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Endogenous Structural Change

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Abstract: A model of cumulative causation is extended to incorporate endogenous structural change. This is achieved via the use of a non-linear feedback of the growth rate on to the income elasticity of demand for exports. The result is a model that, under certain conditions, exhibits chaotic switches between growth eras. Such dynamics are in keeping with the view of the growth process that emerges from the Marx-Myrdal-Kaldor tradition.

JEL Classification: E12, F43, O41

Key words: Growth, Cumulative causation, Non-linear dynamics, Chaos theory

Introduction

The idea that structural change may be endogenous to the growth process is not a new one. Rather, there exists a well-known Marx-Myrdal-Kaldor tradition, notably followed by the likes of John Cornwall, which perceives of an economy's growth performance and its structure, as reflected in its institutions, as being inextricably interrelated through processes of dynamic feedback (see Setterfield, 1999, and Skott and Auerbach, 1995).[1] The resultant vision of the growth process is one in which growth proceeds in a series of eras, the institutions which determine performance during any one era being endogenous to the economy's performance during the previous era and therefore to its institutional history. However, despite its tradition, for a long time progress has been slow in producing a more formal framework that adequately captures this vision. For the most part, economists who doubtless share

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this vision have been satisfied to rely upon what we may christen 'medium-run' analysis. That is to say, the construction of models that, formally speaking, treat structural change 'as if' it were exogenous, but whose authors nevertheless realise that the growth process they model may, in the longer term, set-up irresistible forces for such change. Thus, for example, the cumulative causation model that John McCombie and Tony Thirlwall (1994) outline in their book *Economic Growth and the Balance-of-Payments Constraint*[2] treats certain key parameters as exogenous at a time when it is obvious from the rest of their book that their actual view of such parameters is that they are institutionally determined and 'deeply endogenous'[3] to the growth process. However, in the last few years, progress towards a more formal framework has started to take place in earnest. In particular, we may cite the work of Mark Setterfield (1997) who takes the cumulative causation model outlined by McCombie and Thirlwall and explicitly specifies the key 'exogenous' parameters as deeply endogenous functions of previous growth rates. Having done this, Setterfield asserts that the resultant growth dynamics of this modified model will be in accord with the Marx-Myrdal-Kaldor vision. The aim of this paper is to build on Setterfield's work by modifying the cumulative causation model in McCombie and Thirlwall along similar lines. However, rather than asserting the presence of the desired dynamics, this paper explicitly demonstrates, under certain conditions, the existence of such dynamics. As will come to be seen, it is the use of chaos theory that provides the step forward from Setterfield's paper.

'Exogenous' Structural Change in a Model of Cumulative Causation

The model outlined in McCombie and Thirlwall (1994) is now widely considered to be the 'standard' cumulative causation model (McCombie, 1999; p. 8)[4]. Moreover, it provides for a particularly apt framework for the endogenisation of structural change in pursuit of the Marx-Myrdal-Kaldor vision. This is because it is with Gunnar Myrdal that the phrase 'cumulative causation' originates (Myrdal, 1957) and because it is Nicholas Kaldor who stands as one of the most influential figures in the development of this concept[5]. Indeed, the model was originally developed as an explicit attempt to formalise the mechanism of cumulative causation that Kaldor outlined in his 1970 paper *The Case for Regional Policies*. As is well-known, the model considers a small economy in which growth is export-led[6]. It consists of the four basic relationships given below:[7]

$$y_t = \gamma x_t \tag{1}$$

$$x_t = \eta(p_c - p_{h,t}) + \varepsilon y_c \tag{2}$$

$$p_{h,t} = w - r_t + \tau \tag{3}$$

$$r_t = r_e + \lambda y_t \tag{4}$$

Equation [1] states that the rate of real output growth in the home economy (y) is a positive linear function of its rate of growth of real exports (x). Meanwhile, equation [2] relates real export growth to the relative price competitiveness of the home economy's exports ($p_c - p_{h,t}$) and to a measure of the rate of growth of real incomes in its main export markets (y_c). Equation [3] then postulates that the rate of price inflation of home exports ($p_{h,t}$) is determined via the practice of applying a mark-up on unit labour costs[8], thus implying that $p_{h,t}$ is increasing with both the rate of nominal wage inflation (w) and the rate of mark-up growth (τ), but decreasing with the rate of labour productivity growth (r_t). Finally, equation [4] is Verdoorn's law, which specifies r_t as a separably additive, positive linear function of exogenous labour productivity growth (r_e) and of real output growth. This is justified on the grounds of economies of scale that, following Allyn Young (1928) and Kaldor (1966), are viewed as being primarily 'dynamic' in nature[9]. It is, of course, the feedback that this equation provides for that makes growth in the model cumulative. As for the parameters, γ denotes the elasticity of y with respect to x, η the (absolute) price elasticity of demand for both exports and imports, ε the income elasticity of demand for exports and λ the so-called Verdoorn coefficient.

Of the parameters cited above it is λ and ε that are usually argued to be the main ones through which 'exogenous' structural change enters the model. This is because an economy's ability to realise 'dynamic' economies of scale as reflected by and its non-price competitiveness on world export markets as captured by λ are both likely to be influenced by its institutional set-up. To see how 'exogenous' structural change can be expected to impact upon an economy's growth performance we can therefore examine how changes in λ and ε can be expected to impact upon the equilibrium of the cumulative causation model. We can do this by noting that the model possesses, in Boyer's and Petit's (1991) terminology, both a demand regime and a productivity regime. The demand regime tells us how changes in labour productivity growth stimulate changes in the growth of real demand and thus in real output growth. We can arrive at an equation that describes this regime by first substituting equation [3] into equation [2], and then substituting the resulting expression into equation [1]. Following this procedure gives:

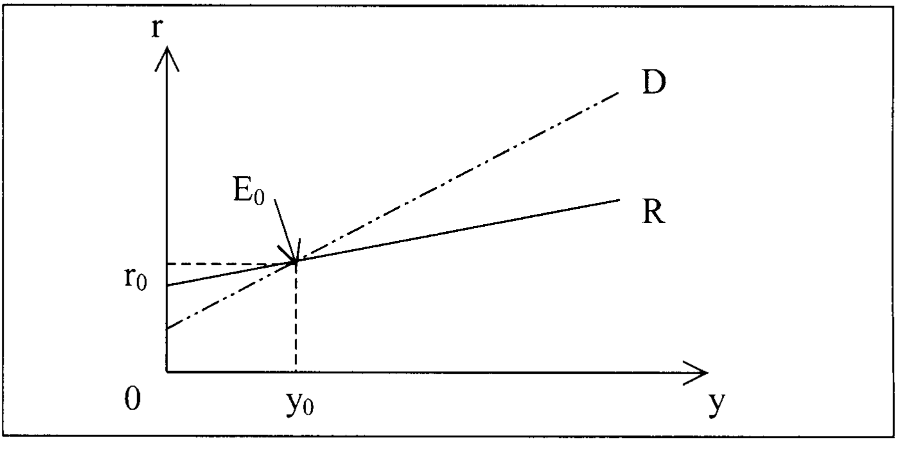
$$y_t = \gamma\left[\eta(p_c - w - \tau) + \varepsilon y_c\right] + \gamma\eta r_t \tag{5}$$

Meanwhile, the productivity regime informs us of how changes in the growth of real demand, and thus in real output growth, impact upon the growth of labour productivity. The equation describing this regime is simply Verdoorn's law, which we reproduce below:

$$r_t = r_e + \lambda y_t \tag{6}$$

Given these equations for the demand and productivity regimes we may depict the solution for the model's equilibrium geometrically, as in *figure 1* below. In this figure the demand regime is portrayed by the curve labelled D and the productivity regime by the curve labelled P. The curve for the demand regime is drawn as steeper than that for the productivity regime to ensure the stability of the resultant equilibrium, depicted by the point E_0[10]. Associated with this equilibrium is the rate of real output growth, y_0, and the rate of labour productivity growth, r_0[11].

Figure 1: Equilibrium in The 'Standard' Cumulative Causation Model



Now, assuming that we are originally in equilibrium, from equation [4] we would expect an 'exogenous' structural change that enters through the Verdoorn coefficient to shift the productivity regime curve. This is shown in *figure 2a* for the case of a positive structural change. The result is to shift the equilibrium point to E_1, thereby causing equilibrium real output growth and equilibrium labour productivity growth to increase to y_1 and r_1 respectively[12]. In contrast, in the case where an 'exogenous' structural change enters through the income elasticity of demand for exports it will, from equation [5], shift the demand regime curve. *Figure 2b* illustrates such a shift, the assumption again being that the 'exogenous' structural change is a positive one. As with the previous case, we see that the result is to increase both the equilibrium rates of real output and labour productivity growth.

Figure 2a: 'Exogenous' Structural Change That Impacts on the Verdoorn Co-Efficient

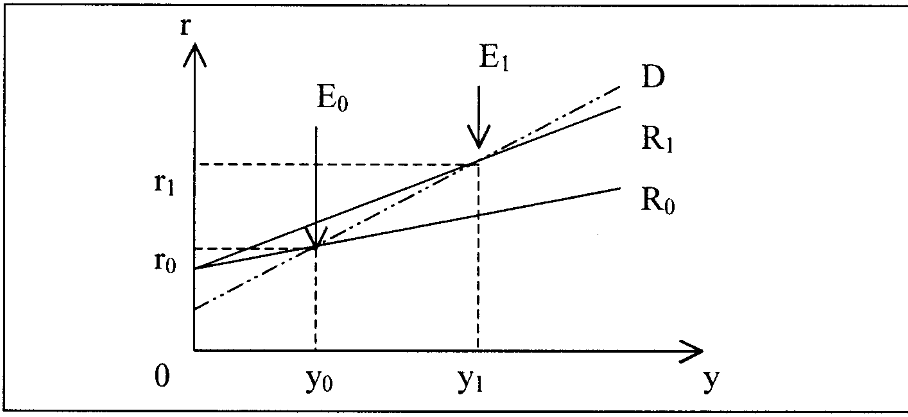
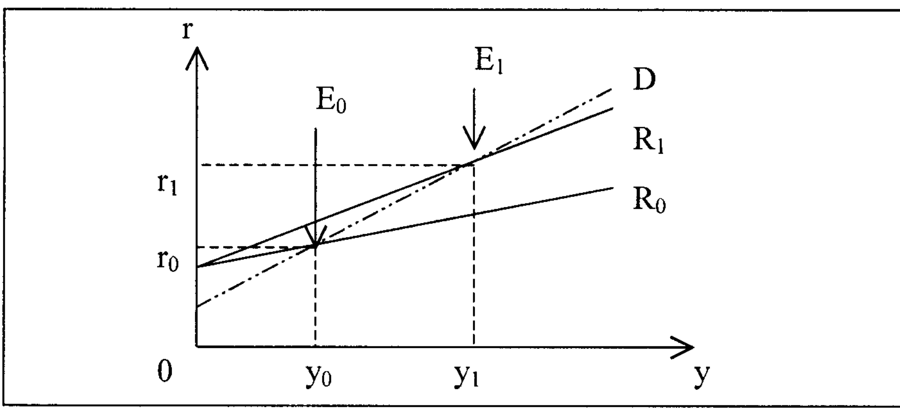


Figure 2b: 'Exogenous' Structural Change that Impacts on the Income Elasticity of Demand for Exports



Extension of the Model to Incorporate Endogenous Structural Change

Towards Endogenous Structural Change

Above we have noted that the Verdoorn coefficient and the income elasticity of demand for exports provide the main channels through which 'exogenous' structural change is able to enter the 'standard' cumulative causation model. The general principles that we shall use to explicitly endogenise these parameters are as follows:

- if an economy's growth performance is poor then pressure will inevitably arise that, through the reform of institutions, eventually translates into successful positive structural change.
- equally inevitably, if growth performance is good then, sooner or later, forces will arise that, in a dynamic context, make for the equivalent of negative structural change.

We can capture these general principles via specifying the following general functional forms for λ and ε:

$$\lambda_t = \lambda(y_{t-1}) \tag{7}$$
$$\varepsilon_t = \varepsilon(y_{t-1}) \tag{8}$$

where $\lim\limits_{y_{t-1} \to 0} (\lambda_t) \geq 0; \lim\limits_{y_{t-1} \to 0} (\varepsilon_t) \geq 0$

$\lim\limits_{y_{t-1} \to 0} (\lambda') \geq 0; \lim\limits_{y_{t-1} \to 0} (\varepsilon') \geq 0$

$\lambda'' < 0; \varepsilon'' < 0$

a prime denotes a partial derivative. In verbal terms these functional forms state that both λ_t and ε_t initially increase, albeit at a diminishing rate, with y_{t-1}. However, eventually, there comes a point where both λ_t and ε_t are at first invariant to y_{t-1} and then fall with y_{t-1}. The time subscripts here refer not to calendar years, but to growth eras. Thus, the growth performance achieved during one era is thought of as leading to endogenous structural change that results in a new era whose institutions, relative to the historical context, represent, depending upon the path of the economy, an improvement or deterioration on the institutions that characterised the previous era. Note that there is nothing to say that all eras will be the same length in calendar time: we might expect some to be relatively short and some to be relatively long, although *a priori* it is impossible to predict which will be which[13, 14].

Some circumstantial evidence that forces such as these are in operation on both λ and ε, and hence on the productivity and demand regimes in the 'standard' cumulative causation model, does exist. Thus, for example, it is well-known that the evidence suggests that, so far as the advanced industrial countries are concerned, the Verdoorn coefficient suffered a downward structural break in the early 1970s following the 'Golden Age of Capitalism', an era that had brought historically unprecedented growth rates to the countries fortunate enough to share in it. Following this downward structural break the advanced countries witnessed a new era of stagflation and poor relative growth performance. However, the poor performance of the 1970s itself set into motion political forces that eventually led to the election of Ronald Reegan in the US and Margaret Thatcher in the UK, both on the promise of fundamental structural reforms to the supply-side of the economy. In the case of the UK, evidence exists that there followed an upward structural break in the income elasticity of demand for its exports (Landesmann and Snell, 1989). More generally, many believe that the institutional reforms implemented by the Reegan and Thatcher administrations, not to mention the succeeding Bush (senior) administration in the US, have contributed to the revival of growth fortunes that, at least until recently, the US and UK economies have been enjoying since the early 1990s and mid-1990s respectively. The argument here being that these reforms have enabled American and UK firms to exploit to the full the dynamic economies of scale associated with new information and communications technologies (ICTs). Although, of course, even if we accept that, not withstanding cyclical downturns, the American and UK economies have entered a new (relatively) high growth era this is not to say that the positive structural break could not have been achieved through alternative institutional changes that would have proved less costly to society as a whole.

The idea that an era of slow growth will give rise to pressure that results in a successful reform of institutions and therefore a positive structural change which can be modelled as an endogenous change in λ and/or ε, seems sufficiently straight forward as not to require further elaboration. Rather the circumstantial evidence presented above seems sufficient to make the point. However, the same cannot be said for the accompanying principle that an era of high growth will, at some point in time, inevitably set in motion forces that will bring about its own demise and that these forces can be captured via endogenous shifts in λ and/or ε. Further elaboration on this point is therefore required. In particular, in a generalisation of Setterfield (1997), we may think of a particular high growth era as being associated with a particular production process, be that a mass-production process, a flexible specialisation process or whatever. The various component parts of this production process will tend to be interrelated *á la* Frankel (1955). Moreover, not only will they tend to be interrelated with each other but also with the various labour and non-labour market institutions that surround them. This, of course, means that the structure of the firm, the institution that embodies the production process, is also interrelated with these other various institutions. Now, as time progresses, we can expect new technologies to make an appearance on the world stage and new consumer tastes to develop in the home economy's export markets. Part of this development of new tastes will be attributable to the new technologies and part to interdependent preferences *á la* James Duesenberry (1949) as consumers in foreign markets move through a 'commodity hierarchy' such as that envisaged by Cornwall (1977) and Luigi Pasinetti (1981). However, the inter-relatedness of the production process as embodied in the structure of the firm and the inter-relatedness of the firm itself with other institutions makes for a natural tendency towards 'lock-in' of that process. Consequently, as the new technologies come along domestic firms might find it difficult to exploit the associated dynamic economies of scale, hence resulting in an endogenous fall in λ[15].

Similarly, the lock-in will result in an inability of firms to keep-up with the movement of their foreign customers through the 'commodity hierarchy'. Hence, although at the start of a high growth era the commodities associated with the accompanying production process might have possessed what foreign consumers viewed as 'superior' characteristics, by the end of the era this is no longer the case. The outcome is an endogenous fall in ε. Of course, such endogenous falls in λ and ε may be avoided if economic actors are sufficiently dynamic. However, it seems unlikely that such dynamism could be maintained, uninterrupted, through successive generations. Also to be noted is that the reasoning outlined here suggests that a failure of institutions to keep pace with world developments is, to all intents and purposes, equivalent to negative structural change in a dynamic context.

From the above it seems inevitable that endogenous structural change will impact both through λ and ε. This impact of endogenous structural change on both the productivity and demand regimes of the cumulative causation model has been emphasised by Boyer and Petit (1991, p. 492). However, to keep the subsequent mathematical analysis tractable without affecting the fundamental insights obtained we shall assume that such change occurs solely through ε[16, 17]. Moreover, we shall assume a specific functional form for ε: namely, the logistic mapping given in equation [8] below. In this mapping, $y = \beta/2$ defines the boundary between a low growth era and a high growth era. Meanwhile, α can be thought of as a measure of the sensitivity of the home economy's institutional structure to endogenous structural change.

$$\varepsilon_t = \alpha(\beta - y_{t-1})y_{t-1} \quad \alpha > 0 \tag{9}$$

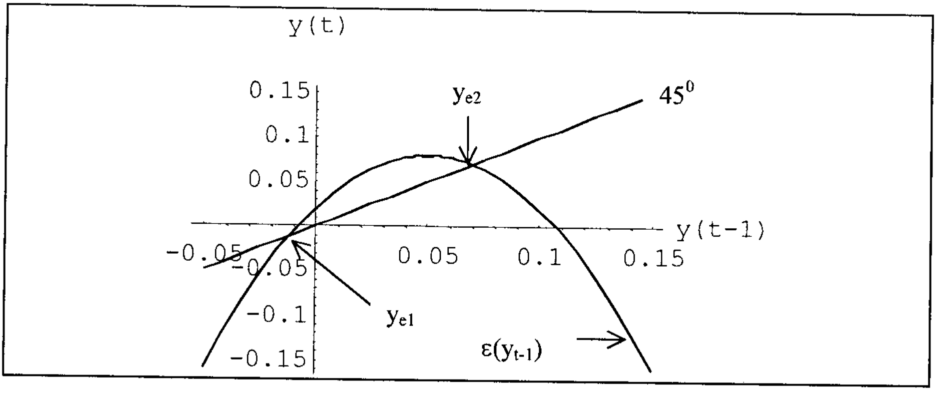
The choice of functional form is not accidental, the dynamical properties of 'simple' logistic mappings have been extensively investigated by mathematicians and play an important role in the development of chaos theory. Moreover, this precise functional form has the added advantage that it produces quite plausible ranges of values for ε. For example, later in our analysis we shall make use of the values $\alpha = 590$ and $\beta = 0.10$. Given these values we see that ε will range from a minimum of zero to a maximum of 1.475. However, despite these reasons for our specific choice of functional form, it is important to note that use of any other sufficiently smooth mapping for ε with a single maximum will produce qualitatively identical results. Indeed, quite remarkably, some important quantitative features of the results would also be identical[18, 19].

Anyway, taking the functional form for ε in equation [8] and combining it with the basic equations, equations [1] to [4], of the 'standard' cumulative causation model allows us to arrive at the following non-linear first-order difference equation for real output growth:

$$y_t = \frac{\gamma\eta(r_c + p_c - w - \tau)}{1-\gamma\eta\lambda} + \left(\frac{\gamma\alpha y_c}{1-\gamma\eta\lambda}\right)(\beta - y_{t-1})y_{t-1} \tag{10}$$

Given its quadratic nature, this system permits two possible steady-state solutions (point attractors). This is illustrated in *figure 3* below.

Figure 3: 'Exogenous' Structural Change that Impacts on the Income Elasticity of Demand for Exports



The figure plots the curve described by the mapping in equation [9] together with a 45-degree line. The 45-degree line provides for a locus of points along which $y_t = y_{t-1}$. From this it follows that the two points at which the curve cuts the 45-degree line are the possible steady-state solutions. We denote the values of real output growth associated with these by y_{e1} and y_{e2} respectively. However, as it transpires, only y_{e2} is ever (locally) stable for economically interesting parameter values. Hence, it is only worth quoting the explicit algebraic solution for y_{e2}. This is given by equation [10].

$$y_{e2} = \frac{(\gamma\eta\lambda + \gamma\alpha\beta y_c - 1) + \sqrt{(1-\gamma\eta\lambda - \gamma\alpha\beta y_c)^2 + 4\gamma^2\alpha\eta(r_c + p_c - w - \tau)y_c}}{2\gamma\alpha y_c} \tag{11}$$

and the condition under which this steady-state solution prevails (i.e. is locally stable) is given by equation [11].

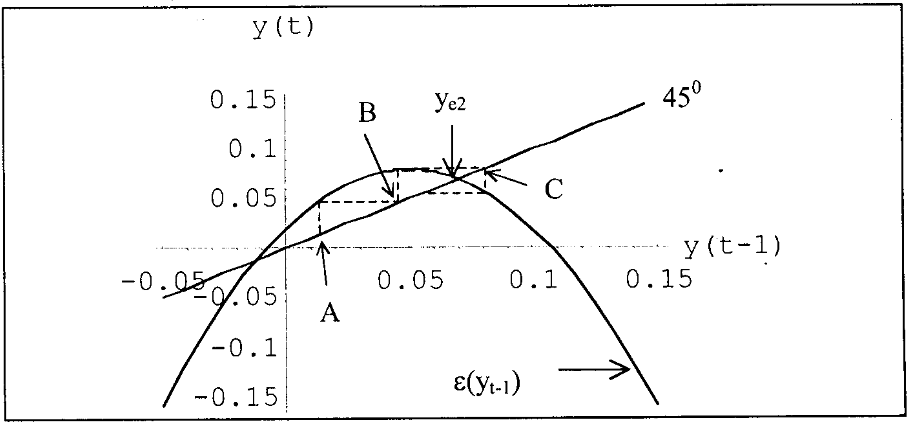
$$\left|\left(\frac{\gamma\alpha y_c}{1-\gamma\eta\lambda}\right)(\beta - 2y_{e2})\right| < 1 \tag{12}$$

which follows from taking a first-order Taylor series approximation of equation [9] around y_{e2} and which can be usefully expressed in the alternative form of:

$$\frac{\gamma\eta\lambda+\gamma\alpha\beta y_c-1}{2\gamma\alpha y_c}<y_{e2}<\frac{1+\gamma\alpha\beta y_c-\gamma\eta\lambda}{2\gamma\alpha y_c} \tag{13}$$

For example, if we take the empirically plausible values of $\gamma = 1$, $\eta = 0.5$, $p_c = 0.05$, $y_c = 0.04$, $w = 0.04$, $\tau = 0$, $r_e = 0.02$, $\lambda = 0.5$ and postulate that $\beta = 0.10$[20], thereby implying that the boundary between low- and high-growth eras is at $y = 0.05$, then y_{e2} is (locally) stable, and, therefore, a valid steady-state solution, provided $\alpha < 456.193$. *Figure 4* illustrates this stability for $\alpha = 450$ using a cobweb diagram:

Figure 4: Convergence to a Locally Stable Steady-State Solution (α= 450)



However, convergence to a (locally) stable steady-state as in *figure 4* only provides for dynamics of the sort we might plausibly associate with the Marx-Myrdal-Kaldor tradition for a finite period of time. Thus, in *figure 4* say that we have an economy that starts off in a low growth era at point A. The result is positive endogenous structural change that increases ε and so takes the economy into a new era that is associated with point B. This era, whilst representing an improvement on the previous era, is still, albeit only marginally, a low growth era. Hence, further positive endogenous structural change follows that takes the economy to point C[21]. It is now enjoying a high growth era and so the next endogenous episode of structural change is negative. However, the resultant era remains a high growth one, as do all subsequent growth eras. Nevertheless, the economy does experience an oscillation of its growth rate as it moves through these subsequent eras. These oscillations are, however, damped and so, eventually, endogenous structural change ceases and the economy remains, exogenous shocks permitting, forever in a high growth era characterised by the steady-state growth rate y_{e2}[22, 23]. For $\alpha = 450$ this growth rate is equal to 7.02 per cent.

Fully Endogenous Structural Change

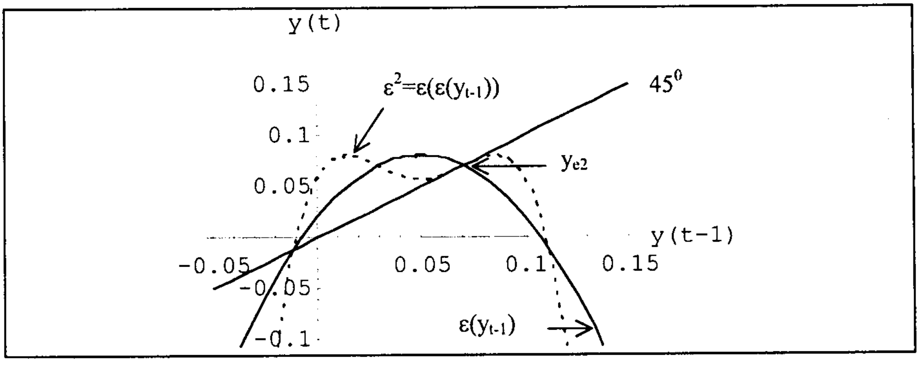
Growth Waves

To obtain endogenous structural change that is on-going even in the very long-run it is necessary to ask what happens as the (local) stability of the steady-state solution, y_{e2}, breaks-down. From equation [12] this will begin to happen when the following equality holds:

$$y_{e2} = \frac{1+\gamma\alpha\beta y_c - \gamma\eta\lambda}{2\gamma\alpha y_c} \tag{14}$$

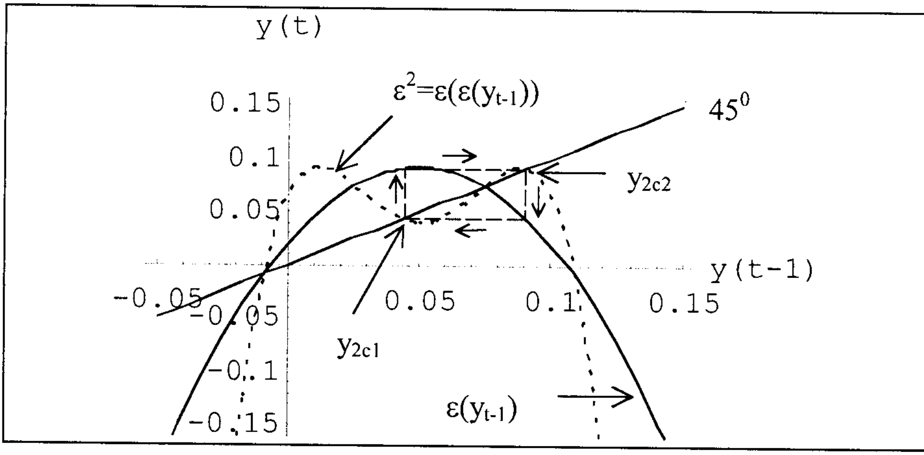
As indicated above, this is equivalent, for our baseline values, to a value of α of 456.193. The corresponding value of y_{e2} is 7.06 per cent. At this point we may say that y_{e2} is semi-stable (Shone, 1998; p. 212). This semi-stability is illustrated in *figure 5* by the fact that the curve $\varepsilon^2 = \varepsilon(\varepsilon(y_{t-1}))$, which plots the corresponding mapping, is tangent to the 45-degree line at y_{e2}. This curve furthermore shows that the nature of the system is such that as the stability of y_{e2} breaks down a stable two-cycle solution appears in its place. That is to say, the equality given in equation [13] defines a pitchfork bifurcation in which the qualitative form of the attractor in our system changes from being a point attractor to a two-period attractor (see Stewart, 1990; p.161).

Figure 5: Semi-stability of y_{e2} (α= 456.193)



The stability of this two-cycle solution can be formally examined by calculating $\varepsilon^{2\prime}(y_{2c1})$ and $\varepsilon^{2\prime}(y_{2c2})$, and y_{2c1} and y_{2c2} denote the two points between which the system cycles. Given our baseline values, this implies that the two-cycle solution is stable providing 456.193 <α< 546.594. *Figure 6* illustrates this for the case where α=540.

Figure 6: Growth Waves in the Extended Model of Cumulative Causation (α = 540)



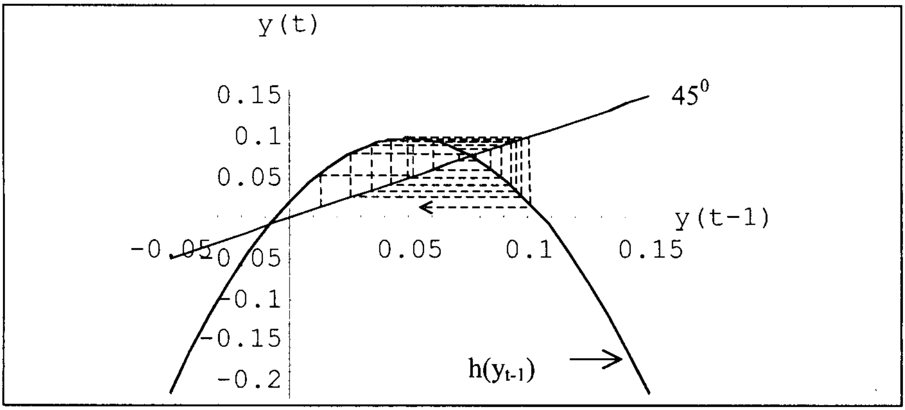
We can clearly see that, once it has settled down, the system cycles between the values y_{2c1} and y_{2c2}. These values correspond to the points at which the curve $\varepsilon^2 = \varepsilon\left(\varepsilon\left(y_{t-1}\right)\right)$ cuts the 45-degree line and, in this particular case, are given by 4.38 per cent and 9.09 per cent respectively. Hence, even in the very long-run, an economy so described exhibits ongoing endogenous structural change. In particular, it experiences alternating low- and high-growth eras, the low-growth era ushering in positive structural change that increases ε and the high-growth era negative structural change that reduces ε. We can envisage such an economy as experiencing waves such as those that might be associated with the various cycles cited by Moses Abramovitz (1961, p. 246). The cycles in question are the so-called Juglar cycle (a relatively short wave of between eight and ten years), the so-called Kuznets cycle (an intermediate wave of between fifteen and twenty-five years), and the so-called Kondratieff cycle (a very long wave of forty to sixty years).

When α = 546.594 the stable two-cycle breaks down only to be replaced by a stable four-cycle. This is because each of y_{2c1} and y_{2c2} themselves pitchfork bifurcate to give two stable solutions. In turn, the stability of this four-cycle lasts until $\alpha \approx 546.594$ at which point further pitchfork bifurcations occur to give a stable eight-cycle, and so it goes on with the stable eight-cycle giving way to a stable sixteen-cycle, the stable sixteen-cycle to a stable thirty-two cycle etc., etc. This process is referred to as double-period cascading: we started-off with a unique point attractor that then underwent a pitchfork bifurcation in which its period doubled to give a two-point attractor, each of whose points then underwent their own pitchfork bifurcations to give a four-point attractor and so forth.

Chaotic Era Changes

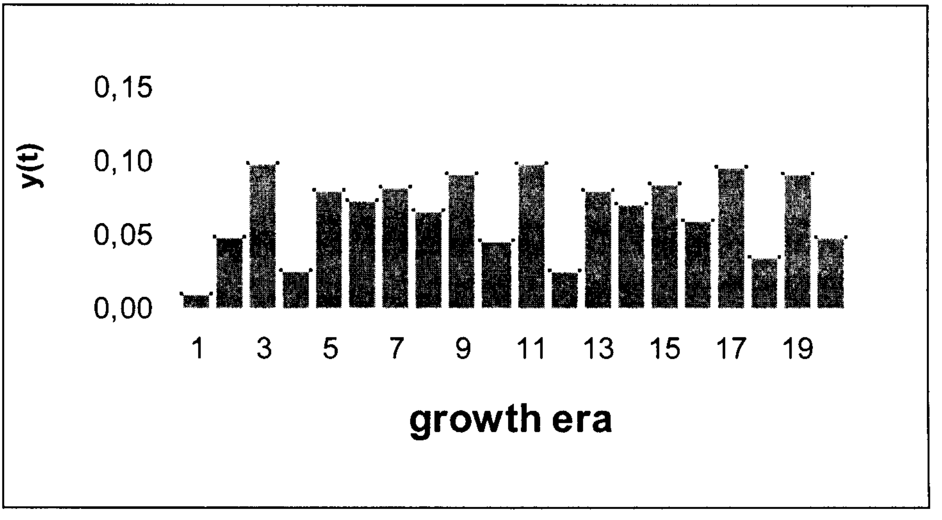
Notice that in the process of double-period cascading the gap between successive values of α at which pitchfork bifurcations occur gets smaller and smaller. Hence, we have seen that the two-cycle is stable for $456.193 < \alpha < 546.594$, but that the four-cycle is stable only for the much smaller range of $546.594 < \alpha < 565.952$[24]. Indeed, we may say that α approaches an upper-limit above which the process of double-period cascading stops. This upper-limit is given by $\alpha \approx 571.232$[25]. The significance of this value is that it defines the point at which the switching of growth eras in our model ceases, even in the very long-run, to follow a regular, although possibly complicated, cyclical pattern, and instead becomes chaotic in nature. *Figures 7a* and *7b* depict this chaotic switching of growth eras for $\alpha = 590$ using a cobweb diagram and a bar chart respectively.

Figure 7a: Cobweb Diagram Showing Chaotic Era Changes in the Extended Model of Cumulative Causation (α= 590)



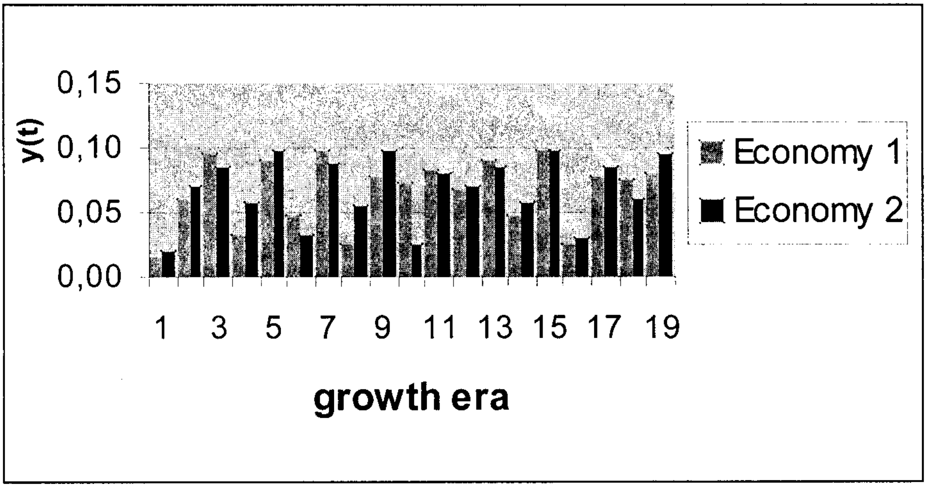
From the figures we can clearly see that the pattern of era changes appears random despite the fact that the non-linear first-order difference equation, equation [9], that underlies the pattern is deterministic. This is precisely what chaos is, the paradoxical situation of seemingly stochastic behaviour in a deterministic system. The consequence of this is that it will not be possible to predict the pattern of growth eras in an economy so described. To do so would require that we know its initial position with infinite precision, which would, effectively, require perfect knowledge of its institutional structure. In turn, this implies that the behaviour of the economy is strongly path-dependent in Kaldor's sense that the state of the economy 'cannot be predicted except as a result of [perfect knowledge of] the sequence of events in previous periods [in this case eras] which led up to it.' (Kaldor, 1972; p. 1244). It also implies that 'history matters'[27] and, moreover, that it matters in a much stronger sense than that apparent in many models, such as the new economic geography models of Paul Krugman *et al.* (see, *inter alia*, Krugman, 1991), where it is reduced to the importance of initial conditions in the presence of predetermined multiple equilibria.

Figure 7b: Bar Chart Showing Chaotic Era Changes in the Extended Model of Cumulative Causation ($\alpha = 590$)



A corollary of the above properties is that in a multi-economy world different economies will tend to evolve through different eras as time goes by. This is especially the case if their initial conditions, which reflect their respective institutional structures are very different. However, even where a set of economies share very similar initial conditions they will, sooner or later, exhibit noticeably different evolutionary paths[28]. Consequently, in a multi-economy world, we would fully expect to witness the phenomena of catching-up, forging ahead, and falling behind that seem to have characterised the comparative growth experiences of different economies in the real world. Such a pattern of catching-up, forging ahead, and falling behind, at least in growth rate terms, is shown for two hypothetical economies in *figure 8* below. To be noted is that both economies are assumed to share identical exogenous variable and parameter values, the only difference between the two being a slight one in initial conditions[29].

Figure 8: Growth Rate Catching-Up, Forging Ahead And Falling Behind



The characteristics of chaotic era changes outlined above seem to capture the most essential elements of the vision of the growth process that arises from the Marx-Myrdal-Kaldor tradition. This is to be compared with the unique steady-state and two-period solutions that do not so fully capture this vision. Even though endogenous structural change may go on for a very long time or indefinitely with these solutions, the ultimate behaviour of the economy is perfectly predictable as the same pattern of growth eras eventually emerges regardless of the initial conditions. Hence, neither is path-dependency present nor does 'history matter'[30,31]. Furthermore, nor would we expect, at least in the very long-run, patterns of catching-up, forging ahead and falling behind for economies identical in all respects bar some slight differences in initial conditions.

Finally, note that although chaotic era changes appear when $\alpha \approx 571.232$, such chaotic era changes cease when α reaches approximately 654.632. After this point, the era changes, sooner or later, explode: low-growth eras leading forever to lower-growth eras or high-growth eras leading forever to higher-growth eras. Furthermore, buried within the chaos interval of $571.232 < \alpha < 654.632$ are little 'windows' of regular behaviour. Thus, for example, for $621.175 < \alpha < 623.730$ a stable 3-cycle pattern emerges. From here, we then witness another process of double-period cascading until chaotic era changes again emerge[32]. Nevertheless, the chaotic behaviour that seems most in keeping with the vision provided by the Marx-Myrdal-Kaldor tradition is characteristic of quite a wide range of plausible parameter values. Indeed, recall that *figure 7* depicts chaotic switching for $\alpha = 590$, a value we earlier noted permits endogenous structural shifts in ε that cause it to fluctuate in the range 0-1.475.

Conclusion

This paper has sought to modify the 'standard' cumulative causation model along the lines indicated by Setterfield (1997) in an effort to capture the vision of the growth process associated with the Marx-Myrdal-Kaldor tradition. In contrast to that encapsulated in much of orthodox growth theory, this vision is one of endogenous structural change arising from processes of dynamic feedback between an economy's growth performance and its institutional structure. Associated with this vision are the ideas that an economy's dynamic behaviour is strongly path-dependent and thus also strongly influenced by history. As we have seen, our model possesses precisely such characteristics once the non-linearity in the feedback of performance during one growth era on the next era's institutions becomes sufficiently strong. This provides for chaotic growth era changes and it is chaos theory that seems to provide the most apt metaphor available for the Marx-Myrdal-Kaldor growth vision.

However, it is appropriate to finish on two notes of caution. First, although we have endogenised structural change in this paper by specifying the income elasticity of demand for an economy's exports as a function of the growth rate of real output in the previous era, the rationale that we have provided for the assumed functional form is only verbal. It would be desirable to more explicitly model how precisely pressure for reform and tendencies for lock-in may give rise to the type of strongly non-linear functional form postulated. Second, there are limits to which the metaphor of chaotic dynamics for the Marx-Myrdal-Kaldor vision can be stretched. In particular, although the structure of behaviour associated with chaotic dynamics appears completely irregular in two-dimensions (i.e. in a plot of y_t on y_{t-1} as in our cobweb diagrams), this is not the case in higher dimensions (i.e. in plots of y_t on more than one lag of y_t). Thus, despite the fact that outcomes are unpredictable what are known as strange attractors (i.e. structured patterns of behaviour that only become apparent in higher dimensions) do exist in the presence of chaos. Within the context of our model this is so because, despite our endogenisation of what is normally treated as exogenous, underlying it still is a set of completely deterministic equations. Yet, we might expect structural change to not only impact upon certain 'exogenous' parameters in a given set of equations, but to change the very nature of the equations themselves. Thus, for example, the demand regime in the 'standard' cumulative causation model, and thus in our model, is what Boyer and Petit (1991, pp. 498-500) classify as a 'pure classical demand regime'. However, this is only one of four possible demand regimes, and perhaps the least empirically relevant of the four for the post-war period, that Boyer and Petit identify. This being the case, we might not expect the underlying structure that, in higher dimensions, chaos theory predicts[33]

NOTES

[1] There are, of course, many important issues on which the views of Karl Marx, Gunnar Myrdal and Nicholas Kaldor differed. However, it seems fair to state that they all shared this point of view regarding the importance of dynamic feedback and therefore to talk of a Marx-Myrdal-Kaldor tradition in this context.

[2] This model is originally attributable to Robert Dixon and Tony Thirlwall (1975; see also Thirlwall, 1974).

[3] This terminology is attributable to Mark Setterfield (1998).

[4] Alternative models of cumulative causation include those by William Beckerman (1962) and Harry Richardson (1978, pp. 147-150). Both are very close to the 'standard' model, which itself has been extended in a variety of directions by a number of different authors. Amongst the most recent examples are papers by Luca De Benedictis (1997), Bernard Fingleton (1999), Miguel León-Ledesma (1999), Mark Roberts (forthcoming), and Ferdinando Targetti and Alessandro Foti (1996).

[5] For a history of the development of the concept of cumulative causation see Phillip Toner (1999).

[6] Small in the sense that real output growth has a negligible effect on real income growth in its main export markets, not in the sense that it faces an infinitely elastic foreign demand for its exports. The two are not irreconcilable, for, as John Williamson and Chris Milner (1991, p. 195) state, 'Many countries that would certainly be described as small by any other economic criterion supply a sufficiently large part of the world market with one or two major products so as not to face an infinitely elastic demand curve.'

[7] Variables omitting a time subscript are assumed to be exogenously determined in the model.

[8] Otherwise known as 'normal cost pricing'. The relationship can also be rationalised by reference to the behaviour of a profit maximising monopolist facing a price elasticity of demand for its output that is procyclical. See Wendy Carlin and David Soskice (1990, pp. 140-143), who also cite evidence on the prevalence of this form of pricing behaviour. Michal Kalecki also assumed such a relationship in some of his work (David Gordon, 1991; p. 521).

[9] The distinction between 'static' and 'dynamic' economies of scale dates back to Frank Knight (1921; discussed in Charles Blitch, 1983).

[10] The stability of the equilibrium point E_0 becomes an issue when we introduce a lag into one or more of the model's basic relationships. Note that the stability condition that the curve for the demand regime be steeper than that for the productivity regime is, from equations [4] and [5], equivalent to the condition $\gamma\eta\lambda < 1$. From this we can deduce that the model predicts a rate of conditional convergence equal to $\beta = 1 - \gamma\eta\lambda$. This is the speed at which an economy approaches its own equilibrium (Roberts, 2001, chapter 2). Boyer and Petit (1991, pp. 490-492) incorrectly argue that the necessary condition for stability in a cumulative causation model is that the curve for the productivity regime be steeper than that for the demand regime.

[11] Algebraically, $y_0 = \gamma[\eta(r_e + p_c - w - \tau) + \varepsilon y_c]/(1 - \gamma\eta\lambda)$ and
$r_0 = \{r_e + \gamma\lambda[\eta(p_c - w - \tau) + \varepsilon y_c]\}/(1 - \gamma\eta\lambda)$

[12] This is assuming that the increase in λ is not so large as to bring about a violation of the stability condition given in note 10.

[13] In endogenising the Verdoorn coefficient along these lines we are responding to Maurizo Pugno's (1999) call to ground Verdoorn's law within a framework of non-linear dynamics.

[14] As will come to be seen, the fact that the functional forms for λ and ε are specified as having a maximum is *crucial* to the generation of dynamics consistent with the Marx-Myrdal-Kaldor tradition.

[15] Pugno (1999, p. 5) cites four studies for the advanced countries that, in his words, 'report clear evidence that the Verdoorn coefficient has been reduced.' Targetti and Foti (1996) report no similar such evidence of a structural break for a sample of developing economies.

[16] Particularly relevant in this context is the concept of general purpose technologies (GPTs). A GPT is a new technological invention (or breakthrough) that has the potential to affect the entire economic, and, we might add, social, system (Aghion, 1998, p. 48; Aghion and Howitt, 1998, p. 244). In particular, a new GPT can be thought of as opening-up a whole new area of potential 'dynamic' economies of scale. However, if an economy is to begin to realise these possibilities its institutions, most notably its firms, must become malleable to change. ICTs provide the most recent example of a GPT. As Timothy Bresnahan (1999) demonstrates, firms in the US have had to undergo massive changes in recent years in their quest to utilise new ICTs. It seems reasonable to predict that economies whose firms have, due to inter-relatedness, been resistant to similar such changes will have experienced recent endogenous falls in their Verdoorn coefficients.

[17] The choice to consider ε rather than λ can be justified on the grounds that, empirically, this parameter exhibits greater cross-country variation than does λ, thereby indicating that this variable is more sensitive to the institutional specifics of an economy and therefore to structural change.

[18] Fabio Fiorillo (1999) also presents a model, which possesses several heterogeneous sectors, in which ε evolves endogenously in a nonlinear manner. The resultant dynamics are strongly path- and time-dependent. However, whereas Fiorillo emphasises purely 'technological' factors in endogenising ε, the emphasis here is more institutional in orientation.

[19] For example, what is known as Feigenbaum's universal constant would continue to characterise the process of double-period cascading. We progress to describe this process later in the paper.

[20] The reason the results are invariant to the mapping is that all sufficiently smooth mappings with a single maximum re-normalise to what Ian Stewart (1990, pp. 204-205) refers to as the Feigenbaum mapping. Importantly, this mapping exhibits the mathematical property of exact self-similarity (Stewart, 1990; p. 205).

[21] These provide baseline values that we shall use throughout the paper.

[22] In passing through the growth eras A, B and C the economy is experiencing a drawn-out process of cumulative causation that is working through non-price competition. Contrast this with the unmodified 'standard' model where cumulative causation only works through the empirically less relevant channel of relative price competition. Given this, it should come as no surprise to discover that our model can be easily extended to include a balance-of-payments constraint, the growth rate obeying Thirlwall's law, without sacrificing any of the interesting dynamics. See McCombie and Roberts (forthcoming) for an outline of this extension.

[23] Of course, the exact pattern of growth eras leading up to the steady-state era would be different to that described for an economy with a different initial condition.

[24] If we were to allow for exogenous shocks then, under the current conditions, the model would exhibit what is known in the NAIRU literature as 'impure' hysteresis or persistence. As Setterfield (1998, p. 290) explains, the argument underlying this concept is that '... the impact of a shock which displaces the dependent variable of a system from its long-run equilibrium may be prolonged as a result of feedback effects on the alleged exogenous (but in fact deeply endogenous) variables of the system ... But these feedback effects will dissipate in the long run ... as the system returns to its former configuration, which constitutes a determinate long-run equilibrium position.'

[25] It is Feigenbaum's universal constant that, in the limit, determines the stability range of α for any particular cycle. If $d_k = (\alpha_k - \alpha_{k-1})/(\alpha_{k+1} - \alpha_k)$ then $\lim_{k\to\infty} d_k = \delta = 4.669202$ where δ is Feigenbaum's universal constant (see Shone, 1998; p. 216). In this schema k is used in reference to the point at which a 2^k cycle first appears. We made use of Feigenbaum's universal constant above to estimate $\alpha = 565.952$ as the approximate value of α at which a stable eight-cycle emerges.

[26] This follows from $\alpha_{k+1} \approx (\alpha_k - \alpha_{k-1})/\delta_k + \alpha_k$, which itself follows from the existence of Feigenbaum's universal constant.

[27] Not to mention the presence of 'pure' hysteresis if we were to allow for exogenous shocks. To be noted is that this 'pure' hysteresis would be occurring in the absence of a unit-root and also in the absence of any adjustment asymmetries in our deeply endogenous parameter, ε. This implies that such hysteresis is likely to be a much more general characteristic of economies than a reading of the standard NAIRU literature (see, for example, Marika Karanassou and David Snower, 1998; p. 834) or even of Setterfield's work (see, especially, Setterfield, 1998) would have us believe.

[28] This brings to mind the historical experiences of countries such as Argentina, Chile, Ireland, New Zealand and Portugal *vis-à-vis* the advanced industrial countries. In 1870 such countries were relatively rich and well-integrated into the world economy, and therefore *a priori* we might have expected them to have experienced similar evolutionary paths to the advanced countries. However, *ex post*, the paths followed have clearly been very different (see Bradford De Long, 1988).

[29] Neither economy is assumed to be an important export market for the other. Relaxing this assumption would cause p_c and y_c in each economy to become endogenous to real output growth in the other economy, thereby further complicating the dynamics.

[30] Of course, if we have, say, a stable 1024-cycle, which does emerge, briefly, during the process of double-period cascading, then to the participants in the economy it will probably appear as if there is path-dependency and as if history matters. Indeed, given that we are thinking in terms of eras, the dynamics associated with the stable steady-state outcome may even give such false impressions to economic actors.

[31] Do not forget, however, that in the mathematics we are taking our discrete time intervals to be eras. As alluded to in section 3.1, even with the deterministic solutions, there is nothing in our analysis that states that all of these eras have to be the same length in calendar time.

[32] Again, the process of double-period cascading is characterised by Feigenbaum's universal constant.

[33] In theory, just how much underlying structure exists to the long-run growth process in higher dimensions is open to the possibility of empirical investigation. The appropriate methodology would not be econometrics, but non-linear signal processing. As Paul Ormerod (1998, p. 84), in a best-selling text on the applicability of chaos theory to economics informs, the essence, in many ways, of such techniques is '... to plot the data in a different way to see if its underlying structure can be revealed.' The problem

with this, of course, in the context of a prediction of chaotic era changes is an acute shortage of possible observations to plot: at most, there would be three/four for the post-war era in the UK.

REFERENCES

Abramovitz, M., (1961). *Thinking About Growth.* Cambridge: Cambridge University Press.

Aghion, P., (1998) 'Inequality and Economic Growth', in: Aghion, P., Williamson, J.G., *Growth, Inequality and Globalization*, Cambridge: Cambridge University Press.

Aghion, P., Howitt, P., (1998) *Endogenous Growth Theory*, Cambridge: Mass., MIT Press.

Beckerman, W., (1962) 'Projecting Europe's Growth'. *Economic Journal*, Vol. 72, No. 288, 912--925.

Blitch, C., (1983) 'Allyn Young on increasing returns'. *Journal of Post-Keynesian Economics*, Vol. 5, No. 3, 359--373.

Boyer, R., Petit, P., (1991) 'Kaldor's Growth Theories: Past, Present and Prospects for the Future', in: Nell, E.J., Semmler, W. (Eds.), *Nicholas Kaldor and Mainstream Economics: Confrontation or Convergence?*, New York: St. Martin's Press.

Bresnahan, T. F., (1999) 'Computerisation and Wage Dispersion: An Analytical Reinterpretation' *Economic Journal*, Vol. 109, No. 456, F390--F415.

Carlin, W., Soskice, D., (1990) *Macroeconomics and the Wage Bargain: A Modern Approach to Employment, Inflation and the Exchange Rate*. Oxford: Oxford University Press.

Cornwall, J., (1977) *Modern Capitalism: Its Growth and Transformation.* New York: St. Martin's Press.

De Benedictis, L., (1997) 'Cumulative Causation, Harrod's Trade Multiplier and Kaldor's Paradox: The Foundations of Post-Keynesian Theory of Growth Differentials', in: Rampa, G., Stella, S., Thirlwall, A.P. (Eds.), *Economic Dynamics, Trade and Growth: Essays on Harrodian Themes*, London: Macmillan.

De Long, B.J., (1988) 'Productivity Growth, Convergence, and Welfare: Comment', *American Economic Review*, Vol. 78, No. 5, 1138--1154.

Dixon, R., Thirlwall, A. P., (1975) 'A Model of Regional Growth Rate Differences on Kaldorian Lines', *Oxford Economic Papers*, Vol. 27, No. 2, 201--214.

Duesenberry, J., (1949) *Income, Saving and the Theory of Consumer Behaviour*. Cambridge, Mass.

Fingleton, B., (1999) 'International Economic Growth: Simultaneous Equation Models Incorporating Regional Effects', *Dept. of Land Economy, University of Cambridge, Discussion Paper No. 96*.

Frankel, M., (1955) 'Obsolescence and Technological Change in a Maturing Economy', *American Economic Review*, Vol. 45, No. 3, 296—319.

Gordon, D., (1991) 'Kaldor's Macro System: Too Much Cumulation, Too Few Contradictions' in: Nell, E.J., Semmler, W. (Eds.), *Nicholas Kaldor and Mainstream Economics: Confrontation or Convergence?*, New York: St. Martin's Press.

Kaldor, N., (1966) *Causes of the Slow Rate of Economic Growth of the United Kingdom*, Cambridge: Cambridge University Press.

Kaldor, N., (1970) 'The Case for Regional Policies', *Scottish Journal of Political Economy*, Vol. 17, No. 3, 337--348.

Kaldor, N., (1972) 'The Irrelevance of Equilibrium Economics', *Economic Journal*, Vol. 82., No. 328, 1237—1255

Karanassou, M., Snower, D., (1998) 'How Labour Market Flexibility Affects Unemployment: Long-Term Implications of the Chain Reaction Theory', *Economic Journal*, Vol. 108, No. 448, 832--849.

Krugman, P.R., (1991) *Geography and Trade*, Cambridge: Mass., MIT Press.

Landesmann, M., Snell, A., (1989) 'The Consequences of Mrs Thatcher for UK Manufacturing Exports', *Economic Journal*, Vol. 99, No. 394, 1--27.

León-Ledesma, M., (1999) 'Accumulation, Innovation and Catching-Up: An Extended Cumulative Growth Model', *Dept. of Economics, University of Kent at Canterbury, Working Paper no. 99/6.*

McCombie, J.S.L., (1998) 'Harrod, Economic Growth and International Trade' in: Rampa, G., Stella, L., Thirlwall, A.P. (Eds.), *Economic Dynamics, Trade and Growth: Essays on Harrodian Themes*, London: Macmillan.

McCombie, J. S. L., (1999) 'Increasing Returns and Economic Growth from a Kaldorian Perspective', presented at the *'1949-1999: A Fifty Years Debate on Verdoorn's Law' Workshop, University of Genova.*

McCombie, J. S. L., Roberts, M., (forthcoming) 'The Role of the Balance of Payments in Economic Growth' in: Setterfield, M. (Ed.), *The Economics of Demand-led Growth*, Cheltenham: Edward Elgar.

McCombie, J. S. L., Thirlwall, A. P., (1994) *Economic Growth and the Balance-of-Payments Constraint.* London: Macmillan.

Myrdal, G., (1957) *Economic Theory and Under-developed Countries*. London: Duckworth.

Pasinetti, L. L., (1981) *Structural Change and Economic Growth.* Cambridge: Cambridge University Press.

Petit, P., (1999) 'Integration and Convergence in the European Union' in: Setterfield, Mark (Ed.), *Growth, Employment and Inflation*, London: Macmillan.

Ormerod, P., (1998) *Butterfly Economics*, London: Faber & Faber.

Pugno, M., (1999) ''Verdoorn's Law' and the Analysis of Steady State Growth: from an Unsatisfactory Marriage to a New Perspective', a communication prepared for the *'1949-1999: A Fifty Years Debate on Verdoorn's Law' Workshop, University of Genova.*

Richardson, H. W., (1978) *Regional and Urban Economics*. Harmondsworth: Penguin

Roberts, M., (2001) *Essays in Cumulative Causation*, unpublished Ph.D. dissertation, Dept. of Land Economy, University of Cambridge.

Roberts, M., (forthcoming) 'Cumulative Causation and Unemployment' in: McCombie, J.S.L., Pungo, M. (Eds), *Verdoorn's Law After 50 Years*, London: Palgrave.

Setterfield, M., (1997) ''History versus equilibrium' and the theory of economic growth', *Cambridge Journal of Economics*, Vol. 21, No. 3, 365--378.

Setterfield, M., (1998) 'Adjustment Asymmetries and Hystersis in Simple Dynamic Models', *The Manchester School*, Vol. 66, No. 3, 283--301.

Setterfield, M., (1999) 'Modelling a Path-Dependent Economy: Lessons from John Cornwall's Macrodynamics' in: Setterfield, M. (Ed.), *Growth, Employment and Inflation*, London: MacMillan.

Shone, R., (1998) *Economic Dynamics: Phase Diagrams and Their Applications.* Cambridge: Cambridge University Press.

Skott, P., Auerbach, P., (1995) 'Cumulative causation and the 'new' theories of economic growth', *Journal of Post-Keynesian Economics*, Vol. 17, No. 3, 381--402.

Stewart, I., (1990) *Does God Play Dice? The New Mathematics of Chaos*. London: Penguin.

Targetti, F., Alessandro, F., (1996) 'Growth and Productivity: a model of cumulative growth and catch-up', *Cambridge Journal of Economics*, Vol. 21, No. 1, 27--43.

Thirlwall, A.P., (1974) 'Regional Economic Disparities and Regional Policy in the Common Market', *Urban Studies*, Vol. 11, No. 1, 1—12.

Toner, P., (1999) *Main Currents in Cumulative Causation: The Dynamics of Growth and Development.* London: MacMillan.

Williamson, J., Milner, C., (1991) *The World Economy: A Textbook in International Economics*, London: Harvester Wheatsheaf.

Young, A., (1928) 'Increasing Returns and Economic Progress', *Economic Journal*, Vol. 38, No. 152, 527--542.