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EQUIVARIANT FIBRANT SPACES

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Abstract. In this paper the concept of a G-fibrant space is introduced. It is shown that any compact metrizable group G is a G-fibrant.

1. Introduction

The general approach to the concept of *a fibrant object* is the following (c.f.[5]): if in a category $\mathcal{C}$ some class Σ of morphisms is specified then an object Y of $\mathcal{C}$ is called Σ-fibrant if for every morphism $s \in \Sigma$, $s : A \to X$, and every morphism $f : A \to Y$ there is a morphism $F : X \to Y$ such that $F \circ s = f$. The classical fibrant objects appear in [9] for the closed model categories where Σ is the class of trivial cofibrations. *A fibrant space* in the sense of F. Cathey is a Σ-fibrant object, where Σ is the class of $SSDR$-maps in the category of metrizable spaces ([6]). In the present paper we provide an equivariant version of a fibrant space.

It is well-known (see [8]) that every compact metrizable group can be represented as an inverse limit of a sequence of Lie groups bonded by fibrations (Proposition 3.3), and therefore it is already a fibrant space in the sense of F. Cathey. On the other hand, due to R. Palais ([7]), every compact Lie group G is a G-ANR (Proposition 3.2) and hence it is a G-fibrant space. These are the basic facts utilized in the proof of our main theorem (Theorem 3.1): every compact metrizable group G is a G-fibrant space. This result justifies the consideration of equivariant fibrant spaces. Also it is clear that equivariant fibrant spaces as well as equivariant $SSDR$-maps can be used in the construction of the equivariant strong shape category following the way of F. Cathey, which is given in [6] for the "non-equivariant" case.

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2. The basic notions

The basic definitions and facts of the equivariant theory, that is the theory of G-spaces and G-maps, can be found in [4]. Throughout the paper the letter G will denote a compact Hausdorff group. By G-$A(N)R$, it is denoted the class of G-equivariant absolute (neighborhood) retracts for all G-metrizable spaces (see, for instance, [2] for the equivariant theory of retracts). In this paper all G-spaces are assumed to be metrizable.

A closed invariant subspace A of a G-space X is called a *G-shape strong deformation retract* of X if there exists a G-equivariant embedding $i : X \hookrightarrow Y$ for some G-AR space Y such that for any pair of invariant neighborhoods U and V of $i(X)$ and $i(A)$ respectively in Y, there is a G-homotopy $H : X \times I \to U$ rel. A such that $H(x, 0) = i(x)$ and $H(x, 1) \in V$ for any $x \in X$.

Note that if for a G-pair (X, A) an embedding $i : X \hookrightarrow M$ satisfies the conditions of the above definition then these conditions hold for any other closed G-equivariant embedding $j : X \hookrightarrow Z$ where Z is a G-AR space.

A closed G-equivariant embedding $s : A \hookrightarrow X$ is called *a G-$SSDR$-map* if s embeds A in X as a G-shape strong deformation retract of X.

A G-space Y is called *a G-fibrant* if for every G-$SSDR$-map $s : A \hookrightarrow X$ and every G-map $f : A \to Y$, there exists a G-map $F : X \to Y$ such that $F \circ s = f$.

Recall that a map $p : E \to B$ is a *G-fibration* if for every G-space X and every commutative diagram of G-maps

$$\begin{tikzcd} X \arrow[r, "h"] \arrow[d, "\delta_0"'] & E \arrow[d, "p"] \\ X \times I \arrow[r, "H"] & B \end{tikzcd}$$

where $\delta_0(x) = (x, 0)$, there exists $\widetilde{H} : X \times I \to E$ such that $\widetilde{H} \circ \delta_0 = h$ and $p \circ \widetilde{H} = H$.

For example, the G-fibrations naturally appear in the following situation. Let U be a G-space. The space U^I of paths $\omega : I \to U$, provided with the compact-open topology, can be treated as a G-space with the action: $(g \star \omega)(t) = g\omega(t)$. Then the projection $p : U^I \to U \times U$, given by $p(\omega) = (\omega(0), \omega(1))$, is a G-fibration.

The following theorem is an equivariant version of Theorem 1.2 of [6].

Theorem 2.1. *Let $s : A \hookrightarrow X$ be a closed G-embedding. Then the following conditions are equivalent:*

(a) *s is a G-$SSDR$-map;*

(b) *for any G-equivariant map $f : A \to Y$, where Y is G-ANR, there is a G-equivariant extension $\widetilde{f} : X \to Y$ such that $\widetilde{f} \circ s = f$, and if $\widetilde{f}_1, \widetilde{f}_2 : X \to Y$ are any two such extensions, then $\widetilde{f}_1 \simeq_G \widetilde{f}_2$ rel. $s(A)$;*

(c) *For any G-fibration $p : E \to B$, where E and B are G-ANR-spaces and any commutative diagram of G-equivariant maps*

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "s"'] & E \arrow[d, "p"] \\ X \arrow[r, "F"] & B \end{tikzcd}$$

there exists a G-equivariant map $\widetilde{F} : X \to E$ such that $\widetilde{F} \circ s = f$ and $p \circ \widetilde{F} = F$.

We shall give the proof of the theorem though it is quite analogous to the proof of its "non-equivariant" case.

Proof. (a) $\Rightarrow$ (b). Clearly, we can assume that $A \subset X$ and $s(a) = a$. Let $X \hookrightarrow M$ be an equivariant closed embedding of X in some G-AR space M (See [3], Proposition 1). Since each G-AR space for metrizable spaces is a G-AE ([3], Proposition 2), there is a G-extension $\widehat{f} : V \to Y$ of f on some invariant open neighborhood V of A in M. By the definition of a G-$SSDR$-map, we can find a G-homotopy $H : X \times I \to M$ such that $H(x, 0) = x$, $H(x, 1) \in V$ and $H(a, t) = a$ for $x \in X$, $a \in A$, $t \in I$. The required extension $\widetilde{f} : X \to Y$ can be given by $\widetilde{f}(x) = \widehat{f}(H(x, 1))$.

Let $\widetilde{f}_1, \widetilde{f}_2 : X \to Y$ be two G-extensions of f. Define a G-equivariant map $F : X \times 0 \cup A \times I \cup X \times 1 \to Y$ by $F(x, 0) = \widetilde{f}_1(x)$, $F(x, 1) = \widetilde{f}_2(x)$, $F(a, t) = f(a)$ for $x \in X$, $a \in A$, $t \in I$. Considering X as a closed invariant subset of a G-AR space M, and therefore $X \times I$ as a closed invariant subset of the G-AR space $M \times I$, we find a G-extension $\overline{F} : W \to Y$ of F on some invariant neighborhood W of $X \times 0 \cup A \times I \cup X \times 1$ in $M \times I$. Clearly, one can choose an invariant neighborhood U of X in M such that $U \times \{0\} \subset W$ and $U \times \{0\} \subset W$. Besides, a standard compactness argument guarantees the existence of an invariant neighborhood V of A in M such that $V \times I \subset W$. Taking a G-homotopy $D : X \times I \to U$ such that $D(x, 0) = x$, $D(x, 1) \in V$ and $D(a, t) = a$ for $x \in X$, $a \in A$, $t \in I$, we can establish G-homotopies $F' : \widetilde{f}_1 \simeq_G h_1$ rel. A, $F'' : \widetilde{f}_2 \simeq_G h_2$ rel. A and $H : h_1 \simeq_G h_2$ rel. A by $F'(x, t) = \overline{F}(D(x, t), 0)$, $F''(x, t) = \overline{F}(D(x, t), 1)$ and $H(x, t) = \overline{F}(D(x, 1), t)$. Thus $\widetilde{f}_1 \simeq_G \widetilde{f}_2$ rel. A.

(b) $\Rightarrow$ (c). Since E is a G-ANR there exists a G-extension $\overline{F} : X \to E$ such that $\overline{F} \circ s = f$. We have $F \circ s = p \circ \overline{F} \circ s$ and by the second part of (b) there is a G-homotopy $H : F \simeq_G p\overline{F}$ rel. $s(A)$. Applying the covering homotopy property we get a G-homotopy $\widetilde{H} : X \times I \to E$, $\widetilde{H} : \tilde{F} \simeq_G \widetilde{F}$ rel. $s(A)$, such that $p \circ \widetilde{H} = H$. So $\widetilde{F} \circ s = f$ and $p \circ \widetilde{F} = F$ as required.

(c) $\Rightarrow$ (a). As above, we can assume that X is an invariant closed subset of some G-AR space M and that A is an invariant closed subset of of X, so $s(a) = a$ for $a \in A$. Let U and V be invariant open neighborhoods of X and A respectively in M. First applying (c) to the G-fibration $V \to *$ and the inclusion $i : A \to V$ we get a G-map $r : X \to V$ such that $r \circ s = i$. Afterwards applying (c) to the commutative diagram

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "s"'] & U^I \arrow[d, "p"] \\ X \arrow[r, "F"] & U \times U \end{tikzcd}$$

where $p(\omega) = (\omega(0), \omega(1))$, $f(a)(t) = a$, $F(x) = (x, r(x))$, we obtain a G-map $\overline{F} : X \to U^I$ such that $\overline{F} \circ s = f$, $p \circ \overline{F} = F$. Now observe that the map $D : X \times I \to U$ defined by $D(x,t) = \overline{F}(x)(t)$ satisfies the conditions of the definition of a G-$SSDR$-map. $\square$

Corollary 2.2. *Every G-ANR is a G-fibrant space.*

3. Main result

The main result of this paper is the following

Theorem 3.1. *Every compact metrizable group G is a G-fibrant space.*

In the proof of this theorem, we shall use the propositions given below.

Proposition 3.2. *([7], Proposition 1.6.6) Let G be a compact Lie group and H be its closed subgroup. Then G/H is a G-ANR space.*

The following result is actually proved in the classical book of Pontrjagin [8]. Note that it can be easily obtained from Corollary 4.4 of [4]: for every neighborhood U of the unit e of a compact group G, there exists a group morphism $\varphi : G \to \mathbf{O}(n)$ such that $\ker \varphi \subseteq U$.

Proposition 3.3. *Let G be a compact metrizable group. Then there exists a decreasing sequence $\{N_i\}_{i \in \mathbb{N}}$ of its normal closed subgroups such that the quotient groups G/N_i are Lie groups, $\bigcap_{i \in \mathbb{N}} N_i = \{e\}$ and*

$$\varprojlim \{G/N_i, q_i^j\} = G$$

where $q_i^j : G/N_j \to G/N_i$, $j \geq i$, are the natural projections.

We omit a routine proof of the following statement.

Proposition 3.4. *Let G be a compact metrizable group and $\{N_i\}_{i\in\mathbb{N}}$ be a sequence of its closed normal subgroups satisfying Proposition 3.3.*

(a) *If X is a G-space, then*

$$X = \varprojlim\{X/N_i, p_i^j\}$$

where $p_i^j : X/N_j \to X/N_i$, $j \geq i$, are the natural projections.

(b) *Let X and Y be G-spaces represented according to (a) as*

$$X = \varprojlim\{X/N_i, p_i^j\} \text{ and } Y = \varprojlim\{Y/N_i, q_i^j\}.$$

If the G/N_i-maps $f_i : X/N_i \to Y/N_i$, $i \in \mathbb{N}$, are such that $q_i^{i+1} f_{i+1} = f_i p_i^{i+1}$, i.e. the diagram

$$\begin{tikzcd} X/N_{i+1} \arrow[r, "f_{i+1}"] \arrow[d, "p_i^{i+1}"'] & Y/N_{i+1} \arrow[d, "q_i^{i+1}"] \\ X/N_i \arrow[r, "f_i"] & Y/N_i \end{tikzcd}$$

commutes for each $i \in \mathbb{N}$, then there exists a unique G-map $f : X \to Y$ such that $q_i f = f_i p_i$ for each i, where $p_i : X \to X/N_i$, $q_i : Y \to Y/N_i$ are N_i-orbit projections.

Proposition 3.5. *Let G be a compact group and N be a closed normal subgroup of G. If $s : A \to X$ is a G-SSDR-map, then the induced map $s/N : A/N \to X/N$ is a G/N-SSDR-map.*

Proof. Let $j_0 : X/N \hookrightarrow Y$ be a closed G/N-embedding of X/N in a G/N-ANR space Y. By Lemma 1 of [1] there exist a G-space Z and a closed G-embedding $\widehat{j}_0 : X \hookrightarrow Z$ such that $Z/N = Y$ and $j_0 \circ p = q_1 \circ \widehat{j}_0$, where $p : X \to X/N$, $q_1 : Z \to Y$ are the N-orbit maps. Let $\widehat{j}_1 : Z \hookrightarrow M$ be a closed G-embedding of Z in a G-AR space M (See [3], Proposition 1). Then by Theorem 1 of [2] M/N is a G/N-ANR space and we get a closed G/N-embedding $j = j_1 \circ j_0 : X/N \hookrightarrow M/N$, where the embedding $j_1 : Z/N \hookrightarrow M/N$ is induced by $\widehat{j}_1$. Moreover, for the closed G-embedding $\widehat{j} = \widehat{j}_1 \circ \widehat{j}_0 : X \hookrightarrow M$ and the N-orbit map $q : M \to M/N$ we have $q \circ \widehat{j} = j \circ p$.

Now let U and V be invariant neighborhoods of X/N and A/N respectively in M/N. Then $\widehat{U} = q^{-1}(U)$ and $\widehat{V} = q^{-1}(V)$ are invariant neighborhoods of X and A respectively in M. Since $s : A \hookrightarrow X$ is a G-$SSDR$-map there exists a G-homotopy $\widehat{H} : X \times I \to \widehat{U}$ rel. A such that $\widehat{H}(x,0) = x$ and $\widehat{H}(x,1) \in \widehat{V}$. It is clear that the induced G/N-homotopy $H : X/N \times I \to U$, defined by $H(N(x),t) = N(\widehat{H}(x,t))$, satisfies the analogous properties and it means that $s/N : A/N \hookrightarrow X/N$ is a G/N-$SSDR$-map. ∎

We need the following version of the covering homotopy theorem (compare [4], Ch. II, Theorem 7.3).

PROPOSITION 3.6. *Let G be a compact Lie group and N be a normal closed subgroup of G. Suppose that for a G-space X, a G/N-homotopy $H : X/N \times I \to G/N$ and a G-map $h : X \to G$ are such that the following diagram commutes*

$$\begin{array}{ccc} X & \xrightarrow{h} & G \\ {\scriptstyle \partial_0}\downarrow & & \downarrow{\scriptstyle q} \\ X/N \times I & \xrightarrow{H} & G/N \end{array}$$

where $\partial_0(x) = (p(x), 0)$ and $p : X \to X/N$, $q : G \to G/N$ are the N-orbit maps. Then there exists a G-homotopy $\widetilde{H} : X \times I \to G$ such that $\widetilde{H}(x, 0) = h(x)$ and $q \circ \widetilde{H} = H \circ (p \times 1_I)$.

Moreover, if A is an invariant closed subset of X such that $H(p(a), t) = H(p(a), 0)$ for any $a \in A$, $t \in I$, then the covering homotopy $\widetilde{H}$ can be chosen so that $\widetilde{H}(a, t) = h(a)$ for any $a \in A$, $t \in I$.

PROOF. Note that the existence of the G-map $h : X \to G$ implies that the action of the group G on X is of a quite simple structure. Indeed, let $S = h^{-1}(e)$, where e is the unit element of the group G, and let $\rho : X \to S$ be a map defined by $\rho(x) = (h(x))^{-1}x$. Then ρ is a retraction such that $\rho(gx) = \rho(x)$ for any $g \in G$, $x \in X$. Now consider the product $G \times S$ as a G-space endowed with the action $g(g', s) = (gg', s)$. It can be easily verified that the map $\varphi : X \to G \times S$, given by $\varphi(x) = (h(x), \rho(x))$, is a G-map and, moreover, it is a G-equivalence because the G-map $\psi : G \times S \to X$, where $\psi(g, s) = gs$, is the inverse map for φ. In fact, we shall use this G-equivalence in the construction of the covering homotopy $\widetilde{H}$.

Let $F : S \times I \to G/N$ be a homotopy defined by $F(s, t) = H(p(s), t)$. For the given invariant closed subset $A \subseteq X$, let $U = A \cap S$, that is $U = \rho(A)$. Then, if $u \in U$, we have $F(u, t) = H(p(u), t) = H(p(u), 0) = F(u, 0)$ for any $t \in I$.

Since N is a Lie group, the projection $q : G \to G/N$ is a locally trivial fibration (see [4], Ch. II, Theorem 5.8) and hence it has the regular homotopy lifting property. In particular, considering the commutative diagram

$$\begin{array}{ccc} S & \xrightarrow{f} & G \\ {\scriptstyle \delta_0}\downarrow & & \downarrow{\scriptstyle q} \\ S \times I & \xrightarrow{F} & G/N \end{array}$$

where $f(s) = e$ and $\delta_0(s) = (s, 0)$ for $s \in S$, one can find a homotopy $\widetilde{F} : S \times I \to G$ which preserves the commutativity of the diagram, that is to say, $q \circ \widetilde{F} = F$, $\widetilde{F} \circ \delta_0 = f$ and, moreover, $\widetilde{F}(u, t) = e$ for any $u \in U$, $t \in I$.

Finally, the required covering homotopy $\widetilde{H} : X \times I \to G$ can be defined by $\widetilde{H}(x, t) = h(x)\widetilde{F}(\rho(x), t)$. The verification of this fact is straightforward.

Indeed, we have $\widetilde{H}(x, 0) = h(x)\widetilde{F}(\rho(x), 0)$, but

$$\widetilde{F}(\rho(x), 0) = \widetilde{F} \circ \delta_0(\rho(x)) = f(\rho(x)) = e,$$

and hence $\widetilde{H}(x, 0) = h(x)$.

Since $q : G \to G/N$ is a group morphism and $q \circ \widetilde{F} = F$, we get

$$q(\widetilde{H}(x, t)) = q(h(x))q(\widetilde{F}(\rho(x), t)) = q(h(x))F(\rho(x), t).$$

By the definition of the homotopy F, we have $F(\rho(x), t) = H(p(\rho(x)), t)$, but H is a G/N-map, and therefore, $q(h(x))H(p(\rho(x)), t) = H(q(h(x))p(\rho(x)), t)$. Reminding that $q : G \to G/N$ and $p : X \to X/N$ are N-orbit maps, we obtain

$$q(h(x))p(\rho(x)) = p(h(x)\rho(x)) = p(x).$$

Thus $q(\widetilde{H}(x, t)) = H(p(x), t)$, that is $q \circ \widetilde{H} = H \circ (p \times 1_I)$.

Besides, if $a \in A$, we have $\rho(a) \in U$, and therefore,

$\widetilde{H}(a, t) = h(a)\widetilde{F}(\rho(a), t) = h(a)e = h(a)$ for any $t \in I$. □

Proof of Theorem 3.1. Let $j : A \hookrightarrow X$ be a G-$SSDR$-map and $f : A \to G$ be a G-map. In order to show that G is a G-fibrant, we must find a G-map $F : X \to G$ such that $F \circ j = f$.

According to Proposition 3.3, we represent the group G as an inverse limit of Lie groups $G = \varprojlim\{G/N_i, q_i^j\}$.

The G-maps j and f induce for each k the G/N_k-maps $j_k = j/N_k : A/N_k \hookrightarrow X/N_k$, $f_k = f/N_k : A/N_k \to G/N_k$. Then for each k the following diagram commutes

$$\begin{tikzcd}
X/N_{k+1} \arrow[d, "p_k^{k+1}"'] & A/N_{k+1} \arrow[l, "j_{k+1}"'] \arrow[r, "f_{k+1}"] \arrow[d, "r_k^{k+1}"'] & G/N_{k+1} \arrow[d, "q_k^{k+1}"] \\
X/N_k & A/N_k \arrow[l, "j_k"'] \arrow[r, "f_k"] & G/N_k
\end{tikzcd}$$

where p_k^{k+1}, q_k^{k+1}, r_k^{k+1} are the natural projections. It is clear that these projections can be treated as orbit projections with respect to the action of the closed subgroup N_i/N_{i+1} of the Lie group G/N_{i+1} on X/N_{i+1}, A/N_{i+1} and G/N_{i+1} respectively. For each k the map j_k is a G/N_k-$SSDR$-map by Proposition 3.5, and G/N_k is a G/N_k-ANR by Proposition 3.2, and therefore there exists a G/N_k-equivariant extension $F_k : X/N_k \to G/N_k$ of f_k such that

$F_k j_k = f_k$. Using these extensions we shall construct by induction G/N_k-maps $T_k : X/N_k \to G/N_k$ satisfying the conditions $T_k p_k^{k+1} = q_k^{k+1} T_{k+1}$ and $T_k j_k = f_k$ for every $k \in \mathbb{N}$. Let $T_1 = F_1$ and suppose that T_k is already found. We have to construct the map T_{k+1}.

One has

$$T_k j_k r_k^{k+1} = f_k r_k^{k+1} = q_k^{k+1} f_{k+1} = q_k^{k+1} F_{k+1} j_{k+1} = F_k^* p_k^{k+1} j_{k+1} = F_k^* j_k r_k^{k+1},$$

where $F_k^* : X/N_k \to G/N_k$ is the G/N_k-map induced by F_{k+1}. Hence $T_k j_k = F_k^* j_k$ because r_k^{k+1} is surjective. According to Theorem 2.1, there is a G/N_k-homotopy $H : F_k^* \simeq T_k \ rel. j_k(A/N_k)$, $H : X/N_k \times I \to G/N_k$.

Now consider the following commutative diagram

$$\begin{tikzcd}
X/N_{k+1} \arrow[r, "F_{k+1}"] \arrow[d, "\partial_0"'] & G/N_{k+1} \arrow[d, "q_k^{k+1}"] \\
X/N_k \times I \arrow[r, "H"] & G/N_k
\end{tikzcd}$$

where $\partial_0([x]) = (p_k^{k+1}[x], 0)$ for $[x] = N_{k+1}(x) \in X/N_{k+1}$.

Taking into account this diagram, we are going to apply Proposition 3.6 to the Lie group G/N_{k+1} acting on X/N_{k+1} and to its closed normal subgroup N_k/N_{k+1}. Note that one can consider the G/N_{k+1}-space X/N_k as the orbit space $X/N_{k+1}/N_k/N_{k+1}$.

By Proposition 3.6, we get a G/N_{k+1}-homotopy

$$\widetilde{H} : X/N_{k+1} \times I \to G/N_{k+1}$$

such that $\widetilde{H}([x], 0) = F_{k+1}([x])$, $q_k^{k+1}(\widetilde{H}([x], t) = H(p_k^{k+1}([x]), t)$ and, for any $t \in I$, $\widetilde{H}([a], t) = F_{k+1}([a])$ if $[a] \in j_{k+1}(A/N_{k+1})$. Putting $T_{k+1}([x]) = \widetilde{H}([x], 1)$ we get the required G/N_{k+1}-map $T_{k+1} : X/N_{k+1} \to G/N_{k+1}$. This completes the inductive step.

The sequence $\{T_k\}_{k \in \mathbb{N}}$, according to Proposition 3.4, determines a unique G-map $F : X \to G$ such that $q_k F = T_k p_k$ for each k. Since $T_k j_k = f_k$ for each k, we can state that $F \circ j = f$. □

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