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Optimal Sizinig of Borehole Heat Exchangers

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The heat pumps coupled to the borehole heat exchangers (BHE) can serve for heating and/or cooling of buildings. Optimal sizing of BHE is important because of high drilling costs and importance of matching borehole thermal capacity with the capacity of the heat pump.

Proper design of such systems is usually based on complex mathematical simulations that have to be performend not only for peak building loads, but also for building loads that are calculated throughout the whole year. For this reason the building loads have to be analysed in more detail than for sizing of a conventional system. Designers of heat pump systems coupled to the BHE should also take into acount the long-term temperature changes in the ground surrounding the BHE, which would influence the overall efficiency of the system during its life cycle. The required BHE length is substantially influenced by the thermal conductivity of the ground, which can be only roughly estimated from local geological data. Experimental thermal response tests provide a good estimate of the ground thermal conductivity and borehole thermal resistance. The high cost of the thermal response test reduces its application to commercial and institutional premises with the large number of BHE.

In this work, the overview of calculation methods for the heat pumps coupled to the BHE is presented, with suggestions for optimal sizing of BHE.

Određivanje optimalne veličine vertikalnog izmjenjivača topline u tlu

Prethodno priopćenje

Dizalice topline koje su povezane s izmjenjivačima topline u vertikalnim bušotinama u tlu (ITVB) mogu služiti za grijanje i/ili hlađenje zgrada. Određivanje optimalnih dimenzija ITVB je važno zbog visoke cijene bušenja i zbog usklađivanja toplinskog učinka ITVB s učinkom dizalice topline.

Ispravno projektiranje takvih sustava je obično temeljeno na složenim matematičkim simulacijama koje treba provesti ne samo za vršna toplinska opterećenja zgrade nego i za promjene opterećenja zgrade koje su izračunate za cijelu godinu. Zbog toga toplinska opterećenja zgrada treba analizirati detaljnije nego je to potrebno kod projektiranja konvencionalnih sustava. Projektanti sustava s dizalicama topline koje su povezane s ITVB trebaju također uzeti u obzir dugoročne promjene temperature tla koje okružuje ITVB, a koje će utjecati na ukupnu djelotvornost sustava u njegovom životnom vijeku. Potrebna duljina ITVB bitno ovisi o toplinskoj vodljivosti tla, vrijednost koje se može samo grubo procijeniti na temelju lokalnih geoloških podataka. Eksperimantalno ispitivanje toplinskog odziva bušotine pruža dobru procjenu toplinske vodljivosti tla, a također i toplinskog otpora bušotine. Visoka cijena provedbe ispitivanja toplinskog odziva bušotine ograničava primjenu takvih ispitivanja samo na velike poslovne i institucionalne zgrade, kod kojih se izvodi veliki broj bušotina. U ovom je radu prikazan pregled metoda proračuna dizalica topline koje su povezane s ITVB, s preporukama za optimalno dimenzioniranje ITVB.

Symbols/Oznake

- a thermal diffusivity of the ground, m²/s
 - temperaturna vodljivost tla
- B distance between adjacend boreholes
 - razmak između susjednih bušotina
- D depth of thermally insulated upper part of the borehole, m
 - dubina toplinski izoliranog gornjeg dijela bušotine
- H borehole length over which heat exchange takes place, m
 - duljina bušotine na kojoj postoji izmjena topline
- q heat transfer per length of the borehole, W/m
 - toplinski tok uzduž bušotine
- $r_{\rm h}$ borehole radius, m
 - polumjer bušotine
- r radial distance, m
 - radijalna udaljenost
- R thermal resistance, (m·K)/W
 - toplinski otpor
- $R_{\rm b}$ thermal resistance between fluid and borehole wall, (m·K)/W
 - toplinski otpor između tekućine i obodne plohe bušotine

- $R_{\rm q}$ thermal resistance due to heat extraction step, (m·K)/W
 - toplinski otpor tla narinutom toplinskom toku
- time, s
 - vrijeme
- t_s steady-state time defined by $t_s = \frac{H^2}{9 \cdot a}$, s
 - kriterij stacionarnosti definiran vremenom $t_s = \frac{H^2}{9 \cdot a}$
- $T_{\rm b}$ temperature at the borehole wall, °C
 - temperatura na obodu bušotine
- T_0 average air temperature at ground surface, °C
 - prosječna temperatura zraka na površini tla
- T_{0m} effective undisturbed temperature in ground, °C
- efektivna temperatura tla prije poremećaja
- vertical coordinate, m
 - vertikalna koordinata
- α geothermal gradient, K/m
 - geotermalni gradijent
- γ Eulers constant, (≈ 0.5772)
 - Eulerova konstanta, (≈ 0.5772)
- λ thermal conductivity of ground, W/(m·K)
 - toplinska provodnost tla
- ρ density of ground, kg/m³
 - gustoća tla

1. Introduction

Heat pumps combined with vertical borehole heat exchangers (BHE) use the ground as a heat source or sink for the purpose of heating or cooling of buildings. The depth of the boreholes usually varies between 40 and 200 m, and the diameter between 75 and 150 mm. A heat exchanger usually consists of one or two U-shaped pipes which are inserted inside the borehole. After the U-pipes are inserted, the borehole is backfilled with thermally enhanced grout. The heat transferring fluid recirculates between the BHE and the heat pump evaporater or condenser. In the heating mode the temperature of the ground surrounding the borehole is decreased. In the cooling mode it is increased. In cold climates, where the heating mode dominates, the fluid extraction temperatures usually fall below 0 °C, requiring the addition of antifreeze to the water.

After many years of operation the heat extraction by the borehole tends to reach a steady-state condition in the ground surrounding the borehole. In steady-state the extracted heat is supplied from two major sources. One is a constant geothermal heat flux, which originates from the deep ground below the borehole, and the other is a radial- and time-dependent heat flux that comes from the surroundings over the ground surface. The contribution of

the ground surface heat flux to the borehole heat extraction becomes significant after many years of operation. During the same period of time the heat acumulated in the ground surrounding the borehole is exhausted, and thermal capacity of the ground becomes unimportant. Reaching a steady-state condition practically means that the long-term temperature change in the ground can be neglected and that the thermal capacity of the ground is not an influencing parameter. In the steady-state the ground becomes a fully reversible heat source.

Where there is a signifficant imbalance between the heat that is extracted from the ground during cold season and the heat that is returned to the ground during warm season, the long-term efficiency of the BHE is fundamental for its proper dimensioning.

The actual heat transfer to or from the borehole varies in accordance with changing building energy requirements. The resulting short time-step fluctuations in the supply and return temperatures of the heat transferring fluid typically vary from $5-10\,^{\circ}\text{C}$ over a given day. The low temperatures of the fluid adversely influence the coefficient of performance (COP) of the heat pump. Proper dimensioning of the borehole depends on both short-term and long-term effects. The short-term fluctuations influence temperature changes not only in the vicinity of the borehole but also in the borehole

itself. At hourly and sub-hourly fluctuations, the thermal capacity of the borehole plays a significant role. From the beginning of constant heat injection the approximated time needed to reach the steady-state within the borehole is up to 12 hours.

Theoretical interpretations of the heat transfer phenomena within the ground surrounding the borehole are usually based on Kelvin's (1861) line-source theory. Practical elaboration of this theory was given by Ingersoll (1948, 1954), though providing only rough approximations to the real heat transfer process.

2. The line-source model

The time dependent ground temperature distribution around the borehole heat exchanger can be expressed in polar coordinates:

$$\frac{1}{a} \cdot \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}$$
 (1)

As the ratio of the BHE length to its diameter is of the order of 10³, the line-source model may be used in most simulations that have been developed for the short and medium time scales where axial effects can be neglected.

Mathematical solution, based on the first principles, for the temperature field development around an instantaneous line-source in an infinite solid was found by Carslaw and Jaeger [1] in the form:

$$T(r,t) = \frac{q}{4 \cdot \pi \cdot \lambda} \cdot \int_{\frac{r^2}{4 \cdot a \cdot t}}^{\infty} \frac{e^{-u}}{u} \cdot du = \frac{q}{4 \cdot \pi \cdot \lambda} \cdot \operatorname{Ei}\left(\frac{r^2}{4 \cdot a \cdot t}\right). \tag{2}$$

It is supposed that for the time $t \le 0$ the temperature in the ground is zero and thus Equation (2) gives the perturbation temperature in the ground. The heat source rate q is specified per meter of the line-source, W/m. It is also called "heat extraction step", as it is defined with Heaviside's step-function:

$$q(t) = q_1 \cdot He(t), \quad He(t < 0) = 0, \quad He(t \ge 0) = 0.$$
 (3)

For values of $(a \cdot t) / r^2$ that are large enough the socalled exponential integral Ei can be approximated with:

$$\operatorname{Ei}\left(\frac{r^2}{4 \cdot a \cdot t}\right) \approx \ln\left(\frac{4 \cdot a \cdot t}{r^2}\right) - \gamma \qquad \frac{a \cdot t}{r^2} \ge 5,\tag{4}$$

where $\gamma \approx 0,5772$ is Euler's constant. The maximum error when applying Equation (4) depends on $(a \cdot t) / r^2$, and is 10% for $(a \cdot t) / r^2 = 5$, but falls below 2,5% for $(a \cdot t) / r^2 \ge 20$, [6].

3. Eskilson's long time step model

Eskilson [2] developed a theory based on the linesource model for the long-time temperature distribution in the ground surrounding the borehole. The thermal process in the borehole is separated from the thermal process in the ground by considering the average temperature of the borehole wall. In this model the effect of groundwater filtration and the thermal disturbance at the ground surface are neglected.

Combining an analytical and numerical approach, Eskilson succeeded in mathematically describing the long-time temperature response of the ground around a single borehole in homogeneous ground on a radial-axial coordinate system. In his model the influence of the thermal capacitance and thermal resistance of the grout, pipe walls, and the heat transferring fluid flow is neglected. The temperature response of the borehole field for any borehole configuration is then obtained by using the principle of superposition.

The effective undisturbed ground temperature T_{0m} (the average temperature of the ground around the borehole before the heat transfer takes place) is usually defined with the undisturbed ground temperature at the depth z = D + H/2.

$$T_{0m} = T_0 + \alpha \cdot \left(D + \frac{H}{2}\right). \tag{5}$$

The vertical temperature profile of the ground indicates a permanent heat flow from dispersed heat sources deep in the ground. The value of the geothermal gradient α is typically from 0,03 to 0,05 °C/m.

The mean annual temperature of the ground surface T_0 is taken to be approximately equal $T_{\rm om}$. The uppermost part of the borehole, corresponding to the depth D, is treated as thermally insulated. The average heat exchange between the ground and the borehole is expressed per meter of active borehole length H, given by

$$q(t) = \frac{1}{H} \cdot \int_{D}^{D+H} 2 \cdot \pi \cdot r_{b} \cdot \lambda \cdot \frac{\partial T}{\partial r} \Big|_{r=r_{b}} \cdot dz.$$
 (6)

In this model the borehole wall temperature and thermal properties of the ground are constants over the borehole length. Even in the case of stratified ground the average values of ground properties over the borehole length can be used as constants. With Equation (6) either the heat flow rate q(t) at the borehole wall can be calculated for a prescribed borehole wall temperature $T_{\rm b}(t)$, or the borehole wall temperature can be calculated for a prescribed heat flow rate. The solution of Equation (6) for the simplest case of constant heat flow rate on the borehole wall is expressed in the form:

$$T_{\rm b}\left(t\right) = T_{\rm 0m} - q_{\rm l} \cdot R_{\rm q} \,. \tag{7}$$

In Equation (7) the positive value of q_1 corresponds to heat extraction, and R_q presents the thermal resistance in the ground due to the heat extraction step.

At first, in the ground near the borehole radial thermal process, dominates and the thermal resistance is a time function. With the line-source approximation of the borehole, Equations (2) and (4), follow the expression for the estimated thermal resistance in the ground

$$R_{\rm q}' \approx \frac{1}{2 \cdot \pi \cdot \lambda} \cdot \left[\ln \left(\frac{\sqrt{4 \cdot a \cdot t}}{r_{\rm b}} \right) - \frac{\gamma}{2} \right].$$
 (8)

The nonradial process around the end regions of the borehole can reasonably be neglected since the ratio of the length to the diameter of the borehole is of the order of 10³. For the same reason the borehole can be theoretically treated as the "line source or sink".

In practice, heat extraction step is realized by starting the flow of the fluid with the temperature lower than the ground temperature. The effective thermal resistance between the borehole wall and the fluid, $R_{\rm b}$, is defined by equation

$$T_{\rm b} - T_{\rm f} = q \cdot R_{\rm b}. \tag{9}$$

In order to realize a high heat flow rate with small temperature drop $T_{\rm b}$ – $T_{\rm p}$ the borehole thermal resistance should be low. This can be realized by the application of grouting material with high thermal conductivity, spacing pipes close to the borehole wall, and ensuring that the fluid flow within the pipes is turbulent.

The lower time limit for Equation (8) is based on the criterion for attaining local steady-state within the borehole and its immediate vicinity. This time limit is associated with the borehole radius, so that from the validity limitation of Equation (4) it follows

$$t_{\rm b} = \frac{5 \cdot r_{\rm b}^2}{a} + \frac{2 \cdot \pi \cdot r_{\rm p}^2 \cdot H}{V_{\rm c}} \approx \frac{5 \cdot r_{\rm b}^2}{a} \tag{10}$$

and usually it is a few hours.

At the steady-state condition the temperature change in the ground due to heat extraction (or injection) caused by the borehole can be neglected. Eskilson found steady-state condition to be approximately reached after time t_s , when the borehole wall temperature becomes virtually constant:

$$t_{\rm s} = \frac{H^2}{9 \cdot a},\tag{11}$$

The steady-state time t_s serves as a criterion for the upper limit time for the validity of Equations (4) and (8), and it is typically a few years. At the steady-state the thermal resistance due to the heat extraction step becomes constant:

$$R_{\rm q} = \frac{1}{2 \cdot \pi \cdot \lambda} \cdot \ln \left(\frac{H}{\sqrt{4.5} \cdot r_{\rm b}} \right). \tag{12}$$

Equations (8) and (12) show a very good accuracy for $t \le 0.1 \cdot t_s$ and $t > 10 \cdot t_s$, respectively. The maximum error of up to 7% occurs in the intermediate interval $0.1 \cdot t_s < t < 10 \cdot t$.

For larger times the thermal process in the ground becomes three-dimensional.

Eskilson expressed the results of his analytical and numerical studies of the transient resistant due to heat extraction step with dimensionless g-functions:

$$R_{\rm q} = \frac{1}{2 \cdot \pi \cdot \lambda} \cdot g\left(\frac{t}{t_{\rm s}}, \frac{r_{\rm b}}{H}\right). \tag{13}$$

g-functions represent the non-dimensionalized resistance between the ground and the borehole wall. In his finite differential analysis Eskilson neglected the borehole thermal resistance and capacitance effects, restricting the minimum time to t_b , according to Equation (10). Numerically calculated, borehole wall temperatures allowed him to calculate the corresponding g-functions, using Equation (7).

Introduction of g-functions, defined by Equation (13), which represent the ground thermal resistance and not the borehole thermal resistance, allowed the application of a single long-time step g-function for any borehole geometry and soil conductivity. The variation of g-function with borehole radius is expressed by:

$$g\left(\frac{t}{t_{s}}, \frac{r_{b}^{*}}{H}\right) = g\left(\frac{t}{t_{s}}, \frac{r_{b}}{H}\right) - \ln\frac{r_{b}^{*}}{r_{b}}.$$
 (14)

Using spatial temperature distribution in the ground, resulting from long time influence of a single borehole, Eskilson applied a spatial superposition principle to determine the long-time step g-functions for multiple borehole systems.

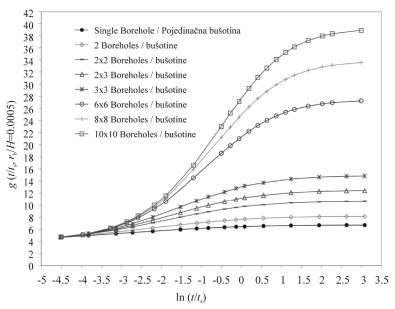


Figure 1. g-functions for a single borehole and multiple borehole configurations [4]

Slika 1. g-funkcije za pojedinačnu bušotinu i za polje bušotina [4]

Figure 1 shows an example of g-functions calculated by Yavuzturk [4] for various multiple borehole configurations, compared to the g-function for a single borehole, for the ratios $r_b / H = 0,0005$ and B / H = 0,1.

4. Short time step models

The local condition around the borehole can substantionally differ from the average values according to fluctuations of the building energy demands. For this reason, Eskilson's model is inadequate for modeling systems with peak-load-dominant loading conditions. Yavuzturk [4] developed a two dimensional numerical

model that calculates the average fluid temperatures in the borehole as a response to heat pulses over subhourly time periods, for a given borehole geometry and known thermal conductivity of homogeneous ground. Numerically, estimated borehole resistance is used to determine the average borehole wall temperature, Equation (9). Short time step g-functions are than calculated with Equations (7) and (13). Figure 2 shows an example of a short time-step g-functions, calculated [4] as extension of the long time-step g-functions.

Introduction of short time-step g-functions requires also new calculating techniques for estimating the borehole thermal response to the arbitrary heat extraction/injection time functions. The methods that are successfully applied in long time-step based models are not applicable in short time-step models because of the huge number of step

functions that have to be superinposed. In order to reduce computation time, various load aggregation schemes are proposed [4, 11].

5. Thermal response test

In practice, the thermal ground properties can be only roughly estimated from the geological data, and the g-functions have to be estimated experimentally. For this purpose the so-called thermal response tests (TRT) are performed. On a test borehole the constant heat pulse (injection or exctraction) is applied, for app. 50 hours. The fluid temperatures are recorded in intervals of 2 to 10

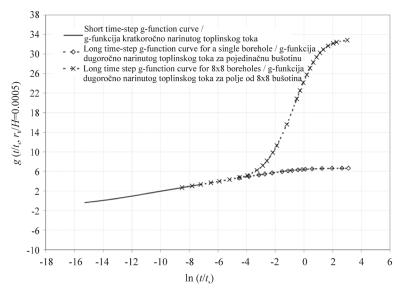


Figure 2. Short time-step g-function curve as an extension of the long time step g-functions plotted for a single borehole and an 8×8 borehole field [4]

Slika 2. g-funkcija kratkoročno narinutog toplinskog toka nadovezana na g-funkciju dugoročno narinutog toplinskog toka za pojedinačnu bušotinu i polje od 8 x 8 bušotina

minutes, and are consequently plotted against ln *t*. From Equations (7), (8) and (9) it follows:

$$T_{\rm f}\left(t\right) = \frac{q}{4 \cdot \pi \cdot \lambda} \cdot \left[\ln\left(\frac{4 \cdot a \cdot t}{r_{\rm b}^{2}}\right) - \gamma \right] + q \cdot R_{\rm b} + T_{\rm 0m}. \tag{15}$$

Supposing that after time t_b the borehole thermal resistance R_b becomes constant, Equation (13) can be rewritten as:

$$T_{\rm f}(t) = k \cdot \ln(t) + C$$
 $k = \frac{q}{4 \cdot \pi \cdot \lambda}$ (16)

The thermal conductivity of the ground can be calculated from the slope k of the curve obtained from the test. As the heat transfer in the ground is not purely conductive, i.e. it comprises the convective component due to the groundwater flow, the values obtained by the TRT are also called "effective thermal conductivities". The groundwater flow is expressed by the Darcyvelocity $m^3/(m^2\cdot s)$, the water flow rate through the free cross-section. It is typically a few meters per second.

The amount of groundwater flow is the main limitation to TRT [7, 8].

6. Heat flow through the ground surface

Radial temperature distribution according to Equation (2) is valid for the endless line-source spaced in an endless solid. Perturbation temperature in an semi-infinite solid can be found [1] by applying the method of images. The zero temperature as boundary condition at the ground surface is obtained [2] by adding a finite mirror line-source above the ground. With such assumptions the resulting time-dependant total heat flow through the ground surface due to the heat extraction is:

$$Q_{\rm s} = q_1 \cdot \sqrt{4 \cdot a \cdot t} \cdot \left(\operatorname{ierfc} \left(\frac{D}{\sqrt{4 \cdot a \cdot t}} \right) - \operatorname{ierfc} \left(\frac{D + H}{\sqrt{4 \cdot a \cdot t}} \right) \right) (17)$$

$$\operatorname{iercf}(x) = \frac{1}{\sqrt{x}} \cdot e^{-x^2} - x \cdot \operatorname{erfc}(x). \tag{18}$$

The fraction of the extracted thermal energy that originates from the ground surface can be expressed by:

$$\eta = \frac{Q_{\rm s}}{H \cdot a}.\tag{19}$$

The transient solution described by Equation (15) is illustrated by the example shown in Figure 3 for a typical borehole with following parameters:

Borehole radius $t_b = 75 \text{ mm}$

Active borehole length H=50 m, 100 m, and 150 m

Thermally insulated upper part of the borehole D=2 m

Effective ground thermal conductivity $\lambda = 2.4 \text{ W/(m\cdot K)}$

Mean ground volumetric thermal capacity $\rho \cdot c_p = 2,55$ MJ/(m³·K)

Design heat extraction rate q = 50 W/m

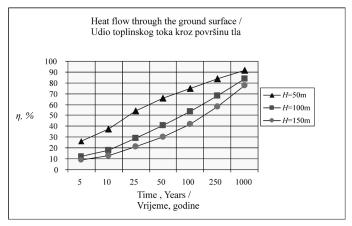


Figure 3. The fraction of the extracted thermal energy that originates from the ground surface for three different borehole lengths

Slika 3. Udio bušotinom izvučene toplinske energije koji potječe od toplinskog toka na površini tla, na primjeru tri dubine bušotine

Heat extraction step q_1 represents the **annual** average heat extraction rate. With annual operating time of 1800 hours follows:

$$q_1 = \frac{1800}{365 \cdot 24} \cdot q = 10.3 \text{ W/m}.$$

The Eskilson's steady time is calculated according to Equation (7) for the three borehole lengths, as shown in *Table 1*.

Table 1. Eskilson's steady time for three borehole lengths **Tablica 1.** Eskilsonov kriterij stacionarnosti na primjeru tri dubine bušotine

H =	m	50	100	150
$t_{\rm s} = \frac{H^2}{9 \cdot a}$	years	7,93	31,7	71,4

Figure 4 shows the average borehole wall temperature that is calculated using the same parameter.

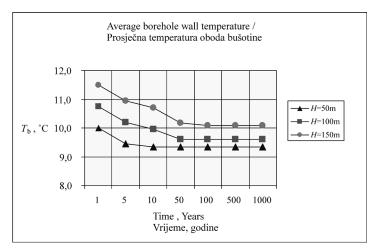


Figure 4. The average borehole wall temperature for three different borehole lengths

Slika 4. Prosječna temperatura oboda bušotine, na primjeru tri dubine bušotine

The yearly averaged heat extraction rate q_1 can be used also in the case of alternating heat extractions and injections. Deeper boreholes are obviously favorable (Figure 4) since higher average temperature on the borehole wall would influence higher average temperature of the fluid in the pipes, and thus greater COP of the heat pump. Designers of GSHP systems should also be aware that the overall performance of the system can continuously be reduced during several decades. In order to ensure the required heat for the heat pump evaporator, lower entering fluid temperature to the BHE and higher pumping rates will be necessary. The pumping power will also be increased because of the higher viscositiv of antifreeze at lower temperature. The higher viscositiy of antifreeze could lead to an impaired heat transfer within the pipes in the case of laminar flow.

7. Conclusion

Optimization of the system consisting of the heat pump coupled to the borehole heat exchanger is based on a detailed analysis of building requirements for heating and cooling, calculated throughout the whole year. Minimizing life cycle costs for the whole system, requires simultaneous modeling of building thermal loads, borehole heat exchanger reaction, and the heat pump efficiency variation with load fluctuation. In cases where the building loads are heating or cooling dominated the attenuation of the borehole thermal capacity after years of operation must be foreseen. In the case of cooling dominated building additional heat sink (e.g. cooling

tower) can help to reduce the load imbalance, but it also must be included in the simulation for optimization. For

> large commercial buildings borehole fields are needed to meet the required thermal capacity. The thermal interaction of the boreholes lowers the ability of borehole to extract or inject heat to the ground. The thermal interaction between the boreholes increases with the number of boreholes in the field and with the time of operation. The proper dimensionig of the borehole heat exchanger depends on the ground thermal properties influencing both the first cost and the life cycle costs. The most important design parameters are the undisturbed ground temperature and its effective thermal conductivity. They can be accurately determined only by "in-situ" performing the thermal response test. To make the ground coupled heat pump systems competitive with conventional systems the borehole heat exchanger must be constructed in such a way as to minimize its thermal

resistance. This can be acheived by spacing plastic pipes close to the borehole wall and applying thermally enhanced grout. The flow of the fluid in pipes must be turbulent. The excessive antifreeze concentration and too large circulation rate of the fluid can cause unnecessary pumping power.

REFERENCES

- [1] CARSLAW, H.S. and JAEGER, J.C.: Conduction of Heat in Solids, Second ed., Oxford University Press, Great Britain, 1959.
- [2] ESKILSON, P.: Thermal Analysis of Heat Extraction Boreholes, Lund-MPh-87/13, Dept. of Mathematical Physics, Lund Institute of Technology, Sweden, 1987.
- [3] CLAESSON, J. and ESKILSON, P.: Condictive Heat Extraction to a Deep Borehole: Thermal Analyses and Dimensioning Rules, Energy Vol. 13, No. 6, pp. 509-527, 1988.
- [4] YAVUZTURK, C.: Modeling of Vertical Ground Loop Heat Exchangers for Ground Source Heat Pump Systems, PhD Thesis, Oklahoma State University, USA, 1999.
- [5] SPITLER, J.D. and CULLIN, J.: Misconceptions Regarding Design of Ground-source Heat Pump Systems, Proceeding of the World Renewable Energy Congress, July 20-25, Glasgow, Scotland, 2008.
- [6] GEHLIN, S.: Thermal Response Test Method Development and Evaluation, Doctoral Thesis,

- Luleå University of Technology, Sweden, 2002.
- [7] SANNER, B. et al.: *Thermal Response Test Current Status and World-Wide Application*, Proceedings World Geothermal Congress, 24-29 April, Antalya, Turkey, 2005.
- [8] SANNER, B. et al.: Technology, Development Status, and Routine Application of Thermal Response Test, Proceedings European Geothermal Congress, 30 May - 1 June, Unteraching, Germany, 2007.
- [9] GEHLIN, S.; HELLSTRÖM, G.: Influence on Thermal Response Test by Groundwater Flow in Vertical Fractures in Hard Rock, Renewable Energy

- 28, pp. 2221-2238, 2003.
- [10] YOUNG, T. R.: Development, Verification, and Design Analysis of the Borehole Fluid Thermal Mass Model for Approximating Short Term Borehole Thermal Response, M.S. Thesis, Oklahoma State University, USA, 2001.
- [11] FISHER, D. E. et al.: Implementation and Validation of Ground-Source Heat Pump System Models in an Integrated Building and System Simulation Environment, HVAC&R Research, Vol. 12, No. 3a, July, 2006.