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Basics of Proportional Systems in Architecture

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Fig. 1 Man caught in the web of measures: body laid on the square. Anthropometrical measures can be seen in the body parts: finger, inch, fist, palm, foot, elbow, yard etc. On the right: Egyptian elbow from second Millennium BC (in Egyptian Museum, Cairo) and my elbow in 2004 AD.

Basics of Proportional Systems in Architecture

Vernacular architecture is an important part of our culture. It is simple, modest, but definitely effective. Science, on the other side, is not primitive, modest, nor simple. Vernacular architecture with its primitive methods is closer to science than we can imagine. The prime man understood that order is the most important, the order that could be found in quantities: line, plane and space. He also started to use first measure units: inch (finger), elbow, foot. The paper presents some examples of the use of proportion systems in architecture.

Vernakularna arhitektura čini značajan dio naše kulture. Ona je jednostavna i skromna, ali zasigurno svrhovita, za razliku od znanosti koja nije ni primitivna ni skromna ni jednostavna. Svojim primitivnim metodama daleko je bliža znanosti nego što možemo zamisliti. Primitivni ljudi shvatici su da je red među stvarima najvažniji, i to red između veličina – pravca, ravnine i prostora. Poceli su uporabljati prve mjerne jedinice: inc (palac), lakat, stopu. U ovom su članku predstavljeni neki primjeri uporabe sustava proporcija u arhitekturi.
INTRODUCTION

MAN AND ARCHITECTURE

Fig. 2 A triangle with sides 3, 4 and 5 units, and with (one) right angle can be constructed in geometry out of 12 units (3 + 4 + 5 = 12). Squares above the sides are 9, 16 and 25. Each side can be calculated with help of Pythagoras’ theorem.

Sl. 2. Trokut sa stranicama od 3, 4 i 5 jedinica i s jednim pravim kutom može se u geometriji konstruirati iz 12 jedinica (3 + 4 + 5 = 12). Kvadrati nad stranicama tako su 9, 16 i 25. Svaka se stranica može izračunati pomoću Pitagorina poučka.

M an is part of nature, but its important part. Any man’s work is therefore natural, so we can also understand architecture as a logic element of nature. But during history there were several theories how to explain this issue. Anselm of Canterbury (1033-1109) explains it as idealistic “realism”, based on Neo-Platonism, and Thomas Aquinas (1225-1274) incorporated it into ecclesiastic doctrine, which lasted until today, also in theology, psychology, aesthetics etc.¹

Do we understand mankind, culture, architecture and mathematics from an anthropological point of view? Egenter understands this theory as inductive and deductive. Historically, deduction based on metaphysics, provided the general axioms on historically established cosmological ideas: creation and nature, the sacred, absolute beauty as absolute aprioris. Inductive methods are essentially based on sensory perception, with certain observations of concrete phenomena.² I agree that the natural sciences reached their enormous success with the inductive method mainly by remaining absolutely clear about the instrumental significance of theoretical procedures. We have to understand the basic stages of development of mankind: where the first step was work and the second was thinking. Understanding is a result of both thinking and experience. Architecture represents just a part of man’s field of work, while the anthropology area is wider. It not only explains man’s work, step by step, but it also explains his thinking, comprehending and work, together with results.

At the beginning, there was only work, and no thought. Fire changed this: man started to think and in the end he became conscious that the time spent on working could be reduced by help of thinking. With more time on his hands he could indulge in other activities: he could draw on the walls of the cave, he could sing, he could improve his speech – culture was born.³ This is an issue of quality, based on quantity. But vice-versa is also possible: quality of brain (thinking) can shorten the time of quantity of hands (work).

Architecture is the science of forming space for all needed purposes: living, working, defending, making culture and fun (if those are not the same). The prime man built the simplest objects: megaliths known as menhirs, table, and dolmen. A row of dolmens composes a corridor or long chamber. Lintel over a number of vertical stone slabs represents the ceiling, mostly of burial chambers. Absence of big stone slabs demands new construction principles. The only possible principle is corbelling, where horizontal layers of stones, stratified, one projected over another, construct a false vault. After some thousands years, Etruscans developed the arch, while all classical Egyptian and Greek architecture used only lintel. In Deir el-Bahiri (near Nile, Egypt), we can find some four thousand years old “arches”. In principle, they resemble an arch, but in fact the construction is typical corbelling, with nice, horizontal stone layers, but cut into a half-circular shape.

Architectural could be divided into theory and practice. While practice implies mostly construction, theory is more complicated. The result is a question of culture and we have to understand the problem: prime man and nature, man in nature, or man in front of nature. During history, nature was teacher and the prime man learned from it. And he survived, in spite of wild animals, which were stronger and bigger.

From the smallest detail to the Universe, we have two elements: chaos and order. Chaos as destruction is not a system, it is only a situation. This situation can not last, it ended in time. “A process is generally called a phenomenon that cannot be described by a finite number of data representing the characteris-

¹ Egenter, 1992: 33
² Egenter, 1992: 55
³ Oliver, 1997: xxii
tic variable." And: "However, in performing this task in actual experiments with digital systems, serious questions arose related to the storing and processing of an infinite number of data contained in the empirical ensembles." On the other side, order is represented by the very construction: if bigger elements are used, longer life is assured.

Nature teaching is very fundamental; it is based on practical examples, but also on stimulating the brain to start thinking, all with the goal of surviving. Man can not stand in front of nature; he stands in the first line only which enables him to survive. Surviving of a man in wild nature is a matter of using the brain. And all his work, architectural included, leads to a better environment, safer and more cultural. There is no dilemma whether architecture is more theoretical or more technical: both elements, hand in hand, compose it. Generally: theory in practice is much more present in our everyday living; in architecture as well.5

**IMPORTANT OF ORDER**

**VAŽNOST REDA**

In architectural work or in architectural contemplation, people feel pleasure or displeasure; they react. The early man understood that order is the most important. Order included in quantities: straight line, plane and space. The first measure units were introduced: inch (finger), elbow, foot, but the problem was that no foot or elbow was the same. The second problem lay in ratio of quantities, mainly of line segments. Thousands years before the first theoretic (Pythagoras for instance), man used the "holy triangle" (3, 4, 5 as "Egyptian") and other physical orders, as well as the equilateral triangle, squares etc. Golden section is widely used in detail as in whole architectural composition. Order helps to co-ordinate complicated elements. Order uses certain methods and procedures in advance planned settings, meant to facilitate the work or to make it better. Order means restrictions in narrow minded spheres, only.

**Practice** related to architectural order is very simple: from orientation, planning, designing, building, and finally putting into function. This sequence can not be changed and none of elements could be neglected. "An important topic became the quest for potential factors which determine the shape of architectural constructions and which can explain the enormous diversity of house forms."7

**Theory** in architecture, by which I mean basic aesthetic theory, is dealing with proportions mostly visible on elevations of buildings. I will mention only four: *Egyptian triangle, square and its diagonal, square root of three, golden section.*

**SOME THEORETICAL EXAMPLES**

**NEKI TEORETSKI PRIMJERI**

**EGIPATSKI TROKUT**

A triangle has three sides and three angles. More important and more usable triangles are right triangles, and on the other side with two same elements: sides and/or angles. Those triangles are symmetrical with respect to the perpendicular bisector of one side. The equilateral triangle is the most usable: it has three equilateral sides, same angles, and it is symmetrical by all the three axes. It is important to say no one triangle is "better" or "more beautiful", they are different.

The triangle appeared. Theory says that the angle between sides measuring three and four units, if the third side’s length is five units, is a right angle or ninety degrees. It is very simple to construct the planes with a right angle. Pyramids are also constructed by help of the right angle, and further more their sides and height are made in proportion 3:4:5. Pyramids were built at the end of the fourth millennium BC and some thousands of years later Pythagoras explained it as squares above the sides. The following is true: $3^2 + 4^2 = 5^2$ ($3^2 = 9$, $4^2 = 16$, $5^2 = 25$, and $9 + 16 = 25$). Or, we can count the shorter side this way: $\sqrt{25} = \sqrt{25 - 16} = \sqrt{9} = 3$. Or maybe the longer side: $\sqrt{25} = \sqrt{25 + 16} = \sqrt{41} = 5$. Great architectures had been constructed with help of this knowledge.

**SQUARE ROOT OF TWO**

**KVADRATNI KORIJEN OD DVA**

The diagonal of a square with a baseline equal to one unit, is equal to the square root of two. This is a very simple statement, but it is not so simple to understand the whole story. The result will be shown at the end of the article, but first the theory: the square has only one baseline and four equal angles. Angles are equal to ninety degrees (four time 90° is equal to 360°). After Pythagoras, the diagonal is equal to addition of the squared sides: $1^2 + 1^2 = (\sqrt{2})^2$, the diagonal = $\sqrt{2}$. If we start to draw a circle on the half of the diagonal, with radius equal to $\sqrt{2}/2$, the circle is drawn around and around the square. Most European paper formats use this ratio: Deutsche Industrienormen (DIN) prepared DIN standards to be used for office papers formats.

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4 Grabec, Sachse, 1997: 277
5 Juvanec, 2002: 15
6 Oliver, 1997: 3
7 Oliver, 1997: 6
SQUARE ROOT OF THREE

Kvadratni korijen iz tri

Square root of three is a bit more complicated than square root of two, but the procedure is the same. We can take an equilateral triangle, a triangle with three sides of the same length. We can mark the side with unit “one”. Pythagoras says:

\[ (\frac{1}{\sqrt{2}})^2 + (\frac{\sqrt{3}}{2})^2 = 1^2, \quad x^2 - (1/2)^2 = (\sqrt{3}/2). \]

The height of an equilateral triangle is equal to the square root of three by two, and it can be observed in the height of great burial monuments in Malta (4500 years BC in hypogeum Hal Saflieni) or in Egypt (Red pyramid in Memphis necropolis 3000 years BC), as well as in stone shelters. Corbelling as principle can be built by help of this in the most usable mode.

GOLDEN SECTION

Zlatni rez

The most usable proportion is the golden section because it embodies natural relationships. Man as a part of nature is also composed in this proportion and therefore from the early man till today he had been using the golden section in his works. The golden section is an infinitive ratio of two sides, where the shorter side is in the ratio to the longer as the longer to the sum of both (a : b = c : a + b). Infinitive means that we can prolong this expression; \( a : b = b : c = c : a + b \). In graphical sense we can construct this ratio in line segments.

The diagonal of two squares (where sides are equal to one) is equal to \( \sqrt{2} \). The whole length is equal to 2, and the height of the rectangle is \( \sqrt{2} \). When man started building he applied lessons learnt from nature and he continued with understanding their sense; today we would call it “theory”. It is very important to know that the exact proportions were already used in the past, but without any theoretical knowledge, only by using simple tools, hands and brains. Man sees and he understands what he sees. Nature is a wonderful teacher, and the punishment for wrong decisions is very severe. That is why we can not find wrong decisions in architectural works: their decay was nature’s punishment. Man repeats what he has seen. A lot of details can be observed in the past, and man used them for his survival. The first detail was definitely a construction element: lintel needed to span a composition.

Here I can use a strange, but true comparison: if a mother-hen can remember up to three chickens, we can identify up to five items (or we merge five by five maximum).

The order in aesthetics contains mostly proportions, the relationships between all composition elements, but mostly those of length. Some of them we can understand: some seem to be very strange (though they are not). Fig. 9 shows some examples related to human body and geometry.

In geometry: proportions are definitely logical, but they can only be understood with knowledge of mathematics. And there is another problem: a man “can see” something. The eye is not as accurate as we would wish, and we have to understand there are some tolerances. Tine Kurent says there is up to five percent of inadequacy to consider.

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In case of a line segment we can see length proportions, in case of a plane figure there are two dimensions (lengths and widths), mostly in form of parallelograms. The circle has no proportions: it has one dimension only, and for proportion we need at least two dimensions. Proportions can be expressed in the case of a square and rectangle by help of diagonals. In architecture the rectangle shape is the most important, while diagonal defines basic proportions of the sides. When man started building he applied lessons learnt from nature and he continued with understanding their sense; today we would call it “theory”. It is very important to know that the exact proportions were already used in the past, but without any theoretical knowledge, only by using simple tools, hands and brains. Man sees and he understands what he sees. Nature is a wonderful teacher, and the punishment for wrong decisions is very severe. That is why we can not find wrong decisions in architectural works: their decay was nature’s punishment. Man repeats what he has seen. A lot of details can be observed in the past, and man used them for his survival. The first detail was definitely a construction element: lintel needed to span a composition.

Man observes. It is a matter of watching, repetition and experience, using brains. If the brains are not so highly developed, nature uses riddles, traps, all for the same purpose:
surviving of a species with more developed talents. Nature has brilliant methods for achieving its goals: stimulation for more capable species, and constraint for those who are not. Comparison between nature’s work and work of a prime man in the example of megaliths is also surprising. We can not understand the way they were built, how some ten or even hundreds of tons of heavy stone slabs had been lifted up to form a composition, without any traces of tools. Here lies another problem: I mentioned “traces” of tools, but there is no evidence they had not have existed. Tools and their traces disappeared in time: and the same methods were “reinvented”, because one can hardly believe that, even brilliant, men knew different constructions in other countries.

The most important of early construction principles is corbelling, using horizontal layers of stones, projecting one above the other. This principle is well-known from more than 4500 years BC, when it appeared in some monuments (naveta on Menorca Island, nuraghe on Sardegna, Etruscan tombs in Italy, Atreus treasury in Greece, Gallarus oratory in Ireland) from second millennium BC to our times. This type of construction still exists in shepherds’ architecture: shelters, wells, bridges. There is another problem to consider: I mentioned “traces”. The fact that there are no traces of tools is not the proof of their non-existence. Important and big architectures were built in dressed stone, for some millennia, and ”un-important” architecture was built in clay, wood or in undressed stone. The lifetime of those constructions is short: some ten years only, without maintenance. The idea of ”reinvented” architecture is not so insignificant.

The closest thing to the man is his body; which he likes and knows — so why not use it for measure units? The most widely spread measure units in use are those of length. As man’s dimensions are the most practical for use he developed the system of anthropomorphic measures: inch (finger), palm, span (between straddling fingers), foot, elbow, yard (step), fathom — the widely spread English system of measures. This system is very useful: with foot or elbow always on disposal, this system is very handy for the man.

There are three important problems in anthropomorphic length units:

1. inadequate system of accretion (1 foot = 12 inches; 1 yard = 3 feet; 1 rod, pole = 5 1/2 yards etc); 40 rods = 1 furlong; 8 furlongs = 1 statute mile) which is definitely misunderstood and complicated.

2. fingers, feet, steps are accommodated to the man’s body: but to what man and from which time? The medieval man was shorter than my father, my father was shorter than me, and I am shorter than my son. For example, one foot can apparently seem to be a very simple and suitable measure unit; but is it really?

Lengths of foot unit differ as follows:

- Gewoenlichen Schuh, Germany\textsuperscript{12} 28.80 cm
- Dezimalschuh, Germany\textsuperscript{13} 29.16 cm
- Greek, Roman foot (pous, pes)\textsuperscript{14} 30 cm
- English foot\textsuperscript{14} 30.48 cm
- Vienna foot\textsuperscript{15} 31.60 cm

\textsuperscript{12} Hippiel, 1994: 5
\textsuperscript{13} Kurent, 1961: 14
\textsuperscript{14} Kurent, 1961: 14
\textsuperscript{15} Kurent, 1961: 14

**ACCOMMODATING THINGS BY THE BODY**

**POSVAJANJE STVARI PREKO TIJELA**

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I found an official announcement of a new measure unit: metre, in Mohorjev koledar (Slovene Diary of St Mohor) from 1886. It is defined as: “1 m: 3 crevlje, 1 palec in 11/10 crtic” (in old Slovene language, which means “1m: 3 feet, 1 inch and 11/10 lines”).

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3. The golden section can not be expressed by use of simple numbers: it is a very sophisticated ratio, but difficult to express in numbers. It can be understood; it can not be defined without knowledge of mathematics.

Complicated mathematical expression of the golden section is shown in Fig. 9. Without mathematical education, this hardly understood expression seems to be very logically applied in woman’s fashion for example (Fig. 8).

Definition of the metre, based on circumference of the Earth, is pretty complicated, but the use, with help of the decimal system, is not.

**ACCOMMODATING PROCEDURES
BY THE BRAIN**

**POSVAJANJE POSTUPAKA MOZGOM**

Today by help of scientific knowledge we understand things (in proportion theory: mathematics, geometry). It was more difficult in former times: Pythagoras knew about geometry more than two thousand years ago, but the use of his knowledge was until, say 1850, limited only to educated people. I have mentioned some theoretical problems, but they have to be explained in their practical aspects of culture and incredible values of the heritage, inherited from our forefathers. A man or a human body is obviously the key.

Anthropology as the scientific study of human beings and their physical, social, material, and cultural development, and especially anthropometrics, are basic elements of scientific approach. Anthropometrics is "the science of measurements as applied to the physical varieties of man of all times and places. Such characteristics as height and weight, the proportions of the body, the rhythm of man’s growth from inception until death, are measured with instruments and the results charted in statistical tables for analysis and interpretations". Those are serious words, and it is hard to understand a mathematical approach without theoretical knowledge. But practice is definitely more comprehensible.

[Lektura: mr.sc. LJILJANA ŠEPIC]
OSNOVE PROPORCIJSKIH SUSTAVA U ARHITEKTURI

Vernakularna arhitektura cini vazan dio naše kultu- re. Ona je jednostavna i skromna, ali zasigurno svr- hovita, za razliku od znanosti koja nije ni primitivna ni skromna ni jednostavna. No, vernakularna je arhitektura svojim primitivnim metodama daleko bliža znanosti nego što možemo pretpostaviti. Pri- mitivni covjek radom je razvijao svoju misao kroz vrijeme i prostor. U početku razvoja naglasak je bio na radu, a manje se razmisljalo jer je osnovna svrha bilo preživljavanje. Zatim je razmisljanjem covjek zaboravio na svoj strah od vatre i poceo uporabljati- ti vatru za svoje potrebe (samoobranu, kuhanje, svijetlo, tehnologiju).

Arhitektura je znanost projektiranja prostora za sve potrebne funkcije: stanovanje, rad, obranu, kul- turu i zabavu. Arhitektura obuhvaća teoriju i prak- su. Dok praksa uglavnom podrazumijeva gradnjenje, teorija je komplicirana, pogotovo na polju mate- matike (naravno, za arhitekte). Pitanje je kulture kako shvatom primitivnog covjeka, njegov odnos prema prirodi, u prirodi i u konfliktu s prirodom. Primitivni je covjek na osnovi svoga tijela i mozga ustanovio prve jedinčine mjere. Razmišljanjem je razvio kulturu teorije estetike, dok je rukama postojale. Antropološke mjere, kao što su "sveti trokut" (3, 4, 5 kao tzv. "egi- patski") i drugi, kao i jednakostranični trokut i kvad- drati. Zlatni rez naveliko se upotrebljava kako u de- talju, tako i u cijelim kompozicijama (Le Corbusier: Modulor i palača UN u New Yorku). Trokuti mogu biti različitih duljina i kutova, a u graditeljstvu su na kvadratni korijen iz četiri plus jedan, a to je (V5). Kvadratni korijen iz tri cini visinu jednako- straničnog trokuta sa stranicom jednakom dva: može se konstruirati pomoću tro tijela jednake duljine − čime je prva igra malih pastira. Tehnički je pogodnije imati osnovicu jednaku jednaku kvad- ratom korijenu osećena plus jedan (npr. dva kvadrata sa stranicama jedna i dva (dva je jednak kvadratnom korijenu iz četiri plus jedan, a to je (V5)) Kvadratni korijen iz tri cini visinu jednako- straničnog trokuta sa stranicom jednakom dva. To je pola veličine kvadratnog korijena iz tri.

Zlatni rez ima kompleksnu matematičku formulu, ali visina tijela od nogu do pupka i od pupka do tije- mena jesu u proporcijama zlatnog reza. Teoretski se zlatni rez može iskazati dijagonalu dvaju kvad- rata, tj. s (V5−1) i (3−V5), gdje je (V5−1) + (3−V5) = 2. Na primjeru ljudskog tijela to ne izgleda tako komplici- rano. Matematička, teorija i praksa i u arhitekturi su bliže nego što možemo pretpostaviti.

Borut Juvanec

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