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A research project ranking method based on independent reviews by using the principle of the distance to the perfectly assessed project

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Abstract. The paper discusses the problem of ranking research projects based on the assessment obtained from $n \geq 1$ independent blinded reviewers. Each reviewer assesses several project features, and the total score is defined as the weighted arithmetic mean, where the weights of features are determined according to the well-known AHP method. In this way, it is possible to identify each project by a point in n-dimensional space. The ranking is performed on the basis of the distance of each project to the perfectly assessed project. Thereby the application of different metric functions is analyzed. We believe it is inappropriate to use a larger number of decimal places if two projects are almost equidistant (according to some distance function) to the perfectly assessed project. In that case, it would be more appropriate to give priority to the project that has received more uniform ratings. This can be achieved by combining different distance functions. The method is illustrated by several simple examples and applied by ranking internal research projects at Josip Juraj Strossmayer University of Osijek, Croatia.

Keywords: multi-criteria decision making, project evaluation, ℓ_p-distance, AHP

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1. Introduction

In this paper, we present a method for establishing a total order of projects based upon the assessment obtained from $n \geq 1$ independent blinded reviewers. The issue of defining the number and importance of the review form features is considered to be an especially important part of the whole procedure, but in this paper this problem will not be taken into consideration. A standard form of the University was used in the example given in Section 5. The weights of the features are defined by using the Analytic Hierarchy Process (AHP) (see e.g. [6, 19, 20]) on the basis of the opinions of a group of experts.

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Each feature was graded by a number from the interval $[1, 5]$, and the perfectly assessed project is considered to be the one whose all features were individually assessed by 5. Much attention is paid in the literature to this issue as well.
After some project is allocated a certain n-tuple of numbers (grades), its rank depends on the distance to the perfectly assessed project. The choice of the metric function is also very often discussed in the literature. In our paper, we used a combination of the Euclidean d_2 distance and the Chebyshev d_∞ distance. Projects rated closer to the perfectly assessed project will be ranked better (see also [23]).
Finally, depending on the funding available, only a few highest ranked projects will be granted.
This paper is organized as follows. The problem is stated and one practical problem of ranking internal research projects at the University of Osijek is discussed in Section 3. The definition of ordering on a set of projects in terms of various distance functions is also introduced. An example indicating the basic problems that might occur is constructed. In Section 4, an analysis of various distance functions, i.e. the Manhattan d_1-distance, the Euclidean d_2-distance, and the Chebyshev d_∞-distance, is performed. The situation when two or more projects are evaluated differently by various reviewers, and yet roughly equally ranked by using some distance function, is especially considered. In that case, we believe priority should be given to projects with more uniform assessments, which can be achieved by combining different distance functions. A real ranking problem of internal research projects at the University of Osijek is described in Section 5, and finally, some conclusions are given in Section 6.

2. Related works

The problem of ranking research projects (see e.g. [4, 11, 13, 22]) as well as ranking departments, institutes and universities (see e.g. [5, 8, 17]) has long been present in the scientific literature.
Most approaches use different multi-criteria decision-making methods, [2, 7, 21], and the AHP method.
In the paper [22], the AHP method is used in order to determine the weights of features. First, by applying adaptive Mahalanobis clustering (see [15]) all projects are grouped into several clusters such that similarly assessed projects are grouped into special ellipsoidal clusters. The cluster of projects assessed as best is specially analyzed and ranked.

3. Problem statement

Let $\mathcal{P} = \{\pi^{(1)}, \ldots, \pi^{(m)}\}$ be a set of projects. Suppose that each project is assessed by $n \geq 1$ independent reviewers based on the review form, in which $k \geq 1$ features $f_1, \ldots, f_k$ (e.g. the quality and relevance of a research proposal, the quality of applicants, etc.; see Example 1) are assessed. The corresponding weight $w_j > 0$ will be associated to each of k features f_j which will be assessed. The weights of project features $w_1, \ldots, w_k$ are defined by the University Management Board by using the AHP method (see e.g. [2, 6, 19–21]). Without loss of generality, let us suppose that $\sum\limits_{s=1}^{k} w_s = 1$. By real numbers $r_{j1}^{(i)}, \dots, r_{jk}^{(i)} \in [1,5]$ we denote grades of features $f_1, \dots, f_k$ given for the project $\pi^{(i)} \in \mathcal{P}$ by the j-th reviewer, with "1" and "5" as the worst and the best rating, respectively. We decided in favor of that grading scale since it is used for the evaluation of student achievement at the University of Osijek. Furthermore, let

$$r_j^{(i)} = \sum_{s=1}^{k} w_s r_{js}^{(i)}, \quad j = 1, \dots, n, \quad i = 1, \dots, m, \tag{1}$$

be the average weighted grade (AWG) of the project $\pi^{(i)}$ obtained from the j-th reviewer. In this way, we are able to associate a vector (point)

$$a^{(i)} = (r_1^{(i)}, \dots, r_n^{(i)}) \in \mathbb{R}^n, \quad i = 1, \dots, m, \tag{2}$$

from n-dimensional vector space $\mathbb{R}^n$ to each project $\pi^{(i)} \in \mathcal{P}$. In this way, instead of the set of projects $\mathcal{P}$, we can observe the set $\mathcal{A} = \{a^{(i)} \in \mathbb{R}^n : i = 1, \dots, m\}$ of points in space $\mathbb{R}^n$.

Example 1. *In 2015, the University of Osijek announced an internal call for proposals for research projects INGI-2015*[‡] *to encourage cooperation between its researchers and prominent researchers from other (especially foreign) universities. 30 candidates from the STEM area and 10 Social Sciences and Humanities candidates submitted their applications to the call. The evaluation was carried out based upon reviews by independent reviewers, one of whom is affiliated with the field of the research proposal in question and the other comes from a different, but related field. Reviewers evaluated features* $f_1, \dots, f_6$ *(given in Table 1) with grades from the interval of real numbers* $[1,5]$. *The University Management Board has defined weights* $w_1, \dots, w_6 > 0$ *of particular features by using the AHP method (see also Table 1). In that way, for each of* $m = 40$ *projects* $\pi^{(i)}$ *the corresponding vector* $a^{(i)} \in \mathbb{R}^2$ *is uniquely determined, whose components are AWGs of all features of the first and the second reviewer*

$$a^{(i)} = (r_1^{(i)}, r_2^{(i)}) \in \mathbb{R}^2, \quad i = 1, \dots, m, \tag{3}$$

(see also Table 4).

	Features	Weights w_i
f_1:	The quality and relevance of the research proposal	0.25
f_2:	The quality of applicants	0.15
f_3:	The quality of guest researchers	0.35
f_4:	Research feasibility study	0.10
f_5:	Financial plan	0.10
f_6:	Inclusion of students	0.05

Table 1: *Elements assessed by reviewers from Example 1 with corresponding weights*

[‡]See: `http://www.unios.hr/ingi2015/`

3.1. Defining the ordering of projects

Furthermore, by $\pi^\star$ we will denote a *perfectly assessed project* to which the point $a^\star = (5,\dots,5) \in \mathbb{R}^n$ is associated in space $\mathbb{R}^n$. The project $\pi^{(i)}$ is considered to be ranked better than the project $\pi^{(j)}$ if the point $a^{(i)}$ is closer to the point $a^\star$ in terms of some distance function, see [23]. In this sense, we introduce the following definition.

Definition 1. *Let $\mathcal{P} = \{\pi^{(1)},\dots,\pi^{(m)}\}$ be a set of projects, $\mathcal{A} \subset \mathbb{R}^n$ a set of corresponding points defined by* (2)*, $\pi^\star$ the perfectly assessed project to which we associate the point $a^\star = (5,\dots,5) \in \mathbb{R}^n$, and let $d\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_+$ be some distance function. The project $\pi^{(i)}$ is said to be better d-ranked than the project $\pi^{(j)}$ and we write $\pi^{(i)} \overset{(d)}{\succeq} \pi^{(j)}$ if and only if there holds $d(a^{(i)},a^\star) \le d(a^{(j)},a^\star)$, i.e.,*

$$\pi^{(i)} \overset{(d)}{\succeq} \pi^{(j)} \Leftrightarrow d(a^{(i)},a^\star) \le d(a^{(j)},a^\star).$$

Furthermore, we say that a set of projects $\mathcal{P}$ is d-ranked if $\pi^{(1)} \overset{(d)}{\succeq} \cdots \overset{(d)}{\succeq} \pi^{(j)} \overset{(d)}{\succeq} \cdots \overset{(d)}{\succeq} \pi^{(m)}$ and j is a d-rank of the project $\pi^{(j)}$.

3.2. Interpretation of the ordering of projects

Let $K_r^{(d)} = \{x \in \mathbb{R}^n \colon d(x,a^\star) \le r\}$ be a hyperball of radius $r > 0$ with the center in the point $a^\star$ in metric space $\mathbb{R}^n$ with distance function $d\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_+$. Obviously, $\pi^{(i)}$ is strongly better d-ranked than $\pi^{(j)}$ ($\pi^{(i)} \overset{(d)}{\succ} \pi^{(j)}$) if the point $a^{(i)}$ is situated in hyperball $K_r^{(d)}$ of a smaller radius. Projects $\pi^{(i)}$ and $\pi^{(j)}$ are equally d-ranked if the corresponding points $a^{(i)}$ and $a^{(j)}$ lie in the same hypercircle $\partial K_r^{(d)}$. In this way, we introduce a weak ordering on the set of points $\mathcal{A}$ and a unique ranking list of projects $\mathcal{P}$ (see e.g. [16]).

The proposed method of project ranking allows the application of various distance functions, and in this paper we will particularly analyze the application of the Manhattan d_1-distance function

$$d_1(x,y) = \sum_{i=1}^{n} |x_i - y_i|, \tag{4}$$

the Euclidean d_2-distance function

$$d_2(x,y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}, \tag{5}$$

and the Chebyshev d_∞-distance function

$$d_\infty(x,y) = \max_{i=1,\dots,n} |x_i - y_i|. \tag{6}$$

Remark 1. *Note that two projects $\pi^{(i)}$, $\pi^{(j)} \in \mathcal{P}$ (see Fig. 1)*

- *have the same* d_1*-rank if* $d_1(a^{(i)}, a^\star) = d_1(a^{(j)}, a^\star)$*, i.e., if the arithmetic means of their grades are equal:* $\frac{1}{n}\sum\limits_{s=1}^{n} r_s^{(i)} = \frac{1}{n}\sum\limits_{s=1}^{n} r_s^{(j)}$*;*
- *have the same* d_2*-rank if* $d_2(a^{(i)}, a^\star) = d_2(a^{(j)}, a^\star)$*;*
- *have the same* d_∞*-rank if* $d_\infty(a^{(i)}, a^\star) = d_\infty(a^{(j)}, a^\star)$*, i.e., if the highest grades obtained for some feature are equal:* $\max\limits_{s=1,\ldots,n} r_s^{(i)} = \max\limits_{s=1,\ldots,n} r_s^{(j)}$*.*

Example 2. *Let* $m = 7$ *and* $n = 2$*. Average grades awarded to projects by two independent reviewers are given in Table 2. In this way, the set* $\mathcal{A} = \{a^{(i)} = (x_i, y_i) \in \mathbb{R}^2 : i = 1, \ldots, 7\}$ *of the corresponding points is determined. The table also gives distances of each project to the perfectly assessed project* $\pi^\star$ *by using* d_1, d_2 *and* d_∞ *distance functions. In addition to the set of points* $\mathcal{A}$*, a few* d_1*-circles suggesting a* d_1*-rank of projects are shown in Fig. 1a. Similarly, Fig. 1b and Fig. 1c contain a few* d_2*-circles and a few* d_∞*-circles suggesting a* d_2*-rank of projects and a* d_∞*-rank of projects, respectively.*

Project	$\pi^{(1)}$	$\pi^{(2)}$	$\pi^{(3)}$	$\pi^{(4)}$	$\pi^{(5)}$	$\pi^{(6)}$	$\pi^{(7)}$
`Rev#1` (x_i)	3.0	2.0	3.4	4.1	4.2	4.9	4.4
`Rev#2` (y_i)	2.8	4.5	4.1	3.4	4.6	3.9	4.4
$d_1(\pi^{(i)}, a^\star)$	4.2	3.5	2.5	2.5	1.2	1.2	1.2
$d_2(\pi^{(i)}, a^\star)$	3.0	3.0	1.8	1.8	0.9	1.1	0.8
$d_\infty(\pi^{(i)}, a^\star)$	2.2	3.0	1.6	1.6	0.8	1.1	0.6

Table 2: *Project grades and distances to the perfectly assessed project* $\pi^\star$

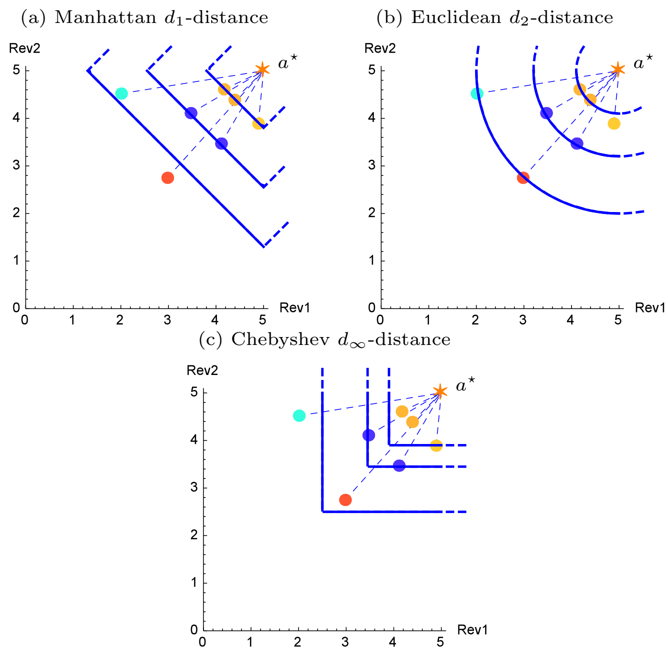

Figure 1: *Distances to the perfectly assessed project $\pi^\star$ represented by the point $a^\star = (5,5)$*

Table 3 gives d_1, d_2, and d_∞ ranking lists of projects from Example 2. Note that projects $\pi^{(3)}, \pi^{(4)}$, i.e. projects $\pi^{(5)}, \pi^{(6)}, \pi^{(7)}$, lie in the same d_1-circle and have the same d_1-rank. Similarly, projects $\pi^{(1)}, \pi^{(2)}$, i.e. projects $\pi^{(3)}, \pi^{(4)}$, lie in the same d_2-circle and have the same d_2-rank. A similar problem also occurs in the application of the d_∞-distance. These problems will be analyzed in detail in the next section.

Rank	Manhattan d_1-distance	Euclidean d_2-distance	Chebyshev d_∞-distance
1	$\pi^{(5)}, \pi^{(6)}, \pi^{(7)}$	$\pi^{(7)}$	$\pi^{(7)}$
2	$\pi^{(3)}, \pi^{(4)}$	$\pi^{(5)}$	$\pi^{(5)}$
3	$\pi^{(2)}$	$\pi^{(6)}$	$\pi^{(6)}$
4	$\pi^{(1)}$	$\pi^{(3)}, \pi^{(4)}$	$\pi^{(3)}, \pi^{(4)}$
5	–	$\pi^{(1)}, \pi^{(2)}$	$\pi^{(1)}$
6	–	–	$\pi^{(2)}$
7	–	–	–

Table 3: *Ranking of projects from Example 2 by using various distance functions*

4. Comparison of the application of various metric functions

As already mentioned in the previous section, two projects $\pi^{(i)}, \pi^{(j)}$ with corresponding points $a^{(i)}, a^{(j)} \in \mathcal{A}$ will be equally d_1-ranked if $\frac{1}{n}\sum r_s^{(i)} = \frac{1}{n}\sum r_s^{(j)}$. It is immediately clear that if the Manhattan d_1-distance function is applied, the rank of some project $\pi \in \mathcal{P}$ will be influenced only by arithmetic means of grades (2), and diversity of individual grades (2) awarded by various reviewers will not affect the d_1-rank of the project at all.

Unlike the d_1-rank, the d_2-rank and the d_∞-rank will depend on grade dispersion (2) referring to the project under consideration.

As an illustration, let us consider two projects represented by points a and a^0 from the plane $\mathbb{R}^2$ (see Fig. 2), which are equally d_1-ranked, i.e., they equally differ from $a^\star$ by the Manhattan distance: $d_1(a^0, a^\star) = d_1(a, a^\star) =: r$. In Fig. 2, it can be seen that the point $a^0 \in \mathbb{R}^2$ represents the project $\pi^0 \in \mathcal{P}$, whose AWGs obtained from both reviewers are mutually equal. Among all projects $\pi \in \mathcal{P}$ for which $d_1(a, a^\star) = r$, the project π^0 attains the best d_2-rank (see Fig. 2a) and the best d_∞-rank (see Fig. 2b).

(a) $\pi^0(a^0)$ attains the best d_2-rank (b) $\pi^0(a^0)$ attains the best d_∞-rank

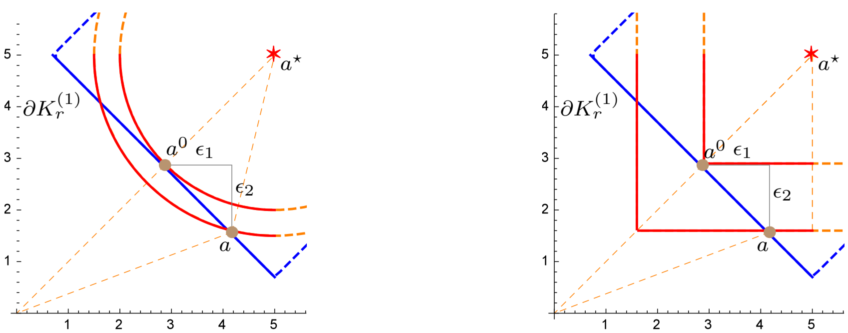

Figure 2: *d_p, $p \geq 2$ distances prefer uniform evaluation grades*

This means that the application of d_2 and d_∞ distances prefers uniform evaluation grades, unlike the Manhattan distance that takes into consideration only the arithmetic means of AWGs obtained from all reviewers. Practically, in case we have projects that are evaluated similarly and we want to give priority to the project with more uniform evaluation grades, we should use either the d_2 or the d_∞ distance, and if we do not want to give priority to such project, we should use the Manhattan distance.

A generalized principle for the case of $n > 1$ reviewers is described in the following theorem.

Theorem 1. *Let $n > 1$ and let $\partial K_r^{(1)} = \{x \in \mathbb{R}_+^n : d_1(x, a^\star) = r,\ r > 0\}$ be part of the Manhattan hypercircle of radius $r > 0$ with the center at the point $a^\star$. Then the shortest d_p, $p \in \{2, \infty\}$, distance from the point $a^\star$ to the hypercircle $\partial K_r^{(1)}$ is attained at the point $a^0 = (5 - \frac{r}{n}, \ldots, 5 - \frac{r}{n}) \in \partial K_r^{(1)}$, i.e.,*

$$d_p(\partial K_r^{(1)}, a^\star) = \min_{a \in \partial K_r^{(1)}} d_p(a, a^\star) = d_p(a^0, a^\star), \quad p \in \{2, \infty\}. \tag{7}$$

Proof. First, let us note that for all $a \in \partial K_r^{(1)}$ there is $\epsilon \in \mathbb{R}^n$, such that

$$a = a^0 + \epsilon = \left(5 - \frac{r}{n} + \epsilon_1, \ldots, 5 - \frac{r}{n} + \epsilon_n\right), \quad \text{where} \quad \sum_{i=1}^{n} \epsilon_i = 0. \tag{8}$$

In order to prove the assertion for $p=2$, let us suppose that $r \in \mathbb{R}_+$ is fixed, define the function

$$\varphi\colon \mathbb{R}^n \to \mathbb{R}_+, \quad \varphi(\epsilon) = d_2^2(a, a^\star) = \left(\frac{r}{n} - \epsilon_1\right)^2 + \cdots + \left(\frac{r}{n} - \epsilon_n\right)^2,$$

and consider the following constrained optimization problem:

$$\min_{\{(\epsilon_1,\ldots,\epsilon_n)\in\mathbb{R}^n : \sum_{i=1}^n \epsilon_i = 0\}} \varphi(\epsilon_1, \ldots, \epsilon_n). \tag{9}$$

The corresponding Lagrange function for problem (9) is

$$L(\epsilon_1, \ldots, \epsilon_n, \lambda) = \left(\frac{r}{n} - \epsilon_1\right)^2 + \cdots + \left(\frac{r}{n} - \epsilon_n\right)^2 + \lambda \sum_{i=1}^n \epsilon_i.$$

From $\frac{\partial L(\epsilon_1,\ldots,\epsilon_n)}{\partial \epsilon_i} = \lambda - 2(\frac{r}{n} - \epsilon_i) = 0$, we obtain $\epsilon_i = \frac{r}{n} - \frac{\lambda}{2}$, $i = 1, \ldots, n$. Finally, because $\sum_{i=1}^n \epsilon_i = 0$, we obtain, $\lambda = \frac{2r}{n}$ i.e. $\epsilon_i = 0$, $i = 1, \ldots, n$. Since the function φ is a strongly convex (quadratic) function, the assertion is proved.
In order to prove the assertion for $p = \infty$, let us define the function

$$\psi\colon \mathbb{R}^n \to \mathbb{R}, \quad \psi(\varepsilon) = d_\infty(a_\varepsilon, a^\star) = \max\left\{\left|\frac{r}{n} - \epsilon_1\right|, \ldots, |\frac{r}{n} - \epsilon_n|\right\}$$

and consider the following constrained optimization problem:

$$\min_{\{(\varepsilon_1,\ldots,\varepsilon_n)\in\mathbb{R}^n :\ \sum_{i=1}^n \varepsilon_i = 0\}} \psi(\varepsilon_1, \ldots, \varepsilon_n), \tag{10}$$

Let $z = \max\{|\frac{r}{n} - \epsilon_1|, \ldots, |\frac{r}{n} - \epsilon_n|\}$. Problem (10) is reduced to a linear programming problem:

$$\begin{aligned}
& z \to \min \\
s.t. \quad & \sum_{i=1}^n \varepsilon_i = 0, && (11) \\
& \frac{r}{n} - \epsilon_i \le z, \quad i = 1, \ldots, n, && (12) \\
-&\frac{r}{n} + \epsilon_i \le z, \quad i = 1, \ldots, n, && (13) \\
& \varepsilon_i \in \mathbb{R}_+. && (14)
\end{aligned}$$

This problem can be solved explicitly. By summing conditions (12) and using (11) we get $r \le nz$ Analogously, by summing conditions (13) and using (11) we obtain $-r \le nz$, and finally $|\frac{r}{n}| = \max\{-\frac{r}{n}, \frac{r}{n}\} \le z$.
Since z can be minimal, it is obvious that optimal $z^\star = |\frac{r}{n}| = \psi(0, \ldots, 0)$.

□

Let us now consider the set of projects $\mathcal{P}^0 \subseteq \mathcal{P}$ which are equally d_2-ranked. As can be seen in Fig. 3a, the Chebyshev d_∞ distance project $\pi^0 \in \mathcal{P}^0$ with uniform evaluation grades $a^0 = (\rho, \rho)$, $\rho \in [1, 5]$ is recognized as best since $d_\infty(a^0, a^\star) \leq d_\infty(a, a^\star)$, for all $a \in \mathcal{P}^0$. At the same time, the project $\pi^0 \in \mathcal{P}^0$ is d_1-ranked worst, and the corresponding vector a^0 has the smallest ℓ_2-norm (see Fig. 3b).

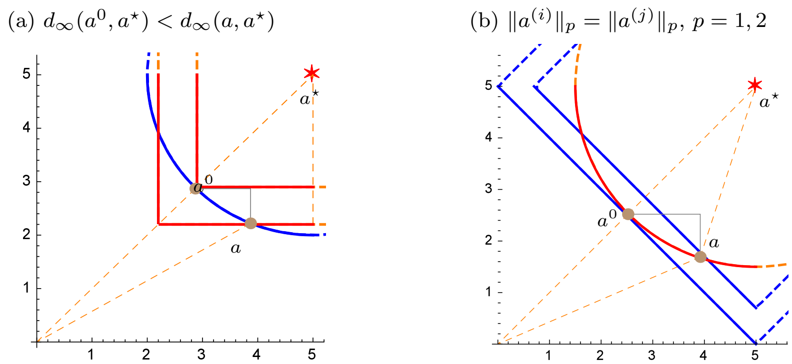

Figure 3: *Two projects equally d_2-ranked*

The following theorem gives a generalization of the aforementioned claims for $n > 1$ and shows that among all projects $\pi \in \mathcal{P}^0$, the highest d_∞-rank is attributed to the project $\pi^0 \in \mathcal{P}^0$ with uniform evaluation grades $a^0 = (\rho, \dots, \rho)$, $\rho \in [1, 5]$. At the same time, the project $\pi^0 \in \mathcal{P}^0$ has the lowest d_1-rank, and the corresponding vector a^0 has the smallest ℓ_2-norm.

Theorem 2. *Let $n > 1$ and let $\partial K_r^{(2)} = \{x \in \mathbb{R}_+^n : d_2(x, a^\star) = r,\ r > 0\}$ be part of the Euclidean hypercircle of radius $r > 0$ with the center in the point $a^\star$.*

(i) The shortest d_∞ distance from the point $a^\star$ to the hypercircle $\partial K_r^{(2)}$ is attained at the point $a^0 = (5 - \frac{r}{\sqrt{n}}, \dots, 5 - \frac{r}{\sqrt{n}}) \in \partial K_r^{(2)}$ i.e.,

$$d_\infty(\partial K_r^{(2)}, a^\star) = \min_{a \in \partial K_r^{(2)}} d_\infty(a, a^\star) = d_\infty(a^0, a^\star). \tag{15}$$

(ii) For all pairs $a^{(i)}, a^{(j)} \in \partial K_r^{(2)}$, there holds

$$d_1(a^{(i)}, a^\star) \geq d_1(a^{(j)}, a^\star) \quad \Leftrightarrow \quad \|a^{(i)}\|_2 \leq \|a^{(j)}\|_2, \tag{16}$$

and particularly, the greatest d_1-distance from the point $a^\star$ to the hypercircle $\partial K_r^{(2)}$ is attained at the point $a^0 = (5 - \frac{r}{\sqrt{n}}, \dots, 5 - \frac{r}{\sqrt{n}}) \in \partial K_r^{(2)}$ i.e.

$$d_1(\partial K_r^{(2)}, a^\star) = \max_{a \in \partial K_r^{(2)}} d_1(a, a^\star) = d_1(a^0, a^\star). \tag{17}$$

Proof. First, let us note that for all $a \in \partial K_r^{(2)}$ there exists $\epsilon \in \mathbb{R}^n$, such that

$$a = a^0 + \epsilon = \left(5 - \frac{r}{\sqrt{n}} + \epsilon_1, \dots, 5 - \frac{r}{\sqrt{n}} + \epsilon_n\right), \quad \text{where} \quad \sum_{i=1}^n \left(r - \epsilon_i \sqrt{n}\right)^2 = nr^2. \tag{18}$$

Namely, the points $a, a^0 \in \partial K_r^{(2)}$ have the same distance from the point $a^\star$, and there holds

$$\left(\frac{r}{\sqrt{n}} - \epsilon_1\right)^2 + \cdots + \left(\frac{r}{\sqrt{n}} - \epsilon_n\right)^2 = \frac{r^2}{n} + \cdots + \frac{r^2}{n}$$

i.e. $\sum_{i=1}^n \left(r - \epsilon_i \sqrt{n}\right)^2 = nr^2$.
Let us define the function

$$\psi\colon \mathbb{R}^n \to \mathbb{R}, \quad \psi(\varepsilon) = d_\infty(a, a^\star) = \max\left\{\left|\frac{r}{\sqrt{n}} - \varepsilon_1\right|, \ldots, \left|\frac{r}{\sqrt{n}} - \varepsilon_n\right|\right\}$$

and consider the following constrained optimization problem

$$\min_{\{(\varepsilon_1,\ldots,\varepsilon_n)\in\mathbb{R}^n\colon\ \sum_{i=1}^n \left(r-\epsilon_i\sqrt{n}\right)^2 = nr^2\}} \psi(\varepsilon_1, \ldots, \varepsilon_n). \tag{19}$$

Let $z = \max\left\{\left|\frac{r}{\sqrt{n}} - \varepsilon_1\right|, \ldots, \left|\frac{r}{\sqrt{n}} - \varepsilon_n\right|\right\}$. Problem (19) is reduced to the following optimization problem

$$\begin{aligned} & z \to \min \\ s.t. \quad & \sum_{i=1}^n \left(r - \epsilon_i \sqrt{n}\right)^2 = nr^2, && (20) \\ & \left|\frac{r}{\sqrt{n}} - \varepsilon_i\right| \le z, \quad i = 1, \ldots, n, && (21) \\ & \varepsilon_i \in \mathbb{R}. && (22) \end{aligned}$$

This problem can be solved explicitly. From (21) we get

$$(r - \epsilon\sqrt{n})^2 \le z^2 n, i = 1, \ldots, n$$

i.e.

$$\sum_{i=1}^n (r - \epsilon\sqrt{n})^2 \le z^2 n^2.$$

Because of (20), the optimal $z^\star$ is

$$z^\star = \sqrt{\frac{1}{n^2} \sum_{i=1}^n (r - \epsilon\sqrt{n})^2} = \frac{r}{\sqrt{n}} = \psi(0).$$

In order to prove assertion (ii), let us suppose that $a^{(i)}, a^{(j)} \in \partial K_r^{(2)}$ are arbitrary.

Then there holds

$$\begin{aligned} d_2(a^{(i)},a^\star) = d_2(a^{(j)},a^\star) &\Leftrightarrow \|a^{(i)}-a^\star\|_2^2 = \|a^{(j)}-a^\star\|_2^2 \\ &\Leftrightarrow \left(a^{(j)}-a^{(i)}\right)(a^\star)^T = \frac{1}{2}\left(\|a^{(j)}\|^2-\|a^{(i)}\|^2\right) \\ &\Leftrightarrow 5\left(\sum_{s=1}^n r_s^{(j)} - \sum_{s=1}^n r_s^{(i)}\right) = \frac{1}{2}\left(\|a^{(j)}\|^2-\|a^{(i)}\|^2\right) \\ &\Leftrightarrow 5\left(d_1(a^{(j)},a^\star)-d_1(a^{(i)},a^\star)\right) = \frac{1}{2}\left(\|a^{(i)}\|_2^2-\|a^{(j)}\|_2^2\right). \end{aligned} \tag{23}$$

From (23) it can be seen that $\|a^{(i)}\|_2 \geq \|a^{(j)}\|_2$, if and only if $d_1(a^{(j)},a^\star) \geq d_1(a^{(i)},a^\star)$, from where there follow (16) and (17). □

Remark 2. *A generalization of results from Theorem 1 and Theorem 2 could be written for an arbitrary d_p $(p \geq 1)$ distance, but the proof would require us to solve a nondifferentiable optimization problem (see e.g. [1, 18]). It should also be noted that the cases d_1, d_2, d_∞ are quite sufficient for the applications in question.*

5. Ranking internal research projects at the University of Osijek

As an illustration, we consider the problem of ranking projects of the internal research program at the University of Osijek (INGI-2015) described in Example 1. The Committee for Research Project Evaluation decided to apply the Euclidean d_2-distance with corrections by the Chebyshev d_∞-distance in terms of Theorem 1 and Theorem 2, i.e., if two projects are approximately equally d_2-ranked, then we give priority to the project with more uniform evaluation grades, i.e. to the project that is d_∞-ranked better.

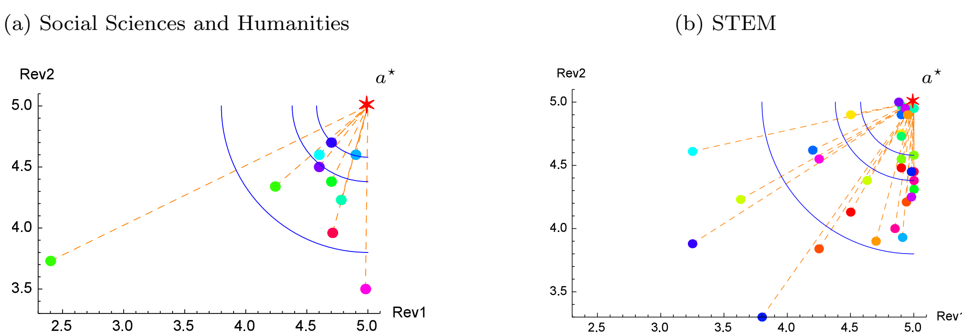

Figure 4: *Ranks of* 10 *Social Sciences and Humanities projects and* 30 *STEM projects*

Let us analyze the problem of ranking $m = 10$ Social Sciences and Humanities project proposals. The grades of features $f_1,\ldots,f_6$ of four best d_2-ranked projects (with d_∞-corrections) and their corresponding $\mathbb{R}^2$ representation are shown in Table 4. A graphical illustration is shown in Fig. 4a.

Rank	Project	Rev	$f_1^{(i)}$	$f_2^{(i)}$	$f_3^{(i)}$	$f_4^{(i)}$	$f_5^{(i)}$	$f_6^{(i)}$	$a^{(i)}$	$d_2(a^i, a^\star)$	$d_\infty(a^i, a^\star)$
1	INGI-1	Rev#1	5	4.2	5	4.5	4.5	4	4.7	0.4	0.3
		Rev#2	4.8	5	4.5	4.8	4.8	4.3	4.7		
2	INGI-2	Rev#1	5	5	5	5	4.5	5	4.9	0.4	0.4
		Rev#2	5	5	5	4	3	4	4.6		
3	INGI-3	Rev#1	4	4	5	5	5	5	4.6	0.6	0.4
		Rev#2	5	4.5	4.3	4.7	4.5	4.3	4.6		
4	INGI-4	Rev#1	5	4	5	5	3	3.5	4.6	0.6	0.5
		Rev#2	4.4	3.8	4.8	4.8	4.3	4.1	4.5		

Table 4: *Properties of the best d_2-ranked Social Sciences and Humanities projects*

Note that projects `INGI-1` and `INGI-2` are equally d_2-ranked (see also Fig. 4a), but the project `INGI-1` is d_∞-ranked better (has more uniform evaluation grades), hence it ranks first. If we tried to differentiate projects `INGI-1` and `INGI-2` by using more decimals in the d_2-rank, then the project `INGI-2` would be placed before the project `INGI-1`. A similar situation takes place with projects `INGI-3` and `INGI-4`.
Similarly, a graphical illustration of $\mathbb{R}^2$ representation of 30 STEM projects is shown in Fig. 4b.

6. Conclusions

Project ranking is a sensitive issue in multi-criteria decision making. During the evaluation process, it can be expected that two or more projects are roughly equally ranked in relation to the selected distance function. We believe that it is not appropriate to rank such projects by using more decimal places, but that the project with more uniform evaluation grades should be positioned better. The paper shows how this can be achieved by combining different distance functions.
The presented method can be applied to other different situations like department ranking inside a university, ranking teachers and associates on the basis of a university survey or on the basis of the quality of scientific research, ranking administrative staff on the basis of a survey, etc.
In our approach, the evaluations of different criteria of each reviewer are first aggregated into a global score and then the projects are ranked with respect to the distance of their global scores to the perfectly assessed project in space $\mathbb{R}^n$, where n is the number of reviewers. Let us mention that instead of this, it is also reasonable to rank the projects directly with respect to the distance between the feature vector and the perfectly assessed project in space $\mathbb{R}^{nk}$, where k is the number of features considered.

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