ISSN 0005–1144
ATKAAF 48(3–4), 121–128 (2007)

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Estimation of Parameters for Synchronous Generator from Sudden Short-circuit Test Data Considering Rotor Speed Variation

UDK 621.313.322.064.1
IFAC 5.5.4

Original scientific paper

Procedure to estimate parameters of synchronous hydro-generator (18 MVA) from sudden short-circuit measurement data is presented. Test performed was under small and slow speed variation and reduced excitation current. Data were collected in the form of sampled time series of measured armature voltage and currents prior to and during the sudden short-circuit test. Neither the rotor speed nor the angle between rotor and stator were measured. Detailed method of initial and final estimation of parameters were also appraised (Prony's method and constrained minimization technique) considering behaviour of the rotor angle and speed during the test.

Key words: constrained minimization, estimation of parameters, Prony's method, sudden short-circuit test, synchronous generator

1 INTRODUCTION

Procedure to estimate synchronous generator parameters from sudden short-circuit test is well known and involves the technique of graphic envelope and tangents construction to estimate them. It has been defined in various standards [1, 2]. However, manual construction of tangents is subjective and in general it is less reliable. Sudden short-circuit test is usually performed with reduced excitation current in relation to the rated no-load value and under constant rated speed of rotation. If the later condition is fulfilled the transformation of armature currents into rotor (d,q) frame results with relatively simple expressions for their direct (d) and quadrature (q) axis components. An example of parameters estimation for a small synchronous machines using mainly (d,q) decomposition has been demonstrated in [3]. Corresponding expressions for armature current can be found in [4] and [5]. In [7] authors recognized many drawbacks of standard graphically-oriented approaches defined in [1] and chose the cost function as norm of weighted-covariance matrix of residuals. However, it is generally concluded that is much easier recognize the constitutive parameters from tailored phase currents responses according to [1].

However, due to turbine control, the rotation speed usually varies in time. The same transformation of armature phase currents requires the knowledge of angle between rotor and stand-still phase (winding) frame of armature or speed of rotation. As mentioned above, the measurement of these variables were not directly performed. Instead, it was decided to estimate the parameters of synchronous machine such as reactances, time constants and angle between rotor and stator (armature) in the same procedure. Recorded instantaneous values of voltages and currents were used to reach the main aim: to produce highly accurate parameter estimates. Initial, rough estimations of parameters were based on Prony's method of analysis of spectral properties of linear signals, [6], combined with Fourier discrete transform. This estimation stage is required to define lower and upper boundaries of parameters more precisely. After establishing these boundaries and positive defined function, constrained minimization can be applied to search the space of possible solutions.

2 SHORT DESCRIPTION OF ESTIMATION PROCEDURE

Assuming slow and small rotational speed variation and large angle deviation the expression (»model«) for the armature current in phase »a« during the three phase short-circuit reads [4, 5]:

$$i_a(t,\Theta)=U_{gq0-}\left[\left(\frac{1}{x''_d}-\frac{1}{x'_d}\right)e^{-\frac{t}{T''_d}}+\left(\frac{1}{x'_d}-\frac{1}{x_d}\right)e^{-\frac{t}{T'_d}}+\frac{1}{x_d}\right]\cos(\gamma(t)+\alpha)-$$

$$-U_{gq0-}\left[\frac{1}{2}\left(\frac{1}{x''_d}+\frac{1}{x''_q}\right)\cos\alpha+\frac{1}{2}\left(\frac{1}{x''_d}-\frac{1}{x''_d}\right)\cos(2\gamma(t)+\alpha)\right]e^{-\frac{t}{T_a}} \quad (1)$$

where:

x_d, x'_d, x''_d and x''_q are synchronous d-axis, transient d-axis, sub-transient d-axis and sub-transient q-axis reactance, respectively expressed in *pu*,

T''_d, T'_d and are sub-transient d-axis, transient d--axis short-circuit time constants,

T_a is armature time constant in s,

U_{gq0} is *pu* armature voltage before the armature was short-circuited and corresponds to the reduced excitation current I_{fd0-},

$\gamma(t)$ is the (electrical) angle between rotor d-axis and axis of the phase »a« and can be represen-ted as: $\gamma(t) = \omega_s t + \delta(t)$ where ω_s is electrical rated angular frequency (in this case: 100 π rad/s) and t is time in s.

For the other two phases (»b« and »c«), instead of α in expression (1), it is necessary to insert $\alpha-\frac{2\pi}{3}$ and $\alpha+\frac{2\pi}{3}$, respectively. The angle α is the angle between rotor d-axis and axis of the phase »a« at $t = 0$.

The set of unknown parameters of synchronous machine is:

$$\mathbf{\Theta}^T = [x_d \;\; x'_d \;\; x''_d \;\; x''_q \;\; T''_d \;\; T'_d \;\; T_a \;\; \alpha] \quad (2)$$

The expression (1) consists of five terms belonging to the next processes and in that order:

1. sub-transient:

$$U_{gq0-}\left(\frac{1}{x''_d}-\frac{1}{x'_d}\right)e^{-\frac{t}{T''_d}}\cos(\gamma(t)+\alpha),$$

2. transient:

$$U_{gq0-}\left(\frac{1}{x'_d}-\frac{1}{x_d}\right)e^{-\frac{t}{T'_d}}\cos(\gamma(t)+\alpha),$$

3. steady state:

$$U_{gq0-}\frac{1}{x_d}\cos(\gamma(t)+\alpha),$$

4. pure armature aperiodic:

$$-\frac{1}{2}U_{gq0-}\left(\frac{1}{x''_d}-\frac{1}{x''_q}\right)e^{-\frac{t}{T''_a}}\cos\alpha \quad \text{and:}$$

5. double frequency due to sub-transient saliency:

$$-\frac{1}{2}U_{gq0-}\left(\frac{1}{x''_d}-\frac{1}{x''_q}\right)e^{-\frac{t}{T''_a}}.$$

The angle $\delta(t)$ is also time dependant, but slow varying variable for which estimation is also needed. The first derivative of $\delta(t)$:

$$\frac{\mathrm{d}\delta}{\mathrm{d}t}=\omega(t)-\omega_s, \quad (3)$$

is angular speed difference (rad/s) between actual $\omega(t)$ and rated value. It is assumed that this difference varies slowly with time and it is relatively small. For the purposes of the case, the angle is synthesized as time polynomial function with satisfactory order n:

$$\delta(t)=\sum_{j=0}^{n}k_j t^j. \quad (4)$$

The order n is estimated on the basis of the preliminary analysis of the dynamics of the fundamental harmonic phase angle of the recorded armature currents in relation to the constant (ω_s) rotating frame. Thus, the overall set of unknown parameters can be presented as:

$$\mathbf{\Theta}^T = [x_d \;\; x'_d \;\; x''_d \;\; x''_q \;\; T''_d \;\; T'_d \;\; T_a \;\; \alpha \;\; k_0 \; k_1 \; ... \; k_n] \quad (5)$$

To estimate these parameters the function $J(\mathbf{W},\mathbf{\Theta},N)$ in the quadratic discretized form:

$$J(\boldsymbol{W},\Theta,N)=\frac{1}{2}\sum_{p=a,b,c}\sum_{i=1}^{N}e_p(i,\Theta)w(i)e_p(i,\Theta), \quad (6)$$

where:

$$e_p(i,\Theta)=i_{pm}(i)-i_p(i,\Theta) \quad (7)$$

is introduced as cost function and shall be minimized with respect to the unknown parameters $\mathbf{\Theta}$.

The quantity $e_p(i,\mathbf{\Theta})$ is the discretized difference between measured phase current $i_{pm}(i)$ and model $i_p(i,\mathbf{\Theta})$ according to the expression (1) at instant i.

The symbol »p« denotes »a«, »b« or »c« phase of the armature currents. Time weights $w(i)$ can be chosen so that is: $w(i) = 1$, $i = 1,2,...,N$ where N is number of time samples. It is important to emphasize that quadratic form (6) with summation indices over the time instants and phases enables unique parameter estimates for all phases.

In the sense of sum of squares, the quality of modelling can be quantified with the next expression (phase »p« = »a«, »b«, »c«):

$$Q_p(\%) = 100\left(1 - \frac{\sum_{i=1}^{N} e_p^2(i,\Theta)}{\sum_{i=1}^{N} i_{pm}^2(i)}\right). \tag{8}$$

3 RESULTS OF ESTIMATION

The tests were made on synchronous hydro-generator 18 MVA, 10.5 kV, $\cos\varphi = 0.85$, 600 rpm, 50 Hz with recording device DEWETRON 41 with 12 bit resolution and 50 kHz sampling rate during the commissioning and field acceptance tests. The estimation was performed with ten times decimated records (sampling rate 5 kHz). The record of transient phenomenon is shown on Figure 1. Captured record contains transient phenomena within 6.984 s and it consists of two parts: pre-trigger state (no-load) lasting 2.203 s (11015 samples) and three phase short-circuit lasting 4.781 s (23907 samples).

3.1 Initial estimation (guess) of reactances and time constants

The initial guess for synchronous reactance (x_{d0}) would have been more accurate if the captured data window were longer. However, the approach to initial estimate would be the same and it can be derived from (1) neglecting all terms except the steady state term (term number 3, page 2). Obviously, this is possible when $t \to \infty$. Thus the initial estimate x_{d0} is:

$$x_{d0} = \frac{U_{gq0-}(pu)}{\frac{1}{6}\sum_{p=a,b,c}(I_{pf\max}(pu) - I_{p_f\min}(pu))} = \frac{0.492}{0.4576} = 1.0752\,pu, \tag{9}$$

where $I_{pf\max}$ and $I_{pf\min}$ are the respective maximum and minimum values of the armature current during the last captured period (20 ms) which is the nearest to the steady state for phase »p«. Similar approach is also adopted for the initial estimate for the sub-transient reactances $x''_{d0} \approx x''_{q0}$. However, instead of $I_{pf\max}$ and $I_{pf\min}$ in (9) $I_{pi\max}$ and $I_{pi\min}$ shall be used. $I_{pi\max}$ and $I_{pi\min}$ are the respective maximum and minimum values of the armature current during the last captured period (20 ms) which is the nearest to the steady state for phase »p«:

$$x''_{d0} \approx x''_{q0} = \frac{U_{gq0-}(pu)}{\frac{1}{6}\sum_{p=a,b,c}(I_{pi\max}(pu) - I_{pi\min}(pu))} = \frac{0.492}{3.9235} = 0.1254\,pu. \tag{10}$$

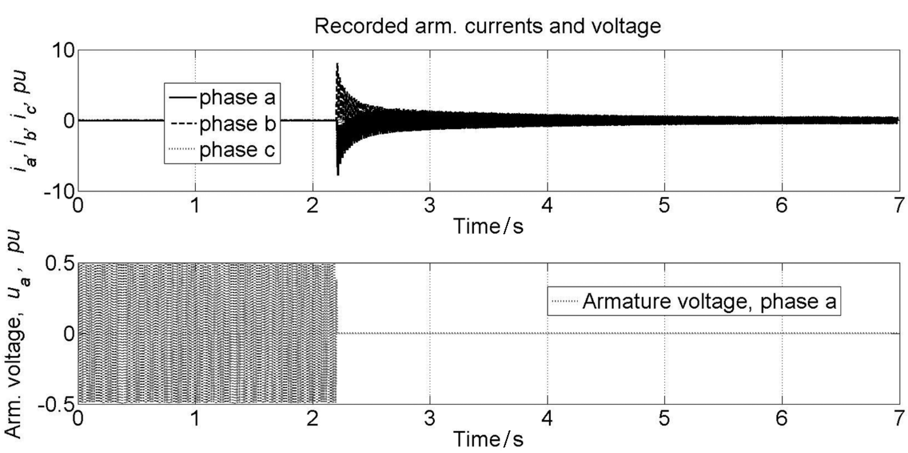

Fig. 1 Recorded armature currents and armature voltage before and after three phase short-circuit was applied (at instant 2.203 s with $U_{gq0-} = 0.492$ pu)

Table 1 **The strongest modes in the »a« phase current**

Mode (s^{-1})	Damping (ζ, *pu*)	Natural frequency (Hz)	Magnitude (*pu*)	Energy of mode (pu^2)
−10.7049 ∓ j626.0795	0.0171	99.6582	0.17062 ± j0.06146	0.003
−111.6684 ∓ j275.019	0.3762	47.2412	0.64669 ∓ j0.26229	0.004
−29.5126	1	–	−0.39523	0.011
−34.6935 ± j308.0515	0.1119	49.3379	0.77175 ± j0.37628	0.020
−0.8823 ± j312.4733	0.0028	49.7319	1.0882 ∓ j0.20952	1.390
−7.4186	1	–	−4.2727	4.922

Very close estimates for these parameters could also be obtained from phase currents fundamental harmonic »dynamics«, see Figure 2, second sub-graph with envelopes for phase currents. These envelopes were calculated using Fourier transform every 20 ms.

The initial estimation of the time constants was made by means of Prony's method of spectral analysis of linear signals [6]. Using the same method, the damping can be taken into account more precisely. As an example, the first 250 ms interval after three phase sudden short-circuit has been taken and analyzed using Prony's method. The expected range for T''_d and T_a are within ten milliseconds and hundred milliseconds, respectively. Subsequently, the appropriate time interval has been chosen. After some hundreds milliseconds the constitutive components associated with this time constants will disappear, (1). Using Prony's method, the strongest modes with the largest energies were found and the results are presented in the Table 1.

Let analyze the last three modes. The strongest of them −7.4186 s^{-1} has the largest energy (4.922 pu^2). Comparing this result with (1), it can be observed that this mode belongs almost to the aperiodic, fourth term in (1). If the angle α is not near to π/2 and the corresponding time constant can be determined: T_{a0} = 1/7.4186 = 0.1348 s. Assumed closeness of x''_d and x''_q, allows the term in (1) with frequency near 100 Hz to be very small. In fact, from the Table 1 it can be observed that this term is very damped and in spite of its significant magnitude, the corresponding energy is very low.

The second mode −0.8823 ± j312.4733 (signal energy 1.39 pu^2) corresponds to the transient second term in (1) and transient short-circuit time estimate is directly: T'_{d0} = 1/0.8823 = 1.1334 s. This term can also be used to estimate the transient reactance. Looking the Table 1, it is possible to reconstruct this component in a very simply way and to take its maximum value. This value is approximately equal to:

$$I'_{c\max} = U_{gq0-}\left(\frac{1}{x'_d} - \frac{1}{x_d}\right).$$

Namely, the initial angle α is not known. Since its value lies between −π and π which corresponds to 10 ms and −10 ms, respectively, this is no longer so important. When this component reaches its maximum value of (T'_{d0} >> 0.01 s), the result of $e^{-(t/T'_d)}\cos(\gamma(t)+\alpha)$ approximately equals to 1. From $T'_{c\max}$ = 2.198 *pu* and x_{d0}, determined earlier, transient reactance can roughly be estimated:

$$\frac{1}{x'_{d0}} = \frac{I'_{c\max}}{U_{gq0-}} + \frac{1}{x_{d0}}, \quad x'_{d0} \approx 0.19\ pu.$$

The third mode −34.6935 ± j308.0515 belongs to the sub-transient component of the first part of (1). Therefore, the sub-transient short-circuit time constant is to be equal to T''_{d0} = 1/34.6935 = 0.0288 s.

3.2 Rough estimate of rotor angle and speed time variation

Assuming that rotor speed deviation is relatively small, the fundamental harmonic of phase current(s) is calculated every 20 ms using discrete Fourier transform as Figure 2 shows. The angle and speed deviation are shown on the first and envelopes on the second sub-graph. This calculation enabled an initial estimation of the »dynamics« of phase angle $\delta_0(t)$ with respect to the constant (50 Hz) rotating frame as a function of time. A numerical time derivative of this angle gives the rough form of the rotor speed as Figure 2 shows. The speed has one minimum and one maximum in the time span shown. The minimal polynomial order that could approximate (fit) it, would be three. The order of angle corresponding to this

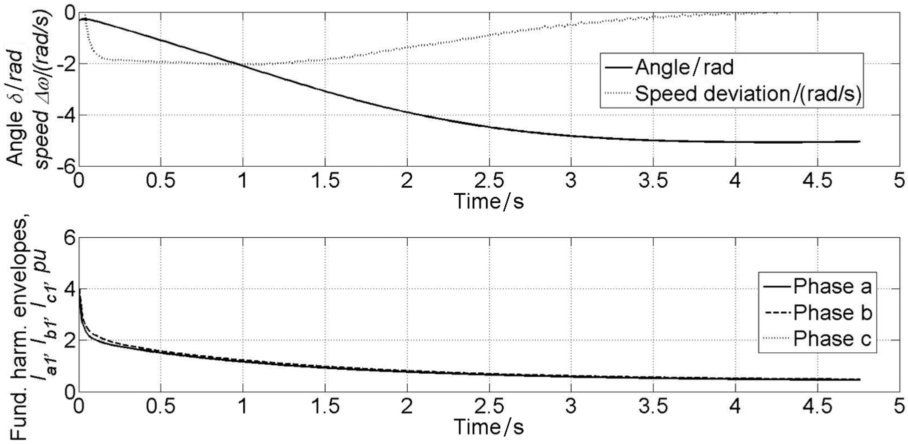

Fig. 2 Rough estimates of rotor angle and speed deviation and fundamental harmonic envelopes of phase currents versus time

speed approximation must be four. Thus, the time polynomial in (4) will in our case be:

$$\delta(t) = k_0 + k_1 t + k_2 t^2 + k_3 t^3 + k_4 t^4. \tag{11}$$

Applying the method of least squares on the phase angle data the coefficients in (11) can be obtained:

$$k^T = [k_0 \;\; k_1 \;\; k_2 \;\; k_3 \;\; k_4] = [-0.1985 \;\; -1.5877 \;\; -0.5849 \;\; 0.2869 \;\; -0.0293].$$

Finally, the initial parameter estimate is:

$$\begin{aligned}\Theta_0^T &= \left[x_{d0} \;\; x'_{d0} \;\; x''_{d0} \;\; x''_{q0} \;\; T''_{d0} \;\; T'_{d0} \;\; T_{a0} \;\; \alpha_0 \;\; k_{00} \;\; k_{10} \;\; k_{20} \;\; k_{30} \;\; k_{40}\right] = \\ &= [1.0752 \;\; 0.19 \;\; 0.1254 \;\; 0.1254 \;\; 0.0288 \;\; 1.1334 \;\; 0.1348 \;\; 0 \;\; -0.1895 \;\; -1.5877 \;\; -0.5849 \;\; 0.2869 \;\; -0.0293].\end{aligned} \tag{12}$$

The lower and upper boundaries of parameters can be stated rather wide:

$$\begin{aligned}\Theta_l^T &= \left[x_{dl} \;\; x'_{dl} \;\; x''_{dl} \;\; x''_{ql} \;\; T''_{dl} \;\; T'_{dl} \;\; T_{al} \;\; \alpha_l \;\; k_{0l} \;\; k_{1l} \;\; k_{2l} \;\; k_{3l} \;\; k_{4l}\right] = \\ &= [0.9 \;\; 0.15 \;\; 0.085 \;\; 0.085 \;\; 0.015 \;\; 1.0 \;\; 0.1 \;\; -\pi \;\; -0.5 \;\; -2.0 \;\; -1.0 \;\; -0.5 \;\; -0.5]. \\ \Theta_u^T &= \left[x_{du} \;\; x'_{du} \;\; x''_{du} \;\; x''_{qu} \;\; T''_{du} \;\; T'_{du} \;\; T_{au} \;\; \alpha_u \;\; k_{0u} \;\; k_{1u} \;\; k_{2u} \;\; k_{3u} \;\; k_{4u}\right] = \\ &= [1.5 \;\; 0.30 \;\; 0.15 \;\; 0.15 \;\; 0.06 \;\; 2.0 \;\; 0.6 \;\; \pi \;\; 0.5 \;\; 2.0 \;\; 1.0 \;\; 0.5 \;\; 0.5].\end{aligned} \tag{13}$$

3.3 Final refinement

The final estimate of the unknown parameters were performed by means of MATLAB built-in constrained minimization routine »FMINCON« with default settings. The function to be minimized is given by (6) and parameters are subjected to constraints (13). The initial guess is given in (12). The evaluated estimates for the machine parameters finally were:

$$\begin{aligned}\Theta^T &= \left[x_d \;\; x'_d \;\; x''_d \;\; x''_q \;\; T''_d \;\; T'_d \;\; T_a \;\; \alpha \;\; k_0 \;\; k_1 \;\; k_2 \;\; k_3 \;\; k_4\right] = \\ &= [1.1336 \;\; 0.2321 \;\; 0.1007 \;\; 0.0922 \;\; 0.022 \;\; 1.2505 \;\; 0.1229 \;\; -0.4458 \;\; 0.1562 \;\; -1.2004 \;\; -0.9203 \;\; 0.3907 \;\; -0.0398].\end{aligned}$$

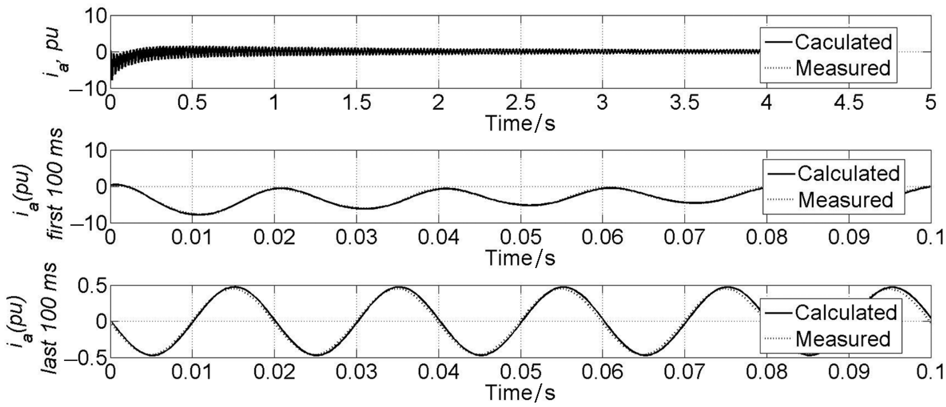

Fig. 3 Calculated and measured current for phase »a«: whole captured phenomenon, the first and the last 100 ms of the phenomenon after the short-circuit was applied

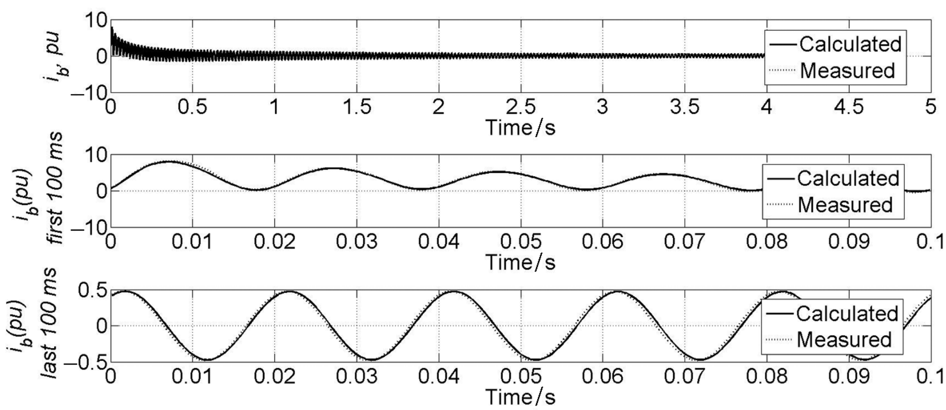

Fig. 4 Calculated and measured current for phase »b«: whole captured phenomenon, the first and the last 100 ms of the phenomenon after the short-circuit was applied

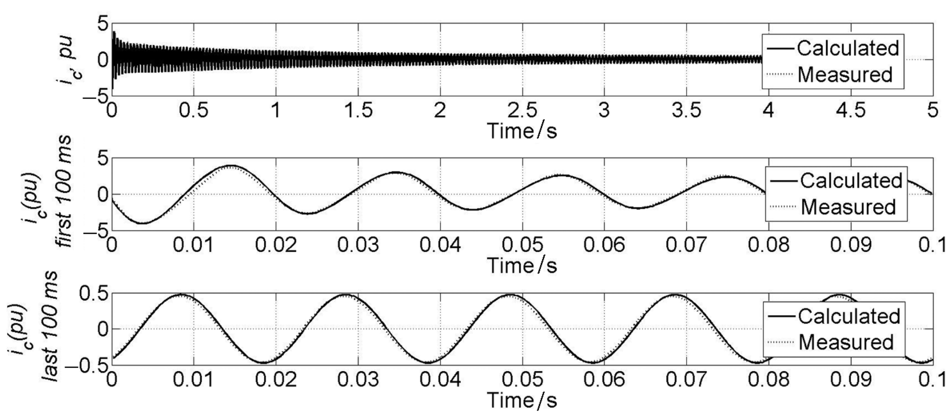

Fig. 5 Calculated and measured current for phase »c«: whole captured phenomenon, the first and the last 100 ms of the phenomenon after the short-circuit was applied

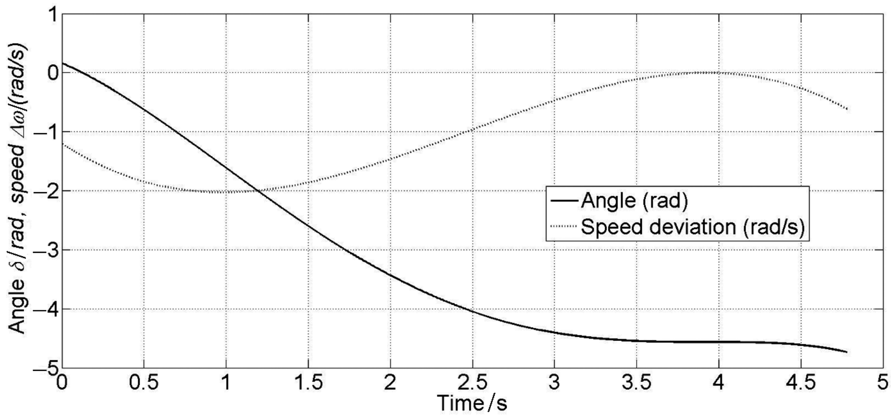

Fig. 6 Final estimated behaviour of rotor angle and speed deviation (in relation to the constant rotating frame) during the three phase short-circuit

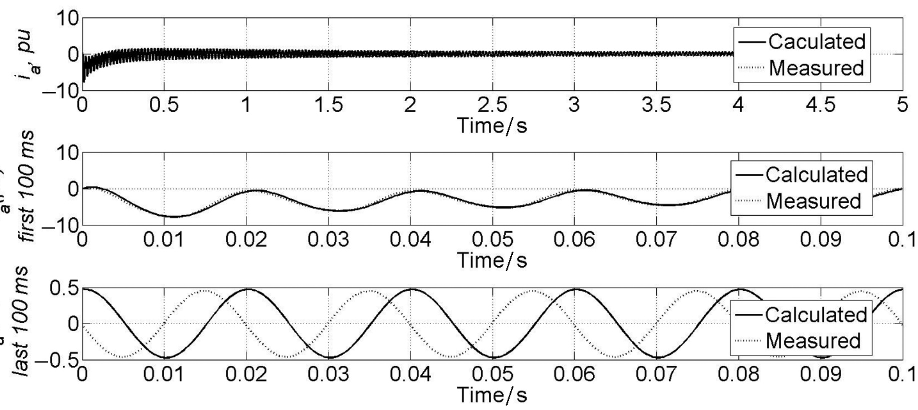

Fig. 7 Calculated and measured current for phase »a«: whole captured phenomenon, the first and the last 100 ms of the phenomenon after the short-circuit instant was applied, case: the speed and angle variation have been neglected in the model of stator phase »a« current

Evaluated minimum of the function $J(\mathbf{W},\mathbf{\Theta},N)$, (6) has been to the value of 81.165 pu^2 and, as it can be seen from (13), no constraints have been violated. The phase shares and modelling quality measures according to (8) were as follows:

- phase a: 18.162 pu^2, Q_a = 99.79 %
- phase b: 29.3221 pu^2, Q_b = 99.66 %
- phase c: 33.2219 pu^2, Q_c = 99.45 %

Corresponding results are presented in figures 3, 4 and 5. The whole phenomenon is shown on the first graph. Beginning of the process (100 ms, after short-circuit) is shown together with the end of the same transient process (also 100 ms) in the second and third sub-graph, respectively.

An excellent agreement between measured and calculated armature currents can be observed. Finally, from Figure 6 the rotor angle and speed behaviour during the test can be observed. At first the angle was determined according to (4) or (11). Using the expression (3) polynomial for δ is derived with respect to the time and the rotor speed deviation was obtained.

Finally, Figure 7 shows the situations when the speed or angle variation in the model of the armature currents is neglected but other parameters were retained as in the Figure 3. Calculated and measured values were synchronized only at the beginning of the process. It is also clear from Figure 6, that the maximum phase difference between model and measurement appears at the instant of t = 1.7 s and it amounts approximately to 180 degrees. Thus, the introduction of the rotor angle deviation into the model has been justified (Figures 3, 4 and 5).

4 CONCLUSION

Procedure to estimate parameters of synchronous hydro-generator using the three phase sudden short-circuit test data has been presented. The test performed was with reduced excitation current and rotor speed variation. Rather simple initial estimates for reactances were made. Satisfactory initial estimates for time constants and some reactances were made using the Prony's method. Initial rotor angle estimate as time function was roughly estimated by means of Fourier discrete transformation. This enabled the determination of the minimum degree of polynomial that satisfactory fits the rotor angle and its speed deviation from rated value. Final refinement of parameter estimates was achieved by means of constrained minimization technique where instantaneous values of all armature currents were used in the criterion function. Subsequently, there was no need for manual construction of envelopes and tangents of transient phenomena (currents) as described in the standard guidelines for synchronous generator parameters estimation. Relatively small variations of rotational speed and large angle deviations during the test were taken into account by means of their estimation from the phase current records. In the whole time span, the excellent match between measured and calculated time responses was achieved. Additional computation time was consumed to achieve the aim mentioned.

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Procjena parametara sinkronog generatora iz podataka mjerenja pokusa udarnog kratkog spoja uzimajući u obzir variranje brzine rotora. Prezentiran je postupak procjene parametara sinkronog hidrogeneratora (18 MVA) iz podataka mjerenja udarnog tropolnog kratkog spoja. Pokus je načinjen u uvjetima polagane i male promjene brzine vrtnje i snižene struje uzbude. Podaci su prikupljeni u obliku uzorkovanih vremenskih nizova mjerenih napona i struja armature prije i tijekom udarnog kratkog spoja. Tijekom pokusa nisu posebno mjereni niti brzina, a niti kut između rotora i statora. Također su razmotrene pojedinosti primijenjenih metoda početne i konačne procjene parametara (Pronyjeva metoda, postupak minimizacije s ograničenjima) uključujući i procjenu ponašanja kuta i brzine vrtnje tijekom pokusa.

Ključne riječi: minimizacija s ograničenjima, pokus udarnog kratkog spoja, procjena parametara, Pronyjeva metoda, sinkroni generator, udarni kratki spoj

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Received: 2007-09-12