Six concyclic points

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Abstract. The theorem about six concyclic points, some of them obtained by means of the symmedians and a median of a triangle, is proved in [1] applying two auxiliary theorems and some complex studies. In this paper the statement of that theorem is a result of some simple considerations.

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Let $\overline{AM}$ be a median and $\overline{AN}$ a symmedian through the vertex $A$ of a triangle $ABC$. The circle $AMN$ meets $\overline{AB}$ and $\overline{AC}$ at the points $E$, $F$ again and the line through $A$ parallel to $\overline{BC}$ meets this circle at the point $P$ again. Let $L$ be the intersection of $\overline{AM}$ and $\overline{EF}$ (Figure 1).

Since $\angle EAN = \angle MAF$, it follows that $|EN| = |MF|$ which implies $EF || MN$ and since $M$ is the midpoint of $BC$, we conclude that $L$ is the midpoint of $EF$.

Since the parallel chords $\overline{AP}$, $\overline{EF}$, $\overline{MN}$ have common bisector through the point $L$ and because the points $A, L, M$ are collinear points, it follows that $P, L, N$ are collinear points too.

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The fact that $D$ is the midpoint of $AC$ results in $DM \parallel AB$. Since the angles $\angle AMF$ and $\angle AEF$ are inscribed in the same arc of the circle and owing to the previously obtained parallelism, we get $\angle AMF = \angle AEF = \angle ABC = \angle DMC$ wherefrom $MF$ is a symmedian of the triangle $ACM$ through the vertex $M$. Similarly, it can be proved that $ME$ is a symmedian through the point $M$ of the triangle $ABM$.

Since the considered circle is uniquely determined by its points $A, M, N$ and because of the unique determination of the intersections of this circle with the sides $AC$ and $AB$ of the triangle $ABC$, we have proved the following theorem which is stated in [1] in the following form.

**Theorem 1.** Let $AM$ be a median and $AN$ a symmedian, through the vertex $A$, of the triangle $ABC$, and $ME$ and $MF$ symmedians through the vertex $M$ of the triangles $ABM$ and $ACM$. Let $P$ be the intersection of the line parallel to the line $BC$ through the point $A$ and line $NL$, where the point $L$ is the intersection of $AM$ and $EF$. Then the points $A, E, F, M, N, P$ lie on one circle.

Since $EF \parallel BC$, the circles $AEF$ and $ABC$ are homothetic with respect to the center $A$, so they touch each other at the point $A$ it means the following statement is valid.

**Corollary 1.** Oprea’s circle from Theorem 1 touches the circumscribed circle of the triangle $ABC$ at the point $A$.

**References**