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Developed procedure for dynamic reanalysis of structures

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Structural dynamic modification techniques can be defined as methods by which dynamic behavior of a structure is improved by predicting the modified behavior brought about by adding modifications like those of lumped masses, rigid links, dampers, beams etc. or by variations in the configuration parameters of the structure itself. The methods of structural dynamic modification, especially those with their roots in finite element models, have often been described as reanalysis. The present paper deals with the problem of improving of dynamic characteristics some structures. New dynamic modification procedure is given as using distribution of potential and kinetic energy in every finite element is used for analysis.
The main goal of dynamic modification is to increase natural frequencies and to increase the difference between them. Some information should be prepared, before setting up the FE model. The first pack of information includes referent peaces of information about the structure: size, material, and boundary conditions.

Razvijena procedura za strukturalnu dinamičku reanalizu

Izvornoznanstveni članak

U ovom radu je razvijena procedura za popravljanje dinamičkog ponašanja strojarskih konstrukcija u eksploataciji. U osnovi ove procedure je distribucija kinetičke i potencijalne energije na glavnim oblicima osciliranja konstrukcije. Inače, tehnika strukturne dinamičke modifikacije može se definirati kao skup metoda pomoću kojih se dinamičko ponašanje konstrukcije može popraviti procjenom modificiranog ponašanja dobivenog dodavanjem modifikacija kao na primjer koncentriranih masa, krutih veza, prigušenja, novih elemenata, i sl. ili promjenom konfiguracijskih parametara u samoj strukturi. Takve metode kod kojih je osnova metoda konačnih elemenata često se nazivaju metode reanalize. Jedan od osnovnih ciljeva u ovom radu, s obzirom na gore spomenuto, jest da se u metode reanalize ugradi još jedna s jasno predstavljenom procedurom korištenja.

1. Introduction

Fundamental items in design requirements for a structure are its purpose and functionality. The development of materials with superior properties in exploitation conditions and introduction of high technology has enabled us to extend the design requirements for now-a-days structures to structural integrity, reliability and life specification, in order to reduce, if not avoid, failure and damage in exploitation. Meeting such complex design requirements is a difficult task. One of the most important influencing factors is external load, in combination with structural response to external load. In the case of more or less uniform loading of static type structural strength and deformation have to be maintained under control. However, situation is more complex when the structure is also exposed to dynamic loading, as is the case in machining tools and manufacturing systems, complex manufacturing systems in mines and electrical energy production, vehicles, rail-road vehicle, aircrafts and helicopters, space engines, especially when operating under high revolution rates. In these cases design requirements must specify dynamic properties such as vibration level, resonance range, response properties, eigenvalues, dynamic stability and modal forms. Very important in that sense are shake (trembling) occurrence and buckling control through optimization.

Symbols/Oznake

$[M]$	- mass matrix of structure, kg - globalna matrica masa strukture
$[K]$	- stiffness matrix of structure, k·N/cm - globalna matrica krutosti strukture
λ_i	- eigenvalues, 1/ s^2 - vlastite vrijednosti
$\{\underline{Q}\}_{\mathrm{i}}$	- eigenvectors, cm - vlastiti vektori
$\Delta\lambda_i$	- changes of eigenvalues, 1/ s^2 - prirasti vlastitih vrijednosti
$\{\Delta\underline{Q}\}_{\mathrm{i}}$	- changes of eigenvectors, cm - prirasti vlastitih vektora
$[\Delta K]$	- changes of stiffness matrix of structure, k·N/cm - prirasti globalne matrice krutosti strukture
$[\Delta M]$	- changes of mass matrix of structure, kg - prirasti globalne matrice mase strukture
E_{k}	- kinetic energy , J - kinetička energija
E_{p}	- potential energy , J - potencijalna energija
$\left\{q_r^s\right\}_e$	- the corresponding r-th eigenvector of, e-th element with s degrees of freedom, cm - odgovarajući r-ti vlastiti vektor, e-tog elementa sa s stupnjeva slobode
$(e_{p,r})_e$; $(e_{k,r})_e$	- potential and kinetic energy of e-th element on its r-th main oscillation mode, J - potencijalna i kinetička energija e-tog elementa na r-tom glavnom obliku osciliranja

Observed deviations in exploitation behavior of a structure have to be eliminated. When structural dynamic problems in exploitation are considered, the so-called re-analysis should be applied. It is defined as an ensemble of methods and techniques that enable introduction of improvements in dynamic response of a structure, applying primarily finite elements method (FEM), by modifying the properties and parameters of a structure.

2. Development of reanalysis procedures

The development of new, simple procedure for structural reanalysis of mechanical system must be capable of modifying its dynamic properties while achieving requested characteristics with conveniently fast convergence of the whole process. The analytical method with a clear concept, based on distribution of kinetic and potential energies is used for modifying the dynamic properties of main vibration (oscillation) forms, occurring in individual components or in grouped assemblies. The analysis of complex structures begins with initial rough analysis of a structure that is followed by the analysis of grouped structural assemblies. The final phase is the precise analysis based on sensitivities of individual elements. The selection of structural parameters for dynamic properties improvement through eventual modification according to energy distribution includes geometry, supporting system and material characteristics. Based on this approach the corresponding algorithm is proposed and applied. One of the important performances of this algorithm is convenience for rational implementation in computer systems, using appropriate software. In this way relevant data for structural system dynamic response during reanalysis can be obtained and considered in the optimization.

Basic theory for determining the existence of solution for frame structure optimization with frequency limits is found in [1]. According to this theory, natural frequencies do not change with uniform frame modification and key limitation for determination of optimal dynamic solution of frame structure modification is mostly that of eigenfrequencies. The optimization criteria for space frame structure with multiple limitations in its natural frequencies is considered in [2]. Knott coordinates and cross sections of elements, although of different nature, have been treated simultaneously in unified design specification for a minimum weight of structure. Optimum first criterion, developed for one limitation based on differentiation of Lagrange function, indicates that at optimum all the variables are of the same efficiency. In order to solve multiple limitations of frequencies global numbers are introduced, avoiding in this way the calculation of Lagrange's multiplicators.

In final stage, the most efficient variables are identified and modified as priority. Using the minimal weight increment, optimal solution can be obtained from initial design solution. The procedure is also effective for repeated values of frequency. In paper [3] the model for modified dynamic structural system is presented, based on reduced appreciative concept of improved method for approximation of eigenvalues and eigenvectors of first order. The expressions for local approximation based on Taylor's series are used as base vectors for eigenparameters perturbance approximation. Reduced system of eigenvalues is generated for each eigenvector using eigenvectors as ~~a~~ base and Ritz's vector approximation of first order. The equations for reanalysis are algebraic [4]. New function to limit eigenvalues approximation in procedure of structural optimization is introduced in [5]. Applied Reyli ratio increases approximation quality for frequency limitations since it approximates eigenforms energy and kinetic energy instead eigenvalues, producing faster and stable convergent solutions.

The application of iterative method for sensitivity determination in reanalysis of structure due to small perturbances of design variables is applied in numerical procedure, discussed in [6]. In this paper the algorithms for displacements and stresses are given, as well as for eigenvalues and forms. Scheme of iteration is modified saving matrix coefficients as constant and using only one decomposition. Implementation of algorithm is simple, and the convergence fast. The extension of the method to the sensitivity of eigenfrequencies with repeated values is convenient to avoid the conditions of matrix coefficients close to bifurcation points, which occurs when non-linear response of a structure is considered.

It should be noticed that dynamic response is given primarily through corresponding eigenfrequencies and main oscillation forms as characteristic (typical) variables. Changing them by changing the design parameters of a structure it is possible to achieve (can bring about) requested structural dynamic response.

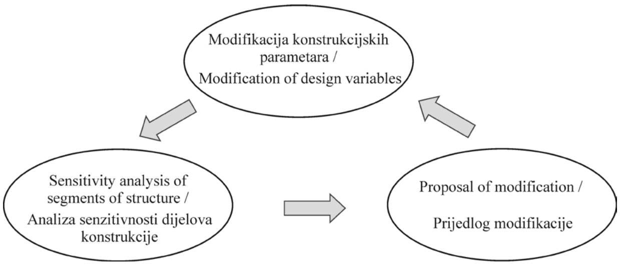

Figure 1. Triangle of dynamics reanalysis
Slika 1. Trokut dinamičke reanalize

Sensitivity analysis is an important point within the *dynamical modification* process. Sensitivity analysis represents a collection of mathematical methods for reanalyzing constructions which is, within dynamical modification, related to sensitivity of eigenvalues and eigenvectors. Therefore, the application of sensitivity analysis is limited to construction of segments for which necessary mathematical relations can be determined. If this is not possible, sensitivity analysis is only partially applicable. Dynamical analysis of complex structures can easily be conducted via finite elements modeling. Therefore, while finite element analysis method is highly adequate for modeling complex structures, one of its major drawbacks lies in the usage of large number of degrees of freedom in calculating the exact eigenpairs. This number can amount to few tens of thousands, or even more. To reduce the calculation time it is possible to divide the complex structure into connected substructures and analyze each one separately. The dynamical behavior of each substructure is represented only by a reduced set of eigenpairs of interest, which contributes to significant problem simplification. A more general problem of structural dynamic analysis has three important aspects. *Firstly*, the observed physical structure is represented by initial finite element model. Modeling is based on numerous idealizing approximations within an exaggerated elaboration of details, which in essence does not significantly improve the accuracy of output data, especially having available powerful computers and appropriate software packages. Optimal alternative is to have the possibility of verifying outputted data that were measured on a prototype or real structure. *Secondly*, the dynamic characteristics of construction under reanalysis are analyzed. What is basically observed are eigenvalues and main forms of oscillations as characteristic variables that can invoke inadequate actual dynamic behavior. *Thirdly*, on the basis of analysis of actual dynamic behavior, modification steps are proposed after which a modified model is obtained. Having in mind that mechanical structures are most often very complex, the most convenient modification steps are not easily obtained. Figure 1 shows a simplified triangle of fine reanalysis. Choosing the structural parts most suitable for reanalysis requires the analysis of sensitivity for separate segments to changes in construction. Most importantly, the best result should be obtained with minimal changes. This most frequently involves the increase in frequency and distance between two neighboring frequencies. Often, frequency requirements (limitations) are imposed in order to avoid coupling (resonance) with the control system. In other cases, structural elements such as bearings or sensitive components of electronic system must have lowest possible amplitudes. Dynamical modification usually focuses on frequency modifications. Reanalysis in structural dynamics, therefore, involves a problem of showing that the modification is necessary and how to achieve the required solution.

2.1. Problem setup

Sensitivity examination on real examples still presents a troublesome task so that analysis of distribution of specified values is performed instead. The distribution of optimization elements is an example of reanalysis which is expressed in percentages of specified quantities from a chosen group of elements. By posing the task of obtaining desired first or other eigenfrequency of a system, construction analysis often requires calculation of a great number of construction alternatives. However, through reanalysis, which is based on balancing the distribution of kinetic and potential energy of all finite elements in the model, it is possible to efficiently obtain the desired eigenfrequencies of a system.

The distribution of potential and kinetic energy over principal modes of oscillation

The matrix form of differential equations of motion for a system that is not subjected to external forces is:

$$[M]\left\{\underline{Q}_i(t)\right\}+[K]\left\{\underline{Q}_i(t)\right\}=\{0\}. \tag{1}$$

Eigenvalue of this differential equation for i-th mode is:

$$[M]\left\{\underline{Q}_i(t)\right\}+[K]\left\{\underline{Q}_i(t)\right\}=\{0\} \tag{2}$$

If this equation is multiplied from left side by transposed value of i-th vector and divided by 2, we obtain the equation of balance for potential and kinetic energy of a construction in main modes of oscillation:

$$\frac{1}{2}\left\{\underline{Q}_i\right\}^T[K]\left\{\underline{Q}_i\right\}=\frac{1}{2}\lambda_i\left\{\underline{Q}_i\right\}^T[M]\left\{\underline{Q}_i\right\}. \tag{3}$$

Therefore, the potential energy of a construction on r-th main oscillation mode, having in mind the previous equation, can be rewritten as:

$$E_{p,r}=\frac{1}{2}\left\{\underline{Q}_r\right\}^T[K]\left\{\underline{Q}_r\right\}. \tag{4}$$

while the kinetic energy in this case becomes:

$$E_{k,r}=\frac{1}{2}\lambda_r\left\{\underline{Q}_r\right\}^T[M]\left\{\underline{Q}_r\right\}, \tag{5}$$

where λ_r- is the r-th eigenvalue, and $\{\underline{Q}_r\}$ - is the r-th eigenvector for the construction.

From equation (3), as well as theoretical overview given in previous paragraph, follows a principle of total energy conservation on main oscillation modes:

$$E_{p,r}=E_{k,r}=E_r. \tag{6}$$

If the construction is discretized to N finite elements, kinetic and potential energy of the entire construction can be rewritten as algebraic sum of energies of all elements in the following way. Given that $\left\{q_r^s\right\}_e$ - is the corresponding r-th eigenvector, of e-th element with s degrees of freedom,

$\left(e_{p,r}\right)_e=\frac{1}{2}\left\{q_r^s\right\}_e^T[k]_e\left\{q_r^s\right\}_e$ - potential energy of e-th element on its r-th main oscillation mode,

$\left(e_{k,r}\right)_e=\frac{1}{2}\omega_r^2\left\{q_r^s\right\}_e^T[m]_e\left\{q_r^s\right\}_e$ - kinetic energy of e-th element on r-th main oscilation mode, then the total kinetic energy of construction on r-th main oscilation mode can be represented with the following sum:

$$E_{k,r}=\sum_{e=1}^{N}\left(e_{k,r}\right)_e=\frac{1}{2}\sum_{e=1}^{N}\omega_r^2\left\{q_r^s\right\}_e^T[m]_e\left\{q_r^s\right\}_e. \tag{7}$$

Analogously, potential energy of construction in r-th main oscillation mode can be also represented with the following sum:

$$E_{p,r}=\sum_{e=1}^{N}\left(e_{p,r}\right)_e=\frac{1}{2}\sum_{e=1}^{N}\omega_r^2\left\{q_r^s\right\}_e^T[k]_e\left\{q_r^s\right\}_e. \tag{8}$$

On the basis of distribution of kinetic and potential energy, expressed in percentages, a group of elements suitable for dynamic analysis can roughly be selected.

2.2. Modification of dynamic parameters

Given that $[\Delta K]$ and $[\Delta M]$ are the corresponding changes in rigidity and mass matrices respectively, then the formula (2) can be applied to the modified system and so called modified[1] equation for the case of free oscillations is: $[K]'\left\{\underline{Q}_i\right\}'=\lambda_i'[M]'\left\{\underline{Q}_i\right\}'$, where we have substituted

$$\begin{gathered}[K]=[K]'-[\Delta K],\ [M]=[M]'-[\Delta M],\\ \left\{\underline{Q}_i\right\}=\left\{\underline{Q}_i\right\}'-\left\{\Delta\underline{Q}_i\right\},\quad \lambda_i=\lambda_i'-\Delta\lambda_i,\end{gathered} \tag{9}$$

where $\Delta\lambda_i$ and $\{\underline{Q}\}_i$ are changes of eigenvalues and eigenvectors, respectively. Now, the equation of the original, nonmodified system $[K]\{\underline{Q}\}_i=\lambda_i[M]\{\underline{Q}\}_i$ can be rewritten as:

$$\begin{aligned}&\left([K]'-[\Delta K]\right)\left(\left\{\underline{Q}_i\right\}'-\left\{\Delta\underline{Q}_i\right\}\right)=\\ &=\left(\lambda_i'-\Delta\lambda_i\right)\left([M]'-[\Delta M]\right)\left(\left\{\underline{Q}_i\right\}'-\left\{\Delta\underline{Q}_i\right\}\right).\end{aligned} \tag{10}$$

The previous equation is the equation of third order with respect to "modified" members, and if the equation of balance for potential and kinetic energy of a construction (3) is rewritten in its "perturbed" form, a fourth order equation is obtained:

[1] A commonly used term in literature for modified equation is "perturbation equation".

$$\left(\left\{\underline{Q}_i\right\}+\left\{\Delta\underline{Q}_i\right\}\right)^T([K]+[\Delta K])\left(\left\{\underline{Q}_i\right\}+\left\{\Delta\underline{Q}_i\right\}\right)= \\ =\left(\lambda_i+\Delta\lambda_i\right)\left(\left\{\underline{Q}_i\right\}+\left\{\Delta\underline{Q}_i\right\}\right)^T([M]+[\Delta M])\cdot \\ \cdot\left(\left\{\underline{Q}_i\right\}+\left\{\Delta\underline{Q}_i\right\}\right) \tag{11}$$

Assuming that the changes in the construction are small, it can be expected that the changes of values of vectors of eigenvalues and eigenvectors will also be small. Therefore, the higher order members in the following equations can be neglected [10]:

$$([K]'-[\Delta K])\left(\left\{\underline{Q}_i\right\}'-\left\{\Delta\underline{Q}_i\right\}\right)= \\ =\left(\lambda_i'-\Delta\lambda_i\right)([M]'-[\Delta M])\left(\left\{\underline{Q}_i\right\}'-\left\{\Delta\underline{Q}_i\right\}\right)$$

$$[K]'\left\{\underline{Q}_i\right\}'-[K]'\left\{\Delta\underline{Q}_i\right\}-[\Delta K]\left\{\underline{Q}_i\right\}'+ \\ +[\Delta K]\left\{\Delta\underline{Q}_i\right\}=\lambda_i'[M]'\left\{\underline{Q}_i\right\}'- \\ -\lambda_i'[M]'\left\{\Delta\underline{Q}_i\right\}-\lambda_i'[\Delta M]\left\{\underline{Q}_i\right\}'- \\ -\Delta\lambda_i'[M]'\left\{\underline{Q}_i\right\}'+\ldots \tag{12}$$

Having in mind the following relationships

$$[K]'\left\{\underline{Q}_i\right\}'=\lambda_i'[M]'\left\{\underline{Q}_i\right\}', \\ [K]'\left\{\Delta\underline{Q}_i\right\}=\lambda_i'[M]'\left\{\Delta\underline{Q}_i\right\}, \tag{13}$$

keeping only the members of first order, the equation (12) becomes a modified equation of first order:

$$[\Delta K]\left\{\underline{Q}_i\right\}'\approx\lambda_i'[\Delta M]\left\{\underline{Q}_i\right\}'+\Delta\lambda_i[M]'\left\{\underline{Q}_i\right\}'. \tag{14}$$

If the previous equation is multiplied from left side by the half of transposed value of i-th eigenvector, the following formula is obtained:

$$\frac{1}{2}\left\{\underline{Q}_i\right\}'^T[\Delta K]\left\{\underline{Q}_i\right\}'\approx\frac{1}{2}\lambda_i'\left\{\underline{Q}_i\right\}'^T[\Delta M]\left\{\underline{Q}_i\right\}'+ \\ +\frac{1}{2}\Delta\lambda_i\left\{\underline{Q}_i\right\}'^T[M]'\left\{\underline{Q}_i\right\}', \tag{15}$$

from which it is possible to express the change of i-th eigenvalue under system modification, which was the final purpose of this procedure:

$$\frac{\Delta\lambda_i}{\lambda_i'}=\frac{\frac{1}{2}\left\{\underline{Q}_i\right\}'^T[\Delta K]\left\{\underline{Q}_i\right\}'-\frac{1}{2}\lambda_i'\left\{\underline{Q}_i\right\}'^T[\Delta M]\left\{\underline{Q}_i\right\}'}{\frac{1}{2}\lambda_i'\left\{\underline{Q}_i\right\}'^T[M]'\left\{\underline{Q}_i\right\}'}. \tag{16}$$

The previous formula can be considered as *basic expression in construction reanalysis* aimed at improving dynamic characteristics. The expression in the nominator represents the difference of increases in potential and kinetic energy between modified and unmodified states. Since the increase in i-th eigenvalue is directly proportional to this difference, each member of nominator is of vital importance for analysis, which will be shown in detail further in the text.

Another important question arises from analyzing the previous formula. The designations " ′ " depict the values which are related to the modified state. Often, due to large size of a certain problem, it is not possible to easily obtain those values. If those changes are small which is a prerequisite for obtaining accurate solution it is possible, with great degree of reliability, to use the expression with values that are related to unmodified system:

$$\frac{\Delta\lambda_i}{\lambda_i}=\frac{\frac{1}{2}\left\{\underline{Q}_i\right\}^T[\Delta K]\left\{\underline{Q}_i\right\}-\frac{1}{2}\lambda_i\left\{\underline{Q}_i\right\}^T[\Delta M]\left\{\underline{Q}_i\right\}}{\frac{1}{2}\lambda_i\left\{\underline{Q}_i\right\}^T[M]\left\{\underline{Q}_i\right\}}, \tag{17}$$

$$E_{p,r}=\frac{1}{2}\left\{\underline{Q}_r\right\}^T[K]\left\{\underline{Q}_r\right\},$$

$$E_{k,r}=\frac{1}{2}\lambda_r\left\{\underline{Q}_r\right\}^T[M]\left\{\underline{Q}_r\right\},\ E_{p,r}=E_{k,r}=E_r.$$

The expression in the denominator of equation (17) represents the kinetic energy of a certain oscillation mode and having in mind equation (3), it also represents the potential energy, for reasons of energy balance in main oscillation modes.

If the modification is performed on e-th finite element, the matrices of mass and stiffness for that finite element become:

$$[k]'_e=[k]_e+[\Delta k]_e=[k]_e+\alpha_e\,[k]_e, \\ [m]'_e=[m]_e+[\Delta m]_e=[m]_e+\beta_e\,[m]_e, \tag{18}$$

where α_e and β_e are values that define the modification of e-th element, and are names *modification coefficients*. In this case, the members of stiffness matrices and mass matrices within the matrices of construction parameters are all equal to zero except for those corresponding to e-th finite element, so that the nominator in equation (17) for r-th oscillation mode becomes

$$\frac{1}{2}\left\{\underline{Q}_r\right\}^T[\Delta K]\left\{\underline{Q}_r\right\}-\frac{1}{2}\lambda_r\left\{\underline{Q}_r\right\}^T[\Delta M]\left\{\underline{Q}_r\right\}= \\ =\frac{1}{2}\alpha_e\left\{q_r^s\right\}_e^T[k]_e\left\{q_r^s\right\}_e-\frac{1}{2}\beta_e\lambda_r\left\{q_r^s\right\}_e^T[m]_e\left\{q_r^s\right\}_e \\ =\frac{1}{2}\left(\alpha_e e_{p,r}-\beta_e e_{k,r}\right), \tag{19}$$

where:

$\left\{q_r^s\right\}_e$ - is the corresponding r-th eigenvector of , e-th element with s degrees of freedom,

$e_{p,r} = \frac{1}{2}\left\{q_r^s\right\}_e^T \left[k\right]_e \left\{q_r^s\right\}_e$ - is the potential energy of e-th element in r-th main oscillation mode without constructional modification,

$e_{k,r} = \frac{1}{2}\omega_r^2 \left\{q_r^s\right\}_e^T \left[m\right]_e \left\{q_r^s\right\}_e$ - is the kinetic energy of e-th element in r-th main oscillation mode without constructional modification.

With this analysis, the formula (17) can be written as:

$$\frac{\Delta\lambda_i}{\lambda_i} = \frac{\frac{1}{2}\left\{\underline{Q}_i\right\}^T [\Delta K]\left\{\underline{Q}_i\right\} - \frac{1}{2}\lambda_i \left\{\underline{Q}_i\right\}^T [\Delta M]\left\{\underline{Q}_i\right\}}{\frac{1}{2}\lambda_i \left\{\underline{Q}_i\right\}^T [M]\left\{\underline{Q}_i\right\}} = \frac{\alpha_e e_{p,r} - \beta_e e_{k,r}}{E_{k,r}}. \qquad (20)$$

Formula (20) shows the influence of separate finite elements to the increase of eigenvalue. If the energy distribution over groups of elements is expressed in percentages for each main oscillation mode, it is possible to obtain rough information that can be used in modification. The basic goal of dynamic modification is to increase the eigenvalues and their distances. The formula (20) is important for understanding the procedure that requires modification of a certain construction. The denominator of previous formula is not changed in the procedure of modification so therefore the main point of analysis is placed on the nominator. The modification of construction assumes the change in only but few segments that are most responsive to change. This sensitivity is expressed in the fact that the change of certain construction parameters of these segments will result in greatest difference in the nominator of previous formula, and consequently in greatest effect on increasing the observed eigenfrequency of the system. Since the observed constructions are already in exploitation, the essence of improving dynamic behavior is to achieve maximum change with minimal "intervention". The question is how to determine segments or substructures of a construction that are most sensitive to small changes in their parameters?

3. The developed procedure of dynamic modification

The problem of dynamic modification of a construction with the goal of improving dynamic characteristics has been a worldwide challenge for many researchers in previous decades.

The methods thereby used are widely different, from strictly mathematical do entirely experimental. Dynamic response of a mechanical structure must be improved by either (i) load control, or (ii) change in dynamic characteristics of a structure. Loads are often the result of interaction of the structure and its environment, so they are not easily controlled. In that case it is important to know that the dynamic response can be improved by redesigning (reanalyzing) the dynamical characteristics of the structure. Having this in mind, the application of the techniques of reanalysis in obtaining the desired conditions for FE model of mechanical structures has showed a rapid improvement in previous decades. There are numerous techniques that are applied in dynamic reanalysis of mechanical structures. One of them is, already mentioned, *sensitivity analysis* that is successfully applied in general as well as in specific dynamical problems. The success of the process of dynamical modification depends on many factors, most important of which are: complexity of a structure including the boundary conditions, and modification method that a research team will choose to apply.

Although many papers have been published in the area of dynamic modifications of constructions, the methodology of modification (reanalysis) of constructions is still under intense development. In this paper a procedure for dynamical modification that can be successfully applied to all types of constructions is presented. Dynamic reanalysis is most often used in real structures that have poor dynamic behavior in exploitation. Successful "reparations" require a proper dynamic analysis and behavior diagnostics of observed structure. Application of results obtained by construction reanalysis achieves, among other results, prolongation of the life cycle of a construction.

Dynamic analysis and diagnostics of model and its groups

Dynamic analysis and diagnosis of model implies the analysis and interpretation of model behavior and its modification. On the basis of analysis of energy distributions in main oscillation modes for all construction elements, following cases are observed, on the basis of which it is possible to derive the algorithm for reanalysis of similar constructions.

1. Elements in which the kinetic and potential energies (and the difference in their increase) are negligible with respect to other elements.
2. Elements in which the kinetic energy is dominant compared to potential energy
3. Elements in which the potential energy is dominant compared to kinetic energy

4. Elements in which the potential and kinetic energy exist and are not negligible in comparison with other elements

Reanalysis algorithm

On the basis of previous analysis it is possible to derive the following algorithm for reanalysis (Figure 2):

Step 1: The observed construction is divided into appropriate number of finite elements for which kinetic $e_{k,r} = \frac{1}{2}\omega_r^2 \left\{q_r^s\right\}_e^T \left[m\right]_e \left\{q_r^s\right\}_e$ and potential $e_{p,r} = \frac{1}{2}\left\{q_r^s\right\}_e^T \left[k\right]_e \left\{q_r^s\right\}_e$ energies are calculated separately, on those main modes that are of interest in the analysis.

Step 2: Comparing the values of potential and kinetic energy over zones or elements, as well as corresponding energy differences, on basis of which the following courses of analysis are formed:

Step 3: In elements for which is true: $e_{pr}\rightarrow 0$, $e_{kr}\rightarrow 0$, there are no possibilities for successful modifications with respect to increasing eigenfrequencies. The influence of these elements to dynamic behavior of construction is weak, but they are suitable for other types of optimizations. Lowering the mass of those elements can lighten the whole construction, without endangering its dynamical behavior.

Step 4: In elements for which holds that $e_{pr}>>e_{kr}$, it is appropriate to increase eigenvalues by increasing stiffness. Previous examples have shown that this increase cannot be arbitrary and the modification parameters for these segments are based on distribution of energy increases.

Step 5: In elements for which holds that $e_{kr}>>e_{pr}$, it is suitable to raise their eigenvalues by decreasing their mass. Mass decrease cannot be arbitrary and is based on distribution of energy increases. Mass decrease is a generally desired type of modification, according to many criteria.

Step 6: Most often, elements appear in constructions for which the values of e_{pr}, e_{kr} are not negligible and are of similar quantities. Then, the situation is more complex and those segments are suitable for reanalysis. The reanalysis procedure is applied on previous examples. First, the modified model is formed that will serve for purposes of comparison. On the basis of differences in increases of potential and kinetic energy $\Delta e_{pr}-\Delta e_{kr}$ between modified and original system, the modification parameters ψ, α, β are calculated for each element independently. It has been shown that modification parameters depend on type of cross sectional area, type of material used, and boundary conditions. By applying the reanalysis formula

$$\Delta\lambda_1 = \frac{\sum_{j=1}^{N}\left[\alpha_j e_{pj}^{(1)} - \beta_j e_{kj}^{(1)}\right]}{\left\{\underline{Q}_1\right\}^T [M] \left\{\underline{Q}_1\right\}}$$

on the basis of obtained modification parameters the increase of eigenvalue can be calculated with great degree of reliability, for a given stage of modification, without running the software for finite elements analysis.

Step 7: When the desired value of increase is achieved, it is possible to conduct the check of modified construction, by running the software for finite element analysis with modified parameters. On the basis of new energy diagrams, evaluation of the modified construction can be obtained. If the difference of energy increases on the designated positions is less than the previous that means that the procedure converges, and vice versa. Convergence is the goal of every optimization procedure.

4. Examples of performed reanalysis

In this analysis thin plates have been considered. The plate, 100 cm x 100 cm in size, and 1 cm thick is considered as initial plate. It was supported by hinges on each side, (Figure 3), or it was fixed along the sides (Figure 4).

The plate thickness was chosen as the design variable in the analysis, with the size 100 cm x 100 cm saved throughout the analysis. The effect of boundary condition was also considered, for both hinged (Figure 3) and fixed supports (Figure 4). Calculation of eigenpairs was performed using software package KOMIPS /13/.

The modification consisted in arbitrary change of initial plate thickness uniformly by 10 %. Potential and kinetic energies for modeled plate have been calculated using the Eqs. (4) and (5) and the differences in increment determined, as presented in Figure 3 and Figure 4. On the basis of graphs obtained for simple supported plate one can conclude that elements near corners of the plate have the biggest positive difference, and that by increasing the stiffness dynamic behavior is improved. The elements in the central part of the plate have the greatest negative difference. It can be noticed that both, potential and kinetic energies have the maximum values on this mode shape, which requires a more detailed reanalysis. However, it can be proved that by increasing stiffness (increasing section thickness), and decreasing mass, the frequency will increase. Analyzing the fixed supported plate, stiffness should be increased on elements corresponding (belonging) to the middle part of the side of square (the purple part on the graph, Figure (4), and at the central part of the plate, it is better to add elements

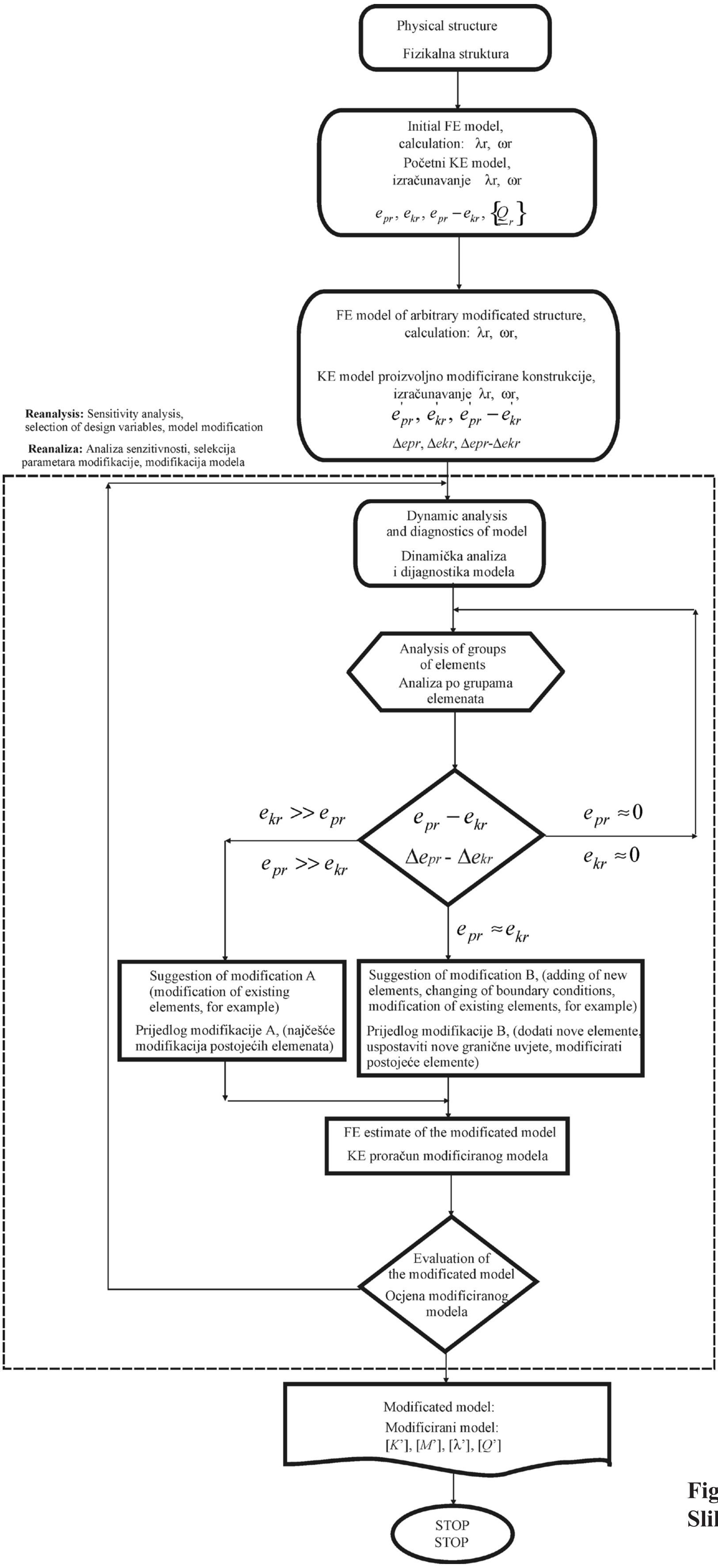

Figure 2. Algorithm of dynamics reanalysis
Slika 2. Algoritam dinamičke reanalize

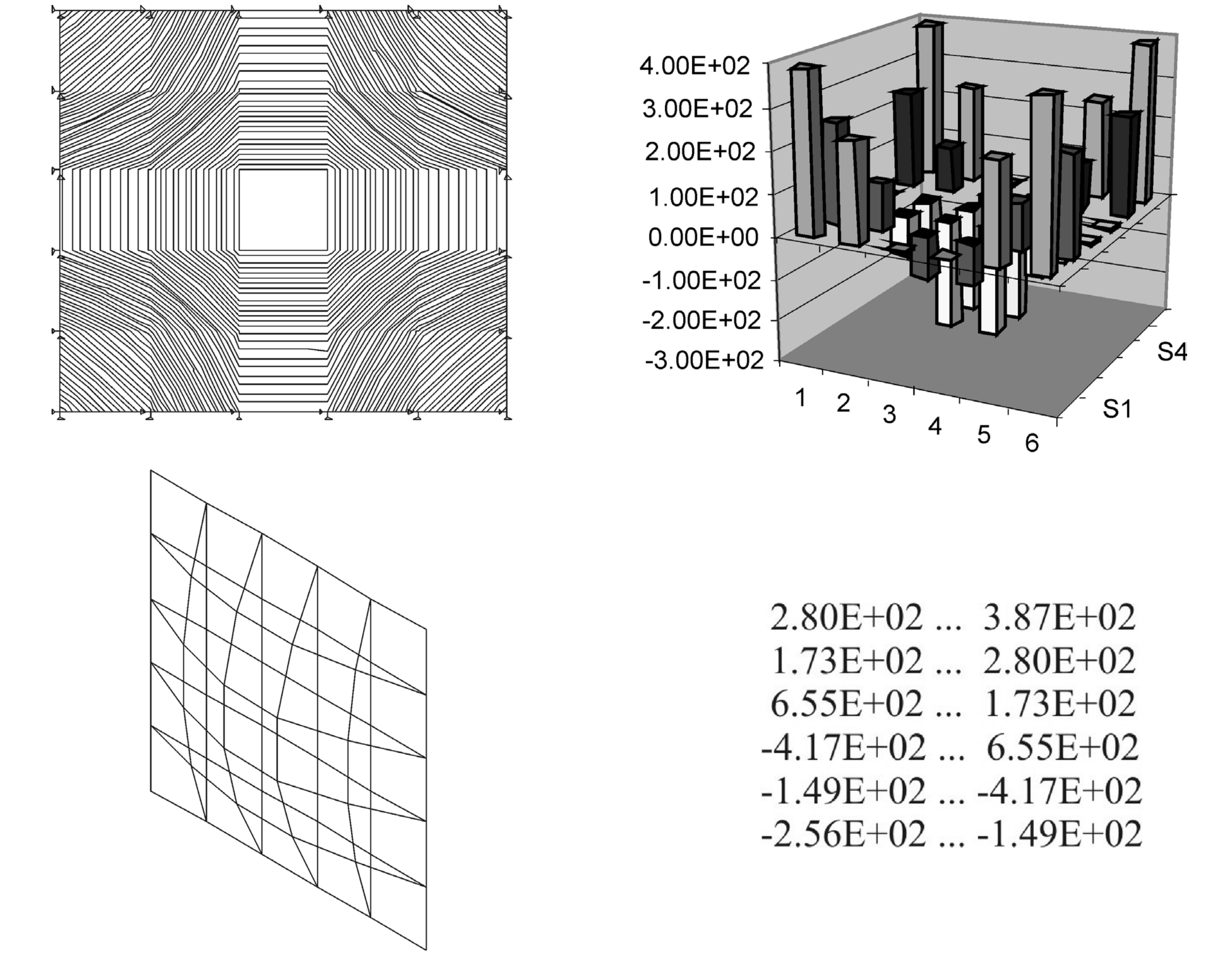

Figure 3. The square plate supported by hinges along the each side. Distribution of potential and kinetic energy increments, kN·cm, for initial and arbitrary modified plate is presented. First frequency of the initial plate is f_{01} = 48.63 Hz, and for arbitrary modified plate f_{01}'= 53.5 Hz.

Slika 3. Kvadratna ploča oslonjena zglobno po rubovima. Predstavljena je raspodjela inkremenata potencijalne i kinetičke energije, kN·cm, za početnu i proizvoljno modificiranu ploču. Prva frekvencija početne ploče je f_{01} = 48.63 Hz, dok je za proizvoljno modificiranu ploču f_{01}'= 53.5 Hz.

with less mass and of greater stiffness. The side walls of a plastic container used for transportation of parcels in aircraft industry prematurely failed due to excessive deflection (Figure 5). In order to solve this problem the dynamic behavior of elements has been analyzed on a proper model. The dimensions of side wall were 295 x 142 x 196.5 mm. Performed analysis produced first three mode shapes (Figure 5) of the plastic wall supported by hinges along the contour.

Two types of models were applied in the analysis. First one was initial, unmodified model, and second one was arbitrary uniformly modified model. It was required that the changes must be small. Distributions of the increment difference of potential and kinetic energies assessed by models in initial state and after arbitrary uniform modification of wall plate are shown in Figure 6 for the first three mode shapes. This design of the wall enabled similar dynamic properties as in the case of thin plate, and the introduction of fixed supports in different locations with the highest growth in kinetic energy enabled significant improvement in dynamic behavior of modified wall.

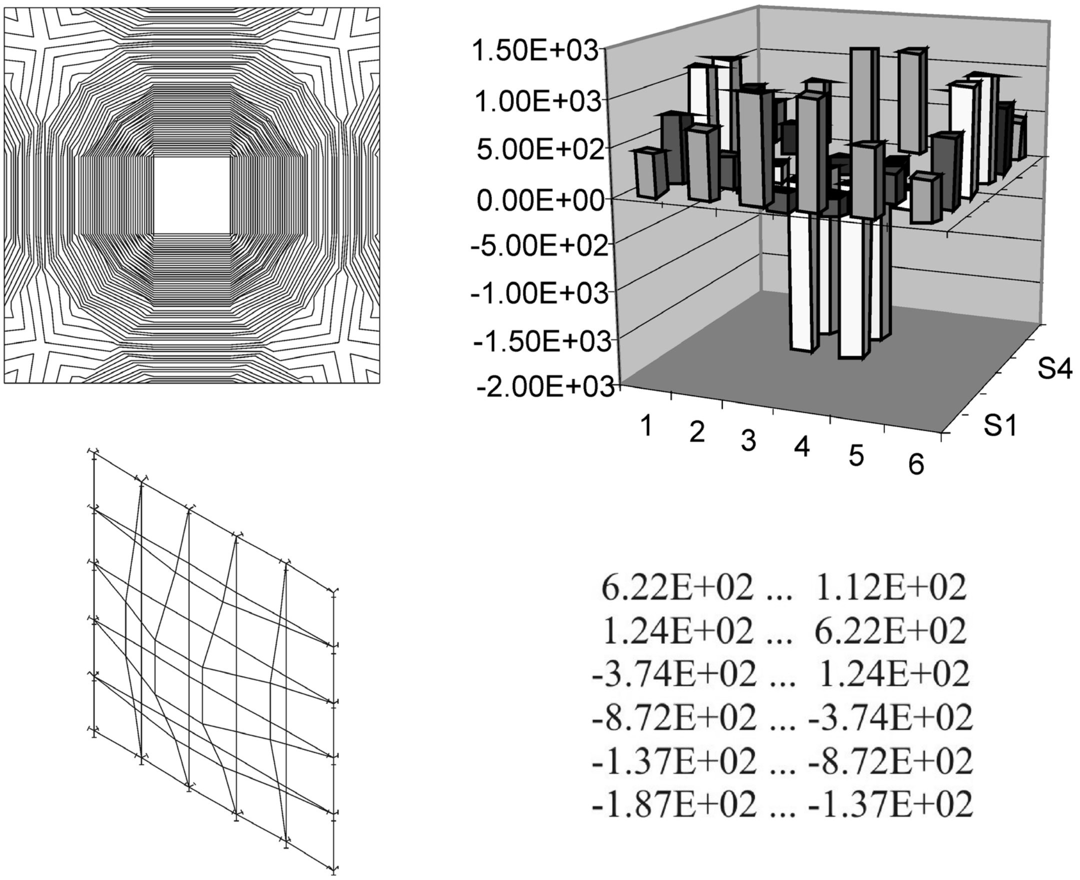

Figure 4. The square plate fixed along the each side. Distribution of potential and kinetic energy increments, kN•cm, for initial and arbitrary modified plate is presented. First frequency of the initial plate is f_{01} = 90.5 Hz, and for arbitrary modified plate f_{01}' = 99.6 Hz.

Slika 4. Kvadratna ploča oslonjena na rubovima. Predstavljena je raspodjela inkrementa potencijalne i kinetičke energije, kN•cm, za početnu i proizvoljno modificiranu ploču. Prva frekvencija početne ploče je f_{01} = 90.5 Hz, dok je za proizvoljno modificiranu ploču f_{01}' = 99.6 Hz.

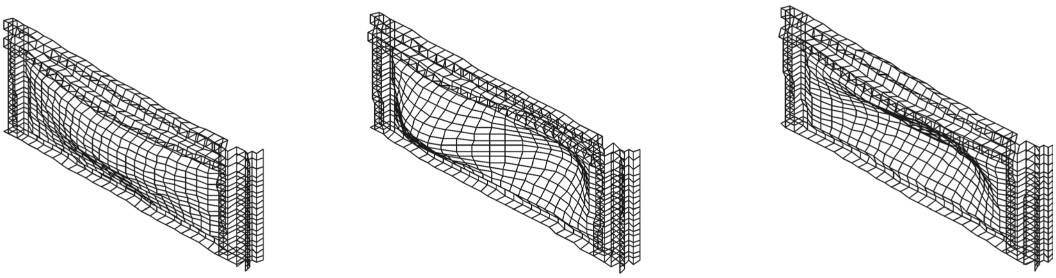

Figure 5. The first three mode shapes of the wall side of the container. Obtained frequencies are: f_{o1} = 57.9 Hz, f_{o2} = 97.4 Hz, f_{o3} = 110 Hz.

Slika 5. Prva tri glavna oblika osciliranja stranica kontejnera. Dobivene frekvencije su: f_{o1} = 57.9 Hz, f_{o2} = 97.4 Hz, f_{o3} = 110 Hz.

5. Conclusion

Studying the dynamic behavior of a construction can predict its response to change in shape, changes in size of its elements or change in materials used. Generally speaking, the aim of system modification with respect to improvements in dynamic behavior is to increase eigenfrequencies and widen the distance between two neighboring frequencies. The specific importance lies in lowest frequencies and those close to system exciting frequencies.

Developed procedure for dynamic modification represents the essence of methodology for improving the dynamic behavior of a construction. Originality of this methodology is that in analyzing the dynamic behavior of construction it uses the distribution of kinetic and potential energy in main oscillation modes. On the basis of analyzing the percentages of distributions for kinetic and potential energy in main oscillation modes, a rough estimate for adequacy is obtainable across zones and construction subgroups. This is especially important for complex constructions. When groups suitable for reanalysis are located, a detailed (fine) analysis of separated subgroup is undertaken. Most often it is necessary to make a modified model which is used for comparison to original, and on the basis of thus derived reanalysis formula, new guidelines are reached. There are clear, mathematically expressed, unambiguous guidelines for further conducting the modification procedure – which is described in the algorithm, and there are segments where the form of modification is not clearly seen. Then, on the basis of analysis of sensitivity to certain changes, a clearer image of further steps is obtained. Based on these cases an algorithm for reanalysis is derived and its essence is in the following. If it is necessary to improve the dynamic behavior of a construction, most often to avoid the resonance with exciting dynamic loads, it is necessary to create the initial finite element model of a given construction and perform the basic calculation of dynamical properties in order to obtain the basic frequencies and main oscillation modes. Kinetic and potential energy of the entire construction can be represented as algebraic sum of the energies of all elements, which is also given here. In order to conduct a rough analysis, the distributions of kinetic and potential energy for construction subgroups in *r*-th main oscillation mode can be expressed in values of percentages. Therefore, on the basis of analyzing the energy distributions in main oscillation modes of main construction elements, it is possible to depict the following cases, on basis of which it is possible to derive the algorithm for reanalysis of similar structures. Following are the characteristic areas:

1. Elements in which the kinetic and potential energies (and the difference in their increase) are negligible with respect to other elements.
2. Elements in which the kinetic energy is dominant compared to potential energy
3. Elements in which the potential energy is dominant compared to kinetic energy
4. Elements in which the potential and kinetic energy exist and are not negligible in comparison with other elements

A great number of examples illustrate the cases mentioned. Also, a great number of empirical correlations are given for certain changes that may lead to desired improvement of dynamic behavior of construction.

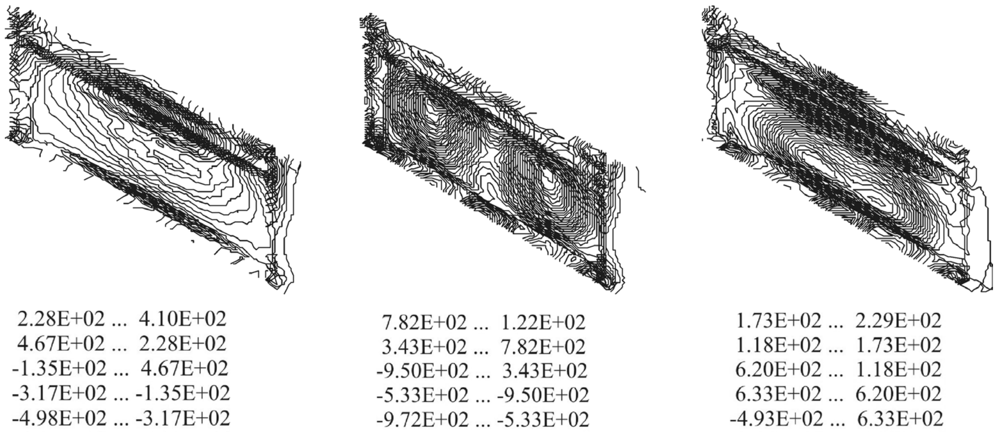

Figure 6. Distribution of potential and kinetic energies increments differences (ΔE_p - ΔE_k), in initial wall design and after arbitrary uniform modification for the first three mode shapes, kN·cm.

Slika 6. Raspodjela inkremenata potencijalne i kinetičke energije (ΔE_p - ΔE_k), za početno projektirani zid i nakon proizvoljne uniformne modifikacije za prva tri moda osciliranja, kN·cm.

The application of developed procedure on real constructions illustrated its practical aspects.

The procedure developed in this paper can be classified as iterative and having great reliability for fast convergence. The convergence of modification procedure assumes relatively fast achievement of proposed goals. Most often the proposed goals are: elevation of eigenfrequencies and increase of distance between two neighboring frequencies. Special importance lies in the lowest frequencies and those whose values are close to excitation frequencies of the system.

Acknowledgments

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