DESIGN OF LYAPUNOV BASED NONLINEAR VELOCITY CONTROL OF ELECTROHYDRAULIC VELOCITY SERVO SYSTEMS

Edvard Detiček, Nenad Gubeljak, Mitja Kastrevc

Development of a hydraulically driven process of steel centrifugal die casting industry requires accurate response of position in time. In the frame of preliminary investigations the analysis and control of electrohydraulic velocity servo system is considered in the presence of flow nonlinearities and internal friction. The nonlinear and uncertainty characteristics make the conventional controller not yield to the system high requirements. Two different nonlinear design procedures are employed: feedback linearization and backstepping. It is shown that both these techniques can be successfully used to stabilize any chosen operating point of the system. Additionally, invaluable new insights are gained about the dynamics of the system under consideration. This illustrates that the true potential of constructive nonlinear design lies far beyond the mere task of achieving a desired control objective. All derived results are validated by computer simulation of the nonlinear mathematical model of the system.

Keywords: computer simulation; electro-hydraulic velocity servo system; integrator backstepping; Lyapunov methods; velocity control

1 Introduction

Wide range of modern industrial applications is encountered in electrohydraulic servosystems because of their ability to handle large inertia and torque loads. They achieve fast responses and high degree of accuracy at the same time. Typical applications include processing of plastics, industrial robots, aircrafts, lunch vehicles, flight simulators, floating cranes [1], testing systems [2] and numerous military applications. Electrohydraulic servo systems can be classified as either a position, velocity or force-torque system, depending on desired control objectives.

This paper investigates velocity closed loop control system of hydraulically driven process of iron centrifugal die-casting in industry. The research is conducted in laboratory environment and by computer simulations.

Basic components of electrohydraulic servo system are a pump that feeds the system with hydraulic fluid from a tank, an accumulator located on the discharge side of the pump that acts as a supporting source of energy, a relief valve to limit maximum operation pressure. The hydraulic axial-piston hydraulic motor and a servo valve are main components of the system. According to electrical input signal, the position of the spool inside the servo valve is controlled. It directs the oil flow through orifices and consequently determines speed of hydraulic actuator as well as direction of the motion. Besides a control, computer and power electronics low noise velocity and pressure sensors are also important elements of such control systems.

For successful closed loop velocity control, development of suitable controller which could reflect such characteristics is very significant, although the dynamics of electro hydraulic servo system is highly nonlinear [3, 4].

The presence of flow nonlinearities and internal friction make the selection of appropriate closed loop controllers with the use of classic linear control theory difficult. Therefore, a nonlinear design procedure is employed here with the use of feedback linearization technique for stabilizing the controller.

Despite the fact that the controller designed by using the technique of feedback linearization successfully achieves the desired objectives theoretically, another controller is designed using the backstepping approach. It avoids unnecessary cancellations that can have a detrimental effect in the presence of parametric uncertainties or non-modelled dynamics. The performance of designed controller is validated by the appropriate simulation of nonlinear mathematical model of the system.

The authors Krstic et al. in their book [5] introduced the fundamental concept of backstepping method. In the extension of this book, the approach focusing on the stabilization problem in stochastic nonlinear systems is also developed. The backstepping control method is also presented in [6÷10] where this technique is explained in detail for regulating and tracking problem.

To transform a given nonlinear system into an equivalent linear system, feedback linearization is employed for changes of coordinates. A major advantage
of feedback linearization approach is related to the cancellations of nonlinearities that are introduced in design process. On the other hand these cancelations can have a negative effect, namely some nonlinearities can improve the system stability. Their cancellation can lead to instability in the presence of modelling uncertainties. To avoid unnecessary cancelations the integrator backstepping approach is used. The method represents a recursive design scheme that can be used for systems in strict feedback form with nonlinearities non constrained by linear bounds. As a design tool, backstepping is less restrictive than feedback linearization and its previously mentioned design flexibility can put "beneficial nonlinearities" to good in use. At every step of backstepping a new Control Lyapunov Function (CLF) is constructed by augmentation of CLF from previous step by a term which penalizes the error between "virtual control" and its desired value, so called stabilizing function. A major advantage of backstepping is the construction of a Lyapunov function whose derivative can be made negative definite by a variety of control laws rather by a specific control law.

2 System description and dynamical modelling

Photography of the experimental rotary electrohydraulic servo system is shown in Fig. 1.

\[ \tau_{sv} \cdot \dot{A}_{sv} + A_{sv} = K_{sv} \cdot u \]  

where \( K_{sv} \) is the servo valve gain and \( \tau_{sv} \) is the servo valve time constant. The constant mentioned can be determined for the manufacturer's catalogue or by certain tests. Due to the fact that the input of the valve is an electric current but the interface card output is in the form of an electric voltage it is in common to use a current to voltage converter. The constant of this converter is considered in the servo valve gain coefficient [11, 12].

For an ideal critical centre, servo valve with a matched and symmetric orifice the input/output flow rate from the servo valve through the orifices (assuming negligible leakage) can be expressed in the following form:

\[ Q_L = C_d A_{sv} \sqrt{\frac{P_s - p_L \text{sign}(A_{sv})}{\rho}} \]  

where \( P_L = p_{C1} - p_{C2} \) is a load pressure or pressure difference between both chambers, \( P_s = p_{C1} + p_{C2} \) is the supply pressure and \( Q_L \) is the load flow. Assuming no external leakage, \( Q_L \) can be considered as the average flow in each path \( Q_L = \frac{Q_{C1} + Q_{C2}}{2} \). Where \( Q_{C1} \) and \( Q_{C2} \) are flow rates to and from the servo valve.

\( C_d \) and \( \rho \) in Eq. (2) indicate the flow discharge coefficient and fluid density, respectively. The \( \text{sign} \) function stands for the change in the direction of fluid flow through the servo valve. Employing the compressibility equation for the fluid flow in the actuator dynamics along with the oil leakage will result in the following expression:

\[ \frac{V_0}{2\beta} \dot{p}_L = C_d A_{sv} \sqrt{\frac{P_s - p_{L_{\text{sign}}}(A_{sv})}{\rho}} - D_m \dot{\Theta} - C_L p_L \]  

where \( \beta \) and \( V_0 \) are, respectively, the fluid bulk modulus and the oil under compression in one chamber of the actuator. \( D_m \) and \( C_L \) represent the actuator volumetric displacement and total leakage coefficient, respectively. By applying Newton’s second low for rotary motion of a hydraulic actuator and neglecting, the Coulomb’s frictional torque:

\[ J_T \dot{\Theta} = D_m p_L - B \cdot \dot{\Theta} + T_L \]  

where \( D_m \) is the volumetric displacement of hydromotor. \( B \) is the viscous damping coefficient, \( J_T \) is the total inertia of the motor and load referred to the motor shaft and \( T_L \) is the load torque, while \( \Theta, \dot{\Theta} \) are angular velocity and acceleration. Combining Eqs. (1) ÷ (4) and choosing \( x_1 = \dot{\Theta}, x_2 = p_L \) and \( x_3 = A_{sv} \) as state variables we can finally describe the system with third order nonlinear state space model.
\[ \dot{x}_1 = -a_1 x_1 + a_2 x_2 + a_3 \]
\[ \dot{x}_2 = -a_4 x_1 - a_3 x_2 + a_6 \left( \sqrt{P_3 - x_2} \right) x_3 \]
\[ \dot{x}_3 = -a_7 x_3 + a_8 u \]

where
\[ a_1 = \frac{B}{J_1}, \quad a_2 = \frac{D_m}{J_1}, \quad a_3 = \frac{T_l}{J_1}, \quad a_4 = \frac{2\beta}{V_0} D_m, \]
\[ a_5 = \frac{2\beta}{V_0} C_L, \quad a_6 = \frac{2\beta}{V_0} C_L, \quad a_7 = \frac{1}{\tau_v} \quad \text{and} \quad a_8 = \frac{K_{sv}}{\tau_v}. \]

A symbolic representation of the applied rotary electrohydraulic servo system is suggested in Fig. 2.

Mathematical description (5), serves as a basis to apply “integrator backstepping” procedure for design of a nonlinear controller.

3 Nonlinear control and backstepping design

This section addresses the problem of designing a controller which provides asymptotic stability of the operating point of interest. Assuming that the full state information is available, the underlying technique for solving this problem is backstepping. Backstepping control is a technique for designing stabilizing controls for special class of nonlinear dynamic systems. These systems are built from subsystems that radiate out from an irreducible subsystem that can be stabilized using some other method. Because of this recursive structure, the designer can start the design process at known stable system and «back out» new controllers that progressively stabilize each outer subsystem. The process terminates when the final external control is reached.

The detailed derivations of finding backstepping control input are covered in the next 4 steps:

First we define negative errors for system (5):
\[ z_1 = x_1 - r \quad \Rightarrow \quad \dot{z}_1 = \dot{x}_1 - \dot{r} \]
\[ z_2 = x_2 - a_1 \quad \Rightarrow \quad \dot{z}_2 = \dot{x}_2 + a_1 \]
\[ z_3 = x_3 - a_2 \quad \Rightarrow \quad \dot{z}_3 = \dot{x}_3 + a_2 \]

**Step 1a:** First negative error and its derivative is:
\[ z_1 = x_1 - r \quad \Rightarrow \quad x_1 = z_1 + r \]
\[ \dot{z}_1 = \dot{x}_1 - \dot{r} \]

**Step 1b:** Define the first candidate of control Lyapunov function (CLF1):
\[ V_1 = \frac{1}{2} z_1^2 \]

Here we introduce first negative control error \[ z_1 = x_1 - r \] the derivative of Lyapunov function is:
\[ \dot{V}_1 = z_1 \dot{z}_1 = z_1 (\dot{x}_1 - \dot{r}) \]

Insert the first equation of system (5) and the second term in Eq. (6):
\[ \dot{V}_1 = z_1 \dot{z}_1 = z_1 \left( -a_1 x_1 + a_2 \left( \sqrt{P_3 - x_2} \right) x_3 - a_3 - \dot{r} \right) \]
\[ = z_1 \left( -a_1 x_1 + a_2 \sqrt{P_3 - x_2} - a_3 - \dot{r} \right) \]

**Step 1c:** Define first virtual control:
\[ a_1 = \frac{1}{a_2} \left( a_1 x_1 + a_3 + \dot{r} - k_1 z_1 \right) \]

With such selection, we achieve the following form of augmented Lyapunov function:
\[ V_1 = a_2 z_1 z_2 - k_1 z_1^2 \]

Partial differentiations of \( a_1 \) with respect to the terms \( x_1, r, \dot{r} \) are performed below:
\[
\frac{\partial a_1}{\partial x_1} = \frac{\partial}{\partial x_1} \left[ \frac{a_1 - k_1}{a_2} \right] x_1 + \frac{k_1}{a_2} = \frac{1}{a_2} \left( a_1 - k_1 \right)
\]
\[
\frac{\partial a_1}{\partial r} = \frac{\partial}{\partial r} \left[ \frac{a_1 - k_1}{a_2} \right] x_1 + \frac{k_1}{a_2} = \frac{1}{a_2}
\]
\[
\frac{\partial a_1}{\partial \dot{r}} = \frac{\partial}{\partial \dot{r}} \left[ \frac{a_1 - k_1}{a_2} \right] x_1 + \frac{k_1}{a_2} = \frac{1}{a_2}
\]

**Step 2a:** Define second error term and its derivative:
\[
\dot{a}_1 = \frac{\partial a_1}{\partial x_1} \dot{x}_1 + \frac{\partial a_1}{\partial r} \dot{r} + \frac{\partial a_1}{\partial \dot{r}} \ddot{r} = \frac{1}{a_2} \left( a_1 - k_1 \right) \dot{x}_1 + \frac{k_1}{a_2} \dot{r} + \frac{1}{a_2} \ddot{r}
\]

Now we define virtual control for the second equation of (5).

**Step 2b:** Define second error term and its derivative:
\[ z_2 = x_2 - a_1 \]
\[ \dot{z}_2 = \dot{x}_2 - a_1 \]
Inserting second equation of system (5) and above derivative, we get:
\[ \dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = \left( -a_4 x_1 - a_3 x_2 + \left( a_6 \sqrt{P_s - x_2} \right) x_3 \right) - \dot{\alpha}_1 \] (16)

**Step 2b:** Selection of the second virtual control. With the augmentation of the first Lyapunov function we define the second Lyapunov function:
\[ V_2 = V_1 + \frac{1}{2} z_2^2 \] (17)

With differentiation of \( V_2 \) we get:
\[
\dot{V}_2 = \dot{V}_1 + z_2 \dot{z}_2 = -k_1 z_1^2 + a_6 \sqrt{P_s - x_2} z_2 z_3 + \\
+ z_2 \left[ a_2 z_1 - a_4 x_1 - a_3 x_2 + a_6 \sqrt{P_s - x_2} \alpha_2 - \dot{\alpha}_1 \right]
\] (18)

**Step 2c** As a second virtual control select in above equation and define:
\[
\alpha_2 = \frac{1}{a_6 \sqrt{P_s - x_2}} \left[ -a_2 z_1 + a_4 x_1 + a_3 x_2 + \dot{\alpha}_1 - k_2 z_2 \right]
\] (19)

With such selection we achieve the following form of augmented Lyapunov function:
\[
\dot{V}_2 = -k_1 z_1^2 - k_2 z_2^2 + \left( a_6 \sqrt{P_s - x_2} \right) z_2 z_3
\] (20)

In the next step we will need the partial derivative of \( \alpha_2 \) with respect to the variables \( x_1, x_2, r, \dot{r}, \ddot{r} \), and introduce the first equation of system (5):
\[
\alpha_2 = \frac{1}{a_6 \sqrt{P_s - x_2}} \left[ -a_2 z_1 + a_4 x_1 + a_3 x_2 + \dot{\alpha}_1 - k_2 z_2 \right]
\] (21)

Finally, we get:
\[
\dot{\alpha}_2 = \frac{1}{a_6 \sqrt{P_s - x_2}} \left[ -a_2 z_1 + a_4 x_1 + a_3 x_2 + \dot{\alpha}_1 - k_2 z_2 \right]
\] (22)

In final step we define the third control, which is no more virtual, namely we introduce here real control variable \( u \). Define the third error term:
\[
z_3 = x_3 - \alpha_2
\] (23)
\[
\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_2
\] (24)

and introduce the third equation of system (5):
\[ V_3 = V_2 + \frac{1}{2} z_3^2 \] (25)

Put into equation of derivative of second augmented Lyapunov function:
\[
\dot{V}_3 = -k_1 z_1^2 - k_2 z_2^2 + \left( a_6 \sqrt{P_s - x_2} \right) z_2 z_3 + \\
+ z_3 \left[ a_2 z_1 - a_4 x_1 - a_3 x_2 + a_6 \sqrt{P_s - x_2} \alpha_2 - \dot{\alpha}_2 \right]
\] (26)

4 Results of simulation studies

In this chapter, computer simulations are implemented to analyse the novel nonlinear controller developed for the servo hydraulic rotary actuator. Results are compared with the implementation of conventional PI-controller. The simulation code is programmed with Matlab-Simulink programming language.

To implement a numerical simulation of rotary servo hydraulic actuator, knowledge of the appropriate system parameters is required. Tab. 1 lists the parameters of laboratory servo hydraulic system.

<table>
<thead>
<tr>
<th>( J )</th>
<th>( D_h )</th>
<th>( B )</th>
<th>( \beta )</th>
<th>( V_h )</th>
<th>( C_s )</th>
<th>( T_i )</th>
<th>( K_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3,4 \times 10^{-1} ) kgm²</td>
<td>( 0,72 \times 10^{-6} ) m²/s</td>
<td>( 1,1 \times 10^{-6} ) Nms/rad</td>
<td>( 9,25 \times 10^{-12} ) m³/Ns</td>
<td>( 2,7121 \times 10^{-5} ) m³</td>
<td>( 0,63 )</td>
<td>( 0,7 ) Nm</td>
<td>( 4,652 \times 10^{-3} ) m³/V</td>
</tr>
</tbody>
</table>

Computer simulation scheme i.e. Simulink model is represented in Fig. 3.

In Fig. 4 the detail from Fig. 3, namely the backstepping controller, is shown.
Using computer simulation of dynamic behaviour of the closed loop system, the three characteristic situations were investigated. Dynamical response of the system on step change in reference velocity is shown in Fig. 5.

Fig. 6 shows the dynamic behaviour of closed loop system when external disturbance occurs, while Fig. 7 presents the situation when external disturbance is removed.
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5 Conclusions

In this paper, a comprehensive investigation was carried out on the nonlinear control of a velocity servo system consisting of an electrohydraulic servo valve and an axial piston hydraulic motor, which has third relative degree and is minimum phase. To avoid the cancelation of all nonlinearities, which can have negative influence due to non-modelled dynamic, the controller has been designed using the backstepping design procedure.

Studies have shown that Lyapunov based approaches are the best strategies for the systems with complex dynamics. This is because the Lyapunov function is based on the system dynamics itself, thus offering more flexibility in building the control signals. Firstly, backstepping relaxes the matching conditions, which means that the perturbation signals or the uncertainties in the system model are not restricted to be shown in the state equations that the input signal of the system contains. Secondly, backstepping avoids the cancellations of usefully nonlinearities and takes the advantage of the usefully ones to increase the system stability and reduce the amplitude of the control signal in order to avoid saturation.

Additionally, invaluable new insights are gained about the dynamics of the system under consideration. This illustrates that true potential of constructive nonlinear design lies far beyond the mere task of achieving a desired control objective. All derived results are validated by computer simulation of the nonlinear mathematical model of the system.

The main objective of the research is to find the control algorithms that enable a greater degree of robustness and stability of the system itself. Backstepping method makes it possible to avoid cancelation of useful nonlinearities, which can have positive effect on system stability.

Investigations were conducted as a preliminary research of closed loop control of hydraulically driven process of iron centrifugal die casting in industry.

6 References


Authors’ addresses:

Edvard Detiček, PhD, Assistant Prof.
University of Maribor,
Faculty of Mechanical Engineering,
Smetanova 17, SI-2000 Maribor, Slovenia
Tel.: +386-2-220-7612, edvarad.deticek@um.si

Nenad Gubeljak, PhD, Full Prof.
University of Maribor,
Faculty of Mechanical Engineering,
Smetanova 17, SI-2000 Maribor, Slovenia
Tel.: +386-2-220-7661, nenad.gubeljak@um.si

Mitja Kastrevc, PhD, Assistant Prof.
University of Maribor,
Faculty of Mechanical Engineering,
Smetanova 17, SI-2000 Maribor, Slovenia
Tel.: +386-2-220-7804, mitja.kastrevc@um.si