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The experimental data concerning growth kinetics of vitamin C (L(+)-ascorbic acid, LAA) crystals in a seeded and cooling batch mass crystallization process realized in a four–compound: L(+)-ascorbic acid–methanol–ethanol–water system are reported. Influences of initial composition of solution and its linear cooling rate on “average, effective” values of crystal linear growth rate were examined. Small divergences between LAA crystal size distributions (CSDs) data from granulometric analysis and Coulter counter were interpreted theoretically and discussed. Linear growth rates of crystals in a batch crystallizer were acquired with a proposed by Nývlt indirect method, based on the analysis of population density n(L) data in a MSMPR (mixed suspension mixed product removal) crystallizer. Size–independent growth (SIG) kinetics was assumed. It can be concluded, that the largest and the most uniform particles of purified, crystalline vitamin C correspond to higher initial concentration of L(+)-ascorbic acid in a solution and lower cooling rate applied.

Key words: Vitamin C, methanol, ethanol, crystal growth kinetics, DT batch crystallizer, size-independent growth (SIG) kinetic model

Introduction

Vitamin C (L(+)-ascorbic acid, C₆H₈O₆, denoted later as LAA), a key ingredient of complex medicaments or multivitamins and natural antioxidant, is an essential compound of pharmaceutical and food industry products. This organic compound influences fundamental metabolic functions of human organism, being responsible, among others, for the structural strength of the blood vessels, metabolism of selected aminos or oriented distribution of specific hormones. According to a Reichstein procedure, LAA is synthesized from D-glucose. However, average content of main product in a post-synthesis mixture in industrial conditions reaches only w = 96 – 98 % level. Increase in chemical purity of LAA is acquired by means of sequencing batch crystallization/dissolution/recrystallization processes from its water solutions. An essential inconvenience of the purification process is a relatively high number of the labour–consuming batch operations, favoring undesirable decomposition of LAA, which – demonstrating strong reducing properties – is quickly oxidized in presence of atmospheric air. This side-process considerably lowers quality of crystal product removed from successive batch crystallization stages, being additionally catalyzed by higher temperature, lower pH and presence of active carbon, copper, iron or other heavy metals.

Introduction into the analyzed binary LAA–water system a third component – aliphatic alcohol (in practice one from C₁ – C₃) offers a new possibility of influencing the process yield and upgrading its quality by reduction of indispensable number of batch crystallization stages. Alcohol(s) (individual or their mixture) presence effects in versatile physicochemical interactions within the discussed system, modifying solubility, metastable zone width as well as nucleation and growth rates of LAA crystals.

Experimental data concerning linear growth rate of vitamin C (LAA) crystals in a four–compound: LAA–methanol–ethanol–water system are presented. The test measurements were carried out in a laboratory batch DT (draft tube) crystallizer of internal circulation of suspension with assumed cooling rate controlled precisely on-line by a PC computer system. Growth rates were estimated indirectly from the CSDs (crystal size distributions) of a batch process product, rearranged in a form of crystal population density functions.

The CSDs were determined with two methods: using laser particle size analyzer COULTER LS-230 and by means of conventional granulometric analy-
sis (PN-67/M-94001). A kinetic model of continuous MSMPR (mixed suspension mixed product removal) crystallizer, mathematically convenient for the kinetic data determination was formally adopted for the estimation of “effective growth kinetics” in the batch process conditions. Values of linear growth rate of LAA crystals evaluated with the use of the most simplified kinetic model (size-independent growth – SIG) are presented. For the purpose of kinetic data elaboration the solubility and spontaneous nucleation temperature data presented in the authors’ earlier work were used.

Kinetics of crystals growth in the batch crystallization processes

Batch crystallization processes are especially convenient for the performance of laboratory experiments. However, in the continuous processes fixed or slightly oscillated within the pseudo steady-state limits the values of process temperature, supersaturation, suspension density, specific surface area of dispersed crystal phase, etc. guarantee temporal stability of resulting kinetic feedbacks inside the system, thus stabilized and physically grounded values of nucleation and growth rates. In batch systems both intrinsic complexity and essential differences in intensity of various partial processes in time occur. Appearance of a new solid phase, enlargement of mass and surface area of isolated crystal population while batch process course as well as dramatic time-variability of supersaturation result in a significant changeability of kinetic parameter values during the process run. Thus possibly precise and detailed enough physical model of a batch mass crystallization process requires simultaneous integration of a relatively complex system of time-dependent population, mass and energy balances, generally in a form of differential equations, complemented by appropriate kinetic dependencies.

In the literature one can find original propositions of the batch kinetic data elaboration. One of the mathematically convenient and relatively simple manners may be an indirect method proposed by Nývlt. It is based on the evaluation of G value from population density function derived from cumulative mass (or volume) CSD obtained in a batch crystallizer with mixer and programmed cooling, thus in a complex unsteady-state polythermic and induced (seeded) process of combined nucleation and crystal growth.

Nývlt suggests that in some process conditions mass CSD produced in a batch crystallizer and presented in a “z–L” coordinate system (see eqs. (1) and (2)):

\[
W(z) = 1 - \left(1 + z + \frac{z^2}{2} + \frac{z^3}{6}\right) \exp(-z) \quad (1)
\]

\[
z = \frac{L}{G\tau} \quad (2)
\]

where: \(W\) – normalized cumulative mass distribution (undersize), \(z\) – dimensionless size, is apparently a straight line – thus qualitatively similar in this coordinate system to a theoretical mass CSD coming from a continuous MSMPR crystallizer. It gives grounds for a formal assumption that the final equations derived from population balance corresponding to a continuous process and applied – formally exclusively – for this regime description can be practically used as a convenient mathematical tool for the elaboration of batch data, yielding in effect the conventional “substitute, effective, average” kinetic parameter values. Moreover, intensity of mixing observed by the authors during own batch experiments justifies formal assumption of MSMPR model as far as concerns the ideal mixing inside the crystallizer vessel (mixed suspension), while analysis of a whole final crystal product (total working volume of laboratory crystallizer creates here an “product suspension sample”) is a formal and practical equivalent of its unclassified, representative removal (mixed product removal).

Eq. (3) representing the SIG (size–independent growth) kinetic model:

\[
n = n_0 \exp(-L/G\tau) \quad (3)
\]

where the individual population density values, \(n_i\) can be calculated from mass \(m(L)\) or volume, \(V(L)\) size distribution data according to the formula below, eq. (4):

\[
n_i = \frac{m_i}{k_i \rho L_i^3 \Delta L_i V_w} = \frac{V_i}{k_i L_i^3 \Delta L_i V_w} \quad (4)
\]

can be thus – according to the method’s assumptions – directly adopted for rough calculation of G value in a batch crystallization process by formal substitution of mean residence time, \(\tau\), with a batch crystallization time, \(t_{cr}\), eq. (5) (see Fig. 1):

\[
\tau = t_{cr} = \frac{T_{cr} - T_f}{R_T} + t_f \quad (5)
\]

This way \(t_{cr}\) value depends both on the physicochemical properties of initial solution introduced into batch crystallizer \((T_{cr} = T_s - \Delta T_{max}, \Delta w_{max} = (dw/dT)_{cr} \Delta T_{max})\) and on technological process conditions applied \((R_T, T_s, t_f)\). Taking advantage of a \(T_{cr} = T_s - \Delta T_{max} = f(w_{LAA}, w_{MeOH}, w_{EtOH})\)
dependency evaluated earlier, \[\tau = t_{cl} = f(w_{\text{LAA}}, w_{\text{MeOH}}, w_{\text{EtOH}}, T_f, R_T, t_f)\] – see eq. (5a):

\[\tau = t_{cl} = \frac{2.932 w_{\text{LAA}} + 0.671 w_{\text{MeOH}} + 0.883 w_{\text{EtOH}} + 174.78 - T_f}{R_T} + t_f\]  

It is, however, not a precise method since it is based on a formal assumption that granulometric composition of the final product, obtained in a very complex batch, non-isothermal and seeded process of nucleation coupled with crystals growth, during which both temperature and supersaturation are the subjects of dramatic changes, is theoretically identical with a product of continuous MSMPR crystallizer working in identical technological conditions assuming mean process temperature of the comparable batch run and excluding seeding. 36–38 The authors’ own research experience suggests, that with this method one can only roughly estimate the – not closely defined – “average”, “effective” crystal growth rate \(G\) (since in a batch mode it is strongly time-dependent value). The resulting kinetic data should be rather used to a general estimation of the production capacity and/or for some rough design calculations, applied e.g. as the nominal values.33 The essence of the assumed theoretical simplification is evaluation of a conventional, entirely computational value – a parameter named “average effective linear growth rate, \(G\)”. Thus, this entirely computational value may find potential application in practice for quick engineering calculations, eventually recognized as a direct and instant indicator for comparison purposes (reference index) while examining various technological options of a batch crystallization process. Moreover, this value adequately represents and reflects a resulting “net-effect” of the complex hydrodynamic and kinetic conditions (nonlinearly changeable during the process time-course).

**Experimental setup and procedure**

**Experimental setup**

The research stand scheme is presented in Fig. 2. Experiments were performed in a laboratory batch DT crystallizer of \(V_w = 0.6 \, \text{dm}^3\) working volume with internal circulation of solution/suspension. It was a hermetic, glass-made cylindrical tank \((V_t = 1 \, \text{dm}^3, D = 120 \, \text{mm}, H = 123 \, \text{mm})\) equipped with a cooling jacket (pipe-in-pipe type heat exchanger) embedded into a circulation profile element wall, coupled with thermostat, where an ice load was applied to cool the heat acceptor – circulating double distilled water – down to a \(T = 275 \, \text{K}\) value. Actual temperature in a crystallizer vessel was monitored by automatic system integrated with technological conditions). From theoretical point of view each batch process configuration, including: assumed initial composition of solution, starting temperature of the process, fixed mode (eventually value) of cooling rate, batch process time, mass and average size (or size distribution) of seeds, etc. can be attributed to a such “average effective linear growth rate, \(G\)”. Thus, this entirely computational value may find potential application in practice for quick engineering calculations, eventually recognized as a direct and instant indicator for comparison purposes (reference index) while examining various technological options of a batch crystallization process. Moreover, this value adequately represents and reflects a resulting “net-effect” of the complex hydrodynamic and kinetic conditions (nonlinearly changeable during the process time-course).

![Experimental setup diagram](image-url)
a PC computer, what enabled one to realize an assumed cooling mode, e.g. linear cooling profile with an accuracy of ± 0.7 K h⁻¹. In the tank axis, inside the circulation tube (DT) (d = 57 mm, h = 53 mm) a three-paddle propeller mixer (dₘ = 55 mm) was installed. Revolution number was assumed constant²¹ (N = 10 ± 0.2 s⁻¹) providing stable and intensive enough circulation of solution (after nucleation – suspension) inside working volume.

**Experimental procedure**

The solutions of various compositions, as bio-chemically active materials, were prepared just before their insertion into the crystallizer using LAA of > 99.7 % purity (GR for analysis and for biochemistry, MERCK, Germany), double distilled water, MeOH (p., POCH Glivice, Poland) and EtOH (p.a., 96 %, ZPS Polmos, Poland). The 0.7 kg of solution of known composition was heated till its actual temperature was ca. 5 K higher than the expected solubility temperature, Tₛ, roughly estimated on the basis of preliminary test results²¹,³⁹ – see Fig. 1. In this moment internal circulation of mixture started. After ca. 15 min of this isothermal circulation the cooling process began with a constant cooling rate, Rₚ, programmed by a computer driven automatic control system. When the solution reached a temperature 1 K lower than the expected solubility temperature, Tₛ, thus crossed lower boundary of metastable zone, dozens of well shaped LAA crystals³⁶,³⁸ were introduced into the mixed system (ca. 0.1 g of total mass, Lₕ = 900 μm). Continuous observation of the cooled system was realized till a moment when characteristic nucleation event was attained, manifested as a sudden turbidity within the bulk solution. Corresponded temperature of suspension can be interpreted as the spontaneous crystallization temperature, Tᵥ,²¹ – a value necessary in eq. (5). When the system attained its assumed final temperature, Tᵢ = 283 K, cooling process was terminated. The resulting, still circulated suspension, was stabilized isothermally in temperature Tᵢ through the defined time tᵢ = 900 s. Then the product crystals were dewatered by a centrifugal separation process, washed with cold ethanol and dried in a temperature T = 298 K without light (photo-decomposition prevention). CSD analysis of the crystal product was carried into effect using two different methods – Coulter counter test and a classical granulometric analysis procedure. The experiments covered influences of saturation temperature, Tᵥ, of initial solution (thus its assumed composition)²¹ and linear cooling rate, Rₛ, on “average, effective linear growth rate” G of LAA crystals.

**Influence of saturation temperature**

Influence of saturation temperature of the mixture, indirectly – solution’s composition – on the growth kinetics, G = f(Tₛ), was investigated assuming a selected, constant value of linear cooling rate, Rₚ = 8.33·10⁻³ K s⁻¹. Batch crystallizer was provided with solutions of the following compounds:
- wₖ LAA = 40, 45 and 50 %,
- wₖ MeOH + wₖ EtOH:
  12 ≤ wₖ MeOH + wₖ EtOH ≤ 40 % for wₖ LAA = 40 %,
  12 ≤ wₖ MeOH + wₖ EtOH ≤ 30 % for wₖ LAA = 45 %,
  12 ≤ wₖ MeOH + wₖ EtOH ≤ 20 % for wₖ LAA = 50 %,
- double distilled water – a complement to 100 %.

The solutions of wₖ LAA = 45 or 50 % were mixed with lower fractions of MeOH+EtOH mixture than it was in case of wₖ LAA = 40 %. Higher fractions of alcohols caused undesired boiling of the solution within temperature range lower than LAA solubility temperature, Tₛ. The 27 diversified compositions of four-compound LAA–MeOH–EtOH–H₂O solutions were tested. For each mixture composition G value was determined on the basis of corresponded CSD data (in n(L) form).

**Influence of linear cooling rate**

For three solutions, selected from 27 ones described above, of the following compositions (w(%)):
- LAA – 40, MeOH – 10, EtOH – 10, water – 40 %,
- LAA – 45, MeOH – 10, EtOH – 10, water – 45 %,
- LAA – 50, MeOH – 10, EtOH – 10, water – 30 %,
additional tests were performed providing the system with a linear cooling rate elevated to a value of Rₚ = 16.66·10⁻³ K s⁻¹ (see Fig. 1), resulting in a G = f(Tᵢ) relationship elaboration possible. These purposefully selected compositions can be of practical significance in pharmaceutical industry. Because presence of alcohol(s) in this system reduces the LAA solubility,²¹ simultaneously reducing the solution’s boiling temperature, in technological practice total fraction of MeOH+EtOH should not exceed w = 20 %.

For experimentally obtained original CSDs of product crystals from two independent sources: Coulter counter and granulometric analysis, population density courses n(L) were computed with the use of eq. (4), see Figs. 3 and 4, creating a base for the calculation of “average, effective” crystal linear growth rate G values with the use of eq. (3). Selected kinetic results are presented in Table 1.
Results and discussion

Our own batch experimental data elaboration was preceded by statistical verification of the fundamental Nývlt’s assumption (linearity in a \( z – L \) coordinate system, eqs. (1) and (2)). After application of the linear regression analysis it was concluded, that all \( R^2 \) values are within the 0.981 – 0.989 range, what practically confirms the usability of eqs. (1) and (2) for the investigated system (similar procedure is presented by Freitas and Giulietti\(^9\)), warranting application of SIG kinetic model of a continuous MSMPR crystallizer, eq. (3), for the own batch experimental data elaboration. The calculation results are presented in Table 1.

Fig. 3 – Course of population density function \( \ln n(L) \) of \( \text{L(+)-ascorbic acid (LAA)} \) crystals produced in a batch cooling crystallizer. Initial mass fractions in solution: LAA – 50, MeOH – 10, EtOH – 10, \( \text{H}_2\text{O} \) – 30 %. Comparison of population density values (n) calculated from mass CSD derived from particle laser analysis (Coulter counter) and from granulometric analysis.

Fig. 4 – Course of population density function \( \ln n(L) \) of \( \text{L(+)-ascorbic acid (LAA)} \) crystals produced in a batch cooling crystallizer. Initial mass fractions in solution: LAA – 40, MeOH – 10, EtOH – 10, \( \text{H}_2\text{O} \) – 40 %. Comparison of population density values (n) calculated from mass CSD derived from particle laser analysis (Coulter counter) and from granulometric analysis.

It can be concluded, that with the increase in LAA mass fraction \( (W_{\text{LAA}}) \) in a LAA–MeOH–EtOH–\( \text{H}_2\text{O} \) system the metastable zone width decreases.\(^2\(^2\)) This phenomenon is not advantageous in continuous mass crystallization processes, restricting the range of sustainable distribution of supersaturation between two principal, competing processes of nucleation and growth, thus guaranteeing kinetic conditions for the stable enlargement of crystal sizes. Contrary, in a batch mode lowering of these boundaries influences the overall process course advantageously since it provides the desirable nucleation event within a shorter time, thus gives a convenient possibility of effective discharging of the generated supersaturation as it comes. This way reduction of

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**Table 1 – Influence of selected process parameters on the mass crystallization of LAA in a DT batch cooling crystallizer**

<table>
<thead>
<tr>
<th>No.</th>
<th>( W_{\text{LAA}} ) %</th>
<th>( R_T \times 10^3 ) K s(^{-1} )</th>
<th>Metastable zone width(^{22} )</th>
<th>( \Delta T_{\text{max}} ) K</th>
<th>( \Delta n_{\text{max}} ) %</th>
<th>( t_{\text{cr}} ) h</th>
<th>Statistical parameters of CSD</th>
<th>Growth rate G ( \cdot 10^8 ) m s(^{-1} )</th>
<th>Growth rate G ( \cdot 10^8 ) m s(^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
<td>8.33</td>
<td>335</td>
<td>37</td>
<td>15.3</td>
<td>1.02</td>
<td>0.214</td>
<td>62.2</td>
<td>0.282</td>
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<td>2</td>
<td>40</td>
<td>16.66</td>
<td>335</td>
<td>37</td>
<td>19.5</td>
<td>0.50</td>
<td>0.137</td>
<td>80.5</td>
<td>0.268</td>
</tr>
<tr>
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<td>45</td>
<td>8.33</td>
<td>343</td>
<td>22</td>
<td>11.6</td>
<td>1.52</td>
<td>0.339</td>
<td>46.7</td>
<td>0.384</td>
</tr>
<tr>
<td>4</td>
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<td>28</td>
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<td>0.301</td>
<td>63.4</td>
<td>0.332</td>
</tr>
<tr>
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<td>8.33</td>
<td>353</td>
<td>18</td>
<td>9.5</td>
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<td>0.448</td>
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<td>0.471</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>16.66</td>
<td>353</td>
<td>24</td>
<td>12.6</td>
<td>1.02</td>
<td>0.406</td>
<td>51.7</td>
<td>0.447</td>
</tr>
</tbody>
</table>

Mass fractions of alcohols in a raw material: MeOH – 10, EtOH – 10 %; distilled water – a complement to 100 %

a) Particle size analyzer COULTER LS-230

b) Granulometric analysis (PN–67/M-94001)
probability of uncontrolled homogeneous nucleation is possible. In practice it leads to more stable conditions of crystals growth, effecting in production of larger particles of higher quality.

Linear cooling rate, \( R_p \), influences the process course considerably, as well. For \( R_p = 8.33 \cdot 10^{-3} \text{ K s}^{-1} \) increases in \( w_{\text{LAA}} \) from \( w = 40 \) to \( 45 \) and from \( w = 45 \) to \( 50 \) \% correspond to reductions in \( \Delta T_{\text{max}} \) equal 7 and 4 K, respectively. For \( R_p = 16.66 \cdot 10^{-3} \text{ K s}^{-1} \) the same increases in \( w_{\text{LAA}} \) give slightly higher reductions in \( \Delta T_{\text{max}} \), i.e., 9 and 4 K.

Increase in \( w_{\text{LAA}} \) from 40 to 50 \% corresponds thus to \( \Delta w_{\text{max}} \) reduction from 15.3 to 9.5 \% for \( R_p = 8.33 \cdot 10^{-3} \text{ K s}^{-1} \) while for \( R_p = 16.66 \cdot 10^{-3} \text{ K s}^{-1} \) it corresponds to a decrease in \( \Delta w_{\text{max}} \), from 19.5 to 12.6 \%. It should be generally noted, that higher values of cooling rate correspond to higher \( \Delta T_{\text{max}} \) and \( \Delta w_{\text{max}} \) values.

It should be mentioned here, that in the system under study, considering the possibility of relatively high supercooling (supersaturation) existence, seed procedure is commonly applied to effectively reduce the metastable zone width in industrial, thus in a design-oriented laboratory-scale conditions. This way secondary nucleation mechanism, induced by the seeding solute crystals, enables one to induce the spontaneous formation of nuclei at the supersaturation level significantly lower than that corresponded to a primary homogeneous or – more likely – heterogeneous nucleation. Catalytic effect of seeds can be theoretically explained by the resulting effect of the following mechanisms: initial breeding, needle breeding, collision breeding, attrition, contact nucleation and fluid shear.

With \( w_{\text{LAA}} \) value increase an enlargement in a necessary batch crystallization time is also observed. Increase in \( w_{\text{LAA}} \) from \( w = 40 \) to 50 \% corresponds to increase in crystallization time \( t_{\text{cr}} \) from 1.02 to 1.98 h \(( R_p = 8.33 \cdot 10^{-3} \text{ K s}^{-1}) \). For higher \( R_p = 16.66 \cdot 10^{-3} \text{ K s}^{-1} \) it corresponds to \( t_{\text{cr}} \) enlargement from 0.50 to 1.02 h. Linear correlations between \( w_{\text{LAA}} \) and \( t_{\text{cr}} \) are clearly observable. On the other hand a twofold increase in linear cooling rate produces, approximately, also twofold decrease in batch crystallization time for any \( w_{\text{LAA}} \) value tested. For \( w_{\text{LAA}} = 40 \% \) increase in \( R_p \) from \( 8.33 \cdot 10^{-3} \) to \( 16.66 \cdot 10^{-3} \text{ K s}^{-1} \) corresponds to a decrease in \( t_{\text{cr}} \) from 1.02 to 0.50 h, for \( w_{\text{LAA}} = 45 \% \): \( t_{\text{cr}} = 1.52 \rightarrow 0.78 \text{ h} \) and for \( w_{\text{LAA}} = 50 \% \): \( t_{\text{cr}} = 1.98 \rightarrow 1.02 \text{ h} \).

Analysis of the changes in coefficient of variation, \( CV \) (where parameter \( CV \) is defined as standard deviation/mean size) suggests that doubling the linear cooling rate value while assuming other process parameter values constant produces in effect increase in \( CV \) value, in particular: \( CV = 62.2 \rightarrow 80.5 \% \) (by ca. 30 \%) for \( w_{\text{LAA}} = 40 \% \), \( CV = 46.7 \rightarrow 63.4 \% \) (by ca. 36 \%) for \( w_{\text{LAA}} = 45 \% \) and \( CV = 31.0 \rightarrow 51.7 \% \) (by ca. 67 \%) for \( w_{\text{LAA}} = 50 \% \). At the same time a clear decrease in mean LAA crystal size is observed, namely: \( L_\text{m} = 0.214 \rightarrow 0.137 \text{ mm} \) (by ca. 36 \%) for \( w_{\text{LAA}} = 40 \% \), \( L_\text{m} = 0.339 \rightarrow 0.301 \text{ mm} \) (by ca. 11 \%) for \( w_{\text{LAA}} = 45 \% \) and \( L_\text{m} = 0.448 \rightarrow 0.406 \text{ mm} \) (by ca. 9 \%) for \( w_{\text{LAA}} = 50 \% \). The observed effects can be explained theoretically by relatively milder, more convenient conditions of mass transfer while generation and instant discharge of supersaturation in case of lower \( R_p \) value.

Combination in trends of change of these both process indicators suggests also, that two essential, superimposing phenomena can be responsible for the final results observed. Increase in homogeneous (locally) and heterogeneous nucleation rates results from a more intensive generation of supersaturation (higher cooling rate), producing also higher values of “effective” crystal growth rate: \( G = 1.74 \cdot 10^{-8} \rightarrow 3.94 \cdot 10^{-8} \text{ m s}^{-1} \) for \( w_{\text{LAA}} = 40 \% \), \( G = 1.41 \cdot 10^{-8} \rightarrow 2.98 \cdot 10^{-8} \text{ m s}^{-1} \) for \( w_{\text{LAA}} = 45 \% \) and \( G = 1.43 \cdot 10^{-8} \rightarrow 2.86 \cdot 10^{-8} \text{ m s}^{-1} \) in case of \( w_{\text{LAA}} = 50 \% \). However, higher kinetic sensitivity of nucleation is expected. Contribution of theoretically possible interactions between imperfect crystal structure resulting from too high growth rates and intensification of secondary nucleation, e.g. needle breeding or polycrystalline breeding mechanisms can not be excluded (see electron microscope images in the work of Wierzbowska et al.). Statistical characteristics of crystal suspension produced in these conditions (composed mainly of crystal fines) indicates both increase in crystal size dispersion (higher CV values) and effective decrease in mean crystal size, \( L_\text{m} \). For a lower cooling rate applied mean crystal size is definitely higher, in spite of nearly two-times longer batch time what undoubtedly influences combined attrition, abrasion and breakage actions. It can be expected, that milder process conditions are of decisive influence, providing more stable and more convenient distribution of supersaturation.

An interesting phenomenon can be observed – for \( w_{\text{LAA}} \leq 50 \% \) the values of crystal linear growth rate are nearly identical for the same values of linear cooling rate, \( R_p \): \( G = 1.41 \cdot 10^{-8} \) and \( 1.43 \cdot 10^{-8} \text{ m s}^{-1} \) \(( R_p = 8.33 \cdot 10^{-3} \text{ K s}^{-1}) \) and \( G = 2.98 \cdot 10^{-8} \) and \( 2.86 \cdot 10^{-8} \text{ m s}^{-1} \) \(( R_p = 16.66 \cdot 10^{-3} \text{ K s}^{-1}) \). Considerably higher values were, however, observed for \( w_{\text{LAA}} = 40 \% \): \( G = 1.74 \cdot 10^{-8} \text{ m s}^{-1} \) \(( R_p = 8.33 \cdot 10^{-3} \text{ K s}^{-1}) \) and \( G = 3.94 \cdot 10^{-8} \text{ m s}^{-1} \) \(( R_p = 16.66 \cdot 10^{-3} \text{ K s}^{-1}) \). For the kinetic data elaborated on the basis of sieve analysis results the observed trends and qualitative conclusions are identical.

The values of crystal linear growth rate observed in a batch process under study are of the
same order as the values presented by Freitas and Giulietti: \( \Gamma = 1.26 \times 10^{-8} - 8.38 \times 10^{-8} \text{ m s}^{-1} (R_T = 2.78 \times 10^{-5} - 13.89 \times 10^{-5} \text{ K s}^{-1}), L_m = 0.123 \times 10^{-3} - 0.297 \times 10^{-3} \text{ m}, \) batch time range \( t_{cr} = 0.45 - 5.92 \text{ h}, \) maximal supercooling range \( \Delta T_{\text{max}} = 6.9 - 26.8 \text{ K} \) and saturation temperature range \( T_s = 321.3 - 334.9 \text{ K}. \) Lower values of \( \Delta T_{\text{max}} \) should be related to lower (even by two orders of magnitude) values of linear cooling rate, \( R_T. \) Also Omar in a seeded (6.25 g mass of seed crystals of \( L_s = 250 \mu \text{m} \) size) batch crystallization experiments reported similar results: in a pure water solution of LAA \( G = 2.4 \times 10^{-8} - 5.4 \times 10^{-8} \text{ m s}^{-1}, \) in a solution composed of \( w = 80 \% \) water / \( w = 20 \% \) ethanol \( G = 1.20 \times 10^{-8} - 3.37 \times 10^{-7} \text{ m s}^{-1}, \) and in a solution composed of \( w = 80 \% \) water / \( w = 20 \% \) propanol \( G = 8.17 \times 10^{-9} - 4.68 \times 10^{-7} \text{ m s}^{-1}. \)

Considering simultaneous, complex action of two alcohols, crystal linear growth rate \( G \) was correlated with either solubility temperature of a batch mixture, \( T_s \) (Fig. 5) or a batch crystallization time, \( t_{cr} \) (Fig. 6). Various proportions in these both alcohols can produce identical effects, externally conveniently represented indirectly by \( T_s \) or \( t_{cr} \) parameters, both being complex functions of \( w_{LAA}, w_{MeOH}, w_{EtOH} \) – contrary to a case of only one alcohol’s influence, where its concentration is an unequivocal factor influencing directly kinetic behavior of these systems.

It is observable, that CSD derived from sieve analysis data provides higher values of mean crystal size, \( L_m \) (see Table 1). Linear crystal growth rate, \( G, \) values calculated on the basis of this CSD have thus overstated values, what should be related to the course of linear segment of \( \ln n(L) \) dependency (see Figs. 3 and 4). These overstated values of population density, \( n, \) compared to the \( n \) recalculated from Coulter counter data result from specificity of granulometric analysis procedure, where theoretically exists a possibility of retention of particles of a size lower than mesh dimension, \( e.g. \) in case of clogging-up of the sieve meshes. Moreover, an essential problem seems to be the influence of non-isomerism of particles and diversity in their cross-dimension values. A possibility of undergoing through the meshes depends on its momentary, accidental configuration in space. This effect is distinctly observed for the elongated particles, like needle-shape LAA crystals \( 21 \) – see scanning electron microscope images in Fig. 7.

In case of laser particle analysis (Coulter counter) accidental exposition of non-isomeric particle with respect to a detector’s light ray is of primary importance. Other possible problems arise from taking into consideration individual volumes of particles. Granulometric analysis provides de facto determination of mass (volume) distribution of a crystal population, while laser analysis provides its specific surface area distribution. Problems resulting from distinctness between analytical procedures applied are further described in a work of Mora et al., \( 41 \) where some relations for a direct comparison of CSD data derived from both discussed sources are also presented and explained in detail.
The resulting kinetic equations in a $G = f(t_{cr})$ form (see Fig. 6), depending on analytical method used for CSD determination, can be presented as empirical correlations, eqs. (6) and (7):

$$G = 1.80 \cdot 10^{-5} t_{cr}^{-0.815} \quad (R^2 = 0.913) \quad (6)$$

or:

$$G = 1.87 \cdot 10^{-5} t_{cr}^{-0.798} \quad (R^2 = 0.988) \quad (7)$$

where: $t_{cr}$ – batch crystallization time (s), defined in general by eq. (5) or, for the particular system under study, by eq. (5a). An $G = f(t_{cr})$ equation is the most general form since $t_{cr}$ is an indirect function of, among others, $T_{cr}$, where $T_{cr} = T_s - \Delta T_{max}$.

Eq. (6) is based on the $G$ values calculated from CSDs (in a form of population density functions) derived from laser particle size analyzer COULTER LS–230, whereas eq. (7) – on the ones from granulometric analysis data. After critical comparison of these two equations it results, that the influence of batch time on the $G$ values is practically identical (see the exponent values in eqs. (6) and (7) as ca. −0.8). Since from granulometric analysis data one obtains overestimated values of the CSD parameters (compare $L_m$ values in Table 1), laser analysis results, eq. (6), are recommended by the authors as more reliable. For the case under study eq. (6) can be further rearranged to the following, detailed form of eq. (6a):

$$G = f(w_{LAA}, w_{MeOH}, w_{EtOH}, T_f, R_l, t_f) = 1.180 \cdot 10^{-5} \cdot \left[ \frac{2.932 w_{LAA} + 0.671 w_{MeOH} + 0.883 w_{EtOH} + 174.78 - T_f}{R_l} + t_f \right]^{0.815}. \quad (6a)$$

From the presented equation it becomes possible to determine an “average, effective” value of crystal linear growth rate, $G$, in a batch process ac-
cording to the Nývlt’s concept. If the profiles of supersaturation discharge during process time are identical in character (or similar, e.g. as parallel curves) in a set of batch experiments (however, for the diversified sets of other process parameter values) it may be – theoretically – possible to use this relative, conventional growth rate as an entirely nominal parameter, convenient for, at least rough, comparative analysis purposes.34

Conclusions

Selected kinetic aspects of crystal growth in a batch, polythermic, isohydrical and seeded process of vitamin C (LAA) mass crystallization from water-alcohols mixtures were raised and discussed. Kinetic model describing mass crystallization process in a continuous, physically idealized MSMPR apparatus, mathematically convenient for the kinetic data determination was formally adopted, after positive verification of the Nývlt’s data determination was formally adopted, after positive verification of the Nývlt’s assumptions and restrictions, to the batch process under study. Size-independent growth (SIG) kinetics was assumed.

It can be concluded that the largest ($L_m$) and more uniform (CV) particles of purified, crystalline vitamin C correspond to higher initial fraction of LAA in a solution ($w_{LAA}$) and lower cooling rate ($R_f$) applied, what is connected with the elongation of batch crystallization time ($t_c$). An observed difference between the extreme $L_m$ values is $\Delta L_m = 0.448 – 0.137 = 0.311$ mm (Coulter counter data) or $\Delta L_m = 0.471 – 0.268 = 0.203$ mm (granulometric analysis data) while the difference between CV values is $\Delta CV = 80.5 – 31.0 = 49.5$.

Because of the considerable, fundamental differences in the batch and continuous process regimes the resulting kinetic data should be – according to the Nývlt’s concept – of formal meaning only, e.g. as a convenient calculation parameter, however providing one with the quantitative characterization of a complex batch, seeded and polythermic process of simultaneous nucleation and crystals growth precisely enough for the engineering design in a form of “average, effective kinetic parameter” values. Moreover, application of this formal approach enables one to make a more objective comparison plane between the kinetic datasets derived from both continuous and batch mass crystallization processes, e.g. in a technology of vitamin C purification.

Comparing own experimental results with Freitas and Giulietti as well as Omar data it can be concluded, that these are of the same magnitude. Observed diversification results probably from different experimental conditions and/or laboratory measurement accuracy. It is worth to mention, that in the process conditions providing relatively low crystal growth rates batch regime secures a more efficient control resulting in larger crystals formation, additionally of lower size dispersion (CV) than it can be obtained in a continuous MSMPR crystallizer.38,42 Thus, the experimental data presented, together with their specific and restricted kinetic interpretation according to the Nývlt’s concept as an “average, effective” $G$ value estimation and correlated in a form of empirical kinetic equation $G(t, p, other technological parameters...)$ can find practical application in the design and/or optimization of modern technologies of LAA purification on the way of multistage batch mass crystallization processes.

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Images of LAA crystals (scanning electron microscope JEOL–5800 LV) were made in Head of Materials Science Laboratory of the Institute of Materials Science and Applied Mechanics, Wroclaw University of Technology, Wroclaw, Poland.

List of symbols

- $d$ – draft tube diameter, m
- $d_{in}$ – agitator diameter, m
- $D$ – crystallizer diameter, m
- $G$ – average, effective linear growth rate of crystals, m s$^{-1}$
- $h$ – draft tube height, m
- $h_p$ – vertical distance between propeller agitator level and crystallizer bottom, m
- $H$ – crystallizer height, m
- $k_c$ – surface shape factor of crystal
- $k_v$ – volumetric shape factor of crystal
- $L$ – crystal characteristic size, m
- $L_{i}$ – mean size of $i$-th crystal fraction, m
- $L_m$ – mean size of crystal population, m
- $L_s$ – seed size, m
- $\Delta L_i$ – size range width of $i$-th crystal fraction, m
- $m_i$ – mass of $i$-th crystal fraction, kg
- $n$ – population density (number of crystals within the defined size range divided by this size range width per unit volume of suspension), m$^{-3}$
$n_i$ – population density of $i$-th crystal fraction, m$^{-3}$ m$^{-3}$
$n_0$ – population density of nuclei (zero-size crystals), m$^{-3}$ m$^{-3}$
$N$ – agitator’s revolution number, s$^{-1}$
$q_v$ – volumetric flow rate of crystal suspension, m$^3$ s$^{-1}$
$R^2$ – correlation coefficient
$R_t$ – linear cooling rate, K s$^{-1}$
$t_{cr}$ – batch crystallization time, s
$t_f$ – time of stabilization of the resulting postprocessing suspension in a final batch process temperature, s
$T$ – temperature, K
$T_{cr}$ – temperature of spontaneous LAA nucleation, K
$T_f$ – final temperature of batch crystallization process, K
$T_s$ – LAA solubility temperature, K
$\Delta T_{max}$ – critical, maximal value of supercooling in the solution (maximum allowable supercooling), defined as $T_s - T_{cr}$, K
$V_i$ – volume of the $i$-th crystal fraction, m$^3$
$V_t$ – total volume of crystallizer, m$^3$
$V_w$ – working volume of crystallizer, m$^3$
$W$ – normalized cumulative mass distribution (undersize)
w – mass fraction, %
$w_{EtOH}$ – initial mass fraction of ethanol in solution, %
$w_{LAA}$ – initial mass fraction of L(+)-ascorbic acid (LAA) in solution, %
$w_{MeOH}$ – initial mass fraction of methanol in solution, %
$\Delta w$ – supersaturation of LAA in solution, %
$\Delta w_{max}$ – critical, maximal supersaturation of LAA in solution, %
z – dimensionless size

Greek letters

$\rho$ – crystal density, kg m$^{-3}$
$\tau$ – mean residence time of crystal suspension, defined as $V_w/q_v$, s

Abbreviations

CSD – crystal size distribution
CV – coefficient of variation (of crystal sizes), %
DT – draft tube (crystallizer type)
EtOH – ethanol
LAA – L(+)-ascorbic acid (vitamin C)
MeOH – methanol
MSMPR – mixed suspension mixed product removal (crystallizer type)
SIG – size-independent growth (kinetics)

References
