Original Paper UDK 164.3: 51
Received March 15th, 2006

Matija Arko
University of Maribor, Department of Philosophy, Koroška 160, SI-2000 Maribor
matija.arko@guest.arnes.si

Is If-then-ism Still an Option?

Abstract

In this paper I am going to try to prove that if-then-ism *is not an option. I will focus on if-then-ism as a strategy to reduce ontological commitments in mathematics. I will start with the definition of if-then-ism in* The Principles of Mathematics*. Then I am going to discuss the Putnam's criticism of if-then-ism. Next I will move on to some arguments of Cian Dorr that support it. In the end, I will talk about ontological parsimony as the general motivation for adopting the if-then-ism. I will discuss ontological parsimony in connection with naturalism and argue that demand for ontological parsimony does not follow from naturalism.*

Keywords
if-then-ism, ontological parsimony, naturalism, philosophy of mathematics

In this essay I will concentrate on some issues concerning "if-then-ism" and try to refute it as a plausible philosophical theory. Russell formulated if-then-ism as a paraphrase strategy which shows that mathematics could be derived out of logic. Russell did not deny existence of abstract objects, what he denied was necessary existence of abstract objects.[1] Mathematical claims are necessary true, whereas according to him no existential claims are necesarily true. So statements of pure mathematics are not existential:

"All propositions as to what actually exists, like the space we live in, belong to experimental or empirical science, not to mathematics." (Russell, B., 1992, p. 5)

Russell's reasons for if-then-ism are certainly not connected with ontological parsimony, because in this period his ontology was very rich:

"Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as one, I call a term. This, then, is the widest word in the philosophical vocabulary. I shall use as synonymous with it the words unit, individual, and entity. The first two emphasize the fact that every term is one, while the third is derived from the fact that every term has being, i.e. is in some sense." (Russell, B., 1992, p. 43)

Later there were some proposals to use if-then-ism in order to avoid ontological commitments to abstract mathematical objects (numbers).

Russell starts with the following definition of "if-then-ism" in *The Principles of Mathematics*:

"1. PURE Mathematic is the class of all propositions of the form 'p implies q', where p and q are propositions containing one or more variables, the same in two propositions, and neither p nor q contains any constants except logical constants." (Russell, B., 1992, p. 3)

1
See: Colin Cheyne, "Existence Claims and Casuality", *Australasian Journal of Philosophy*, Vol. 76, No. 1, March 1998, pp. 37–38

If-then-ism nicely fits into the general project of logicism. If if-then-ists are able to translate the whole mathematics into statements which have the form of implications, then the project has achieved its goal.

This connection explains why pure mathematics should not ontologically commit one to anything. It is not the task of logic to make any existence claims. Logic just helps one to infer from one sentence to another, and mathematics viewed as a branch of logic has the same role. Mathematical statements are true, but mathematics remains ontologically neutral. It does not commit us to mathematical objects nor denies their existence. Since the logicists' project failed, if-then-ism is no longer a method with which one can support this project and I will not discuss if-then-ism in that connection.[2]

In this paper I will focus on if-then-ism as a strategy to reduce ontological commitments in mathematics. First I am going to discuss some arguments of Cian Dorr that support if-then-ism, then I will move on to the Putnam's criticism of if-then-ism. In the end, I am going to talk about ontological parsimony as the general motivation for adopting the if-then-ism.

Let me now present an argument of Cian Dorr in favour of if-then-ism. Dorr is ready to accept abstract objects only in case if they are indispensable for good explanations. But he believes that benefits one gains by believing in the existence of abstract objects are illusory. He thinks sciences are paradigmatic case of how one should gain knowledge. In other words, he is an adherent of naturalism. When a philosopher has to decide between realism and anti-realism concerning abstract objects she just has to compare which theory is better in accounting for and systematising the given (scientific) data, in the same way, as one compares different scientific theories.

Dorr then takes three theories T, $T´$ and $T_{\dagger}$. T is an ordinary scientific theory, $T´$ is an if-then-istic theory which avoids ontological commitment to abstract objects, defined by following principle:

"If it were the case that [mathematical axioms] and the concrete world were just as it actually is, it would be the case that T." (Dorr, C., 2005, p. 9)

So what Dorr suggests is if-then-istic interpretation of mathematical statements. The other feature of his analysis is fictionalism. His if-then-ism is different from Russellian. Russell thinks that mathematics is a class of true propositions that are deduced from logic, whereas Dorr denies that mathematical statemens are true. That explains why he uses counterfactuals in the above citation. Platonistic reading of mathematical statements is not true in the actual world, but merely on some other possible world.

Let me return to the above mentioned three theories. $T_{\dagger}$ is also a if-thenistic theory. $T_{\dagger}$ avoids ontological commitment to sub-atomic particles:

$T_{\dagger}$ "As far as atoms and larger entities are concerned, it is just as if T." (Dorr, C., 2005, p. 10)

Let me first take a look at T and $T´$. $T´$ has, according to Dorr, exactly the same consequences for the concrete world as T itself. The realist has to give an argument that $T´$ provides a worse explanation than T.

Dorr next considers $T_{\dagger}$. $T_{\dagger}$ is not a good theory. Science gives us good reasons to believe that there are subatomic particles whereas $T_{\dagger}$ denies this fact. A theory like $T_{\dagger}$ demands, if it is true, further explanation. Any further explanation of $T_{\dagger}$ would have to appeal to the existence of subatomic particles.

T' and $T_{\dagger}$ are similar theories because they are formed in a similar way. Both of them are derived from other, stronger theory, T by the application of a complex operator. So one might think that both of them are bad. But unlike $T_{\dagger}$, T' does not demand further explanation. There are alternatives to standard platonistic scientific theories, which do not entail existence of abstract mathematical objects, like numbers. The most famous example is Fieldian physics (see Field, H., 1980) But Fieldian and other attempts are not carried out completeley. They have not formulated all the science in nominalistic terms. Besides, there are several critiques to such strategies. So their success is not yet beyond doubt.

Dorr points out the relevant difference between T' and $T_{\dagger}$ in operators which were applied to the original theory T. The operator in case of $T_{\dagger}$ is a kind of possibility-operator. So $T_{\dagger}$ says that there is some world where T is true, while the facts about atoms and larger things are just as they are in the actual world. The operator in T' can also be expressed in terms of possible worlds. The theory states that in every world where mathematical axioms are true, and which is like the actual world in other respects, T is true. So it has the form of *necessity*-operator.

In case of $T_{\dagger}$ we use the possibility operator which could be expressed as existential quantification over posible worlds, whereas in case of T' one uses universal quantification over possible worlds.

"… existential quantification is a distinctive source of theoretical badness…On the other hand, universal quantification doesn't seem to be a source of badness in the same way. When we talk in our physical theories about the 'x', 'y' and 'z' co-ordinates of particles, we certainly don't mean to suggest that there is a distinguished, physically real co-ordinate system, concerning which we could sensibly ask how far we are from its origin. Rather, we are implicitly making a universally quantified claim, to the effect that the theory in question holds true for every acceptable co-ordinate system." (Dorr, C., 2005, p. 13)

Therefore, Dorr claims that existential quantification is the source of all trouble, whereas universal quantification is not problematic. In other words, he believes that existential quantification commits one ontologically to things one does not want to commit to, and this is the source of its weakness. In case of T' one uses universal quantification over possible worlds and no ontological commitment occurs.

His argument for mathematical if-then-ism could be reconstructed in a following way:

1. premise: Only universal quantification over possible worlds is involved in mathematical if-then-ism.
2. premise: Universal quantification over abstract objects (numbers, possible worlds) does not ontologically commit one to the existence of abstract objects.
3. (hidden) premise: Abstract objects (numbers, possible worlds) are not to be accepted in ontology.
4. conclusion: Mathematical if-then-ism is an acceptable solution.

I would like to stress once again that Dorr's kind of if-then-ism is a fictionalist kind of if-then-ism (Mathematical axioms are not true, but if they were

2
For a nice introuduction into that problem, see: Ernest Nagel and James R. Newman, *Goedel's Proof*, Routledge, London 1993

true, then such and such statements would follow from them). Mathematics is a fiction, and all the content which scientist needs for her work must consequentially be statable without invoking mathematical entities. Dorr tries to achieve that by avoiding existential quantification over abstract objects.

Fictionalism is subject to Putnam's criticism. Putnam denies any possibility of being anti-realist concerning abstract mathematical entities, and at the same time being realist concerning physical theories. In science one cannot avoid existential quantification over abstract objects. Not believing in existence of numbers and at the same time using mathematics for scientific purposes, according to Hilary Putnam, is like

"… trying to maintain that God does not exist and angels do not exist while maintaining at the very same time that it is an objective fact that God has put an angel in charge of each star and the angels in charge of each of a pair of binary stars were always created at the same time!" (Putnam, H., 1975, p. 74)

Fieldian nominalist answers to that challenge by claiming that mathematics is conservative. This means roughly that mathematics does not add nothing to the nominalistic content. It merely helps to deduce purely nominalistic claims from initial claims that can also be stated purely nominalistic. Instead of focusing attention on this issue, I will try to spell out my objections.

My first objection to Dorr's argument concerns his interpretation of if-then-ism. Mathematical if-then-ism is nothing else but mere translation of all mathematical statements in the form of *formal* implication. Dorr just replaces one form of implication with the other (universal quantification over possible worlds). This new interpretation does not make things clearer. If someone does not find if-then-ism in standard Russellian manner to be atractive, why would she change her belief because of this new modal interpretation? Actually, this new interpretation makes things even worse because of the accepted fictionalism.

My second objection to Dorr's argument concerns its third premise. There are two reasons why nominalists do not accept abstract objects. First of them is that they seem to be epistemologically inaccesible, because one cannot have any causal connection with them. The second objection is that one can do without abstract objects. That means that abstract objects violate the principle of Ockham's razor. I will try to answer to that challenge.

Traditionally, the principle of Ockham's razor is understood as a rule that forbids the multiplication of entities beyond actual need. Hence, one is justified to accept only those entities that are necessarily needed for the construction of theories. It has been argued that this principle demands scarce ontology. This view could be summarized with next slogan: the parsimonious the ontology the better the theory. For the most part, this has been the standpoint supported by declared naturalists (Quine, Field, Dorr). For naturalists, any kind of interference of ontological principles with science is unacceptable. So it is quite natural to ask what is the role of Ockham's razor in science. Since naturalists reject interference of ontological principles with science, one must regard Ockham's razor as a methodological principle. Here, I wish to mention that Mark Colyvan (see Colyvan, M., 2001, pp. 76–77) speaks of four methodological principles:

a) simplicity and economy of the theory;
b) explanatory and unificatory power of the theory;
c) boldness and fruitfulness;
d) formal elegance.

Ockham's razor belongs to the first group of principles. Nominalists understand the Ockham's razor principle as a principle which aims at reduction of ontological commitments, regardless of the above-mentioned methodological principles. According to Fieldian nominalism, we should avoid numbers if we can, without paying any attention to the fact that, for example, if-then-istic conception of mathematics is much more complicated than the Platonistic one. Ockham's razor is, in the first place, a methodological principle aimed at successful, simple, uniform and explanatory powerful theories. Its main task is to avoid the entities that do not have any role in explanation or prediction and not to minimalize the number of entities needed by the theory. Abstract objects take important part in scientific explanation of different natural phenomena. So Ockham's razor does not forbid their use.

If someone has been able to point at an attempt of avoiding certain kinds of entities, there were usually other reasons for such avoidance as, for instance, lack of explanatory power, threat of inconsistency etc. I believe that the nominalists have been largely overstating the importance of ontological parsimony.

If Ockham's razor principle is read only as a demand for parsimonious ontology, then this principle is incoherent with naturalism, leading into the minimalist prejudice that does not follow from the needs of science. For example, Fieldian physics can not be regarded as advancement in development of science. Because of its complicated and long-winded structure is useless for practical needs of science. It also means no progress in explanatory and unificatory power of physics. Demand for parsimonious ontology is incoherent with naturalism, because it means interference of ontological principles with science.

Complicated paraphrase strategies do not contribute to the explanatory and unificatory power of mathematics. If-then-ist´s account of mathematics is in conflict with sincere assertions of mathematicians who claim that certain mathematical entity exists, it is very implausible to say that a mathematician actually does not know what he asserts (see Resnik, 1980). On the other hand, physicist's claims about the existence of some physical entity are not regarded by if-then-ist in the same way. The platonistic logical analysis of mathematics is the same as in case of referring to concrete objects. One analyses the sentence "3 is prime" in the same way as the sentence "John is fat". Both of the sentences have subject-predicate form. There is no double explanation. A theory which offers a unified explanation of several different phenomena is to be favoured over a theory which has the same explanatory power but offers different explanations for different phenomena.

In this paper I tried to show that if-then-ism is not an option becuse of the following reasons:

1. A science without abstract objects (science that contains the if-then-istic paraphrase of mathematics or Fieldian nominalized science) cannot be regarded as advancement according to methodological principles. Complicated paraphrase strategies do not contribute to the explanatory and unificatory power of mathematics – on the contrary, they make theories less simple and less usefull. Morever, complete science stated exclusively in nominalistic terms is still a goal that has not been achieved yet. Mathematics that contains abstract objects is still a practical necessity.

2. Arguments of Cian Dorr contained contentious premise, namely his demand for ontological parsimony. If one treats the demand for ontological parsimony as ontological principle, then his methodes are not compatible with naturalism (Interference of ontological principles in science). On the other hand, ontological parsimony understood as methodological principle does not commit one to the acceptance of anti-realism concerning abstract objects.

References:

Balaguer, M. (1998). *Platonism and Anti-platonism in Mathematics*. New York: Oxford University Press.

Colyvan, M. (2001). *The Indispensability of Mathematics*. Oxford: Oxford University Press.

Dorr, C. (2005). "There are no abstract objects"; draft version (February 23, 2005). Forthcoming in: *Blackwell Great Debates in Metaphysics*, ed. John Hawthorne, Theodore Sider and Dean Zimmerman, Oxford: Blackwell.

Field, H. (1980). *Science Without Numbers. A Defence of Nominalism.* London: Basil Blackwell Publisher.

Quine, W. V. (1953). "On What There Is"; reprinted in: W. V. Quine, *From a Logical Point of View. Nine Logico-Philosophical Essays*. Cambridge, MA: Harvard University Press.

Russell, B. (1992). *Principles of Mathematics.* London: Routledge; first published in 1903.

Putnam, H. (1975). "What is mathematical truth?"; reprinted in: H. Putnam (1979), *Mathematics, Matter and Method. Philosophical Papers*, vol. 1, 2nd edn., Cambridge: Cambridge University Press.

Matija Arko

Ist der *Wenn-Dann-Ismus* immer noch eine Option?

Zusammenfassung

Im Folgenden werde ich über den Wenn-Dann-Ismus als Strategie zur Verringerung unserer ontologischen Verpflichtungen in der Mathematik sprechen. Ich beginne mit der Definition des Wenn-Dann-Ismus in Principles of Matemathics*. Danach setze ich mich mit Puntnams Kritik des Wenn-Dann-Ismus auseinander. Es folgen einige Argumente von Cian Dorr, die diese Kritik unterstützen. Zuletzt soll von der ontologischen Sparsamkeit als allgemeinem Motiv für die Aneignung des Wenn-Dann-Ismus die Rede sein. Ich werde von der ontologischen Sparsamkeit („parsimony") in Zusammenhang mit dem Naturalismus sprechen und zugunsten der These argumentieren, dass die Bevorzugung der ontologischen Sparsamkeit nicht aus dem Naturalismus hervorgeht.*

Schlüsselwörter

Wenn-Dann-Ismus, ontologische Sparsamkeit, Naturalismus, Philosophie der Mathematik

Matija Arko

Le si-alors-sinon, est-il toujours une option?

Sommaire

Je parlerai dans cet article du si-alors-sinon comme d'une stratégie de nos obligations ontologiques dans les mathématiques. Je commencerai par la définition du si-alors-sinon dans les Principia mathematica*. Ensuite, je discuterai la critique du si-alors-sinon par Putnam. Suivront certains arguments de Ciana Dorra qui l'appuient. Finalement, il sera question de la « parcimonie » ontologique en tant que motif d'adoption du si-alors-sinon. Je traiterai de la parcimonie ontologique en liaison avec le naturalisme en essayant de démontrer que la parcimonie ontologique n'est pas une exigence découlant du naturalisme.*

Mots clés

si-alors-sinon, parcimonie ontologique, naturalisme, philosophie des mathématiques