Network providers need very effective optimization tool for good utilization of scarce link capacities during exploitation. In the case of multiple link capacities with mutual traffic correlation such problem could be more demanding. The sizing problem is explained for satellite link, but it could be applicable for other transmission resources. Dimensioning of getway link (G-link) can be realized only by new constructions (new channel equipment) on the Earth side. Mathematical model for optimal capacity sizing of different link elements (capacity types) is explained, minimizing the total cost (expansion, conversion and maintenance). Instead of nonlinear convex optimization technique, that could be very exhausting, the network optimization method is applied. With such approach an efficient heuristic algorithm for three different capacity types is being developed. Through numerical test-examples this approach shows the significant complexity savings, but giving us very close to optimal result. However, in real circumstances some adding limitations on capacity state values have to be introduced. In comparison to other options it is obvious that heuristic option M_H (with only one negative value per capacity state) shows the best ratio between complexity reduction and result deterioration.

**Keywords:** multiple link capacity expansion, capacity expansion/conversion problem, transmission capacities with mutual traffic correlation, satellite link resource management

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1. **Introduction**

1.1 **Sizing of G-link**

This paper concentrates on GEO (Geo-stationary Orbit) satellite networks that take an important role in the public and private Internets. But satellites are multiple access systems with very limited transmission capacity compared to terrestrial network nodes. A GEO (Geo-stationary Orbit) satellite covers about one third of Earth so many ground segments, Network Control Centre (NCC) or Land Earth Stations (LES) are dispersed around, interfacing the terrestrial network. The capacity on GEO satellite is defined with a number of spot beams that cover the region and each getway (LES) is associated with at least one beam. Generally, getway can support one or more beams and it depends on traffic intensity. Each beam has multiple channel carriers and they are partitioned among getways based on estimated traffic demands; see [2]. Also, we know that the traffic to/from ground segment is limited with simultaneous usage of channel capacities on the getway up/down links (G-link).

Taking into account continuous increase of traffic demands from SPs, each getway (LES) has to expand the G-link capacity during exploitation but it could be very expensive. Periodically, new (adding) channel equipment has to be installed on the ground side and it is the only way how network can ensure more simultaneous connections (channels) toward terrestrial network (for certain location on the ground).

Each getway link (G-link) is a very high-speed point-to-point bi-directional TDM link that ensures a large number of simultaneous channels. Although the satellite capacity is limited, every new generation of satellite enables more working channels on it. It is a very good occasion for significant expansion of the G-link capacity. During the next exploitation period (5 - 6 years) the G-link has to be expanded but in a rational manner; see [3]. Expansions can be done anytime, at early beginnings, in the middle or at the end of planning period. An example for the expansion of only one capacity type can be seen in Fig 1.
In this paper we are looking for the optimal expansion/conversion sequence to decide at which moment and in what amount of the appropriate capacity type (type of channel) the new constructions/conversions have to be done. Network providers (NP) want to ensure both: minimal expansion/maintenance/conversion cost and fulfillment of traffic demands. If reductions or conversions (dotted lines) are possible the shortages or idle capacities can be efficiently eliminated.

1.2 Differentiation of capacity types
Razlikovanje vrste kapaciteta

In the context of QoS provisioning we have basic differentiation of voice, data stream/FTP and messaging traffic on the same link. In the future we need better granularity of quality levels that influences capacity allocation very much. Today it is desirable to dynamically allocate the channel capacity to service provider request on BoD (Bandwidth on Demand) principles, so the differentiation of QoS service classes is necessary; see [4].

a) Many broadband services, as VoIP (Voice over Internet Protocol) on BoD (Bandwidth on Demand) principles, require the implementation of QoS mechanisms. In broadband DVB-S/DVB-RCS networks (Digital Video Broadcast – Return Channel Satellite) different capacity request categories are defined 5 and 6. These are Constant Rate Assignment (CRA), Rate Based Dynamic Capacity (RBDC), Volume Based Dynamic Capacity (VBDC) and Free Capacity Assignment (FCA), each with distinct and well-defined properties. Users of satellite systems are usually price-sensitive, so that their satisfaction is determined by both the quality and the price for it. One such parameter is capacity category and they are not independent of QoS scalability. Regarding to allocation mechanisms the assignment is made in the following order: CRA first, followed by RBDC, then VBDC and any left-over distributed as FCA. In that case capacity types are ranked by the quality level.

b) Similar problem can be seen in VSAT (Very Small Aperture Terminal), the systems covering big cruiser ships. We can optimize the link budget profile with dynamically changed capacity arrangement. We can differentiate the priority traffic classes and the best effort traffic so the bandwidth optimization leads to variable channel types and their amount in time. To ensure TCP over BoD the bandwidth has to be dynamically changed during an hour, day, week etc: see [7] and [8]. Such optimization in bandwidth arrangement (throughput) cuts the expenses drastically.

c) Another example for capacity type differentiation is in mobile satellite networks with global coverage. Traffic to-and-from mobiles is served in LES and can be routed through different satellites. Normally, more than one GEO-satellite is visible from each LES (e.g. Inmarsat network). If LES works with two or more satellites each link is of different capacity type but they are in firm correlation, serving the traffic from the same world region; see [13] and [15]. For example, if unused (idle) capacity on the link exists the traffic can be redirected through another satellite covering the same region, or in other words: channel equipment on the ground can be re-used from one satellite link to another. In that case we are talking about conversions and the capacity types are not ranked.

Many similar capacity expansion/conversion problems exist where multiple link capacities show mutual traffic correlation. Conversions can be allowed anytime and in all possible directions (combinations) or with some restrictions. With capacity conversion we can prevent money loss caused by bad utilization of capacity and with much lower expenses than through expansion.

1.3 Capacity planning over finite time period
Planiranje kapaciteta za određeno vremensko razdoblje

From previous discussion it is clear that the capacity link expansion problem (CEP) of different capacity types exists. In satellite link capacity planning NPs want to satisfy given traffic demands (by SPs) at the lowest possible cost. They need an optimal decision in which moment and in what amount of each capacity type expansions (new constructions) have to be done.

Two strategies can be applied: fulfillment of traffic demands can be a must or traffic loss can be allowed at a pre-defined rate. In both cases we need an optimal decision in which moment and to what amount of specified capacity type the expansion has to be done.

We expect that traffic demands rise in time which can be addressed through introduction of more channel equipment (modem units); see Fig. 1. But in some periods traffic demand increment can be zero (that means stagnation) or can be negative (decreasing). In both cases idle capacities can appear and may cause significant loss of money. So the optimal expansion of link capacity must be carefully done.

The reduction (disposal) of link equipment is not an expansion/maintenance/conversion cost and fulfillment of traffic demands. If reductions or conversions (dotted lines) are possible the shortages or idle capacities can be efficiently eliminated.

Figure 1 Capacity planning for only one capacity type over the finite time period
Slika 1. Planiranje kapaciteta za samo jednu vrstu opreme i za konačni broj perioda
The CEP (Capacity Expansion Problem) is formulated as minimization of the cost function over a set of variables confined within a constraint set. The expansion/conversion problem for three or more (multiple) different capacity types with allowed shortages is a very complex problem. Although it can be solved with many different approximation techniques it is still the subject of many scientific papers. Some papers that have significant influence on this paper are: Suk-Gwon & Gavish [9], Castro & Nabona [12] and Lee & Luss [16].

After the brief explanation of the capacity expansion problem (CEP) the mathematical model is formulated in section 2 of this paper. In section 3 the algorithm approach and basic heuristic solution are explained. In section 4 different heuristic options are proposed and compared through numerical test-examples. Discussion is related to algorithm complexity and computational savings, as well as about efficiency of algorithm options and their possible application.

2 Mathematical model for CEP
Matematički model za CEP

The CEP for a finite planning period is similar to a multi-period inventory problem but it also has elements of multi commodity assignment problem. Because of nonlinear cost functions over time, showing the effect of economy of scale, we can apply any nonlinear optimization technique. Instead of a nonlinear convex optimization (NP-complete), that can be very complicated and time-consuming, a network optimization method can be efficiently applied. The main reason for such approach is the possibility of discrete capacity values for limited number of capacity types (channel type) so the optimization process can be significantly improved. In that sense CEP-problem can be formulated as Minimum Cost Multi-Commodity Flow Problem (MCMCF); see [12]. It can be easily represented by multi-commodity single (common) source multiple destination network and the flow diagram can be seen in Fig. 2.

Each satellite link capacity (called commodity) is expanded over time to serve demand types. Commodity \( i \) for \( i = 1, 2, ..., N \) is designed primarily to serve demand of type \( i \), but it can be converted to satisfy demand \( j (j \neq i) \). In such conversion process some limitations can be introduced. Fig. 2 gives a network flow representation of CEP for \( N \) capacity types and \( T \) time periods. The \( t \)-th row of nodes represents the capacity states for each period \( t \) for appropriate capacity type \( i \). At each node \((i,t)\) there is an external traffic demand increment \( r_{i,t} \), possibly negative. For convenience, the \( r_{i,t} \) is assumed to be integer. Horizontal links (branches) are representing capacity flows between two time periods. An unused capacity (surplus) can be utilized in the next time period but maintenance of idle capacity could be too expensive.

On the contrary, shortages (negative values) can produce unsatisfied demands. Period \( T+1 \) may be introduced when all capacity states values are zero; see (2.4). It means that after planning period all traffic demands must be satisfied. The sum of traffic demand for capacity type \( i \) between two periods:

\[
R_i(t_1, t_2) = \sum_{t=t_1}^{t_2} r_{i,t}
\]  

(2.1)

![Figure 2 A network flow representation of the capacity expansion problem](image-url)
The sum of demands for the whole planning period and for all capacity types has to be positive or zero:

$$\sum_{i=1}^{N} R_i(1,T) \geq 0 \quad (2.2)$$

It means that we do not expect reduction of total capacity after the whole planning period; in other words we presume the increase of capacity. Horizontal links that connect nodes represent the capacity \( I_i \) between any time period \( t \) and \( t+1 \). Positive value represents inventory surplus and negative value represents shortage of capacity. The vertical links represent link expansions (new constructions) with \( x_{ij} \), or link capacity conversions with \( y_{ij} \), in the appropriate time period. Common node \( O \) is the source for all capacity expansions.

Capacity points are noted with \( t \) at time period \( t \) in which the relative amount of capacity for each capacity type \( I_i \) is known (in defined bounds) and which at least one capacity state value \( I_i \leq 0 \).

$$\alpha_t = (I_{1,t}, I_{2,t}, ..., I_{N,t}) \quad (2.3)$$
$$\alpha_t = \alpha_{t+1} = (0, 0, ..., 0) \quad (2.4)$$

Let \( C \) be the number of capacity points \( (C = C_r, r = 1) \), and the total number of capacity points (the sum of them) is \( C \). Reduction of \( C \) can be done through imposing appropriate capacity bounds or by introducing adding constraints.

Let \( G(E, A) \) denote a network topology, where \( E \) is the set of nodes representing possible satellite link capacity states (capacity points) in each time period \( T \) and \( A \) the set of links (arcs), representing the capacity changes between time periods. Vertical set of nodes for each time period is represented by the capacity point \( \alpha_t \).

On diagram from Fig. 2 each horizontal link is characterized by link weight value. Each weight \( w_{ij} \) depends on the capacity situation on the satellite link between two time periods, defined by the capacity expansion values \( x_{ij} \), conversion values \( y_{ij} \) and traffic demands \( r_{ij} \). It means that the link weight (cost) is the function of used capacity: lower amount of used capacity (smaller bandwidth) gives lower weight (cost). The lack of capacity can decrease the link weight values the same as idle capacity can increase it. If the link cost corresponds to the amount of used capacity, the objective is to find optimal expansion strategy that minimizes the total cost incurred over the whole planning period. The main condition is that given traffic demands must be fully satisfied without shortages on the end of planning period.

The CEP is formulated as minimization of the cost function over a set of variables confined within a constraint set. In general it is the multi-constraint optimization problem (MCP) but in this paper we are concerned with one-dimensional link weight vectors for \( T+1 \) links on the path \( \{w_{ij}, t \in A, i = 1, ..., N\} \). We have only one capacity constraint for each capacity type and for each time period denoted with \( L_i, (L_{1}, L_{2}, ..., L_{N}) \). Definition of the single-constrained problem is to find an expansion sequence from the first to the last node in the network from Fig. 2, such that:

$$w = \min \sum_{i=1}^{N} \sum_{t=1}^{T} w_{i,j}(I_{i,t}, x_{i,t}, y_{i,t}, r_{i,t}) \quad (2.5)$$

where: \( I_{i,j} \leq L_{i,j} \). \( \quad (2.6) \)

A path obeying the above condition is said to be feasible. Note that there may be multiple feasible expansion solutions. The total cost over time includes cost for capacity expansion \( c_{ij}(x_{ij}) \), capacity conversion cost \( g_{ij}(y_{ij}) \), idle capacity and capacity shortage cost \( h_{ij}(I_{i,t}) \), and penalty costs in the form of joint set-up cost \( p(z_{i}) \). If the link weights correspond to the costs the objective is to find optimal expansion policy that minimizes the total cost incurred over the whole planning period. The CEP problem can be formulated as follows:

$$\min \left\{ \sum_{i=1}^{T} \sum_{j=1}^{N} \left[ c_{ij}(x) + h_{ij}(y_{ij}) + g_{ij}(y_{ij}) \right] + p(z_{i}) \right\} \quad (2.7)$$

so that we have:

$$I_{i,t+1} = I_{i,t} + x_{ij} + \sum_{j=1}^{N} y_{ij} - \sum_{j=1}^{N} y_{ij} - r_{ij} \quad (2.8)$$
$$I_{i,t} = I_{i,t+1} = 0 \quad (2.9)$$

for \( i = 1, 2, ..., T; j = 1, 2, ..., N; j = i + 1, ..., N(i<j) \).

Normally, the objective function is a nonlinear function, consisting of many different costs functions. They are usually represented by nonlinear variable cost functions. It can be assumed that all cost functions are concave and non-decreasing, reflecting economies of scale.

3 Algorithm development
Razvoj algoritma

It this approach the network optimization method is efficiently applied. The network optimization can be divided in two steps. In the first step we are calculating the minimal weights \( d_{i,j}(\alpha_{i,j}) \) between all pairs of capacity points for the whole planning horizon. The calculation of each weight value is called: capacity expansion sub-problem (CES). The number of all possible CES values depends on the total number of capacity points. It requires solving repeatedly a certain single period expansion problem (SPEP).

\[ \text{Figure 3 The CEP problem can be seen as the shortest path problem for an acyclic network of calculated CESs} \]

Slika 3. CEP-problem se može sagledati i kao traženje najkraćeg puta u acikličkoj mreži pod-problema.

In the second step we are looking for the shortest path from the first to the last node in the network with the former calculated weights between node pairs (capacity points);
see Fig. 3. Suppose that all values (CES) are known, the optimal solution for CEP can be found by searching for the optimal sequence of capacity points and their associated capacity state values for each time period. On that level the CEP problem can be seen as a shortest path problem for an acyclic network in which the nodes represent all capacity point values, and branches (links) represent CES values; see Fig 3. Then Dijkstra’s or Floyd’s algorithm or any similar algorithm can be efficiently applied; see Lee and Luss [16].

In CEP we have to find many weight values \( d_u(\alpha, \alpha) \) that emanate two capacity points, from each node \((u, \alpha)\) to node \((v+1, \alpha_u)\) for \(0 \leq v \leq u \geq T+1\). We can calculate \( d_u(\alpha, \alpha): \)

\[
\min \left\{ \sum_{i=1}^{N} \sum_{i=1}^{N} c_{ij}(x_{ij}) + h_i(t_{i+1}) + \sum_{j=1}^{N} g_{ij}(y_{ij}) \right\} \tag{3.1}
\]

where \( I_{i+1} = I_{i} + D_i(u, v) - R_i(u, v) \) \tag{3.2}

\[
R_i(t_1, t_2) = \sum_{i=1}^{N} r_{ij} \tag{3.3}
\]

\[
D_i(u, v) = \sum_{i=1}^{N} x_{ij} + \sum_{j=1}^{N} \left( f_{ij} - y_{ij} \right) \tag{3.4}
\]

for \(i = 1, 2, ..., N; t = 1, ..., T\)

\( D \) value represents the amount of total capacity changes for capacity type \(i\) for appropriate time period. The total cost between two capacity points for periods \(u\) and \(v\) includes many costs: the capacity expansion cost \( c_{ij}\), idle capacity or capacity shortage cost \( h_i\), capacity conversion cost \( g_{ij}\), and joint set-up cost \( p\). The last can be used as a strategic penalty cost with strong influence to optimization process.

3.1 Basic algorithm option

To compute the sub-problem value \( d_u\), it is convenient to describe the problem as a single source multi-commodity and multi-destination network. To solve CES it requires solving repeatedly a certain single period problem. Let \( SPEP_t(t, D, ..., D)\) be a Single Period Expansion Problem associated with period \(t\) for capacity type \(i\), \(i + 1\), ..., \(j\) and corresponding values of capacity change intention \( D\), \( D_i\), ..., \( D\). A detailed explanation of various SPEP solutions (strategy) for three different capacity types can be found in previous work of Krile [13].

The computational effort is \(O(T^N N^{2N-1})\). If there are no limitations on capacity state values the complexity of such algorithm approach depends on traffic demands \(R\) and it is pretty large, increasing exponentially with \(N\).

3.2 Heuristic approach

Heuristički pristup

To reduce the complexity we can introduce the basic and adding properties based on extreme flow theory, see Zangwill [17]. With such algorithm approach we can ensure the close to optimal result, using significantly less effort in computational procedure. In all numerical test-examples it can obtain the best possible result (close to optimal expansion sequence). The key for algorithm improvement is in the fact that extreme flow theory enables separation of these extreme flows which can be a part of an optimal expansion solution from those which cannot be. Any of them, if it cannot be a part of the optimal sequence, is set to infinity.

One may observe that the absence of cycles with positive flows implies that each node has at most one incoming flow from the source node. This result holds for all single source networks. That means that the optimal solution of \( d_u\) has at most one expansion for each capacity type. If conversions exist (as a part of partial expansion option) such flow can be a part of extreme flow only if there are no cycles with positive flows. All basic and adding constraints for the CES problem with three capacity types are identified in the paper by Krile [14] and Krile & Kos [15].

The total number of capacity points \( C_i\) is the measure of the complexity for the CEP-problem. Through many numerical test-examples we compared this heuristic with algorithm on exact approach. Results of one test-example are given in Table 1. For all of them the best possible expansion sequence is achieved. The basic heuristic solution (denoted with Basic_A) with no limitation on capacity state value shows the savings in percents on average near 45 % that is proportionally reflected on computation time savings; see diagram in Fig. 5.

4 Testing of different algorithm options

Testiranje različitih heurističkih varijanti

In real application we normally apply definite granularity of capacity state values. For traffic demand increment values \(R\) we put only discrete values (only integer). It reduces the number of the capacity points significantly. Because of that the minimal step of capacity change (step_I) has strong influence on the algorithm complexity.

However, in real situation we cannot accept Basic_A option because the existence of negative capacity value for more than one capacity type per capacity point (in one period) can block the whole system (traffic congestion). So we have to introduce some limitations on the capacity state values, talking about algorithm options:

a) Only one negative capacity value in the capacity point. Such option is denoted with M_H (Minimal-shortage Heuristic option);

b) Total sum of the link capacity values (for all quality levels) is positive A_H (Acceptable Heuristic option);

c) Total sum is positive and only one value can be negative. Such option is denoted with R_H (Real Heuristic option);

d) Algorithm option that allows only non-negative capacity state values is denoted with P_H (Positive Heuristic option);

e) Only null capacity values are allowed. A trivial heuristic option (denoted with T_H) allows only zero values in capacity point (only one capacity point).

We compared the efficiency of algorithm in above mentioned options. We made many similar numerical test-examples for \(N=3\) and \(T=6\). One of test examples is shown in graphical form in Fig. 4. Elements of cost functions and final results are given in Table 1.
On the top of Fig. 4 we can see traffic demands in the form of traffic increments for each time period and for each of three capacity types. The best expansion sequence is achieved with M_H algorithm option (left side). Further decrease of complexity (near 20 %) occurred; see Table 1. On bottom diagram we can see the presence of capacity surplus (idle capacity) between periods 1 and 2. Also, the shortage exists between periods 4 and 5 with amount of -10 for the second capacity type.

On the right side of Fig 4. we can see the result gained by algorithm options A_H (similar as with R_H), but small deterioration of result exists (<10 %). In that case shortages don’t exist, but complexity is significantly decreased.

Different algorithm options are compared in Fig 5. The average values (trends) of the best possible result (the minimal cost) and algorithm complexity can be seen on the same diagram. Only for few test-examples the best possible expansion sequence can be found no matter the algorithm option we use. For the most test-examples the algorithm option M_H can obtain the best possible result with average

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**Figure 4** On the left side we can see the best possible expansion sequence. On the right side the near-optimal result gained by heuristic option of much lower complexity.

**Figure 5** Trends of algorithm complexity and result deterioration. Values are given in % relatively to referent algorithm.

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complexity savings of more than 75%. For other algorithm options the reduction of complexity is obvious, but significant deterioration of result appears. Only for some test-examples the final results are still in acceptable limits. In most cases the trivial algorithm option \( T_{H} \) shows the significant deterioration and it is not acceptable for real use. A good fact for all algorithm options is that efficiency rises with the increase of value \( T \); see Fig 6.

At the same time it can ensure the fulfillment of traffic demands with minimal exploitation loss caused by the lack of capacity. From further testing results it is obvious that some algorithm options (heuristics) are very effective, with acceptable deterioration of result. They can be efficiently applied to short-term or medium-term satellite link capacity planning for finite number of time periods.

In this paper such CEP algorithm for multiple capacity type allocation on satellite links with mutual traffic correlation is discussed. The network optimization is applied instead of well-known programming technique because the problem recognizes only discrete capacity state values. In this paper the algorithm for three capacity types is tested, representing the method of CEP algorithm development for \( M \) different capacity types. In real application we have to introduce some limitations on the capacity state values, talking about algorithm options. It reduces the algorithm complexity significantly, but still it is so high. In the most test-examples the algorithm option \( M_{H} \) can find the best possible expansion sequence (minimal cost) but with reasonable algorithm complexity. At the same time it can ensure the fulfillment of traffic demands with minimal exploitation loss caused by the lack of capacity. From further testing results it is obvious that some algorithm options (heuristics) are very effective, with acceptable deterioration of result. They can be efficiently applied to short-term or medium-term satellite link capacity planning for finite number of time periods.

### Conclusion

Zaključak

Today, satellite network provider (NP) needs an efficient resource management tool for better utilization of capacities. Continuous increase of traffic (arrangement from service providers - SP) makes NP to install more channel equipment at ground segment. It has to be done in optimal way through a finite planning period, consisting of number of intervals. That problem is in firm correlation with resource allocation to each SP and their planning strategy. Inappropriate bandwidth reservation (lack of bandwidth) could result in congestion possibilities. On the other hand, too much capacity (idle) can cause the huge expenses. In case that different capacity types are in firm correlation it can increase the complexity of the optimization process very much.

In this paper such CEP algorithm for multiple capacity type allocation on satellite links with mutual traffic correlation is discussed. The network optimization is applied instead of well-known programming technique because the problem recognizes only discrete capacity state values. In this paper the algorithm for three capacity types is tested, representing the method of CEP algorithm development for \( M \) different capacity types. In real application we have to introduce some limitations on the capacity state values, talking about algorithm options. It reduces the algorithm complexity significantly, but still it is so high. In the most test-examples the algorithm option \( M_{H} \) can find the best possible expansion sequence (minimal cost) but with reasonable algorithm complexity. At the same time it can ensure the fulfillment of traffic demands with minimal exploitation loss caused by the lack of capacity. From further testing results it is obvious that some algorithm options (heuristics) are very effective, with acceptable deterioration of result. They can be efficiently applied to short-term or medium-term satellite link capacity planning for finite number of time periods.

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