The paper deals with the statistical analysis of the selected parameter of the blast furnace process. It was proved that analyzed measurements have been autocorrelated. ARIMA control chart was selected for their analysis as a very useful statistical process control (SPC) method. At first the methodology for control limits setting in ARIMA control charts considering the time series outlier analysis as an integral part of the ARIMA model building was designed. Then this proposal was applied to the statistical analysis of the selected blast furnace process output parameter with the aim to compare two production methods.

**Key words:** ARIMA control chart, blast furnace process, environmental aspects, time series outliers analysis


**Ključne riječi:** ARIMA kontrolni dijagram, visokopečni procesi, ekološki aspekti, izlazne vrijednosti vremenske sekvence

**INTRODUCTION**

Statistical process control (SPC) can be defined as a method of monitoring, controlling and, ideally, improving a process using statistical analysis. It is used to identify and remove undesirable variation that exceeds the natural (common) variation with the aim of reaching statistical stability of a process.

Correct setting of control limits in control charts is one of the main conditions for the successful application of statistical process control and for meeting its basic goal, i.e. verifying statistical stability of the analyzed process.

The control limits setting has been solved in many publications (for instance [1-3]). But no publications distinguish between autocorrelated and nonautocorrelated data from the point of view of this problem.

The standard methodology for the control limits setting can be summarized into the following steps:

1. Data collection.
2. Computation of control limits using appropriate formulae.
3. Control chart construction.
4. Control chart analysis.
6. Control limits recalculation.

Steps 5 and 6 are realized only when the analysis in step 4 has revealed the process nonstability (there were some points out of the limits or some nonrandom patterns in the control chart). Process regulation (step 5) consists of specifying the assignable causes of the process nonstability and acceptance and realization of adequate corrective actions. Without this step it is not recommended to go to the next step. Step 6 is obviously worked out via excluding out-of-control points and control limits recomputation using the remaining points [4]. These steps are repeated until the control chart starts to signalize the statistical stability of the analyzed process.

Standard way of the control limits recomputation (excluding them from the data set) is not suitable for autocorrelated data and when ARIMA modeling is applied.

Autoregressive integrated moving average (ARIMA) models represent very useful class of linear models that form the heart of the stochastic model-based Box-Jenkins methodology for the time series analysis [4].

In practice time series are often influenced by uncontrolled or unexpected events that can lead to occurrence
of large unusual observations called outliers. Outliers can largely affect selection of a suitable model, estimation of model parameters, forecasting, properties of the model residuals including (more detailed information concerning reasons for the outlier detection and adjustment can be found for instance in [5, 6]). For that reason outlier analysis must be an integral part of the model building phase. The algorithm of the outlier detection and adjustment is discussed in [6] and it is also mentioned in [4].

In the frame of statistical process control (SPC) an approach based on time series modeling using ARIMA models has proved to be useful in dealing with autocorrelated data (see for instance [3, 7, 8]). Some criticism of the residual control charts has appeared in the paper by MacGregor [9]. He reminds that the residual charts are inefficient at detecting shifts in the mean, especially when the autocorrelation is large and positive. But he does not consider that the absence of the time series outlier analysis can be one of the main insufficiencies of SPC using the control charts based on the residuals from ARIMA models. Wright, Booth and Hu in [10] suggest using the joint estimation outlier detection method as an SPC method. But they deal especially with short-run autocorrelated data and they do not solve the problem of control limits setting in the ARIMA residuals control chart.

This paper deals with the proposal of the methodology for the control limits setting in ARIMA control charts which considers the time series outlier analysis as an integral part of the model building and which contains the outlier analysis as a step before the control limits setting.

**USED METHODOLOGY**

Demonstration of applicability of the SPC methods to comparison of the iron ore production variants from the point of view of the selected output parameter was the main goal of these research activities. It was proved that analyzed measurements are autocorrelated. For that reason it was decided to use ARIMA control chart. But before applying it to the statistical analysis of the selected blast furnace output parameters control limits setting had to be solved because the standard way (recomputation without out-of control points) is not suitable and the SPC literature does not offer solution of this problem. For that reason the procedure for control limits setting in ARIMA control chart was proposed and then it was applied to the statistical analysis of the selected parameter.

The proposal of the procedure

Proposed algorithm for control limits setting when using ARIMA modeling is described using the flow chart (see Figure 1).

As it can be seen on the figure after identification of the initial ARIMA model and estimation of its parameters (assuming that there are no outliers in the analyzed time series) the outliers’ identification and assessment have to be realized. When some outlier is identified its cause must be searched for and adequate corrective action must be realized.

When final overall outlier model is identified residuals from this model should be verified. When they are normally distributed and independent with constant variance central line (CTR) and control limits (upper control limit UCL and lower control limit LCL) for the selected control chart (classical Shewhart control charts, exponentially weighted moving average (EWMA) or cumulative sum (CUSUM) control charts for individual measurements) can be computed from these residuals and ongoing statistical process control can start.

When using the classical Shewhart control chart for individual observations formulae for computation of CTR, UCL and LCL are as follows:

\[
CTR = \bar{\sigma} (\bar{e} = 0),
\]

\[
UCL = \bar{e} + \frac{3}{\sqrt{128}} R_k,
\]

\[
LCL = \bar{e} - \frac{3}{\sqrt{128}} R_k,
\]

**Figure 1. Flow chart of the proposed procedure for control limits setting in ARIMA control chart**
where:
\[ \overline{e} \] - mean of residuals,
\[ R_m \] - average moving range.

When residuals do not meet all assumptions some different time series model than ARIMA (for instance TAR model, linear or nonlinear volatility model) or some nonparametric method can be applied.

**Application of suggested algorithm**

Suggested algorithm will be shown on the analysis of the selected output parameter of the blast furnace process, i.e. the amount of \( \text{H}_2 \) in the output blast furnace gas in % (daily measurements). During the analyzed period (2004 and 2005) there were applied two production methods different in additional fuel (let us mark these different methods I (using the tar) and II (using the oil)). The comparison of stability of these two production methods from the point of view of this selected output parameter has been set as a goal of this analysis. For the both methods the fitting ARIMA model was identified (it was selected from several acceptable models based on the residuals verification and the value of criteria AIC and BIC for the selection of the best one) and estimated using data from the 2004 year. After residuals verification control limits for the classical Shewhart control chart for individual measurements were computed using residuals of this model. These control limits were used for the ongoing process control in 2005.

The time series outlier analysis was realized using software SPSS 15 and all the rest of statistical analysis control charts including was made in STATGRAPHICS Plus, Version 5.0.

**RESULTS AND DISCUSSION**

Autocorrelation of \( \text{H}_2 / \% \) for the production method I was confirmed by tests for randomness. As the best model for this time series there was identified ARIMA \((1,0,0)\). More information about this model can be found in Table 1.

<table>
<thead>
<tr>
<th>Parameters estimation</th>
<th>Outliers</th>
<th>Outlier estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant = 2,807</td>
<td>Additive</td>
<td>0,904</td>
</tr>
<tr>
<td>( \rho_1 = 0,805 )</td>
<td>Additive</td>
<td>0,751</td>
</tr>
<tr>
<td></td>
<td>Transient</td>
<td>0,981</td>
</tr>
<tr>
<td></td>
<td>Magnitude</td>
<td>0,726</td>
</tr>
<tr>
<td></td>
<td>Decay factor</td>
<td>0,738</td>
</tr>
<tr>
<td></td>
<td>Innovational</td>
<td>0,735</td>
</tr>
</tbody>
</table>

Causes of outliers were discussed and possible corrective actions were considered.

In the next step residuals from this final overall outlier model were verified. Tests confirmed that residuals are normally distributed, independent and with constant variance (see Table 2).

<table>
<thead>
<tr>
<th>Verified assumption</th>
<th>Test</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normality</td>
<td>( \chi^2 )</td>
<td>0,49</td>
</tr>
<tr>
<td></td>
<td>Kolmogorov</td>
<td>0,65</td>
</tr>
<tr>
<td></td>
<td>Kolmogorov-Smirnov</td>
<td>&gt; 0,1</td>
</tr>
<tr>
<td></td>
<td>Anderson-Darling</td>
<td>0,55</td>
</tr>
<tr>
<td></td>
<td>Skewness</td>
<td>0,89</td>
</tr>
<tr>
<td></td>
<td>Kurtosis</td>
<td>0,22</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>Runs above and below median</td>
<td>0,34</td>
</tr>
<tr>
<td></td>
<td>Box-Pierce</td>
<td>0,78</td>
</tr>
<tr>
<td>Constant variance</td>
<td>Bartlett</td>
<td>0,78</td>
</tr>
</tbody>
</table>

Results of the residuals verification of the final ARIMA model enabled to use these residuals for setting of control limits and verifying the process stability. Shewhart control chart for individuals was applied to these residuals (see Figure 2). It can be seen that process in 2004 can be considered to be statistical stable (in control) and that control limits were set correctly and they can be applied to the process in a future (see Figure 3).

As it can be seen on Figure 3 in 2005 process could not be considered to be in control. It reflects that some
discussed corrective actions have not been actually realized.

Analysis of the portion of $H_2$ in the blast furnace gas by the production method II was done in the same way as the previous one. As the best final model for the time series of data from 2004 year there was identified ARIMA (0, 1, 2). There were no outliers identified. Shewhart control chart for individual measurements from the residuals of this model has been constructed (see Figure 4). Figure 4 shows that the process (production method II) in 2004 can be considered to be statistical stable (in control) and that control limits were correctly set and they can be applied to the process in a future (see Figure 5).

As it can be seen on Figure 5 in 2005 process could not be considered to be in control. It reflects the same as the technology I, i.e. some mentioned corrective actions have not been actually realized.

Comparing Figure 3 and Figure 5 it can be seen that technology II is less stable than technology I (4 points out of limits as compared to 2 points out of limits). In addition expressed with standard deviation of the original $H_2$ measurements variation of technology II is larger then variation of technology I.

CONCLUSION

Comparison of two iron production methods different in the used additional fuel from the point of view of the selected output parameter of the blast furnace process was the goal of the statistical analysis demonstrated in this paper. As the main instrument for this comparison ARIMA control chart was selected. By the application of ARIMA control chart the problem with correct control limits setting had appeared. As a result the algorithm for control limits setting in ARIMA control chart was proposed and applied to the selected parameter. The main ideas of this algorithm can be formulated as follows:

- Time series outliers’ analysis must be realized as an integral part of the ARIMA model building.
- Outliers’ analysis must precede control limits setting.
- Causes of the identified outliers must be searched for and adequate correct actions must be realized.

Then control limits of the final ARIMA model can serve as criteria for the process statistical stability evaluation and for comparison of the analyzed production methods.

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REFERENCES


Note: The responsible for English language is D. Noskievičová, Czech Republic.