ABSTRACT
AC currents in multiple layers in the transformer window can increase copper losses significantly due to the proximity effect. Traditionally used Dowell's curves show that the phenomenon starts at copper thickness as low as 1/5 of the skin depth which is just 1.7 mm at 60 Hz. Many designs deviate from assumptions beyond Dowell's solution, which leads to suboptimal design. Finite Element Analysis software allows accurate modeling of high frequency phenomenon but is still considered too tedious to use and requires expert operators for accurate results. New generation products like EMS from EMWorks combine powerful simulation capabilities with easy to use interface appropriate for hands-on engineers in everyday use.

KEYWORDS
Finite Element Analysis, AC copper losses, skin effect, proximity effect

Magnetic component design

3D Electromagnetic Simulation Allows Reduction of AC Copper Losses

1. Introduction
Finite Element Analysis (FEA) software has been used by electrical engineers for several decades. It is a great tool for simulating electromagnetic fields in chokes and transformers allowing accurate computation of the spatial distribution of the current, flux density, associated losses and resulting temperature rise as well as the impact on efficiency of the whole magnetic component. By manipulating dimensions and geometrical arrangements we can yield the most compact, efficient and lowest-cost structure.

Unfortunately, commercial 3D FEA software gained reputation as expensive, tedious and requiring highly skilled and specialised operator to yield sufficiently accurate and reliable results. In this situation many practicing designers were forced to resort to simplified methods with results left to chances.

FEA vendors have been busy for years trying to improve the ease of use, accuracy, stability and versatility of their tools with slow but systematic progress. Some of them became truly practical design tools not only for a PhD working on a science project but also for hands-on designers with general knowledge of magnetic components. Examples for this article were generated using EMS from EMWorks.
2. AC copper losses due to the skin and proximity effect

The nature of AC copper loss challenge in power transformer design is well known. AC currents in windings induce eddy currents. These currents create an uneven current distribution leading to thermal problems and the necessity to redesign the transformer.

In general, currents tend to alter their distribution in a way which minimises the overall amount of energy extracted from the source, both real and reactive. Reduction of the reactive component associated with the energy stored in the magnetic field inside the wire pushes the current towards the surface of the conductor.

Because the conduction losses are proportional to the square of the current density, this uneven current distribution leads to an increase of total losses. Resulting current crowding in a single conductor is most conveniently characterised by skin depth which is defined as a depth below the surface at which the current density has fallen to 1/e. Effective resistance of a wire with AC current is equal to that with DC current uniformly distributed across the skin depth. See Fig. 1.

Skin depth can be calculated according to the formula:

$$\delta = \sqrt{\frac{2\rho}{\omega \mu}}$$

where: $\delta$ - skin depth, $\rho$ - resistivity of a conductor, $\mu$ - permeability of a conductor, $\omega$ - angular frequency

Skin depth for copper at 60 Hz is equal to 8.5 mm. It can be calculated that a single wire with the diameter equal to 3 skin depths (25 mm at 60 Hz) will experience copper loss increase only by about 10%. So it may seem that AC losses are generally not the problem. This, however, applies only to a single wire. When multiple wires conducting AC current are located near each other, we can observe accumulation of the eddy current, layer upon layer. This phenomenon is called a proximity effect and can lead to AC copper losses rising at an astonishing rate even with the wire diameter significantly below the skin depth. Fig. 2 presents current density distribution with just few wires conducting in the same direction. Current density pattern and associated copper losses become impossible to derive analytically. Outer surface current crowding is much stronger than with the case of a single wire, while portions of the cross section become “idle” (Wires in Fig. 1 and 2 carry the same net current and use the same colour scale).

Copper losses with AC current may be surprisingly high. Structures with many layers should be carefully checked against proximity effect.
This pattern is caused by the fact that the internal group of wires (4 central one) pushes its current to the boundary of the group, located near the second layer. This current, in turn, induces additional eddy current in the second layer (12 external wires), which adds to its own eddy current. So in layer 3, current density would triple. The exact nature of this phenomenon may be observed with the EMS plot looking at current density vectors. See Fig. 3.

The current flowing in one of the central wires (the upper one in Fig. 3) induces additional eddy current in the external wire (the lower one in Fig. 3) on top of the eddy current created by its own current. Two eddy currents combined are so strong that the current flows “backwards” near the surface of the external wire. At the bottom side eddy currents flow in the same direction as the main winding current and the combined current density doubles.

This phenomenon leads to a very fast rise in copper losses. The total dissipation in layer 2 is five times greater than in layer 1 (four times than doubled current density on the opposite side + one on the near side).

Current crowding and losses will accelerate if more layers are added. The dissipation in the 3rd layer will be 13 times greater (32+22), in the 4th layer – 25, etc.

Naturally, the problem is so severe because of the wire being very large in diameter, larger than twice the skin depth. If the conductor layer is thinner, than eddy currents are suppressed. It can be viewed as currents flowing in the opposite direction being forced to occupy the same space and cancelling each other, which brings current distribution back to a more uniform one.

Considering how induced currents tend to accumulate, it can be expected that residual eddy currents may add up to a significant level even with the wire diameter below the skin depth.

### 3. Dowell’s curves – estimating AC copper losses

In 1966 P. Dowell [3] made a small number of simplifying assumptions corresponding to a typical geometry of a core and winding in a transformer and solved Maxwell equations for the problem of proximity losses. Converting analytical solution to a graphical form created ubiquitous Dowell’s curves shown in Fig. 4.

Dowell’s curves allow estimating copper losses with multiple layers conducting AC current. They vividly show exponential growth of AC losses when the diameter of conductor is too large.
Dowell’s curves are easy to use. All that is required is skin depth, conductor dimensions and layer arrangement.

Horizontal axis corresponds to the ratio between the thickness of the copper layer and the skin depth. 0.1 corresponds to the copper layer thickness equal to 1/10 of the skin depth, where proximity losses are completely suppressed. 10 corresponds to the copper layer 10 times thicker than the skin depth, where proximity losses are in full strength. (If round wire is used, the diameter for the calculation of the ratio should be multiplied by 0.83).

Vertical axis corresponds to the ratio of the losses with AC current to those with the DC current (where skin and proximity effect are absent and current density is uniform).

Individual curves correspond to the increase of the resistance depending on the position of the wire in the layer stack. Curve m=1 shows a coefficient for the first layer counting from the core up: curve m=2 for the second, etc., see Fig. 5.

Curves with a fractional number pertain to a situation where adjacent layers conduct the current in the opposite direction, for example on the boundary between primary and secondary. M=0.5 shows losses with a single layer sandwiched between two layers conducting current in the opposite direction, etc.

Using this form of Dowell’s curve, one must remember that copper losses will be different for each layer and the losses for all layers have to be combined. Alternatively, one may use another version of the Dowell’s curve with coefficient corresponding to the total losses of all layers. (Caution is advised as these two versions look similar!).

Because AC losses go up so rapidly with the increased number of layers, not only can the additional wire thickness become useless, but the losses can also actually go up! This can be derived from Fig. 4. For example, let us assume we have 3 layers of copper with the thickness of 0.5 skin depth. From Fig. 4 we can read that the resistance will be approximately 1.1 times higher than for DC. If we double the wire diameter, its DC resistance drops 4 times but AC loss coefficient shoots up to 3. As a result, our effective AC resistance will be \( \frac{1}{4} \cdot 3 = 0.75 \) of the DC case. But if we quadruple the wire diameter, then using Dowell’s curve, we can calculate: \( \frac{1}{16} \cdot 20 = 1.25 \) more losses than for DC. With greater number of layers AC copper losses rise even faster, as can be seen from Fig. 4.

Figure 4: Dowell’s curves showing the ratio between AC and DC resistance [1]

Figure 5: Stack order number “m” from Dowell’s curves for the case of a transformer with wires arranged in 3 layers.

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Depending on the number of layers, the optimal conductor thickness variation is nicely shown in another version of Dowell’s curves shown in Fig. 6.

Curves in Fig. 6 are derived from the same analytical formulas. The vertical axis now shows how many times copper losses with AC current are higher comparing with a DC losses in a single wire with the diameter equal to a skin depth. Losses on the left are high because copper layer is thin. They are also identical for all layers because proximity losses are completely suppressed. As we move to the right initially, the losses go down because with more copper current density drops. When the wire thickness is increased too much, the losses start to rise again. It is where the proximity phenomenon starts to dominate. Far to the right, the losses become constant as the eddy currents have all the space they need to fully develop and any extra copper is just inert.

The optimal layer thickness depends on its position in the stack: for \( m=0.5 \) (primary – secondary sandwich) the optimal thickness is equal to 3 skin depths, while for \( m=15 \) it is only 0.3 (2.5mm for 60 Hz).

It should also be noted that using several strands of smaller wire instead of one thi-
Electromagnetic Finite Element Analysis software can simulate complex patterns of AC currents in practical structures. Accuracy is much higher than with Dowell’s curves

4. Finite element analysis assets

Armed with the Dowell curves we can design a transformer with minimal AC losses but practical application is more complicated than it seems. For example, different number of primary and secondary turns leads to the fractional number of layers in the Dowell scheme. The thickness of individual layers, depending on their proximity situation, should also be varied in such a way that we hit the minimum from Fig. 6. For windings carrying both AC and DC, the optimal thickness is increased. The presence of higher harmonics, in turn, requires additional reduction of the copper thickness (skin depth goes down with the frequency).

Moreover, in practical transformer geometry assumptions behind the Dowells model are frequently not accurate leading to significant errors. The most important are:

1. Interaction between the windings of different phases
2. Interaction with the eddy currents in the core
3. Non-negligible and varying distance from the layers to the core
4. Additional losses in the wires in the extreme locations of the layer
5. Current crowding due to passing from one layer to another
6. Current crowding in the winding terminations
7. Current crowding on the inner side of the wire wound around the core
8. Flux fringing from the gaps in the core

The impact of these factors can be seen from the simulation of a current distribution in a planar winding of a transformer for 20 kHz switching power supply. It is a bit extreme example but vividly shows how the actual current distribution can become very complex due to proximity effect.

Fig. 7 shows completely different proximity effects in cross section in the plane X and plane Y as well as on the surface. The windings inside the window close to the core tend to have the lowest losses, while the windings in the middle are exposed to the flux fringing from the central core gap and experience the highest losses.

The Dowells curves are a very useful tool but assumptions behind the model frequently oversimplify the impact of the actual transformer geometry. This method can result in suboptimal design with significant AC losses. Using new generation of 3D Finite Element Analysis software we can achieve much higher accuracy because all features of the geometry are incorporated. Results are presented in the convenient form of 3D plots, losses in all conductors, self-inductance, coupling, etc.
Additional option is the possibility of thermal and electromagnetic analysis performed simultaneously. Core and copper losses are automatically used to generate a 3D temperature distribution, giving us immediate insight into the hot spots inside the structure before we run hardware measurements.

The basic thermal simulator is not a CFD (Computational Fluid Dynamics) type of software so we cannot check the impact of the airflow or circulating cooling fluid. The most practical approach is to assume a uniform thermal flux density on the object surface corresponding to the typical cooling conditions. For example, 10 W/K-m² can be used as a typical cooling from the surface located in still air.

Thermal simulation allows pinpointing trouble spots. High temperature of the central windings is caused by the flux fringing from the core gap. Temperature rise needed to remove the heat through the insulator or the core is also simulated.

Fig. 7 was generated with fine mesh across the whole structure to observe all details of the proximity effects in the windings. Model with 18 million elements was simulated overnight on a single processor portable BOX workstation. This should be enough even for complex structures with strong proximity effects. Such complex models are necessary only for final verification and "global" results. Optimisation of the details is usually performed defining fine mesh only in selected areas. "Selective" simulations with several hundred thousands of elements zap through in few minutes.

**Conclusion**

Proximity losses in transformer windings can be estimated with limited accuracy using analytical formulas and Dowell’s curves. New generation, easy to use Finite Element Analysis software allows much higher accuracy allowing design modifications reducing AC copper losses. FEA takes into account 3D geometrical features of the magnetic component and complex behaviour of multilayer windings with AC current.

EMS project with parameterised model of the transformer convenient for experiments with the proximity effect is available for download from www.envelopepower.com.

**References**

[4] 3D magnetic simulation project with parameterised model of the transformer convenient for experiments with the proximity effect is available for download from www.envelopepower.com.

**FEA software allows simultaneous simulation of electromagnetic power losses and resulting temperature distribution. It is a great quick check on potential thermal issues**

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Peter Markowski has been involved with power conversion design since graduating in 1990 with an advanced degree in power electronics. For most of his career he worked for Emerson, formerly Artesyn and Computer Products as a product designer and advanced technology engineer. This year he started the consulting business Envelope Power LLC offering complete power supply design and 3D electromagnetic simulation. Peter is the author of 16 U.S. patents and several applications encompassing various aspects of the power conversion engineering.