A LINEAR THEORY FOR LEE WAVE ROTORS

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Abstract: A linear theory for the prediction of rotor formation under trapped lee waves has been developed. The theory is based on the assumption that the flow can be divided into two layers: an outer region in which the flow is inviscid, and an inner region close to the ground where turbulent stresses are important. The flow in the inner region is driven by the pressure gradient due to lee waves aloft. A simple mixing-length turbulence closure is assumed and for a given lee-wave pressure field, analytic solutions to the inner-region flow are obtained. Results from the linear theory are compared with a series of two-dimensional numerical simulations in which a sharp temperature inversion is present upwind of a ridge. Lee waves may form on the inversion, which when of sufficient amplitude, give rise to flow separation and rotors underneath the wave crests. The point at which linear theory predicts flow separation underneath the wave crests is shown to correspond, approximately, to the occurrence of lee-wave rotors in the numerical results and a flow regime diagram based on linear theory is constructed and compared to that obtained from the simulations.

Keywords - *rotors, linear theory*

1. INTRODUCTION

Rotors associated with lee wave activity represent potential hazards to aviation and local populations in mountainous areas. Since computing constraints and the chaotic nature of the flow limit the usefulness of a deterministic forecast of the flow field, a forecast of flow *type* may prove more useful. Vosper (2004) developed a linear theory to examine the properties of lee waves occurring in the presence of a sharp upstream temperature inversion (conditions frequently experienced on East Falkland Island, South Atlantic, in northerly flow (Mobbs at al., 2005)). When compared to idealised two-dimensional (2-D) non-linear simulations, the theory successfully predicted the occurrence of lee waves, and their wavelength. In this study, we present a linear theory for the formation of lee-wave rotors. The predictions of the theory are compared with results of 2-D simulations. Vosper (2004) showed that the occurrence of different flow types could be summarised in a flow regime diagram. Three-dimensional (3-D) simulations are used to test the applicability of the the regime diagram as a forecasting tool for flows over realistic terrain.

2. LINEAR THEORY

In the idealised problem considered, the upstream wind speed is constant with height (above the boundary layer). The profile of potential temperature is such that a neutral layer exists in contact with the surface, capped by a sharp temperature inversion at a height $z = z_i$, above which the atmosphere is stably stratified with constant Brunt Väisälä frequency, $N = 0.01s^{-1}$. The inviscid linear theory developed by Vosper (2004) predicts the occurrence of lee waves at Froude numbers below a critical value given by,

$$F_i^2 \leq \tanh(Z)/Z \qquad (1)$$

where Z is the dimensionless inversion height, Nz_i/U, and the Froude number is expressed, $F_i = U/\sqrt{g'z_i}$. g' is a reduced gravity which depends on the change in potential temperature across the inversion.

In this study we extend the inviscid theory presented by Vosper (2004) to include a representation of the boundary-layer processes. The methodology follows that of Jackson and Hunt's (1975) theory for turbulent flow over a hill in that the flow is divided into an 'outer region', where Vosper's inviscid lee wave solution is valid, and a thin, turbulent 'inner layer' adjacent to the ground which responds to the outer region pressure field. A variant of the inner-layer solution due to Mason and King (1985) is considered, in which the unperturbed inner-layer wind is assumed to have a typical logarithmic surface-layer profile. The turbulence within the inner layer is represented by a simple mixing length closure and the flow perturbations in this layer are a result of the lee-wave pressure gradient, advection and the perturbation turbulent shear-stress gradients. Flow separation is considered to occur underneath the lee wave field when the perturbed streamwise surface wind component calculated falls below 0 ms^{-1} downstream of the mountain.

3. MODEL SIMULATIONS

Simulations were performed using the Met Office BLASIUS model (Wood and Mason 1993). High resolution simulations in 2-D were carried out for the purpose of comparison with the linear theory results. A range of inversion heights and strengths were used. Further high resolution simulations in three dimensions involved the use of a realistic terrain map representing East Falkland, South Atlantic, (maximum height < 700 m) in order to test the applicability of the flow regime diagram for 2-D flows to flows over realistic terrain.

4. RESULTS AND CONCLUSION

Fig. 1 shows the flow in a 2-D simulation where $F_i = 0.6$, $H/z_i = 0.5$. Lee waves occur, and areas of flow reversal indicate they are accompanied by rotors. The 2-D simulations are summarised in Fig. 2. The dot-dot-dashed line on the plot represents the linear theory prediction of where flow reversal occurs. The line encloses nearly all of the non-linear simulations in which lee-wave rotors occur. Flow reversal is also present in the hydraulic jump simulations. The linear theory does not capture this since the predicted lee wave amplitude diminishes as the flow becomes more non-linear at low F_i (Vosper, 2004). The dotted line in Fig. 2 denotes where the ratio of lee wave wavelength to hill width is equal to unity, and is a good indicator of where, as F_i decreases, this non-linearity becomes important.

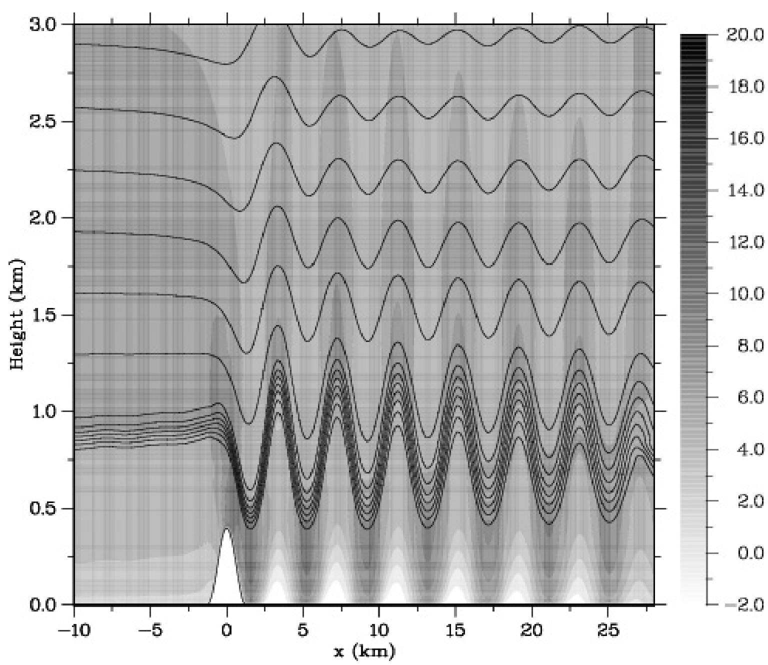

Figure 1: The flow fields in 2-D simulations after an integration time of $96L/U$ when $F_i = 0.6$, $H/z_i = 0.5$, corresponding to an inversion height and strength of $z_i = 800$ m, $\Delta\theta$=6.53 K respectively. Quantities shown are the x velocity component (shaded contours, units ms^{-1}) and potential temperature (line contours, interval=1 K). Flow is from left to right.

The results of 3-D simulations resemble the flow types occurring in 2-D, but with additional 3-D flow features and unsteadiness (Sheridan and Vosper, 2005a,b). For instance, Fig. 3 shows the winds at 10 m

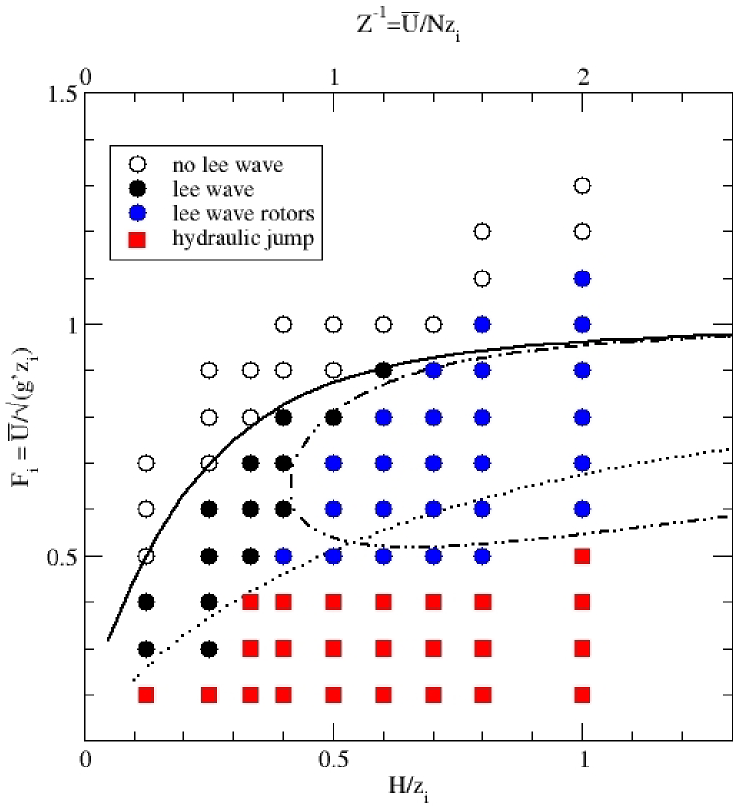

Figure 2: A flow regime diagram showing the F_i and H/z_i dependence of the flow in the 2-D simulations. Also shown are the following linear theory predictions: the critical Froude number for lee waves on the inversion (solid line), the line where the ratio of lee wave wavelength to hill width is equal to unity (dotted line), the line where the streamwise wind perturbation exceeds the streamwise component of the background wind at the surface when surface friction is accounted for using the model of Mason and King (1985) (dot-dot-dashed line).

downstream of a part of the ridge where $H/z_i \sim 0.5$, when $F_i = 0.6$. The 'banding' of the wind and areas of flow reversal indicate lee waves and rotors occur, as was the case in the 2-D flow with these upstream parameters. The flow in the 3-D simulations is largely determined by the profiles of wind and temperature and maximum height of orography directly upstream, suggesting that the regime diagram for 2-D flows may be used to predict the flow type in complex terrain.

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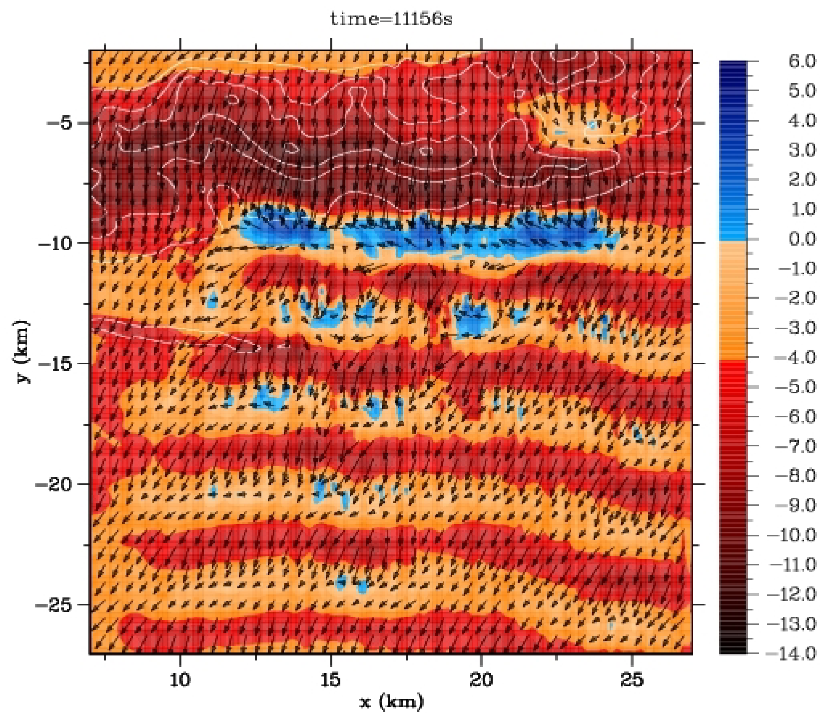

Figure 3: The flow in a 3-D simulation where $F_i = 0.6$, $z_i = 800$ m, in a region downstream of the Wickham range where the height of upstream orography is ~ 400 m. The integration time is as shown. Quantities shown are the 10 m horizontal wind vectors, the southerly flow component (colour contours, units ms^{-1}) and the terrain height (line contours, interval 100 m).