EMBEDDED RETAINING WALL LIMIT STATE ANALYSIS ACCORDING TO EUROCODE 7

Krešo Ivandić, Matija Bandić, Božo Soldo

The paper deals with the problem of criteria for choosing the proposed calculation approaches in Eurocode 7 with respect to current technical regulations and regular engineering practice. A new procedure for the implementation of a systematic analysis and mutual comparison of different project approaches is proposed. Parametric analysis of embedded retaining wall limit states is performed. Combining of partial coefficients, according to proposed design approach, results in a visible wide span of equivalent values of the safety factor. Conclusions and recommendations tied to the application of the optimal calculation method are also given as well as the original mode of the analysis results.

Keywords: design approaches, embedded retaining wall, equivalent safety factor, Eurocode 7, limit states, parametric analysis

Analiza graničnih stanja potporne konstrukcije prema Eurokodu 7

Razmatra se problem kriterija odabira predloženih proračunskih pristupa u Eurokodu 7 u odnosu na dosadašnju tehničku regulativu i uobičajenu inženjersku praksu. Predlaže se novi postupak za provedbu sustavne analize i međusobne usporedbe različitih projektnih pristupa. Provodi se parametarska analiza graničnih stanja potporne konstrukcije. Kombiniranjem parcijalnih koeficijenata, prema predloženim projektnim pristupima, vidljiv je široki raspon ekvivalentnih vrijednosti faktora sigurnosti. Prilažu se zaključci i preporuke, vezane za upotrebu optimalnog načina proračuna. Također daje se originalni način prikaza dobivenih rezultata analiza.

Ključne riječi: ekvivalentni faktor sigurnosti, Eurokod 7, granična stanja, parametarska analiza, potporna konstrukcija, projektni pristupi

1 Introduction

Through coming into force of the Concrete Construction Technical Regulations [1], to the European standards were also added [2] and [3] (Attachment 1.4, Standards) in civil engineering, i.e. construction and geotechnical regulations. Based on this, a need appeared for revising current technical regulations [4] and critical view of new standards. A certain number of comparable analyses on as large as possible number of geotechnical tasks are needed to be performed. This will allow inter-comparison of new approaches, but also confronting current procedures.

1.1 Limit state analysis

Introduction of European norms (Eurocode 0.9) is aimed at regulating the application of proof, where the designed construction will be safe enough not to reach the so-called "limit states". Each design must be proven for a range of ultimate limit states and a range of serviceability limit states. It must be removed enough from levels at which it may break/fracture or deform, and be safe enough to hold out against unforeseen events.

Through the use of limit states in construction, the outdated concept of permissible stress design has been replaced. The use of partial safety factors results in the same level of safety, regardless of load combinations, material characteristics and problem geometry. Calculable material characteristics are determined based on the implicit supposition on ensuring a limitless number of samples tested in controlled conditions. Loads are predictable with the known probability distribution. Using partial safety factors the distribution of reliability and/or risk distribution on the main calculation elements is determined.

Probability methods assume that the effects made through the actions being performed on the construction and its resistance, with taking into consideration material characteristics and problem geometry, have a certain probability of distribution emergence. Taking into consideration these active dispensation probabilities (action and resistance) allows the calculation of break-up probability. In such a calculation partial safety factors are not applied; all actions, material characteristics and resistance are part of the calculation with their own probability distribution. The application of partial safety factors (European norms) is aimed at building a bridge between calculations used so far and pure probability analysis. In that context, the use of Eurocode 7 (EC7) introduces a principle of limit state design into geotechnical engineering. The calculation of foundation and over-foundation construction is thus merged with phasing out of existing conceptual barriers.

1.2 Eurocode 7 and limit states

Geotechnical engineering is faced with a problem of quantification of the reliability level. The reason for this is a limited number of samples, with frequently unknown probability distribution of examined values. Taking that into consideration, the use of one of the newly adopted systems is not primarily based on predetermined probability patterns as is the case in construction. Adoption of proposed procedures should be argued through coordination with results collected based on long-term experience from engineering practice, or on statistical calculation of experimental data and practical on-site observations. This statistical calculation should be performed using the method
of probabilistic reliability theory [5].

The application of EC7 introduces the practice of using partial safety factors. Factored in are external load (temporary, permanent, favourable and unfavourable action), soil characteristics (parameters of non-drained and drained shear rigidity, bulk density), and resistance (bearing capacity, sliding). The means of factoring, i.e. the individual combination, is defined in advance according to appropriate proposed design approach. The total of 112 values of partial factors and 34 co-relation factors has been ascertained, giving a total of 146 values. Remaining controversial is the criterion used in choosing each individual approach. There is confusion caused by different approaches being used for the same physical task. Non-critical choice of factors within the chosen approach leads to the lack of assessing the physical nature of the analyzed task. Results of performed analyses should be coordinated with experiences from “typical” problems from practice, which can then be used as reference values. The question arises, what to do with tasks which are not typical, such as new types of actions, non-standard structural sizes, etc.? The criterion of individual approach evaluation does not exist for such non-standard situations.

The paper offers an analysis of one standard problem in geotechnical engineering. Equivalent value of the safety factor leads to certain conclusions tied to inter-comparison and criteria of choice of each proposed design approach.

## 2 Current practice of retaining structures design computations procedures

Dosadašnji postupci proračuna potpornih konstrukcija

In the classic limit analysis, the allocation of strain corresponds to limit conditions of soil break-up, due to the rotation of the solid matter. Strains behind the retaining structure achieve their minimal value (active limit strain), while they achieve their maximum at the front (passive limit strain). The designed retaining structure must be proven against reaching certain corresponding limit states. This condition is met using the safety factor. Some of the possible ways of factoring are shown in Fig. 1 according to [6]:

a) The gross pressure method, $FS$ on total passive resistance;

b) The net pressure method, $FS$ on net passive resistance diagram;

c) The revised or Burland and Potts method, $FS$ on own weight beneath the dig level, bilaterally;

d) The strength method, $FS$ on appropriate soil parameters

The calculation d) is conceptually the closest to EC7. The difference is that in EC7 external actions and resistance in various predetermined combinations are factored in with soil parameters. The example used in the paper is processed using method a). This is a special case of the LRFD – Load and resistance factor design method [7, 8], where, in this case, only the passive resistance is reduced. This approach is the one most frequently used in practice.

External actions on the retaining structure contain active pressure and passive resistance, created by soil weight and external load, as well as pore pressure both from the active and the passive sides. The paper contains the analysis for a water-free profile, with external uniform load.

### 3 Design approaches in EC7

Projektni pristupi u Eurokod 7

The proposed EC7 approaches do not contain explicit calculations of the global safety factor; factoring is performed using appropriate coefficients. Three design approaches are used: DA1, DA2 and DA3 (Design approach 1, 2 and 3) with various factoring combinations to external actions, material characteristics and resistance (A – Action, M – Material, R – Resistance). Fig. 2 and Fig. 3 show the values of partial coefficients for all design approaches for the limit analysis of the retaining structure in coarse-grained soil. Double-underlined letters show those values actually having the influence in certain approaches, i.e., those not equal to 1.

Tab. 1 shows the schematic of partial combination factors ($\neq 1$) for design approaches (DA1-J, DAi):

<table>
<thead>
<tr>
<th>Pr.</th>
<th>DA1</th>
<th>DA1 2</th>
<th>DA2</th>
<th>DA3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ko.</td>
<td>A1+ M1+ R1</td>
<td>A1+ M2+ R1</td>
<td>A1+ M1+ R2</td>
<td>A1+ M2+ R1</td>
</tr>
</tbody>
</table>

Table 1 Design approaches partial factor combinations

Tablica 1. Kombinacije parcijalnih faktora za projektno pristupe

Figure 1 Safety factor definition according to [6]

Slika 1. Definicija faktora sigurnosti prema [6]
Where in DA3:
- $A_1 = \text{structural action}$
- $A_2 = \text{geotechnical action}$

**4 Passive resistance like resistance or action**

EC7 does not explicitly state how to take passive resistance into consideration. With gravitational structures and shallow foundations it can be neglected, similarly to the classic calculation method. With analysis of retaining structures this is not possible because it represents a fundamental part of the analysis and not a certain influence on the increase of the safety factor. The question is if it should be considered as resistance or external action. The answer can mean significant differences in actual proof against limit states GEO and STR. The analysis of the influence of treating passive resistance as actual resistance, then as external favourable and unfavourable action, and finally without the effect of factoring, was performed in [9]. The conclusion of the author is that passive resistance is at the same time considered to be unfavourable action and resistance. In this case the characteristic passive resistance is factored using the quotient $\gamma_c/\gamma_u = 1.5/2.5 = 0.6$.

Arguments for such an approach are supported through ensuring consistency of adding to numerical analyses, but also to classic procedures, of net pressure or the revised Burland and Pots method.

The need for this type of analyses is determined by the fact that the EC7 does not explicitly specify the means of treating passive resistance, but also because of the introduction of the so-called Single Source Principle in [2] (§2.4.2(9) Note). It states the following: In some cases it is permissible to assume that unfavourable (or destabilising) and favourable (or stabilising) permanent actions emerge from the same source. If that is the assumption, then one partial coefficient may be applied to the sum/total of these actions/effects. It is not specified when and in which cases this principle can be used. If it can be applied to retaining structures, then passive resistance can be treated as external unfavourable action, since it stems from one source, which is the soil to the retaining structure.

In classic analyses passive resistance is calculated as resistance, where it is the only thing factored. If one considers all of the above, examples have passive resistance calculated as resistance, not as external action.

**5 Numerical example**

Fig. 4 shows an example in which a parallel analysis of proposed design approaches with the classic calculation method was performed. The EC7 does not strictly define the means of ascertaining soil pressure. Attachment C [2] offers formulae and corresponding diagrams. The paper chose the Rankine model, which excludes friction, thus allowing the analysis of the simplest possible case. Calculations have so far been performed for the ultimate limit state, for unfavourable action of permanent and periodical load, and for passive resistance. Serviceability limit state was not considered. The retaining structure is treated with free earth support [9].
Conditions for reaching the necessary values are:

1. $M_{\text{act}} = M_{\text{det}}$, equality of design moments on point 0 (GEO – geotechnical limit state, rotation around the anchor point), followed by the embedment depth $d_e$, where $M_{\text{act}}$ – action of design moments in the direction of rotation, $M_{\text{det}}$ – action of design moments in direction opposite to the direction of rotation

2. $F_s = H_{\text{act}} - H_{\text{det}}$, sum of design horizontal forces (GEO - geotechnical limit state on the horizontal shift, and for the anchor also the STR – structure limit state of the anchor), where $F_s$ – design force in the anchor, $H_{\text{act}}$ – horizontal design pressure, $H_{\text{det}}$ – horizontal design resistance,

3. $V_{\text{d}} = 0$, place of action where the design transversal force $V_{\text{d}} = 0$, and the design bending moment is at its maximum $M_{\text{max,d}}$ (STR – structural limit state of the retaining structure).

6. Equivalent Safety Factor

Comparation of individual approaches is determined using the following sequence: 1. Embden depth $d_e$, 2. Design force in the anchor $F_{\text{act}}$, 3. Maximum design bending moment $M_{\text{max,d}}$.

Conditions of determining equivalent factors of safety, for $d_e$, $F_{\text{act}}$ and $M_{\text{max,d}}$:

1) $F_{S_0} = M_{\text{act}}/M_{\text{det}}$ – for the rotation around the head of the anchor, where $M_{\text{act}}$ – action of characteristic (gained using characteristic, unfactored soil parameters and unfactored resistance) moments in the direction of rotation, $M_{\text{det}}$ – action of characteristic moments in the direction opposite to the direction of rotation

2) $F_{S_0} = H_{\text{act}}/(H_{\text{det}} - F_s)$ – for translation of retaining structure, where $H_{\text{act}}$ – horizontal characteristic pressure, $H_{\text{det}}$ – horizontal characteristic resistance, $F_s = H_{\text{act}} - H_{\text{det}}$, design force in the anchor for each of the proposed approaches

3) $F_{S_0}$ from the condition of given embedment depth $d_e$ and given design maximum moment $M_{\text{max,d}}$ for examined variant of design approach.

Equivalent safety factor $F_{S_0}$ is gained using the condition for determining the three unknowns: $z'$ – depth where the horizontal force is equal to zero, i.e. where the maximum moment appears, $F_s$ – design force in the anchor, $F_{S_0}$ – equivalent safety factor from the equation of equal design moments (Fig. 5). For all three safety factors, the embedment depth $d_e$ is equal. For the security factor $F_{S_0}$, the embedment depth and the force in the anchor are equal. For $F_{S_0}$, the depth $d_e$ and the bending moment $M_{\text{max,d}}$ are equal, but not the point of the maximum moment and the force in the anchor.

Next in line are conditions for the calculation of the three unknown values for the case in which the maximum design moment is beneath the level of the excavation. In case of this moment being beneath the dig level, another part of the design passive resistance must be included at the appropriate depth.

1) $q_{\text{eq}} \cdot z' \cdot K_A + \gamma \cdot z' \cdot K_A \cdot z' \cdot 0.5 - F_d = 0 \tag{1}$

Condition of horizontal force balance at the depth $z'$. 

2) $F_d = H_{\text{eq}} - H_{\text{act}}/F_{S_0} \tag{2}$

Force in the anchor from the condition of equilibrium of horizontal forces

3) $M_{\text{max,d}} = \gamma q_{\text{eq}} \cdot (z' \cdot K_A - 0.5 - \gamma \cdot z' \cdot K_A) \cdot 0.33 \cdot (z' - d_e) \tag{3}$

Location of the maximum moment where $K_A = \tan(45^\circ - \varphi/2)$ coefficient of active pressure, other according to earlier labels.

Fig. 5 also shows the quality allocation of external parts, external actions and sought values. Due to consistency, labels are according to EC7. Three statistical values are being determined: embedment depth $d_e$, force in the anchor $F_{\text{act}}$ and the maximum bending moment $M_{\text{max,d}}$. The design passive resistance $H_{\text{pass}}$ (corresponds to the classic mobilised resistance $E_{\text{pass}}$) is reached by factoring the characteristic resistance $H_{\text{act}}$ (corresponds to the classic total resistance $E_{\text{act}}$). The values of external action on the active side $H_{\text{act}}$ – from the action of soil weight (corresponds to $E_{\text{act}}$), $H_{\text{pass}}$ – from the action of the characteristic external external load $q_{\text{eq}}$ (corresponds to $E_{\text{pass}}$ from $q$), $F_d$ – design force in the anchor (corresponds to $S$), $M_{\text{max,d}}$ – design maximum bending moment (corresponds to $M_{\text{pass}}$).

$FS = 1.5$ was chosen as reference value of the global safety factor in the classic approach. For each friction angle and chosen $FS$ there is a unique combination of $d_e$, $F_d$, $M_{\text{max,d}}$ and $M_{\text{max,d}}$ which are used as parallel values.
The results of analysis with comments
Rezultati provedenih analiza s komentarima

The results of performed analyses are presented according to gained statical values and equivalent safety factors. A common standardised view of values is also given. Finally, the amount of deviation of each individual calculation from the classic analysis is determined.

7.1 Depth of embedment, anchor force, max. bending moment
Dubina ukapanja, sila u sidru, maksimalni moment savijanja

Figures 6-8 show the values of embedment depth \( d \), anchor force \( F_a \) and max. bending moment \( M_{max,d} \) for proposed design approaches and for the classic calculation method and \( FS = 1,5 \).

Fig. 6 shows a general expected trend of decreasing the depth \( d \) with the increase in the value of the friction angle for all methods of calculation. In relation to the classic calculation method, it is smaller for all approaches except for DA2 for smaller values of the friction angle (up to 30°). With the increase of the friction angle, DA1_2 and DA3 trend changes, i.e. depths increase with respect to the classic calculation. At the same time, the relative differences in ratio \( d \) decrease when the friction angle increases.

Fig. 7 gives the smallest values of design anchor forces \( F_a \) for all approaches. As is the case with \( d \), the highest design anchor forces are reached with DA2. The difference with respect to \( d \) is that for all approaches except for DA2 there is no significant decrease in the difference of the anchor force with the increase of the friction angle.

Similarly to Fig. 7, in Fig. 8 the values of maximum design moments decrease with the increase of the friction angle. Moments are also minimal for the classic calculation. The highest values are measured for DA2.

The highest values of examined statical values are accomplished for DA2, for the entire span of friction angles. The reason for this is the multiplication of partial factors on external unfavourable permanent and temporary action with resistance on the passive side. Increase of sizes is, with respect to other approaches, approximately 50 %. Other approaches DA1_1, DA1_2 and DA3 give results which fall within relatively narrow parameters. Deviation with respect to the classic method of calculation is the smallest for \( d \). For \( F_a \) and \( M_{max,d} \), deviations with respect to the classic method of calculation are significant, especially for DA2.

7.2 Equivalent safety factors
Ekvivalentni faktori sigurnosti

Figures 9–11 show the values of equivalent safety factors according to described criteria of equal embedment depths, and pairs of embedment depth–design anchor force and embedment depth–maximum design bending moment.

Equivalent safety factors \( FS \), are constant for all friction angles for DA1_1 and DA2 (Fig. 9). The reason for this is that multiplication with partial factors is performed after the calculation of characteristic external actions and resistance. In that case, equivalent characteristic resistance in the classic calculation can be extracted and divided by the equivalent safety factor. In other words, characteristic
actions are constant for the classic calculation and for DA1_1 and DA2. For DA1_1, $F_{S_a} = 1.6$ appeared as the influence of the equal external periodical load and its factor $\gamma_0 = 1.5$ on the partial factor $\gamma_r = 1.35$ for permanent unfavourable external action. A similar case is put forth by DA2, where $F_{S_a} = 1.9$. In DA1_2 and DA3 the factoring is performed before the calculation. Therefore, unlike for DA2, where $F_{S_a}$ cannot be extracted. $F_{S_a}$ for DA1_2 and DA3, characteristic actions cannot be extracted. $F_{S_a}$ for DA1_2 and DA3 has the tendency of rising with the increase of the friction angle. For smaller friction angles, the $F_{S_a}$ is less than 1.5, while for $\phi = 40^\circ$ and more, its value grows to beyond 1.7.

In Fig. 11, similar to anchor forces, equivalent safety factors $F_{S_a}$ from the conditions of equality of design maximum moments show a tendency to rise with the increase of the friction angle. Their absolute values in this case are lower than in the case of using the design anchor force. For DA1_1, $F_{S_a}$ is mostly lower than 1.5, except for higher friction angles. For other approaches, $F_{S_a}$ tends to be relatively large as the friction angle increases.

7.3 Normalized values

Normalizirane vrijednosti

Figures 12-14 show the values of equivalent safety factors divided by 1.5, in co-relation with relative embedment depths, or embedment depths for individual approaches divided by embedment depths in the classic method of calculation $d_i/d_{d,1.5}$, within the examined span of friction angles. This allows a direct comparison of individual approaches for the two criteria with respect to the classic solution. The classic solution with $FS = 1.5$ is presented by the point $<1, 1>$ in the coordinate system. The farther the values get from the point in this system, the more the result of individual approach analysis deviates from the classic calculation. Another visible aspect is the comparison of the equivalent safety factor for a certain statical value and its change, all according to values found in the classic calculation.

Fig. 12 shows the allocation of relative embedment depth values $d_i$. For DA1_1 and DA2, the relative ratio of safety factors is constant, since the equivalent $F_{S_a}$ is constant. The difference is the trend, where for DA1_1 the relation (ratio) rises, while with DA2 the relative values of embedment depths fall as the friction angle increases. For DA1_1, the values are the closest to those found in the classic calculation, but with safety factor values smaller than 1.5. DA2 shows the tendency of decreasing the embedment depth as the friction angle gets wider, warning the values were noted for $\phi = 35^\circ$. For DA1_2 and DA3, relative values are less than one for smaller angles, while they grow for larger angles - but never over 1.2.
Maximum moment increase is also noticed in DA1_2 and increase of the friction angle. A relatively large range of 1.5–2, but also showing a tendency of decreasing with the maximum moments is the highest in DA2, the range being wider, with mild growth of the anchor force. For DA1_1, grows as the friction angle gets larger, with mild growth of the anchor force in tune with the equivalent safety factor values, these approaches the increase of the soil friction angle smaller friction angles for all approaches except DA2. For DA1_2 and DA3, relative values grow at the same time. A wide range of equivalent values of the safety factor for DA1_2 and DA3, relative values are above the ones gained through the classic calculation method for the safety factor FS=1.5. The ordinate in the diagram represents the anchor force and the safety factor significantly deviate from the classic calculation method. Increasing the friction angle points to also increases the differences with respect to the reference classic calculation method. Increasing the friction angle result in the anchor force better in tune with the equivalent values. For DA1_2 and DA3: 0.9–1.3. For DA1_1 the range is the lowest: 0.95–1.0.

7.4 Deviation measure
Mjera odstupanja

Diagrams shown in Figures 15–17 determine the measure of deviation of relative values in diagrams in Figures 12–14. Individual values represent absolute distances from the point (1, 1), i.e. the unique solution gained using the classic calculation method for the safety factor FS=1.5. The ordinate in the diagram represents deviation under common criteria of the equivalent safety factor and corresponding calculated statistic value.
offers, which is also true for approaches DA1_2 and DA3 for friction angles smaller than 30°.

Fig. 16 shows somewhat different trends with respect to burying-in depth. Relative differentials grew for all approaches; more significantly for DA1_2 and DA3 and somewhat less for DA1_1. The DA2 shows the most even approaches; more significantly for DA1_2 and DA3 and burying-in depth. Relative differentials grew for all approaches, but also through their direct comparison. Individual approaches result in somewhat more conservative values than classic structures. In general, the new approaches result in wider angles demand the opposite. For DA2, the decrease of friction angle widens, deviations increase, except for DA2, where the trend is the opposite of the aforementioned. In conclusion, DA2 offers overly conservative solutions, also be necessary. Larger deviations in differences among the approaches shown in this paper are not expected. For smaller values of pressure angles, all approaches except DA2 fall within relatively close confines, with small deviations with respect to the classic analysis. As the friction angle widens, deviations increase, except for DA2, where the trend is the opposite of the aforementioned. In conclusion, DA2 offers overly conservative solutions, regardless of the value of soil shear rigidity parameters. When using other proposed procedures, it is possible to reach optimal results through adequate choice of characteristic values. For less wide friction angles more conservative characteristic values may be chosen, while wider angles demand the opposite. For DA2, the decrease of external action partial factor values is recommended, thereby moving closer to the classic method of calculation. The way of choosing the mentioned characteristic values and partial factors should be defined in revised current technical regulations through appropriate National annexes of EC7.

9 References

Literatura

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Analiza graničnih stanja potporne konstrukcije prema Eurokodu 7

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