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SHEAR DEFORMATION LAMINATE THEORY USED FOR SANDWICHES

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Preliminary Note – Prethodno priopćenje

The shear deformation laminate theory is very useful for the calculation of the sandwich composites. Sandwich can be defined as a special laminate with three layers and therefore can be modeled using shear deformation laminate theory by neglecting of membrane and bending deformations in the core and the shear deformation in the facings.

Key words: sandwich plate, shear deformation theory, stress resultant analysis, symmetric and asymmetric laminate

Deformacijska teorija smika laminata rabljenih za sendviče. Deformacijska teorija smika laminata je rabljena za proračun sendvičevih kompozita. Sendvič se može definirati kao specijalni laminat s tri vrste, i glede toga može se modelirati pomoću smičuće teorije laminata pri zanemarivanju membranske i zavojne deformacije u jezgri i smika na površini sendviča.

Ključne riječi: sendvička daska, deforacijska teorija smika, analiza rezultata naprezanja, simetrički i antisimetrički laminati

INTRODUCTION

One special group of laminated composites used extensively in engineering applications is sandwich composites. Sandwich panels consist of thin facings and core. The facings are made of high strength material while the core is made of thick and lightweight materials. The motivation for sandwich structure elements is twofold. First, for beam or plate bending the maximum normal stresses occur at the top and the bottom surface. So it makes sense using high-strength materials at the top and the bottom and using low and lightweight strength materials in the middle. Second, the bending resistance for a rectangular cross-sectional beam or plate is proportional to the cube of the thickness. Increasing the thickness by adding a core in the middle increases the resistance. The maximum shear stress is generally in the middle of the sandwich requiring a core to support shear. The advantages in weight and bending stiffness make sandwich composites attractive in many applications [1, 2].

GENERAL ASSUMPTIONS

A sandwich can be defined as a special laminate with three layers (Figure 1). The thin cover sheets, i.e. the layers 1 and 3, are laminates of the thicknesses symmetric sandwiches h_1 for the lower skin and h_3 for the upper skin. The thickness of the core is h_2. In a general case h_1 does not have to be equal to h_3, but in the most important practical case of symmetric sandwiches $h_1 = h_3$ [3, 4].

The assumptions for macro-mechanical modeling of sandwiches are:

1. The thickness of the core is much greater than that of the skins, $h_2 >> h_1,\ h_3$.
2. The strains $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ vary linearly through the core thickness
$$\{\varepsilon(x,y,z)\} = \{\bar{\varepsilon}(x,y)\} + z\{\kappa(x,y)\},\quad -\frac{h_2}{2} \le z \le +\frac{h_2}{2} \tag{1}$$
3. The sheets only transmit stresses $\sigma_x, \sigma_y, \tau_{xy}$ and the in-plane strains $\varepsilon_x,\ \varepsilon_y,\ \gamma_{xy}$ are uniform through the thickness of the skins. The transverse shear stresses $\tau_{xz},\ \tau_{yz}$ are neglected within the skin.
4. The core only transmits transverse shear stresses τ_{xz}, τ_{yz}, the stresses $\sigma_x, \sigma_y, \tau_{xy}$ are neglected.
5. The strain ε_z is neglected in the sheets and the core.

STRESS RESULTANTS AND STRESS ANALYSIS

In the stress analysis we can obtain the stress resultants of the sandwich (Figure 2).

There are the normal forces in cover sheets $i = 1,3$

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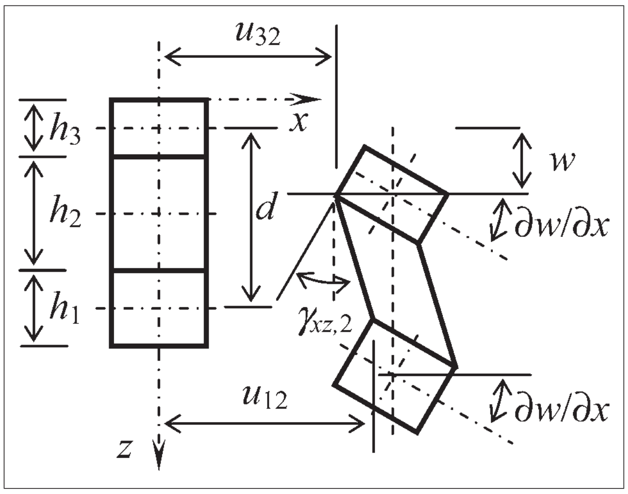

Figure 1 Geometry of deformation in the (x,z) plane

$$N_{xi} = D_{Ni}\left(\frac{\partial u_i}{\partial x} + \nu_i \frac{\partial v_i}{\partial y}\right),$$

$$N_{yi} = D_{Ni}\left(\nu_i \frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y}\right),$$

$$N_{xyi} = \frac{D_{Ni}(1-\nu_i)}{2}\left(\frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x}\right)$$

where:

$$D_{Ni} = E_i h_i \,/\, (1-\nu_i^2) \tag{2}$$

The bending moments can be written as:

$$M_{xi} = -D_{Mi}\left(\frac{\partial^2 w}{\partial x^2} + \nu_i \frac{\partial^2 w}{\partial y^2}\right),$$

$$M_{yi} = -D_{Mi}\left(\nu_i \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right), \tag{3}$$

$$M_{xyi} = -D_{Mi}(1-\nu_i)\frac{\partial^2 w}{\partial x \partial y}.$$

The shear forces

$$V_{xzi} = -D_{Mi}\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right)$$

$$V_{yzi} = -D_{Mi}\left(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \partial y}\right)$$

where:

$$D_{Mi} = E_i h_i^3 \,/\, 12(1-\nu_i^2) \tag{3}$$

The shear stresses in the core are written:

$$\tau_{xz} = G_2 \gamma_{xz2} = \frac{G_2}{h_2}\left(u_1 - u_3 + d\frac{\partial w}{\partial x}\right),$$

$$\tau_{yz} = G_2 \gamma_{yz2} = \frac{G_2}{h_2}\left(v_1 - v_3 + d\frac{\partial w}{\partial y}\right). \tag{4}$$

The equilibrium equations for internal forces are following:

$$\frac{\partial N_{xi}}{\partial x} + \frac{\partial N_{yxi}}{\partial y} + \frac{\partial V_{zxi}}{\partial z} = 0,$$

$$\frac{\partial N_{xyi}}{\partial x} + \frac{\partial N_{yi}}{\partial y} + \frac{\partial V_{zyi}}{\partial z} = 0, \quad i = 1,3$$

$$\frac{\partial V_{xz}}{\partial x} + \frac{\partial V_{yz}}{\partial y} + p = 0 \tag{5}$$

where:

$$\frac{\partial V_{xz1}}{\partial z} = -\tau_{zx}, \quad \frac{\partial V_{xz3}}{\partial z} = \tau_{zx},$$

$$V_{xz} = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} + \frac{\partial V_{xz}}{\partial z},$$

$$V_{yz} = \frac{\partial M_{yx}}{\partial x} + \frac{\partial M_y}{\partial y} + \frac{\partial V_{yz}}{\partial z}, \tag{6}$$

$$\frac{\partial V_{xz}}{\partial z} = \tau_{xz} h_2, \; \frac{\partial V_{yz}}{\partial z} = \tau_{yz} h_{2.}$$

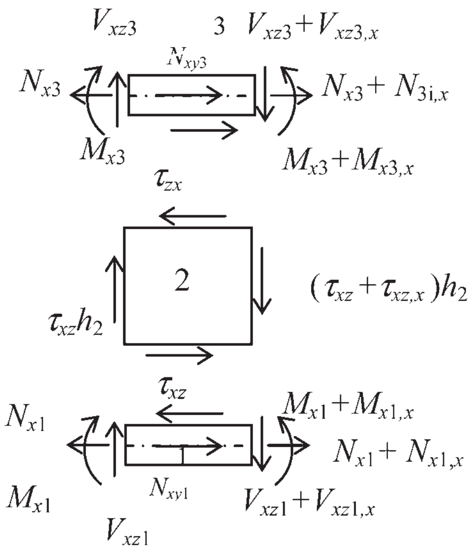

Figure 2 Internal forces at the sandwich element in the (x, z) plane

NUMERICAL EXAMPLE

Simply supported sandwich plate with the dimensions L =1m, H=0,8m (Figure 3) and thickness h=0,016m is loaded in the bending plane q=15kPa. The material constants of the core are given in the Table 1. The facings are made of laminate:

a) symmetric [0/90/90/0]
b) asymmetric [0/90/0/90],

with material characteristic of each layer given in the Table 2.

Table 1 **Material constants of the core**

G_2/MPa	ν_2
16,154	0,3

Table 2 **Material constants of the laminate layer**

E_1/GPa	E_2/GPa	G_{12}/GPa	ν_{12}
140,766	12,335	6,457	0,38

Table 3 **Effective material constants**

E_x/GPa	E_y/GPa	G_{xy}/GPa	ν
76,9787	76,9787	25,9537	0,483

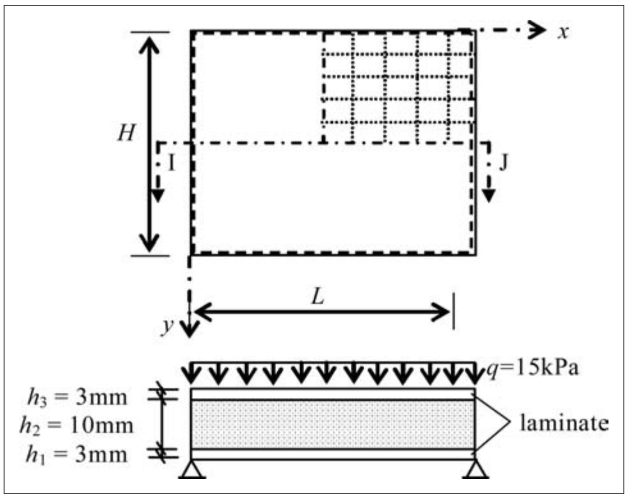

Figure 3 Geometry of simply supported sandwich plate

RESULTS AND DISCUSSION

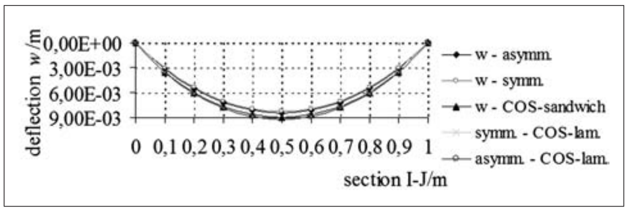

Figure 4 Variation of deflection *w*/m across section I-J/m

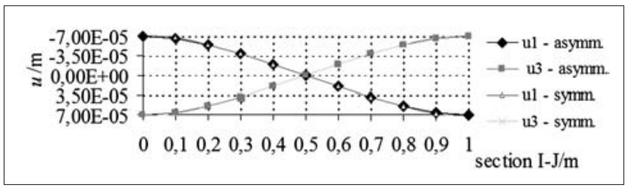

Figure 5 Variation of displacement *u*/m across section I-J/m

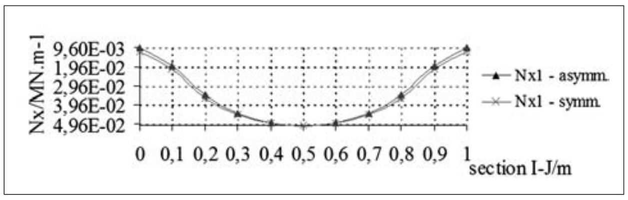

Figure 6 Variation of normal force N_x/MN · m^{-1} in the bottom layer across I-J/m

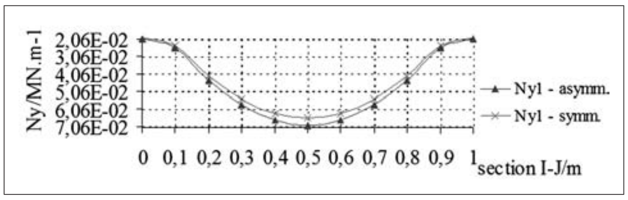

Figure 7 Variation of normal force N_y/MN · m^{-1} in the bottom layer across I-J/m

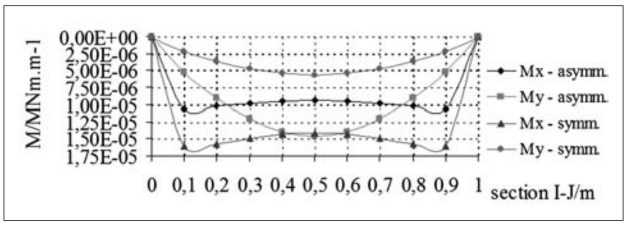

Figure 8 Variation of bending moments Mx / MNm · m^{-1} and M_y / MNm · m^{-1} across section I-J/m

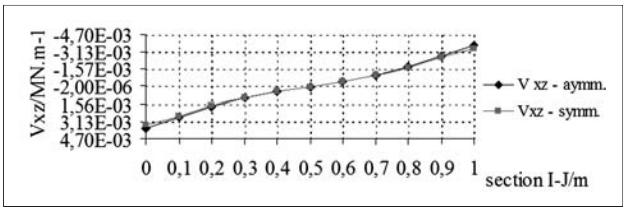

Figure 9 Variation of shear force V_{xz}/ MN · m^{-1} in the core across section I-J/m

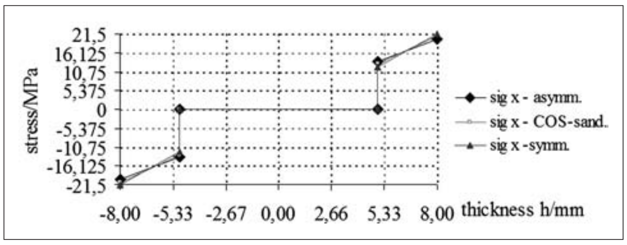

Figure 10 Variation of normal stress σ_x/MPa across thickness *h*/mm – laminate equivalent

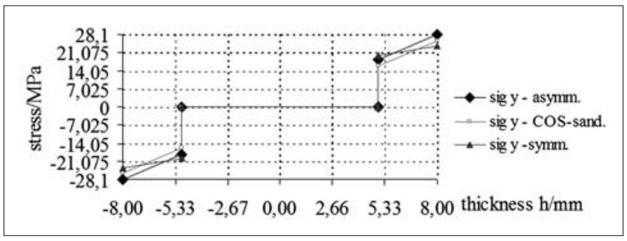

Figure 11 Variation of normal stress σ_x/MPa across thickness *h*/mm - laminate equivalent

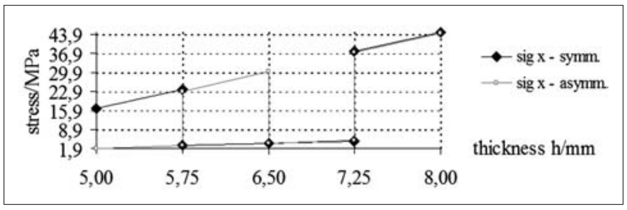

Figure 12 Variation of normal stress σ_x/MPa across thickness *h*/mm – real laminate facing

In the Figure 4 there is shown the variation of deflection *w* across the length *L* and differences between all calculations. It means calculations:

- with the help of FEM program (symmetric and asymmetric stacking of layers),
- with the help of program COSMOS/M (symmetric and asymmetric stacking of layers in laminate finite element and with laminate equivalent in sandwich finite element).

In the Figure 5 is shown the variation of displacement *u* across the length *L* in the first and third layer (symmetric and asymmetric stacking of layers) in FEM program. By the symmetric laminate we can not observe the coupling effect, however by the asymmetric laminate the coupling effect is observed. Therefore we notice the slight differences between values of normal forces (Figures 6, 7). We also observe the evident differences in the bending moments diagram (Figure 8) as result of the change of the *z*-coordinate of the laminate layers from the mid-plane. Because the resistance of the sandwich is proportional to the cube of the thickness, the differences between bending moments are greater than between normal forces, because normal forces do not depend on *z*-coordinate. In the Figure 9 there is shown the variation of shear force V_{xz} in the core across length *L* by the symmetric and asymmetric layers stacking of the sandwich facings. In the Figure 10 and 11 there is shown the variation of normal stress σ_x and σ_y, respectively, across sandwich thickness by using the laminate equivalent in FEM program (symmetric and asymmetric stacking of layers) and in program COSMOS/M.

CONCLUSION

In the paper we solved the symmetric sandwich plate with the help of finite element method program (FEM) and program COSMOS/M [5, 6]. In the case a) the facings of the sandwich are made of the symmetric [0/90/90/0] laminates, in the case b) of the asymmetric [0/90/0/90] laminates. In the program COSMOS/M we solved the plate with the help of laminate finite element SHELL4L with nine layers and sandwich finite element SHELL4L with three layers. In the case of three layers sandwich we made the laminate equivalent with effective elastic properties (Table 3). The normal stresses in the laminate equivalent are continuous through the thickness of facings (Figures 10, 11), whereas the normal stresses in the real laminate facings are discontinuous, because of differences between layer angles (Figure 12).

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Note: The responsible translator for English language is Martina Pľuchtová, Košice, Slovakia.