Abstract

Markov switching model captures the sudden changes in the observed series using exogenous variable which is unobserved and follows a stochastic process. This research fits Markov switching model to quarterly real GDP growth rates in Croatia for the period 2000:1 to 2016:2 in order to analyze changes in mean over time. Research results show that Croatian GDP growth rates are regime dependent. Markov switching model with two regimes detects shifts in Croatian GDP growth rates. Consistently with the previous similar researches, the research results indicate long lasting recession period and sluggish Croatian economy.

Key words: Markov model, regimes, GDP, Croatia

1. INTRODUCTION

Modelling nonlinear economic time series using regime switching models has gained in popularity in the past two decades. Sudden shifts or breaks in an economic time series violate the linearity and so make an observed series nonlinear. It is often the case due to arising economic or financial crises within the observed time series. One possibility could be the splitting of the observed time series. The splitting of the observed time series as a result can have two series of different properties, one containing the period before the crises and another one containing the period after the crises and then estimate the separate
models for separate series exhibiting different behaviour patterns or regimes. So the result would be the piecewise linear model. As a matter of fact, this approach would provide a valid result for estimates of the changes but information regarding development of the series or its evolution would be lost. To adequately capture the information on development and evolution of such time series exhibiting different properties in a different time periods Hamilton (1989) proposed Markov switching autoregressive (MS-AR) model. MS-AR model is one of the most popular regime switching models that explains development of the observable time series depending on the unobserved regime variable. The main aim of this paper is to fit Markov Switching model on the real gross domestic product (GDP) growth time series in Croatian economy and determine the switching points between expansion and recession periods. Škare and Stjepanović (2013) examined output fluctuations in Croatia under fractional integration framework and found macroeconomic shocks in Croatian real output highly persistent. Krznar (2011) observed the time period from the year 1998 up to 2010 and identified the end of the first recession in mid-1999 and the start of the second recession in the third quarter of 2008. This paper further examines real GDP growth rate in Croatia and trying to identify its structural changes including the period after the year 2010.

The paper consists of five parts. After the first, introductory part, the second part provides a brief literature overview. The third part of the paper presents the proposed empirical strategy and methodology and the fourth part presents results of the study. The fifth part refers to the conclusion.

2. **BRIEF LITERATURE OVERVIEW**

Hamilton (1989) extended Markov switching regression developed by Goldfeld and Quandt (1973) and applied the extended model to the US real GNP growth rates. Hamilton (1989) illustrated that real GDP growth in US is characterized as an autoregressive model with a switching mean. The mean has been found to switch between low and high growth regimes and the estimated time of the low growth regime corresponded to the dates of recessions in US as provided by the Business Cycle Dating Committee of the National Bureau of Economic Research. Following aforementioned Hamilton’s paper, regime switching model is frequently applied to model real GDP growth rates and to identify the shifts and phases of the business cycles (Beaudry and Koop, 1993; Tiao and Tsay, 1994; Potter, 1995; (Pesaran and Potter, 1997; Chauvet, 1998; Van Dijk and Franses, 1999; Kim and Nelson, 1999; Öcal and Osborne, 2000; Kaun, 2002; Kim, Morley and Piger, 2005). Kaun (2002) points out that GDP growth rates stay at a relatively lower level and less persistent during contractions while during expansions typically fluctuate around a higher level and are more persistent. The model proposed by Hamilton (1989) is univariate model, since the real GDP is the only variable the model takes into account. To take into account other relevant economic indicators, Stock and Watson (1989) proposed
the kind of its multivariate form. Stock and Watson (2002) using principal components analysis on a large number of predictors developed the series that afterward was modelled using Markov switching model. Chauvet (1998) suggests dynamic factor model version. Afterwards, Forni, Hallin, Lippi and Reichlin (2005) proposed the generalized dynamic factor model. Besides the structural change in mean, there is a possibility to allow structural change in the variance of residuals as well (McConnell and Quiros, 2000; Albert and Chib, 1993; Chauvet and Piger, 2003). Kim (2009) proposed the Markov-switching model with endogenous explanatory variables.

In addition to its application in explaining GDP dynamics, the Markov switching models have been successfully applied in a wide range of economics and finance. Rapach and Wohar (2002) studied regime changes in international real interest rates. Chib et al. (2011) and Bianchi (2013) fitted the Markov switching model and provide evidence for regime shifts in the inflation response. Ricci-Risquete, Ramajo and De Castro (2016) using Markov switching approach address the issue of Time-varying effects of fiscal policy in Spain. So there is a number of Markov switching model varieties and its application is extensive. These models have been successfully applied to GDP growth in larger and less volatile economies than the Croatian. The contribution of this paper is to estimate Markov switching model fitted for real GDP growth rates of the small open Croatian economy.

3. EMPIRICAL STRATEGY AND CORRESPONDING METHODOLOGY

In economics, the average growth rate of GDP tends to be higher in expansions than in recessions. Furthermore, expansions tend to last longer than recessions. Therefore the GDP growth rate dynamics might be state dependent. To address this issue nonlinear models are often applied. The vast majority of empirical researches assume that the GDP growth follows a random walk with drift whose stationary differences evolve according to an autoregressive Markov switching process (Clements and Krolzig, 2004; Kim, Morley and Piger, 2005; Chauvet and Hamilton, 2006; Camacho and Quiros, 2007).

The two state Markov-switching model assumes that expected real GDP growth rates differ in period of recession and period of expansion:

\[
\begin{align*}
E(y_t) &= \mu_1 \\
E(y_t) &= \mu_2
\end{align*}
\]

where:

\(y_t\) - denotes the observed time series at time \(t\),
\(\mu_1\) - denotes the expected mean of the series during the period of expansion and
\( \mu_2 \) - denotes the expected mean of the series during the period of recession.

In other words:

\[
E(y_t) = \mu_{st}
\]  
(3)

where: \( s_t = \{0,1\} \) indicate state of the economy i.e. expansion or recession.

Consequently, Markov switching model can be formulated in the equation (4):

\[
y_t = \mu_{st} + \varepsilon_t
\]  
(4)

Economic time series often exhibit the strong dependence between the past observations. In that case autocorrelation among residuals in the estimated model is present and obtained estimates may not be valid. Assuming the residuals being AR(1) process leads to the equation (5):

\[
\varepsilon_t = \rho \cdot \varepsilon_{t-1} + u_t
\]

\[
y_t = \mu_{st} + \rho \cdot (y_{t-1} - \mu_{st-1}) + u_{st,t}
\]  
(5)

The essential idea of the model is that the observable time series vectors depend on the unobserved regime variable. The residuals of the estimated regression models are normally distributed. Variance of the residuals in the equation (5) \( (u_{st,t}) \) may be regime dependent or constrained to be the same in both regimes.

The first order assumption states that the probability of being in a regime depends on the previous state. So the first order assumption may be expressed by the equation (6).

\[
P(s_t = k | s_{t-1} = i) = p_{i,k}
\]  
(6)

Even though is not required, it’s often assumed that transition probabilities are not time dependent, time invariant or constant. In that case, the equation (7) holds.

\[
p_{i,k}(t) = p_{i,k}
\]  
(7)

Transition probabilities to stay in the same state are given by equations (8) and (9):

\[
P(s_t = 0 | s_{t-1} = 0) = p
\]  
(8)

\[
P(s_t = 1 | s_{t-1} = 1) = q
\]  
(9)

Following total probability theorem, transition probabilities to change the state can be expressed in the equations (10) and (11):

\[
P(s_t = 1 | s_{t-1} = 0) = 1 - p
\]  
(10)

\[
P(s_t = 0 | s_{t-1} = 1) = 1 - q
\]  
(11)
The estimates are obtained by maximising a conditional log likelihood function \( \ln[f(y_t|y_{t-1},...,y_1)] \).

In this research two state Markov process is applied and therefore two transition probabilities need to be obtained. The property of the transition probabilities can be evaluated recursively so that each step begins with filtered estimates of the regime probabilities for the previous period. This procedure is called one-step ahead prediction of the regime probabilities. Eventually given the parameter estimates, the inference on \( s_t \) is made following Hamilton (1989). So following Hamilton (1989), current value of \( s_t \) depends only on the immediate past value. It is unknown in which state the process is but the probabilities can be estimated. The process can switch between states repeatedly over the sample. So, state dependent parameters, transition probabilities and the expected duration of states will be estimated.

4. **EMPIRICAL RESULTS AND DISCUSSIONS**

The data sample consists of real GDP growth rates, comparison with corresponding quarter of previous year from first quarter 2000 up to second quarter 2016 are taken from Croatian Bureau of Statistics (quarterly gross domestic product by expenditure approach, constant previous year prices). Table 1 shows the descriptive statistics of the sample data.

Table 1 shows the descriptive statistics for Croatian real GDP growth rates. Descriptive statistics show wide range of Croatian real GDP growth rates. Minimum recorded real GDP growth rate is recorded in the first quarter of 2009.
and amounts -8,62%, while the maximum recorded real GDP growth rate is recorded in the first quarter of 2007 and amounts 6,85%.

In what follows Markov switching model formulated in the equation (4) is estimated and parameter estimates are summarized in Table 2.

Table 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RECESSION</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-1,55</td>
<td>-2,14</td>
<td>0,03</td>
</tr>
</tbody>
</table>

| EXPANSION |             |         |         |
| C         | 4,29        | 14,24   | 0,00    |

Source: Author

Out of Table 2, the one can see the regime specific mean estimates. Estimated mean for the recession regime is negative and amounts -1,55% while the estimated mean for the regime of expansion amounts 4,29%. Krznar (2013) obtained similar results i.e. 4,4% for the regime of expansion and -2,8% for the regime of recession. However, diagnostics checking indicate existence of serial correlation in residuals (figure 2 and figure 3 in the appendix). Correlogram indicates similar pattern for the regime of expansion and regime of recession. Looking at the autocorrelation function (ACF) and partial autocorrelation (PACF) plots one can see that PACF "cuts off" after lag one while the ACF decays more slowly for the regime of expansion as well as for the regime of recession. Following ACF and PACF plot the residuals might be AR(1) process and Markov-Switching AR(1) model given by equation (1) is estimated. Table 3 shows the estimates for Markov-Switching AR(1) model.

Table 3

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RECESSION</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-1,68</td>
<td>-0,66</td>
<td>0,50</td>
</tr>
</tbody>
</table>

| EXPANSION |             |         |         |
| C         | 7,17        | 2,36    | 0,01    |

<table>
<thead>
<tr>
<th>Common coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
</tr>
<tr>
<td>LOG(SIGMA)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transition Matrix Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>P11-C</td>
</tr>
<tr>
<td>P21-C</td>
</tr>
</tbody>
</table>

Source: Author
Table 3, the one can see the regime specific mean estimates. Estimated mean for regime of recession is negative and amounts -1.68 while the estimated mean for regime of expansion amounts 7.17. Regime specific mean estimates are followed by common coefficients estimates and transition matrix parameters representing the logistic coefficients for the regime probabilities. Furthermore, diagnostics checking indicate normality of residuals (Figure 4 in the appendix) and no serial correlations among residuals (Table 6 in the appendix). Hamilton (1994) shows that if the roots lie inside the unit circle, the estimated AR process is stable. Since in the presented model no root lies outside the unit circle and the modulus is less than one (amounts 0.93), AR meets the stability requirement.

Following the fitted Markov-Switching AR(1) model, Figure 1 shows the Real growth rate time series dynamics in Croatia.

Figure 1 Real growth rate time series (rgrts) and recession (Regime 1) in Croatia

Source: Author

Figure 1 illustrates that the recession period in Croatia that started in the third quarter of 2008 ended in the third quarter of 2015.

Figure 1 shows smoothed and filtered probabilities for recession (Regime 1) and expansion (Regime 2) in Croatia.
Figure 2 Smoothed and filtered probabilities for recession (Regime 1) and expansion (Regime 2) in Croatia

Source: Author

Smoothed and filtered probabilities for recession (Regime 1) and expansion (Regime 2) in Croatia indicates end of the recession in the third quarter of 2015.

Table 4 shows the matrix containing the probabilities of transition between the states.

<table>
<thead>
<tr>
<th></th>
<th>Expansion</th>
<th>Recession</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expansion</td>
<td>0.98</td>
<td>0.02</td>
</tr>
<tr>
<td>Recession</td>
<td>0.02</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Source: Author

The time-varying probabilities in Table 3 show considerable state dependence in the transition probabilities. Probability of staying in expansion \( P(s_t = 1 | s_{t-1} = 1) \) amounts 0.985269, while the probability of staying in recession \( P(s_t = 2 | s_{t-1} = 2) \) amounts 0.983828. Here presented research results are consistent with Škare and Stjepanić (2013) who found long memory property of GDP growth rates in Croatia.

Expected duration for each state is presented in Table 4.
Table 5

<table>
<thead>
<tr>
<th>Regime</th>
<th>Expansion</th>
<th>Recession</th>
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</thead>
<tbody>
<tr>
<td>Expected duration</td>
<td>67.88213</td>
<td>61.83626</td>
</tr>
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</table>

The expected duration of the regime of expansion and regime of recession is approximately the same indicating that GDP growth rates will remain in the origin state for a long time before moving to the second state. Conclusively the research results clearly indicate long lasting recession period in Croatia and therefore needs for structural changes in Croatian economy.

5. **CONCLUDING REMARKS**

The Markov switching model fitted in this paper captures the sudden changes in Croatian real GDP growth rates using exogenous variable which is unobserved and follow a stochastic process. There are several conclusions that can be drawn out of the research presented in this paper. Firstly, there is regime switching structure in the Croatian quarterly real GDP growth rate. Secondly, Croatian real GDP growth rate is well fitted by proposed Markov-Switching AR(1) model and the filtered probabilities can be extracted. The estimated results show start of the recession in the third quarter of 2008 and in the first quarter of 2009 real GDP growth rate reached its minimum in amount of -8.62%. The recession ended in the third quarter of 2015. Thirdly, shocks affecting real output in Croatia have long lasting effects. The research results are consistent with previous similar research pointing out on the long memory property in the Croatian economy. Long lasting recession period identified out of this research clearly points on needs for structural changes in Croatian economy.

**REFERENCES**


APPENDIX

Figure 3 Autocorrelation and partial correlation in residuals of model represented by equation 4 (regime of recession)

Figure 4 Autocorrelation and partial correlation in residuals of model represented by equation 4 (regime of expansion)
Figure 5 Normal Q-Q for the MS-AR(1) model

Table 6

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob*</th>
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<td>0.096</td>
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<td>28</td>
<td>-0.204</td>
<td>-0.115</td>
<td>43.277</td>
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STRUKTURNJE PROMJENE U REALNIM STOPAMA RASTA HRVATSKOG BDP-a

Sažetak

Markovljev model promjene režima bilježi nagle promjene u promatranom nizu uz pomoć egzogene varijable koja je neopaziva i slijedi stohastički proces. U radu se Markovljev model promjene režima primjenjuje na tromjesečnim stopama rasta realnog BDP-a u Hrvatskoj za razdoblje od 2000:1 do 2016:2 kako bi se analizirale promjene u očekivanim vrijednostima realnih stopa rasta. Rezultati istraživanja pokazuju da stope rasta realnog BDP-a u Hrvatskoj ovise o režimima. Markovljev model promjene s dva režima uočava pomake stopa rasta spomenutog BDP-a u. U skladu s rezultatima prethodnih istraživanja, rezultati istraživanja pokazuju tromost Hrvatskog gospodarstva i dugotrajnu recesiju.

Ključne riječi: Markovljev model, režimi, BDP, Hrvatska

JEL klasifikacija: C13, C32