Photobioreactor design is a subject of great relevance for the attainment of a sustained development in modern technology, and has also considerable interest from the basic scientific and technologic point of view. The aim of the present review paper is presenting and comparing some of the recent attempts by the authors of modelling photosynthesis in reactors. A short inspection of the kinetic models proposed for photobioreactor design is done, and some examples of the integration of such kinetic models and bioreactor fluid dynamics in the modelling of photobioreactors are presented.

Key words:
Photosynthesis, photoinhibition, mathematical modelling, photobioreactors

Introduction

Photosynthesis has been studied for a very long time, being this justified by the vital importance it has for the very existence of life on earth. The exploitation by man of the many opportunities that photosynthesis offers for the production of valuable biochemicals is based on the accumulated knowledge on the matter, and it is one of the most interesting and challenging problems in biochemical engineering. Moreover, the production of bio-fuels from algal biomass is one of the most ambitious aims in the quest for a sustainable ecological balance worldwide and has gained lately much attention. In Fig. 1, a sketch is shown to indicate the place of algal culture for bio fuels in the overall fuel-energy scheme. The overall aim is the diminution of environmental contamination, which would be attained by recycling part of the CO₂ produced in the utilization of fossil fuels for energy generation.

In spite of the huge interest in bio fuel production from algae, the economic aspects of the process are still to be satisfactorily solved. Therefore, the most important engineering aim in this area is the development of a process that provides biomass rich in chemical energy at the lowest cost. Assuming the best algal species for the process is identified and selected, the next quest remaining is an optimal design of the bioreactor. Such optimization requires a deep knowledge of the system, and a mathematical model that represents it satisfactorily. The model should therefore be able to represent the basic characteristics of algal kinetics, being still simple enough to allow a relatively easy computational approach.

The kinetic model

A satisfactory kinetic model is the base for any bioreactor calculation, design or optimization. In the case of photosynthetic cells, much is known about the basics of the photochemical and biochemical mechanisms involved, and much of it can be found in standard textbooks and periodicals. However, the picture emerging after inspection of this basic knowledge renders the description of a system that is much too complicated for direct utilization in engineering calculations. On the other hand, the actual behaviour of the photosynthetic cultures is very complicated, it includes many variables, and the different steps in the processes have time-constants that differ in orders of magnitude. Therefore, it is difficult to represent the behaviour of a culture by simple kinetic expressions. This is specially so when the dynamic behaviour of the cultures has to be considered, as is the case of the integration of fluid-dynamics with photosynthesis that will be ex-
plained further on. Because of this, all of the mathematical models of photosynthesis available in the literature are based on the lumping of a large amount of biochemical reactions into simpler steps or into hypothetical concepts, which aim at representing the behaviour of the actual biochemical apparatus. The selection of a model is thus the result of the compromise between the “loyalties to biology”, that is, to the elements of the biochemical steps that are quite known in the photosynthetic process, and the computational burden resulting of a complex mathematical formulation.

The minimal requirement from a mathematical model is the prediction of the P-I curve. That is the dependence of the Photosynthesis rate on irradiance, with the easily measurable parameters usually called \( \alpha \), the initial photosynthesis rate, and \( P_m \) the maximal photosynthesis rate, at certain irradiance \( I_m \). One of the earliest approaches is the Aiba equation, adopting the form of a substrate-inhibited enzymatic reaction to account for photoinhibition (Aiba, 1982).

\[ \frac{\mu}{K_1 + I + K_f I^2} \]

This single equation gives a satisfactory result for photo-adapted systems operation at steady state. The only variable, illumination \( I \), is extra-cellular, and the parameters can easily be found empirically, fitting to experimental data.

On the other hand, several much more sophisticated models aiming at the representation of the dynamic behaviour of photosynthetic cells and their capacity of adaptation to different illumination intensities have been proposed lately. Those models include as variables not only the irradiance, but also some intra-cellular variables as chlorophyll concentration, extent of light-damaged protein D1 in Photosystem II, nitrogen and carbon content in the cell (Geider et al., 1998; Harmon and Challenor, 1997; Pahlow, 2005; Smith et al., 2007; Marshall et al., 2000).

There is still another group of models of photosynthesis that can be situated between the previous two extremes. Those are the models using the concept of Photosynthetic Unit (PSU), called also Photosynthetic Factories (PSF) (Prezelin, 1981; Megard et al., 1984; Eilers and Peeters, 1988; Zonneveld, 1997, 1998, Camacho Rubio et al., 2003). The PSF is defined as the sum of light trapping system, reaction centres and associated apparatus, which are activated by a given amount of light energy to produce a certain amount of photoproduct. In spite of the Gargantuan lumping, this definition keeps open the possibility of giving a fair representation of many of the characteristics of the actual photosynthesis apparatus, and even enables to integrating into the model some measurable intracellular variables, as the concentration of chlorophyll a, cytochrome, D1 protein, etc. (Zonneveld 1997, 1998; Camacho Rubio et al., 2003). The PSF has three states, the open state (indicating that photons can enter the PSF) called \( x_1 \), the activated state (closed) called \( x_2 \), and the inhibited, or non-functional state called \( x_3 \). The PSF in resting or open state can be stimulated and transferred to the activated state when it captures a number of photons required for excitation. The PSFs in activated state have two possible paths, either receiving additional photons and become inhibited, or passing the gained energy to acceptors to start the photosynthesis at a rate controlled by enzymatic systems, and return to the open state. The inhibited PSF can eventually recover, returning to the open state. Those transitions have been schematized in Fig. 2.

The three possible states of the PSU are shown, together with the rates of the different steps. \( r_1 \) is the rate of light energy capture, \( r_2 \) the rate of biomass production, \( r_3 \) the rate of PSU inhibition, \( r_4 \) the rate of inhibited PSU recovery, and \( r_5 \) the rate of energy spending in cell maintenance.

\[ \frac{d\text{Chl}}{dt} = k_1 \cdot (\text{Chl}_{max} - \text{Chl}) - k_2 \cdot \text{Chl} \]
where $Chl$ is chlorophyll-a content in algal cells, $Chl_{\text{max}}$ the maximum value of chlorophyll-a content in algal cells, $k_1$ and $k_2$ are rate coefficients of chlorophyll-a synthesis and degradation respectively.

Thus, in spite of huge extent of simplification, the concept of PSU retains the prospect of representing some of the characteristic of the photosystem that most counts in photobioreactor behaviour: the fast response to sudden changes in illumination, the saturation of the reaction centres in PSII, the interconnection of the fast photon-associated reactions, the slower dark reactions leading to biomass synthesis, the photo damage due to high photon flux density (PFD) and the recovery of the damaged D1 proteins.

### Photosynthesis in the bioreactor

The most unique aspect of light as a substrate is that its availability depends not only on the rate of light input, but also on space (distance from the illuminated face). The exponential decay of the irradiance as the distance from the illuminated face increases creates three zones with different regimes of growth in each. A first zone, which extends from the illuminated wall till the point where the light energy arriving just balances the energy needed for growth at the maximum rate ($I_s$). In this zone, light is the limiting substrate and the photosynthetic rate will be proportional to $I$. The third, poorly illuminated zone where growth will be negative because of lack of enough light.

In each stage a simple exponential decay of irradiance is usually assumed. More sophisticated approaches have been proposed and elaborated as well (Cassano et al, 1995; Cornet et al, 1995, 1998; Pottier et al., 2005).

Several mathematical models of photobioreactors based in this scheme of light decay were proposed. The general problem of photo-reactor design considering light attenuation has been extensively discussed by Bernardez et al. (1987). Several mathematical descriptions of photobioreactors have taken into consideration the distribution of light in the volume of the culture, either using an averaged value of the irradiance, or averaging the growth rate (Dermoun et al., 1992; Evers, 1991; Frohlich et al., 1983; Molina Grima et al., 1993; Molina Grima et al., 1996).

None of the above takes into explicit account the fluid dynamics in the bioreactor. The importance of this element can be supported considering the time constants that appear in the analysis of the photosynthetic process and the dynamics of the reactor, as shown in Fig. 3. This figure is based on Lam et al. (1986) and Lam and Bungay (1986). The figure shows the chain of processes that lead from photon capture to organic molecule synthesis, covering an extremely extended range of time constants. In the figure, the time constants of the steps occurring inside the cell are indicated in parallel to the characteristic times of the fluid dynamics in the bioreactor. It can be seen that the time scale of CO2 fixation in the process, which corresponds to biochemical dark reactions in the cell, is of the order of

![Comparison of the time constants of the processes occurring in the cell during photosynthesis and of the fluid dynamics in bioreactors](image-url)
magnitude of the time constants for bioreactor dynamics. This is required, from the point of view of process dynamics, to make possible the interaction of these two processes. The aim in the present paper is presenting and comparing some examples in integrating kinetics and fluid dynamics in the modelling of photobioreactors.

**Simulated illumination/darkness cycles**

In an actual bioreactor, suspended photosynthetic cells move in a more-or-less chaotic way from the high illumination zones to the less illuminated ones. The simplest way of mimicking this type of cell history would be assuming that a certain cycle is repeated time and again by the cell. This cycle is taken as representative for the liquid flow in the photobioreactor as a whole. This approach can easily be modelled mathematically and the calculation is straightforward. Moreover, such a system can be actually built in the laboratory and true measurements can be done of the main variables. Such experimental device has proven to be an extremely useful tool for basic studies on photobioreactor design. Studies of this type, where thin cultures are used in order to avoid self shading by the cells and the light intensity perceived by all the cells in the culture is the same and is measurable, have been first used by Lee and Pirt (1981). Terry (1986) first proposed a methodology for the definition of the illumination cycles. While this has been generally accepted, caution should be used respect the conditions of cycle frequency and illumination where light/dark cycles result in efficiency gain (Janssen et al, 2000, 2001, 2002). Latter on, Wu and Merchuk (2001) combined light/darkness cycles with Eilers and Peeters PSU model (1988). In order to do so, a thin-film photobioreactor where the cells passed repeatedly over an overall 45 s period with varying proportions of light and darkness was built. Fig. 4 shows schematically how the amount of PSU in state 2 would periodically change after the system had entered the pseudo steady state. In the figure it has been assumed that the PSU reaches light saturation during the illuminated period, but this is not necessary the case in general.

These experiments were used to calibrate the mathematical model, and the kinetic constants obtained allowed the calculation of Fig. 5. The figure shows the results produced by the model in terms of the observed specific growth rate, \( \mu \), as a function of \( t_l/t_c \), the fraction of cycle time spent in light conditions, for a fixed value of irradiance \( I \). Each line shows how \( \mu \) would change, for the same cycle length of 45 seconds, as the light time increases from zero to 100 %. The observed growth rate, \( \mu \), increases monotonically with illumination fraction for lower values of \( I \). For \( I = 220 \, \mu \text{Em}^{-2}\text{s}^{-1} \) the curve shows a plateau when the illumination fraction approaches unity, and for higher irradiances a clear maximum appears. The simulation also shows that for higher values of \( I \) the location of the maximum keeps shifting towards lower values of \( t_l/t_c \). In other words, the higher the irradiance, the longer is the dark period that can be afforded by the system without loss of growth. This is due to the growth inhibition caused by high irradiance, and its reversible character. For greater values of \( I \), more PSFs reach the closed state, and the dark period allows the repair of damage leading to more PSF in the productive states.

The information presented in Fig. 5 allowed the drawing of what the authors called “island of existence” (Wu and Merchuk, 2001), the area on
the plane illumination fraction/Irradiance where a photobioreactor could operate at steady state.

While those results are of great interest, they correspond to an over-simplified scheme, since not only is the whole illumination story of the cells represented by one single cycle, but the illumination changes are extreme, from maximal \( I \) to complete darkness. In the following section, some of the attempts of a more realistic description will be reviewed.

**The bubble column (BC)**

Bubble columns are one of the most popular types of reactors because of reasons related to construction and operation simplicity (Deckwer, 1985). This type of reactors is frequently used to carry out photosynthetic processes. Inside the BC, the light history of the suspended photosynthetic cells is controlled by the fluid dynamics. Therefore, a description of the fluid dynamics in the bubble column is required for an adequate representation of the process. Several flow models for bubble column reactor have been postulated, such as circulation cell model (Joshi and Sharma, 1979), cylindrical eddies model (Zehner, 1986), single internal loop model (Hills, 1974; Ueyama and Miyauchi, 1979), radial distribution model (Clark et al., 1987), and others. More recently, Camacho Rubio et al. (2004) provided a method for simultaneously quantifying axial and radial dispersion coefficients. The latter was shown to be important for establishing the frequency of light-dark cycling of the fluid in bubble column photobioreactors and may be useful for the design of other column-type photoreactors.

Hydrodynamic parameters such as gas holdup and the bubble size can affect irradiance inside the bubble columns (Sánchez Mirón et al., 1999) and consequently the light history of cells. Under outdoor conditions, presence of gas bubbles generally enhances internal irradiance when the sun is low on the horizon. Near solar noon, the bubbles diminish the internal column irradiance relative to the ungassed state. The effect of aeration on internal irradiance diminishes as the gas flow rate is reduced; however, even at low superficial gas velocities corresponding to a fractional gas holdup below 1%, the irradiance level is affected by up to 15 % relative to gas-free operation. Therefore, for best performance, bubble columns need to be operated at the highest feasible aeration rates consistent with the shear tolerance of the microalgae; however, the aeration rate must not be so high as to produce a gas holdup level that prevents light transmission through the column.

Wu and Merchuk (2002) simulated algal growth in a BC of a given radius \( R \). The main information required for such a simulation is the fraction of the time that a photosynthetic element spends at each light intensity. This was represented by means of a typical trajectory. The trajectory was assumed to follow a circulation cell of the type defined by Joshi and Sharma (Joshi and Sharma, 1979). In order to simplify the calculations, the relationship between the time \( t \) and radial position \( z \) of the cell was assigned to be a cosine function:

\[
t = \frac{R}{2} \left( 1 - \cos \frac{2\pi}{T} t \right)
\]  

The cycle time \( T \) was obtained using the surface renewal model proposed by Danckwerts (1951). The rate of renewal of elements at the wall of the bubble column was evaluated using available data on heat transfer rate through the walls of a bubble column and the Colburn analogy (Colburn, 1933). Limiting this analogy to heat and mass transfer, it can be expressed as (Foust et al., 1979)

\[
\frac{1}{h} \rho C_p \cdot \frac{2}{Pr^3} = k_L \cdot Sc^2
\]  

Equation (4) relates the mass transfer coefficient \( k_L \) to the heat transfer coefficient, \( h \), and the physical properties of the system. The heat transfer coefficient in bubble columns was evaluated by the following equation, which has proven to be successful over a wide range of reactor dimensions and liquid properties (Zaidi et al, 1990):

\[
St = 0.1 \cdot (Re \cdot Fr \cdot Pr^2)^{\frac{1}{4}}
\]  

and thus the mean surface residence time can be estimated.

The authors assumed the distribution of contact times originally proposed by Danckwerts in his surface renewal model (1951):

\[
\Phi(t) = s \cdot e^{-st}
\]  

This distribution was discretized considering three fractions with three different contact times: one equivalent to all the elements having residence time shorter than \( t_1 \), the second equivalent to all elements having contact times between \( t_1 \) and \( t_2 \), and the third one equivalent to all the elements with contact times longer than \( t_2 \) (Fig. 6). These fractions will be:

\[
F_1 = 1 - e^{-st_1}
\]
\[
F_2 = e^{-st_1} - e^{-st_2}
\]
\[
F_3 = 1 - F_1 - F_2 = e^{-st_2}
\]  

Mean renewal time representative of each of these fractions can be calculated as:
The light intensity as a function of culture depth was estimated by using Lambert-Beer law:

$$I(t) = I_0 \cdot e^{-(k_{1} + k_{2})t}$$  \hspace{1cm} (10)

This light history of the photosynthetic cells was integrated with the modified (maintenance added) Eilers & Peeters model (1988). The results are presented as a function of “Ground Productivity”. Instead of plotting the biomass concentration of a single reactor, an assembly or “farm” of photobioreactors was considered. The total biomass obtained at the end of a batch culture per unit area of ground required and per unit time (ground productivity, $P_g$) was considered. It was assumed that in addition to the area required for the column itself, which depends on $D_c^2$, there is a certain distance required between adjacent column installations, $L$, to allow man passage for operation maintenance and to reduce self-shading among adjacent columns. According to these assumptions, the productivity per unit area can be defined as:

$$P_g = \frac{D_c^2 \cdot H \cdot (x_f - x_0)}{(D_c + L)^2 \cdot \Delta t} = P_D \cdot H \cdot \frac{\Delta x}{\Delta t}$$  \hspace{1cm} (11)

The effect of the light intensity and column diameter on the growth is studied and shown in Fig. 7. The simulation is for the superficial gas velocity of 0.0032 m/s. The figure shows the area productivity versus column diameter at different PFD. For each PFD, $P_g$ firstly increases with the increase of column diameter, reaches a peak, and then goes down. When the effect of irradiance on $P_g$ for a constant $D_c$ is examined, the picture depends on the $D_c$: For $D_c < 0.2$ m, the ground productivity increases as the PFD increases till 1000 $\mu$Em$^{-2}$s$^{-1}$. The curve of PFD 1500 $\mu$Em$^{-2}$s$^{-1}$ shows a lower value of $P_g$. The curve corresponding to an irradiance of 2000 $\mu$Em$^{-2}$s$^{-1}$ is much lower at small column diameters, and $P_g$ begins to increase only when the diameter is around 0.2 m. This fact is important for photobioreactor design when considering the design of a plant. Diameters that could be unpractical at the low irradiance usually utilized in laboratories, may give good productivity in the range of PFD of natural illumination.

A closer examination of Fig. 7 reveals that the maximum of each curve occurs at different column diameter. This indicates that the simulations can be an important tool for the detailed design of the photobioreactor. Fig. 8 is even more revealing, because it shows the influence of the gas superficial velocity $J_G$ on $P_g$ as a function of the PFD for three diameters. It is important to stress that those two variables, gas superficial velocity and tube diameter, appear in the model via the consideration of the flow patterns in conjunction with the kinetic model.

A further refinement of the concept of photobioreactor farm was proposed by Sánchez Mirón et
The maximum number of the vertical column reactors that may be accommodated in a given area depends on the height of the column which, together with the position of the Sun, establishes the maximum extent of column shadow on the ground. The length of the shadow from the column’s base is given by

\[ L_s = \frac{H}{\tan \theta_i} \]  

(12)

where \( H \) is the height of the column and \( \theta_i \) is the angle of incidence of the direct solar radiation. The angle of incidence-the inclination of the Sun from the normal to the vertical axis of the bubble column-depends on the geographic latitude, \( \phi \), the day of the year \( N \), and the solar hour \( h \); the angle of incidence is given as (Liu and Jordan, 1960):

\[ \theta_i = 90^\circ - \cos^{-1} (\cos \delta \cdot \cos \phi \cdot \cos \omega + \sin \delta \cdot \sin \phi) \]  

(13)

where \( \phi \) is the geographic latitude. The angles \( \omega \) and \( \delta \) are related to the solar hour and the day of the year (Liu and Jordan, 1960), respectively, as follows:

\[ \omega = 15 \cdot (12 - h) \]  

(14)

\[ \delta = 23.45 \cdot \sin \left( \frac{360 \cdot (284 + N)}{365} \right) \]  

(15)

The loci of the maximum extent of the shadow of a 2.6 m tall bubble column are plotted in Fig. 9 for representative days in winter, spring, and summer seasons at a given geographic location. The maximum extent of the shadow in January is about 9 m, whereas the maximum extent in July is about 1.5 m. These distances are measured north-south between parallel east-west lines passing through the base of the vertical column and the tip of the column’s shadow. Ideally, parallel east-west rows of bubble columns should be spaced by at least the maximum length of the shadow in winter. This would assure that the reactors are never mutually shaded, however, a more optimal setup would place the rows of reactors closer, about midway between the high extremes of the shadow length in the summer and the winter. Consequently, there will be no mutual shading in the summer but some shading would occur during the winter. In a single east-west row of columns the columns could be spaced quite close together. Close spacing within east-west rows has no impact on illumination, but it improves efficiency of land use. The optimal column height will also depend not only on technical considerations such as wind speed and strength of optically transparent materials used, but also on the impact of height on fluid dynamics.
The Air Lift Reactor (ALR)

ALRs are completely different from BCs in their fluid dynamics. Instead of the quasi chaotic movement in the BC, the ALR offers an overall ordered flow through defined ducts that are built with this purpose in mind. If the configuration of concentric tubes is chosen and the light source is at the external wall, the internal draft tube delimits an area that is inherently the darkest. Wu and Merchuk (2004) simulated algal growth in the ALR combining an analytical solution of the equations and finite elements calculation, assuming that the downcomer was divided into several radial regions according to the prevailing PFD. Each interval has a constant PFD and the change in irradiance from region to region is ΔI. Fig. 10 illustrates the light intervals. Fig. 11 shows how the model was able to fit successfully the results obtained in a bench-scale ALR, including the effect of gas superficial velocity $J_G$.

In Fig. 12, the combined effects between the two main design variables of an ALR, $A_r/A_d$ (area ratio) and $H_d$ (column height) are presented. Those variables influence the flow patterns in the reactor and determine in fact the illumination history of the cells. The solution obtained by Wu and Merchuk (2004) show that there is an area on the $A_r/A_d-H_d$ plane where net growth in not possible, and in consequence both variables should be manipulated by the designer to place the photobioreactor inside the zone of sustained growth. This is another version of the previously mentioned “island of existence”. The plane where the map is drawn in this case is however related only to design variables that determine the illumination cycles, and will correspond to given conditions of light and gas flow rate.

Tubular photobioreactors

Tubular photobioreactors consist of straight, coiled or looped transparent tubing arranged in various ways for maximizing sunlight capture. Phototrophic cultures are circulated through the tubes by various methods; use of airlift circulators is especially common. Molina Grima et al. (1999, 2000) developed a method for relating the light/dark frequency to prevailing hydrodynamics and irradiance level in a tubular photobioreactor. The light zone is also defined as that in which the light intensity is at saturation value, or greater, and the dark zone is one where the light intensity is below the saturation threshold. Dependence of culture biomass productivity ($P_b$) on light/dark cycle frequency ($\nu$) was found to be a Monod type growth equation given by:

$$P_b = \frac{P_{b\ max}}{K_v + \nu}$$

where $P_{b\ max}$ is the maximum biomass productivity, $\nu$ the light/dark cycle frequency, and $K_v$ the frequency for half the maximum productivity. Both
$P_{b\text{max}}$ and $K_v$ showed a linear dependence on the day-averaged irradiance measured on the reactor’s surface ($I_0$) given by:

$$P_{b\text{max}} = a + b \cdot I_0$$

(17)

$$K_v = c + d \cdot I_0$$

(18)

where $a$ to $d$ are constants.

The frequency was calculated as

$$v = \frac{1 - \phi_f}{t_d}$$

(19)

being $t_d$ the time spent in the dark zone and $\phi_f$ is the fractional culture volume that is illuminated (i.e. the photic volume fraction) for a known level of external irradiance, biomass concentration, and absorption coefficient of the biomass, that can be estimated from the light profiles; $\phi_f = V_f/(V_f + V_d)$. The values of $t_d$ were calculated as

$$t_d = \frac{d_r \cdot (\theta - \sin \theta)}{2 \cdot U_R \cdot \sin \theta}$$

(20)

where $d_r$ is the tube diameter, $U_R$ is the radial velocity and $\theta$ is a specific angle necessary for calculating $\phi_f$. Since, cells move radially with the fluid because of momentum transport between the turbulent core and the more quiescent boundary layer adjacent to walls, a radial velocity was approximated as the characteristic velocity of turbulence in the center of the tube:

$$U_R = 0.2 \left( \frac{U_l^2 \cdot \mu}{d_r \cdot \rho} \right)^{1/8}$$

(21)

which allows the calculation of $U_R$ in the turbulent core as a function of the superficial liquid velocity ($U_l$), the tube diameter ($d_r$), and the density ($\rho$) and viscosity ($\mu$) of the culture broth.

Biomass productivity data are shown in Fig. 13 as a function of light/dark cycle frequency day-averaged irradiance is shown. Experimental data were obtained from two outdoor placed tubular photobioreactors with tube internal diameters of 0.053 m and 0.025 m and operated as continuous cultures at various dilution rates. Scale-up capability was also proved using the criterion of keeping constant the light/dark cycle frequency in reactors of different diameters.

Other attempts have been done following the approach presented here (Luo and Al-Dahann, 2004; Pruvost et al, 2002, Muller-Feuga et al, 2002, 2003, Potier et al, 2005). The present paper is not meant to be an exhaustive report, but to enlighten the main principles of the approach of integrating kinetic models of photosynthesis and flow patterns that define the light history of the cells in photobioreactors.

**Summary**

While many of the photobioreactors in operation today are based more on invention than on engineering design, the knowledge basis available on the modelling of photosynthetic systems has advanced sensibly in the last time. The public interest on sustainability provides a positive vector pressing toward the application of this knowledge. One of the most important elements that has to be integrated into the design is the fluid dynamics and the influence of the flow patterns on the yield of the photosynthetic systems. Several examples of the ways in which the integration of fluid dynamics and photosynthesis kinetics can be carried out are presented in the present review.

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**Nomenclature**

$Chl$ – chlorophyll-a content in algal cells (g m$^{-3}$)

$Chl_{max}$ – maximum value of chlorophyll-a content in algal cells (g m$^{-3}$)

$C_p$ – Heat capacity (cal · g$^{-1}$ K$^{-1}$)

$d_r$, $D_c$ – tube diameter, (m)
Greek symbols

- \( \alpha, \beta, \gamma, \delta \) – kinetic constants (Table 1)
- \( \Delta \) – interval
- \( \phi_f \) – fractional culture volume that is illuminated (i.e. the photic volume fraction).
- \( \phi \) – geographic latitude
- \( h \) – solar hour
- \( \mu \) – viscosity of the culture broth (g m\(^{-1}\) s\(^{-1}\)), specific growth rate (s\(^{-1}\))
- \( \mu_{\text{max}} \) – Maximal specific growth rate (s\(^{-1}\))
- \( \delta \) – declination the angular position of the Sun at solar noon with respect to the plane of the equator, north positive.
- \( \Phi \) – Surface renewal distribution function, (s\(^{-1}\))
- \( \theta \) – specific angle necessary for calculating
- \( \theta_i \) – angle of incidence of the direct solar radiation
- \( \nu \) – Light/dark cycle frequency, (s\(^{-1}\))
- \( \rho \) – density of the culture broth, (g m\(^{-3}\))
- \( \omega \) – angle corresponding to the solar hour

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