to eigenvalues of graph Laplacian. The delays in communication network are assumed to be time-dependent. Stability without uncertainties and robust

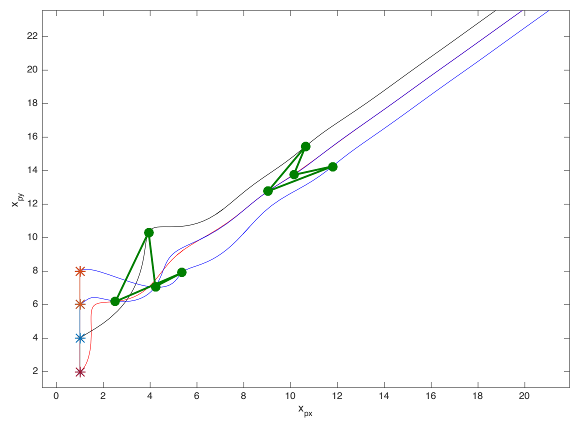

Fig. 7. Trajectories of vehicles ($\tau(t) = 0.15 + 0.15\sin(t)$, $D = 0.2I$)

stability with uncertainties of vehicles formation are considered. The sufficient conditions are given to guarantee that with time-varying delays and uncertainties the vehicles formation can asymptotically converge to predefined formation. Several theorems are concluded on the basis of linear matrix inequality theory and free-weighting matrix method. Several theorems are concluded. Simulations show validity and effectiveness of the results.

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