Six concyclic points

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Abstract. *The theorem about six concyclic points, some of them obtained by means of the symmedians and a median of a triangle, is proved in [1] applying two auxiliary theorems and some complex studies. In this paper the statement of that theorem is a result of some simple considerations.*

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Let $\overline{AM}$ be a median and $\overline{AN}$ a symmedian through the vertex A of a triangle ABC. The circle AMN meets $\overline{AB}$ and $\overline{AC}$ at the points E, F again and the line through A parallel to $\overline{BC}$ meets this circle at the point P again. Let L be the intersection of $\overline{AM}$ and $\overline{EF}$ (*Figure 1*).

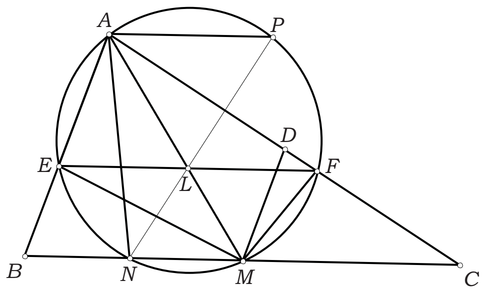

Figure 1.

Since $\measuredangle EAN = \measuredangle MAF$, it follows that $|EN| = |MF|$ which implies $EF||MN$ and since M is the midpoint of $\overline{BC}$, we conclude that L is the midpoint of $\overline{EF}$.

Since the parallel chords $\overline{AP}$, $\overline{EF}$, $\overline{NM}$ have common bisector through the point L and because the points A, L, M are collinear points, it follows that P, L, N are collinear points too.

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The fact that D is the midpoint of $\overline{AC}$ results in $DM||AB$. Since the angles $\measuredangle AMF$ and $\measuredangle AEF$ are inscribed in the same arc of the circle and owing to the previously obtained parallelism, we get $\measuredangle AMF = \measuredangle AEF = \measuredangle ABC = \measuredangle DMC$ wherefrom $\overline{MF}$ is a symmedian of the triangle ACM through the vertex M. Similarly, it can be proved that $\overline{ME}$ is a symmedian through the point M of the triangle ABM.

Since the considered circle is uniquely determined by its points A, M, N and because of the unique determination of the intersections of this circle with the sides $\overline{AC}$ and $\overline{AB}$ of the triangle ABC, we have proved the following theorem which is stated in [1] in the following form.

Theorem 1. *Let $\overline{AM}$ be a median and $\overline{AN}$ a symmedian, through the vertex A, of the triangle ABC, and $\overline{ME}$ and $\overline{MF}$ symmedians through the vertex M of the triangles ABM and ACM. Let P be the intersection of the line parallel to the line BC through the point A and line NL, where the point L is the intersection of $\overline{AM}$ and $\overline{EF}$. Then the points A, E, F, M, N, P lie on one circle.*

Since $EF||BC$, the circles AEF and ABC are homothetic with respect to the center A, so they touch each other at the point A it means the following statement is valid.

Corollary 1. *Oprea's circle from Theorem 1 touches the circumscribed circle of the triangle ABC at the point A.*

References

[1] N. Oprea, *Sase puncte conciclice*, Lucrările Sem. Creat. Mat. **7**(1997–1998), 77–82.