CRITICAL EVALUATION AND PROPOSED REFINEMENT OF THE TROEN AND MAHRT (1986) BOUNDARY LAYER MODEL

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Abstract: A widely used bulk model of the atmospheric boundary layer (BL) was proposed by Troen and Mahrt (1986) (hereinafter TM). The TM parameterization scheme was conceived for use in models where only a coarse resolution in the BL can be achieved (e.g. climate models and weather prediction models up to the meso-β scale). Nevertheless, this parameterization scheme is still widely adopted in high resolution local area models, even in complex terrain areas. In various LES-based tests of BL parameterizations, the TM model is typically found to fail in predicting the entrainment rate, often overestimating it in a BL with strong shear, and underestimating it in conditions of free convection. As K-closures can still be of interest in NWP, possible refinements to the TM bulk model, correcting its shortcomings, are discussed, with the further aim of reducing as much as possible the number of empirical constants in favour of conceptually based parameterisations.

***Keywords** – boundary layer, bulk parameterization schemes, entrainment, mixing height.*

1 OUTLINE OF THE TROEN AN MAHRT (1986) SCHEME

In the TM model, the Monin-Obukhov similarity theory (MOST) is used to represent surface turbulent fluxes, with a modification of the traditional Businger-Dyer (cf. Sorbjan, 1989) similarity functions to comply with the free convection limit. A structure for the eddy diffusivity K within the BL is prescribed, according to bulk dynamic stability criteria and to matching conditions with asymptotic scaling at the top of the surface layer (SL), along with a countergradient γ_c to account for non-local top-down heat transport due to large eddies. The basic framework of the TM model can be summarized in the following formulae:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K\left(\frac{\partial \theta}{\partial z} - \gamma_c\right)\right] \qquad Ri_c = \frac{g}{\theta_0}\frac{\theta_h - \theta_s}{u_h^2}h \qquad K = k\, z\, w_s\left(1 - \frac{z}{h}\right)^p \tag{1.a,b,c}$$

where θ is potential temperature and u is the horizontal wind velocity component (mean values with respect to turbulent fluctuations), while θ_0 is a reference value for θ, z is the vertical coordinate, g is the acceleration due to gravity, k is von Karman's constant, h is the whole BL depth and the subscripts "h" and "s" refer to quantities evaluated at BL top and at ground level respectively.

The scale velocity w_s is a combination of the two relevant velocity scales in the convective BL, namely shear velocity u_* and convective velocity w_*, in the form: $w_s \equiv (u_*^3 + 7\varepsilon k w_*^3)^{1/3}$. Its definition is justified on the basis of MOST, assuming the similarity function $\phi_h(\zeta) = (1-7\zeta)^{-1/3}$, while ε is the ratio between the SL depth and the BL depth (arbitrarily set at $\varepsilon = 0.1$).

In TM model θ_s is computed by adding to the potential temperature of the lowermost model level a "surface excess temperature" θ_t,= C H_0/w_s, where H_0 is the surface heat flux and C = 6.5 is an empirically determined constant. The countergradient γ_c is evaluated as $\gamma_c = \theta_t/h$ (constant throughout the layer).

Equation 1.b provides a criterion to determine the BL height h on the basis of the bulk Richardson number: a critical value Ri_c is chosen in the range 0÷1 (usually 0.5), and Ri_b in the BL is repeatedly calculated increasing h, until the condition $Ri_b = Ri_c$ is met. Once h is determined, Equation 1.c can be used to prescribe a K-profile, with a shape specified by the coefficient p (cubic profile with the usual assumption p = 2). Defined this way, K vanishes outside the BL and approaches the value suggested by MOST as z decreases.

2 SHORTCOMINGS OF THE TM MODEL

The TM model has been widely adopted in mesoscale modeling. The Ri_b criterion used to estimate h, and thus constrain the K–profile is physically based, robust, and computationally cheap. Nevertheless, the TM model has a few conceptual limitations:

a) the K-profile does not fulfill the asymptotic free convection scaling $K \approx z^{4/3}$ in the SL;
b) there is no sound justification for the parameterization of γ_c and θ_s, as well as for setting p = 2;
c) the criterion in Equation 1.c devised to estimate h may break down in the case of neutral stratification or no wind: these conditions cannot be properly described using bulk formulas.

Moreover, idealised 1–D runs lasting 12 hours allow to point out undesired features in the BL structure as reconstructed by the TM model (Figure 1):

d) the estimate of h depends on the vertical resolution of the model;
e) h decreases in the latest stage of the runs, although the surface heat flux is still positive.

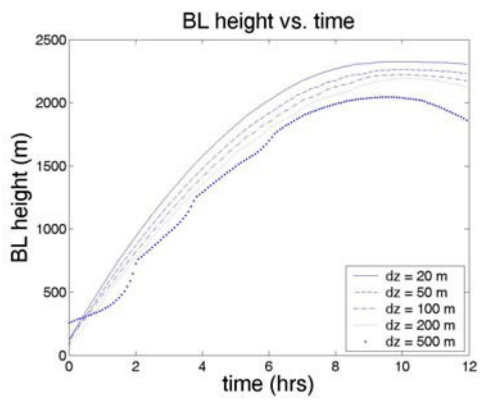

Figure 1. BL height evolution as estimated by the TM model, at different vertical resolutions (dz); the surface heat flux H_0 varies according to $H_0 = H_{max} \sin \omega t$ (here $2\pi\omega^{-1} = 24\ h$, $H_{max} = 0.2\ K\ m\ s^{-1}$).

Finally, other shortcomings of TM scheme can be pointed out on the basis of a comparison with the output of LES simulations (Ayotte et al., 1996):

f) overestimation of the entrainment rate in a BL with strong shear.
g) underestimation of the entrainment rate in conditions of free convection.

3 REFINEMENT OF THE TM MODEL

To overcome these problems, a few adjustments to the TM model can be proposed:

a) In order to prevent an overestimation of the shear production of turbulence, which is typical when Equation 1.b is applied in conditions with strong shear, Vogelezang and Holtslag (1992, VH) proposed to consider bulk gradients in the outer layer only (i.e. only above the SL); the inclusion of an additional term related to u_* (βu_*^2, with β of order 100) allowed them to handle near neutral conditions. Following their approach, we compute Ri_b (assuming Pr = 1) as:

$$Ri_c = \frac{g}{\theta_0} \frac{\theta_h - \theta_{sl}}{(u_h - u_{sl})^2 + \beta u_*^2} (h - h_{sl}), \tag{2}$$

where the subscript "sl" identifies quantities referred to the SL top.

b) h_{sl} has to be estimated in order to evaluate Ri_c form Eq. 2. To this purpose, following Kader and Yaglom (1990, KY), we assume the SL as the layer where u_* is a relevant parameter in similarity analysis. The SL then coincides with the dynamic and dynamic–convective sublayers as defined by KY, and its depth can be estimated based on the ζ ranges of these sublayers. The notions of minimum Obukhov length L_{min} and minimum friction velocity v_*, (cf. Businger, 1973), parameterized in terms of the nondimensional roughness z_0/h (cf. Schumann, 1988), allow to prevent h_{sl} from vanishing under free convection. The upper limit for the dynamic–convective sublayer is taken as $\zeta = -1.6$. The resulting criterion for h_{sl} is:

$$h_{sl} = 1.6 \max\{-L , -L_{min}\} = \max\{-1.6L , 0.56(z_0 h)^{1/2}\} \tag{3}$$

Interestingly, h_{sl} decreases with increasing convective conditions, in accordance with previous analyses e.g. by Grachev et al. (1997). Average values of z_0 and h yield a free convection minimum h_{sl} of 3÷8 m.

c) θ_{sl} can be conveniently extrapolated, starting from the known value θ_z at a height z (e. g. the lowest model level) by means of a suitable similarity function. Under free convection conditions (where, as in the above definition for h_{sl}, u_* .does no more enter as a scaling variable) we have, $\partial\theta/\partial z \approx H_0^{2/3}(g/\theta_0)^{-1/3}z^{-4/3} = -\zeta^{-1/3}$. Following Carl et al. (1973) we adopt $\phi_h(\zeta) = \Pr(1-16\zeta)^{-1/3}$ (which respects the free convection limit). Integration is straightforward:

$$\theta_{sl} = \theta_z + \Pr\frac{H}{kw_*}\left[\ln\frac{x-1}{\sqrt{x^2+x+1}} + \sqrt{3}\arctan\frac{2x+1}{\sqrt{3}}\right]_{x(sl)}^{x(z)} \tag{4}$$

where $x = (1-16\zeta)^{1/3}$. Then the excess temperature can be directly evaluated as $\theta_t = \theta_{sl} - \theta_z$.

d) Once the estimation of h is fixed, the problem of specifying K can be addressed. This is usually parameterized as the product of a length scale and a velocity scale. As an example, in the diabatic SL similarity analysis yields $K = k\, z\, u_* \phi_h^{-1}$ holds; z is the proper length scale, while a combined velocity scale $w_s \equiv (u_*^3+7\varepsilon k w_*^3)^{1/3}$ was derived by TM from $u_*\phi_h^{-1}$. Defined this way, w_s is constant with height and does not respect the free convection scaling for K. An alternative formulation, which is height-dependent and provides the correct scaling is:

$$w_s = \left[\left(1-\frac{z}{h}\right)u_*^3 + 16kw_*^3\frac{z}{h}\right]^{1/3} \tag{5}$$

Also, TM extend SL similarity by multiplying $K = k\, z\, w_s$ by a factor $(1 - z/h)^p$. The usual assumption of a heat flux linearly decreasing with height, $H = H_0(1-z/h)$, is retrieved with p = 1: however this determines unphysical θ gradients at the BL top. Instead TM choose p = 2, which allows a smooth transition between the BL and the free atmosphere and yields a cubic profile for K (as is often suggested in literature). The issue of finding proper velocity and scale lengths, suitable to extend similarity considerations to the whole BL depth, needs to be addressed in detail. A local similarity approach based on a bottom-up decomposition, as suggested by Sorbjan (1989), is currently being explored.

e) A variety of justifications for a countergradient flux correction have been proposed, based on an analysis of the budgets of second order moments (Sorbjan, 1989). De Roode et al. (2004) support the hypothesis that for a dry CBL with an entrainment-to-surface flux ratio of about 0.2, γ_c may be considered constant with height. From a mathematical viewpoint, Stevens (2000) points out that role of γ_c is essentially that of allowing realistic θ and heat flux profiles, i.e. with the θ-gradient and the heat flux profile vanishing at different levels. 1D tests of the TM model suggest that this latter feature is reproduced as well, simply by introducing a background stratification and a vertically varying eddy viscosity. The inclusion of γ_c is instead needed to obtain a near neutral θ-profile in the mixed layer (as desired), but it also produces a shallower BL.

4 CONCLUSIONS AND OUTLOOK

Several adjustments to the traditional framework of a bulk BL model are proposed. The sensitivity of the model to each of them needs to be evaluated, and the modelled BL structure should be compared against measurements (very few of them area available in the literature) or LES results.

Three points that deserve further discussion are:

1. a generalization of the Ri_b–criterion to estimate h, in order to handle free convection cases in a neutral environment: the approach of combining bulk gradients in the outer layer and velocity scales related to buoyancy and momentum fluxes in the SL seems to be promising;
2. the definition of the most appropriate length and velocity scales in order to define a physically based K-profile along the whole BL depth;
3. a proper parameterization of countergradient fluxes.

Further developments will be the extension of various concepts to the stable case ($\zeta \geq 0$) and to the moist atmosphere. The appropriateness and possible limitations of adapting this parameterization scheme to model the BL development over complex terrain, in view of its use for realistic topography (as is currently done in operational use of some NWP models) also needs to be carefully assessed.

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