DETERMINING OPTIMUM SHIP CAPACITY BY APPLICATION OF INVENTORY THEORY IN FREIGHT MANAGEMENT

SUMMARY

The determination of the optimum ship capacity is very important for short and long-term planning of freight management and ship investment. In view of ship operations, maritime transportation depends on the number, size, type and speed of ships in a certain shipping line. For long-term ship-operating situations the ship which has the optimum size can provide the maximum profit. The optimum ship capacity problem can be formulated and solved by the application of inventory management theory to maritime transportation. The optimum ship capacity is determined by minimising the transportation cost objective function. This function depends on the ship capacity variable and has the following parameters: the total amount of cargo transported in unit time between two ports; the distance between two ports; accumulating time; loading and unloading time per ship; investment cost; holding cost per item per unit of time. When the above model is mathematically solved, it is seen that while the optimum ship capacity increases with traffic quantity, transportation distance between two ports, fixed costs and loading and unloading time; it decreases with cost per cargo unit per unit of time.

In this paper, using the inventory management theory, an optimum ship capacity model is developed. For long-term planning, it is important for the ship owners to decide the size of the ship they will purchase. In the second part of the paper, assuming that the size of the ship is found in the first part, short-term planning will be discussed, which consists mainly finding the optimum ship load.

Keywords: Maritime, Transportation, Capacity, Freight, Ship, Inventory

1. INTRODUCTION

Maritime transportation continues to play an important role in the movement of certain types of commodities and is still the dominant form of transportation in international trade. Since maritime transportation is facing fierce competition in freight
transportation, in order to increase the market share and utilise maritime transportation effectively, operation management principles and optimisation and planning techniques should be adopted. Maritime transportation’s unique characteristic is that it is the most energy-efficient and lowest-cost form of transportation (Katz, 1981). The amount of energy needed to overcome friction and to propel a vessel through the water is significantly less than the amount needed on land. This advantage in efficiency is most pronounced for large vessels designed to transport great quantities of cargo. The disadvantage of maritime transportation is that it is slow which makes it more suitable for the movement of low-value commodities such as ore and grain, and liquids such as petroleum. Other modes of transportation with higher speeds are more suitable for commodities with a high value per unit weight such as manufactured goods.

In this paper, it will be shown that it is possible to minimise the total unit transport cost per ton-mile by optimising ship capacity and load in operating ships. The optimum ship capacity and load may be determined by suitable allocation of resources and available facilities (Garrod and Miklius, 1985). In this process, the traffic volume between two ports is considered to be the available data for the demand side. According to the data on the supply side, it is possible to determine the feasible ship capacity and load, ship speed and power and port facilities. In this way, the specified set of available ship operation decisions can be assigned to the resources of each class of traffic. Maritime decisions may be roughly divided into ship and port-operation decisions.

In this paper, the optimum ship capacity model has been developed from inventory theory in operations management. The main purpose in the long run, is to order or purchase ships of optimum size. In section “Formulation of the Ship Load”, the same problem is applied to short-term decisions. In this case, it is assumed that there is an existing ship, and the optimum ship load is formulated for a certain transport line and traffic load conditions.

2. INVENTORY THEORY AND TRANSPORTATION

According to inventory theory, a batch-processing task is planned in production and operating systems. A batch flow system might exist where it is necessary to transfer items from stock to customers in batches rather than in unit quantities.

Likewise, certain transport systems might operate on a batch processing arrangement. Transportation might be feasible only when a certain number of customers are present and also certain quantity of cargo has accumulated in the terminal (Wild, 1981). Thus a batch size decision problem might be initiated by the growth of the queue of customers and the accumulation of cargoes.
2.1. **Determination of batch size**

The normal approach here is to determine batch sizes in which, in some way total cost or unit cost is minimised. Certainly other approaches exist, e.g. maximisation of profits, but the cost minimisation approach is the most important, and is the only approach which will be dealt with in this paper.

Batch quantities which are too large will result in high stock levels and cause a large amount of capital to be tied up in stock which might otherwise invested elsewhere. Additionally, unusually high stock levels will incur other costs, such as the stock cost, keeping insurance and depreciation. On the other hand, batch quantities which are too small will result in the need for a large number of batches to be processed for a given period of time, thus causing large set-up costs.

Perhaps the most important point to note is that, in practice, batch size is rarely constant. It will often be appropriate to split batches during processing and/or to use a batch size, which differs from the theoretical optimum. The discussion of batch size should be taken in this context and the formulae, which are developed, will be used with discretion rather than rigidly.

Figure 1 shows the relationship of these costs and batch size. Clearly the problem is to determine the batch size which minimises total variable costs. Here, the aim is to determine the economic batch size $Q^*$ associated with minimum total to variable cost $C^*$.

First, the deterministic model in which demand, etc., is considered to be known and constant will be dealt with (Dervisiotis, 1981).
To determine the optimum batch size, the cost components, which vary according to batch size, are considered. These costs are as follows:

- stock-holding cost/unit of time:
  \[ \frac{1}{2} C_2 Q \]

- set-up, receiving and storing cost per unit of time:
  \[ C_o \frac{D}{Q} + C_1 D \]

Then, total cost of set-up and holding,
\[ C_r = C_o \frac{D}{Q} + C_1 D + \frac{1}{2} C_2 Q \]  \hspace{1cm} (1)

And then unit cost function, is obtained by dividing the above formula by \( D \):
\[ C = \frac{C_o}{Q} + C_1 + \frac{1}{2} C_2 \frac{Q}{D} \]  \hspace{1cm} (2)

where
\( Q \) = Process batch quantity
\( C_o \) = Set-up or preparation cost/batch
\( C_1 \) = Set-up and holding cost/item
\( C_2 \) = Stock-holding cost item/unit of time
\( D \) = Consumption or demand rate
\( C \) = Unit cost
Differentiating the unit cost function with respect to $Q$ and setting this equal to zero, the size of the batch, which minimises cost, is obtained:

$$Q^* = \left[ 2D \frac{C_1}{C_2} \right]^{1/2}$$  \hspace{1cm} (3)

Substituting $Q^*$ into cost function $C$ gives the minimum total cost per unit of time associated with this processing policy, i.e.

$$C^* = C_1 + \left[ \frac{2C_1C_2}{D} \right]^{1/2}$$  \hspace{1cm} (4)

The above simple model might be extended to include the possibility of stock shortages or stock-outs. This introduces additional cost factor, the cost of shortages. The cost of such shortage in terms of loss of profit, etc., can be introduced into the formula, since it will influence the choice of batch size.

3. OPTIMUM SHIP CAPACITY AND LOAD PROBLEM

3.1. The importance of optimum ship capacity and load for long-term planning

Ship operation in maritime transportation depends on the number, size, type and speed of ships in a certain shipping line (Galal, 1978; Lovric, 1978). In a certain shipping line, optimum ship capacity and load are especially important for short-term and long-term planning decisions which minimise unit transport costs (McCann, 2001). In the shipping line, optimum ship capacity and load depend on the transport distance, traffic over a certain time between two ports, and also investment and constant costs of ships. Therefore, for long-term ship operating situations, the ship which has the optimum size can provide the maximum profit or minimum loss to the shipping organisation. Also the optimum size of a ship can balance the costs of traffic accumulation time, handling and waiting times as related to the direct travel costs and port costs (Jansson and Shneerson, 1982; Jansson and Shneerson, 1985). So the operation situation of the optimum ship load, which has balanced the waiting and accumulating time costs of cargo, can attract cargo from other transportation systems.

Users’ costs depend on all firms operating in the market collectively and are external to any individual firm. Thus, users’ costs includes waiting, and accumulation time costs in addition to shipping company costs.

If these considerations are accepted, then the ship operation and construction
policies in the long-term planning concept can be determined according to the optimum ship capacity in the maritime transportation.

3.2. Formulation of the optimum ship capacity problem

Although costs per ton at sea decrease with ship size, at the same time cost per ton will increase at the port; thus the sum of costs per ton in port and at sea will be minimised at a point which corresponds to the optimum ship size. So the long-term average cost is minimised by optimising simultaneously the size, the frequency of service and the handling and accumulating conditions of cargo (Özen, Güler and Türkay, 1993).

In this formulation, the daily long-term average cost function generally includes the daily port costs and the daily sea costs as explained below:

The daily port costs incurred in port is the sum of daily fuel costs, operating costs, capital costs, crew costs, loading and unloading costs, accumulating and waiting-time costs, handling costs, and other operating costs and port charges.

Similarly the daily costs incurred at sea is the sum of the daily fuel costs, capital costs, crew costs and operating costs.

In this formulation, the ship size variable also depends on the number of operating ships in certain traffic conditions.

Now, let’s consider that there is transit distance between two ports, and there is a certain amount of cargo traffic per unit time. The transportation of this cargo between two ports must be economical with optimum operation supply conditions, and optimum ship capacity is the ship capacity, which minimises the unit cost function. Therefore, the unit cost function, which depends on the ship capacity variable, is formulated for a certain route and/or regular maritime transportation.

In formulation, if the ship capacity, which defines cargo capacity per ship, is denoted by $X$, and the number of ships operating per year is denoted by $n$, then it follows that $n = D/X$ where $D =$ Total annual demand.

3.3. Constant ship cost

As stated in the above section, constant ship cost is made up of the following costs: Ship investment cost, crew cost, maintenance and repairment cost. For each cost, there is a constant part and also another part which depend on the size of the ship:

- Ship investment cost per day is:
In the above formula:

\[ C_s = \frac{1}{300} \left( \frac{(1+i)^N}{((1+i)^N - 1)} \right) (P_p + P_{is} X) \]

\[ = C_{as} + C_{is} X \]

\( P_o = \) Constant part of capital cost of ship,

\( P_{is} = \) Capital unit cost of ship which is a coefficient for varying ship capacity/size.

- Ship crew cost per day can be expressed in a similar way;

\[ C_{sp} + C_{sp} X \]

- Ship maintenance and ship repair costs per day is;

\[ C_{om} + C_{om} X \]

Hence, annual constant ship cost for a certain line can be written as follows;

\[ [C_{as} + C_{sp} + C_{om} + (C_{in} + C_{sp} + C_{om}) X] \left( \tau_o + \tau X + \frac{L}{24V} \right) \frac{D}{X} \]

\[ = (C_o + C_1 X)(\tau_o + \tau X + \frac{L}{24V}) \frac{D}{X} \] (5)

Where:

\( \tau = \) Unit handling time per unit cargo in ports, as day,

\( L = \) Travel distance of sea, as sea miles,

\( V = \) Speed of ship, as sea miles per hour,

\( \tau_o = \) Average waiting and fixed handling time per ship in ports, as day,

\( D = \) Total demand in a year.

3.4. Fuel and diesel oil consumption cost
Cost of fuel and diesel oil consumption at sea and in ports may be formulated as follows:

- In ports the fuel and diesel oil consumption cost is
  \[ C_e (e_{op} + e_{ip}X) (\tau_o + \tau) \]

- At sea the fuel and diesel oil consumption cost is:
  \[ C_e (e_{os} + e_{is}X) \frac{L}{24W} \]

Hence, annual total fuel and diesel oil consumption cost at sea and in ports is:

\[ C_e ((e_{op} + e_{ip}X)(\tau_o + \tau) + (e_{os} + e_{is}X) \frac{L}{24W}) \frac{D}{X} \]  

(6)

Where:

- \( C_e \) = Average price of fuel and diesel oil, as $/ton
- \( e_{op}, e_{os} \) = Constant consumption of fuel and diesel oil, per ship in port and at sea respectively, as ton/day
- \( e_{ip}, e_{is} \) = Unit consumption parameters of fuel and diesel oil per unit cargo and ship capacity in port and at sea respectively, as ton/unit-day.

3.5. Port and handling costs in ports

Cost components of a ship in port and cargo handling can be defined as follows:

\[ [C_s + (C_{as} + C_s)(\tau_{as} + \tau)] \frac{D}{X} \]  

(7)

Where:
$C_b$ = Cost of ship using the port, as $ per ship,
$C_m, C_p$ = Unit operating costs of machine and personnel, respectively, handling the
cargo related to ship, as $ per day per ship,
$\tau_{oh}$ = Constant time of cargo handling

3.6. Time cost of cargo

Accumulating, waiting, handling and travel time costs of cargo between two
ports, generally, can be defined as the following costs.
The accumulating time cost of cargo corresponds to the stock holding cost in
inventory theory as explained previously. So,

- For $T=1$, accumulating time cost of cargo is;

$$C_2 \frac{1}{2D} X^2$$

- The waiting and handling time cost of cargo is;

$$C_2 (\tau_{oh} X + \frac{1}{2} \tau X^2)$$

- The travel time cost of cargo between two ports is;

$$C_2 \frac{L}{24V} X$$

Hence accumulating, waiting, handling and travel time costs of cargo between two
ports, generally, can be defined as follows.

$$C_2 \left[ \frac{1}{2D} X^2 + \tau_{oh} X + \frac{1}{2} \tau X^2 + \frac{L}{24V} \frac{D}{X} \right]$$

(8)

Where:

$C_2$ = Unit time cost per unit of cargo

3.7. Cost function in long-term and optimum ship capacity
Now the total annual cost function between two ports can be written as the summation of the equations (5), (6), (7) and (8).

\[
CT = [(C_o + C_sX)(\tau_o + \tau X + \frac{L}{24V})] + C_s((\varepsilon_{op} + \varepsilon_{os}X)(\tau_o + \tau X) + (\varepsilon_{co} + \varepsilon_{cs}X) \frac{L}{24V}) + C_o + (C_m + C_p)(\tau_{oh} + \tau Y) + C_2\left(\frac{1}{2} + \tau\right)X + \tau_{oh} + \frac{L}{24V}X] \frac{D}{X}
\]

Unit cargo cost function can be obtained by dividing the annual total cost by \(D\) as:

\[
C_u(X) = \frac{CT}{D} = \frac{C_o + C_s \tau_o + \tau X + \frac{L}{24V}}{D} + C_s\left(\frac{\varepsilon_{op} + \varepsilon_{os}X}{X} + \frac{L}{24V}\right) + C_o + (C_m + C_p)(\frac{\tau_{oh}}{X} + \tau) + C_2\left(\frac{1}{2} + \tau\right)X + \tau_{oh} + \frac{L}{24V}
\]

By differentiating the unit cost function with respect to \(X\), optimum ship capacity \(X_o\) is found in the following form, as shown in Figure 2.

\[
X_o = \left[\frac{(C_o \tau_o + \frac{L}{24V}) + C_s(\varepsilon_{op} \tau_o + \varepsilon_{os} \frac{L}{24V}) + C_o + (C_m + C_p) \tau_{oh}}{(C_1 + C_s \varepsilon_{os}) \tau + C_2 \left(\frac{1}{2} + \tau\right)}\right]^{\frac{1}{2}}
\]

Figure 2. Relationship between ship load and unit cost

As shown in the resulting formula, the optimum ship capacity/size varies directly with investment and constant costs, constant handling and waiting times, constant handling costs, transport distance, and traffic load while it varies inversely with
traffic cost per unit time, unit traffic handling time and ship unit traffic costs. This is the direct result of the fact that the marginal cost of these factors changes with size.

In this model, the parameters, which don’t have significant impact on the values of the optimum ship capacity, may be omitted for calculation simplicity.

4. FORMULATION OF THE OPTIMUM SHIP LOAD

The aim of short-term planning (1-2 years) is to load the existing ship to optimal capacity in accordance with short-term fluctuating ship traffic which is taken as $d$. This is more suitable for tramp (irregular) shipping rather than liner shipping. For short-term planning decisions, optimum ship load should be determined according to the parameters of the existing ship capacity, which is constant, although this was not so in the long-term shipping formulation.

In short-term operating planning, since the ship capacity is known, it is assumed to be constant and parameters of ship are as shown below:

\[
C_{ss} = C_{sp} + C_{sp}X_{os}
\]

\[
C_{sp} = C_{op} + C_{op}X_{os}
\]

\[
C_{sm} = C_{om} + C_{om}X_{os}
\]

\[
e_{po} = e_{po} + e_{po}X_{os}
\]
\[ e_{so} = e_m + (e_a - e_{wi})X_m \]

These depend on ship capacity \( X_{so} \) and if \( e_{wi} \) is the energy and diesel oil consumption parameter as ton per day per unit load, then the total cost function can be determined as follows:

\[
CT(X) = (C_{ss} + C_{ap} + C_{sm})(\tau_o + \frac{L}{24V} + \tau X) + C_e(e_{po} + e_{wo})(\tau_o + \tau X) + C_e e_{po} (\tau_o + \tau X)
\]

\[
+ C_e (e_{wo} + e_{so} X) \frac{L}{24V} + C_b + (C_{ms} + C_{p})(\tau_{oh} + \tau X) + C_s \left( \frac{1}{2} \frac{1}{d} + \tau \right) X
\]

\[
+ \tau_{oh} \frac{L}{24V} X \right) \frac{d}{X}
\]

Hence unit cost function depending on variable ship size \( X \) is found as follows:

\[
C_r' (X) = \frac{CT(X)}{d} = \left( (C_{ss} + C_{ap} + C_{sm})(\tau_o + \frac{L}{24V}) + C_e(e_{po} + e_{so} \frac{L}{24V}) + C_b + (C_{ms} + C_{p})\tau_{oh} \frac{1}{X} + (C_{ms} + C_{p})\tau + C_e(e_{po} + e_{so} \frac{L}{24V}) + C_s \left( \frac{1}{2} \frac{1}{d} + \tau \right) \frac{X}{2}
\]

\[
+ C_e (e_{po} \tau_o + e_{so} \frac{L}{24V}) + C_b
\]

In this formula, if above arrangements can be assumed;

\[
C_1 = (C_{ss} + C_{ap} + C_{sm} + C_e e_{po} + C_m + C_{rs})(\tau + C_e e_{so} \frac{L}{24V})
\]

\[
C_{mp} = C_{ss} + C_{ap} + C_{sm}
\]
The above function can be rewritten as:

\[ C_{at}(X) = (C_{zmp}(\tau_o + \frac{L}{24V}) + C_{at} + C_{bp}) / X + C_1 + \frac{1}{2} C_2 (\tau + \frac{1}{d}) X \]

Differentiating this unit cost function with respect to \( X \), the optimum load of a ship operating in certain transport line and traffic load conditions can be determined as follows:

\[ X_o = [2 \frac{C_{zmp}(\tau_o + \frac{L}{24V}) + C_{at} + C_{bp}}{C_2(\tau + \frac{1}{d})}]^{\frac{1}{2}} \]

From this formulation, it is seen that the optimum ship load curve changes with traffic and travel time, and this is shown in Fig. 3 and Fig. 4 respectively.

**Figure 3. Optimum ship load curve related to traffic amount**

**Figure 4. Optimum ship load curve according to travel time**

Hence, minimum unit total cost, \( C_{min} \) can be calculated as follows.

\[ C_{min} = C_{at}(X_o) = C_1 + [2(C_{zmp}(\tau_o + \frac{L}{24V}) + C_{at} + C_{bp} C_2(\tau + \frac{1}{d}))]^{\frac{1}{2}} \]

This minimum total cost \( C_{min} \) changes with traffic and travel time as shown in...
5. CONCLUSION

The model presented in this paper optimises ship capacity and load by using optimum batch size inventory theory in maritime transportation for a given shipping line. The model is general in structure, but is specific in the estimated parameters such as distance between two ports, traffic and ship type. So, it is obvious that for
different parameters with respect to types of ships and rate characteristics, the model will produce numerically different results.

The cost function which optimises simultaneously operator’s and user’s costs makes it possible to trace out the long-term average costs of serving in a given ship-
ping line.

It is possible to use ships with smaller capacities by decreasing the waiting and handling periods in ports.

The optimum ship load in changing cargo traffic conditions can be a criterion for users, operators and designers.

In application, statistical data and parameters, which are specified by statistic analysis and calculations, should be used. The result should be obtained by iteration method.

As a result, the presented model enables an economic ship transportation with minimum and balanced operators and users’ costs. The alterations of shipping line conditions and parameters allow the enlargement of the area of ship capacity and load, which can meet the real operating and transportation demand spectrum.

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UTVRĐIVANJE OPTIMALNOG BRODARSKOG KAPACITETA PRIMJENOM TEORIJE INVENTARA U UPRAVLJANJU BRODSKIM PROSTOROM

SAŽETAK

Utvrđivanje optimalnoga brodskog kapaciteta vrlo je značajno i za kratkoročno i za dugoročno planiranje upravljanja teretom i brodskim investicijama. Sa stajališta poslovanja broda, pomorski prijevoz ovisi o broju, kapacitetu, vrsti i brzini brodova na određenoj liniji. U dugoročnom poslovanju brodova, brod optimalne veličine donosi i najveću dobit. Problem optimalnoga brodskog kapaciteta može se formulirati i riješiti primjenom teorije upravljanja osnovnim sredstvima na pomorski prijevoz. Optimalni kapacitet broda određuje se smanjivanjem objektivne funkcije prijevoznih troškova na najmanju moguću mjeru, a ta funkcija ovisi o varijabli brodskoga kapaciteta i obuhvaća naredne parametre: ukupnu količinu prevezenoga tereta u jedinici vremena između dviju luka, akumulirano vrijeme, vrijeme trajanja ukrcaja i iskrcaja po brodu, investicijske troškove, troškove održavanja po pojedinim točkama u jedinici vremena.

Kad se matematički riješi navedeni problem, vidi se da dok optimalni brodski kapacitet raste s porastom prometa, općim troškovima i trajanjem ukrcaja i iskrcaja, on pada s troškovima po jedinici tereta u jedinici vremena. U ovome se radu, temeljem teorije upravljanja osnovnim sredstvima, razvija model optimalnoga brodskog kapaciteta. Za dugoročno planiranje je važno da brodari utvrde kapacitet broda koji se namjerava nabaviti. U drugome dijelu rada, pod pretpostavkom da je veličina broda određena u prvome dijelu, raspravlja se o kratkoročnom planiranju koje se bavi uglavnom optimalnim opterećenjem broda.

Ključne riječi: pomorski, prijevoz, kapacitet, teret, brod, osnovno sredstvo

COME STABILIRE LA CAPACITÀ OTTIMALE DELLA NAVE APPLICANDO LA TEORIA INVENTORIALE NELLA GESTIONE DEI NOLI

SOMMARIO

Per programmare a breve e a lunga scadenza la gestione dei noli e d’impiego di una nave è di massima importanza stabilire la portata ottimale della stessa. Dal punto di vista operativo, il trasporto marittimo dipende dal numero delle navi, dalle loro dimensioni, dal tipo e velocità e dal
l’impiego in una determinata linea marittima. In situazioni operative di lunga scadenza una nave di dimensioni ottimali può assicurare il massimo del profitto. Il problema della portata ottimale della nave può essere espresso e risolto grazie alla teoria della gestione inventoriale applicata al trasporto marittimo. La portata ottimale della nave viene determinata riducendo la funzione oggettiva dei costi di trasporto. La funzione dipende dalla variabile di portata della nave e comprende i seguenti parametri: totale complessivo del carico trasportato per unità di tempo tra due porti; distanza tra i due porti; totale complessivo del tempo; tempo impiegato per il carico/scarico per nave singola; costi operativi della nave; costo di ritenuta per collo per unità di tempo. La soluzione matematica del suindicato modello ci indica come la portata ottimale della nave tende a crescere con il volume del traffico, la distanza tra due porti, i costi fissi, il tempo di carico/scarico e a diminuire con il costo per unità di carico per unità di tempo. La prima parte del saggio elabora il modello di portata ottimale della nave grazie all’applicazione della teoria di gestione inventoriale. Nella programmazione a lunga scadenza per gli armatori che intendono acquistare una nave è di rigore un scelta accurata delle sue dimensioni. La seconda parte del saggio descrive la programmazione a breve scadenza che consiste principalmente nella operazione di carico ottimale della nave.

Parole chiave: marittimo, trasporto, capacità, noleggio, nave, inventario