V. GROZDANIĆ

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THREE-DIMENSIONAL MATHEMATICAL MODEL OF THE SOLIDIFICATION OF LARGE STEEL INGOTS

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Three-dimensional mathematical model of the solidification of low-carbon steel ingot in a chilled mould has been formulated and investigated in the paper. The model is based on theoretical knowledge and it is supplemented with experience data. In the mathematical model the thermo physical data are incorporated which are temperature dependent, what gives nonlinearity to the model. The model is solved by means of the explicit finite difference method - the method of enthalpy. The solidification algorithm has been solved with computer SPERRY 1106 in program language ASCII FORTRAN. The time of solidification, the optimum stripping time, the thickness of the solid skin in any direction, and the potentional prints of possible defects in ingot may be obtained on the basis of the model.

Key words: *3-D mathematical model, solidification, steel ingot*

Trodimenzijski matematički model skrućivanja velikih čeličnih ingota. U radu je oblikovan i istražen trodimenzijski matematički model skrućivanja čeličnog ingota u kokili koji se temelji na teorijskim znanjima a dopunjen je iskustvenim podacima. U matematički model uključeni su toplotehnički podaci koji ovise o temperaturi, što modelu daje nelinearnost. Model je riješen numerički pomoću eksplicitne metode konačnih razlika - entalpijskom metodom. Algoritam skrućivanja programiran je u programskom jeziku ASCII FORTRAN za računalo SPERRY 1106. Na temelju modela može se odrediti vrijeme skrućivanja odnosno optimalno vrijeme stripovanja kokila, debljina čvrste kore u bilo kojem smjeru, te potencijalna možebitnih eventualnih defekata u ingotu.

Ključne riječi: *trodimenzijski matematički model, skrućivanje, čelični ingot*

INTRODUCTION

Till 1970 steel was the most frequently poured in chilled moulds [1]. But after 2003 year the share of poured steel in ingot moulds was cca 12 % [2]. Pouring steel in ingot moulds loses importance because of introduction of the continuous casting [3]. Pouring of ingots in the chilled mould will be continuing when need large steel ingots are. The industrial use of increasingly large ingots for the production of vary heavy forgings necessitates an understanding of the phenomena witch in a direct relationship with the solidification of metal, because of their influence on the quality of the manufactured product. The major influences are chemical and structural witch are in direct connection with process of the progress of solidification [4]. So, the first step is establishing mathematical model of the solidification of ingot.

V. Grozdanić, Faculty of Metallurgy University of Zagreb, Sisak, Croatia

Mathematical modeling of the solidification of ingot in a chilled mould has great practical interest, because it may be with high confidence determined the time of the ingot solidification, i.e. the time when is possible stripping of the chilled moulds, what leads to the optimization of the process of ingot manufacture. To obtained that, with correct established mathematical model it is necessary to use adequate thermo physical properties of materials (which are temperature dependent) [5], proper numerical method of solving partial differential equation of heat conduction, and adequate high speed computer of great memory like

Table 1. **Chemical composition of steel ingot and chilled mould [4 - 7]**
Tablica 1. **Kemijski sastav čeličnog ingota i kokile [4 - 7]**

	Chemical composition / %					
	C	Si	Mn	Al	S	P
Steel ingot	0,25	0,40	0,60	0,10	-	-
Chilled mould	3,3–4	1,45–2,10	0,5–1	-	0,10	0,20

SPERRY 1106. Partial differential equation of the unsteady heat flow is solved numerically by means of so-called the enthalpy method, which is proved to be good for the calculation of the freezing process of relatively large steel ingots, like is shown in Figure 1.

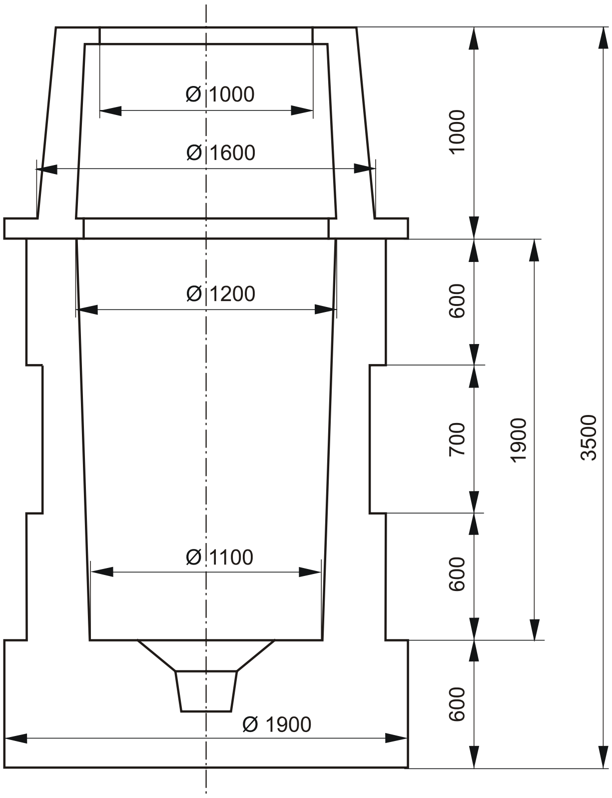

Figure 1. **Schematic presentation of calculated 20 t steel ingot and its mould**
Slika 1. **Shematski prikaz 20 t čeličnog ingota i kokile**

Chemical composition of steel ingot and chilled mould is presented in Table 1. [4 - 7].

MATHEMATICAL MODEL OF THE SOLIDIFICATION

Partial differential equation of the unsteady heat flow

The partial differential equation of the unsteady heat flow in the cylindrical coordinate system is shown by Equation (1) [8]:

$$c_p\rho\frac{\partial\vartheta}{\partial t}=\frac{\partial}{\partial r}\left(k\frac{\partial\vartheta}{\partial r}\right)+\frac{k}{r}\frac{\partial\vartheta}{\partial r}+\frac{\partial}{\partial z}\left(k\frac{\partial\vartheta}{\partial z}\right) \quad (1)$$

Assuming that small changes in temperature do not influence the physical properties of the material, Equation (1) can be simplified to Equation (2):

$$\frac{\partial\vartheta}{\partial t}=a\left(\frac{\partial^2\vartheta}{\partial r^2}+\frac{1}{r}\frac{\partial\vartheta}{\partial r}+\frac{\partial^2\vartheta}{\partial z^2}\right) \quad (2)$$

where

$$a=\frac{k}{\rho c_p} \quad (3)$$

Introducing the function ϕ as shown with Equation (4), Equation (1) can be converted into Equation (5):

$$\phi=\int_{\vartheta_d}^{\vartheta}\frac{k}{k_d}d\vartheta \quad (4)$$

$$\frac{\partial\vartheta}{\partial t}=a\phi\left(\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}\right) \quad (5)$$

Equation (5) can be rewritten as Equation (7), by using the relation shown in Equation (6):

$$\frac{\partial H}{\partial t}=\frac{\partial H}{\partial\vartheta}\cdot\frac{\partial\vartheta}{\partial t},\quad c_p=\frac{\partial H}{\partial\vartheta} \quad (6)$$

$$\frac{\partial H}{\partial t}=\frac{k_d}{\rho}\left(\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}\right) \quad (7)$$

Since the physical properties of the ingot and the chilled mould vary according to the changes in temperature, the heat transfer inside them can be calculated by Equation (7). It is, however, practically impossible to make the numerical calculation directly. Thus the difference analogue has been applied to Equation (7), as shown with Equation (8).

If the heat content of the mesh point (i, j) at the time t is expressed by $H_{i,j,n}$ and the heat content of the same mesh point after the time Δt is shown by $H_{i,j,n+1}$, Equation (7) becomes as Equation (8):

$$H_{i,j,n+1}=H_{i,j,n(r)}+\left(F_{i,j,n(r)}+F_{i,j,n(z)}\right) \quad (8)$$

The terms of $F_{i,j,n}(r)$ and $F_{i,j,n}(z)$ in Equation (8) can be rewritten as (9) and (10):

$$F_{i,j,n(r)}=A\left[\frac{\phi_{i-1,j,n}-\phi_{i,j,n}}{(\Delta r)^2}-\right.$$
$$\left.-\frac{\phi_{i,j,n}-\phi_{i+1,j,n}}{(\Delta r)^2}+\frac{\phi_{i-1,j,n}-\phi_{i+1,j,n}}{2r\Delta r}\right] \quad (9)$$

$$F_{i,j,n(z)}=A\left[\frac{\phi_{i,j-1,n}-\phi_{i,j,n}}{(\Delta z)^2}-\frac{\phi_{i,j,n}-\phi_{i,j+1,n}}{(\Delta z)^2}\right] \quad (10)$$

where

$$A = \frac{k_d \Delta t}{\rho} \tag{11}$$

For the calculation of the heat conductions in the ingot and the mould by the difference equation, Equation (8), firstly it is necessary to know the temperature of the molten steel in the mould and that of the mould wall at the time of pouring as the boundary conditions. These temperatures can be calculated by Equation (12):

$$\sqrt{a_M}\, c_{pM} \rho_M \Delta\vartheta_M = \sqrt{a_s}\, \rho_s \left(c_{ps} \cdot \Delta\vartheta_s - \Delta H_f\right) \tag{12}$$

Calculation of the heat radiation and absorption at the ingot and mould surfaces

During the freezing process of the molten steel, the heat radiation or absorption is continuously taken place at the ingot and mould boundary surface. The relevant equations employed are listed below.

1. Radial heat radiation

$$H_{i,j,n+1} = H_{i,j,n} + A\left[\frac{\phi_{i-1,j,n} - \phi_{i,j,n}}{(\Delta r)^2} - \frac{W}{k_d \Delta r} + \frac{W}{2rk_d} + \frac{\phi_{i,j-1,n} - \phi_{i,j,n}}{(\Delta z)^2} - \frac{\phi_{i,j,n} - \phi_{i,j+1,n}}{(\Delta z)^2}\right] \tag{13}$$

2. Upward heat radiation

$$H_{i,j,n+1} = H_{i,j,n} + A\left[\frac{\phi_{i-1,j,n} - \phi_{i,j,n}}{(\Delta r)^2} - \frac{\phi_{i,j,n} - \phi_{i+1,j,n}}{(\Delta r)^2} + \frac{\phi_{i-1,j,n} - \phi_{i+1,j,n}}{2r\Delta r} + \frac{\phi_{i,j-1,n} - \phi_{i,j,n}}{(\Delta z)^2} - \frac{W}{k_d \Delta z}\right] \tag{14}$$

3. Downward heat radiation

$$H_{i,j,n+1} = H_{i,j,n} + A\left[\frac{\phi_{i-1,j,n} - \phi_{i,j,n}}{(\Delta r)^2} + \frac{\phi_{i,j,n} - \phi_{i+1,j,n}}{(\Delta r)^2} + \frac{\phi_{i-1,j,n} - \phi_{i+1,j,n}}{2r\Delta r} + \frac{\phi_{i,j+1,n} - \phi_{i,j,n}}{(\Delta z)^2} - \frac{W}{k_d \Delta z}\right] \tag{15}$$

4. Heat radiation at the upper corner

$$H_{i,j,n+1} = H_{i,j,n} + A\left[\frac{\phi_{i-1,j,n} - \phi_{i,j,n}}{(\Delta r)^2} - \frac{W}{k_d \Delta r} + \frac{W}{2rk_d} + \frac{\phi_{i,j-1,n} - \phi_{i,j,n}}{(\Delta z)^2} - \frac{W}{k_d \Delta z}\right] \tag{16}$$

5. Heat radiation at the lower corner

$$H_{i,j,n+1} = H_{i,j,n} + A\left[\frac{\phi_{i-1,j,n} - \phi_{i,j,n}}{(\Delta r)^2} - \frac{W}{k_d \Delta r} + \frac{W}{2rk_d} + \frac{\phi_{i,j+1,n} - \phi_{i,j,n}}{\left(\Delta z\right)^2} - \frac{W}{k_d \Delta z}\right] \tag{17}$$

6. Radial heat absorption

$$H_{i,j,n+1} = H_{i,j,n} + A\left[\frac{\phi_{i+1,j,n} - \phi_{i,j,n}}{(\Delta r)^2} - \frac{W}{k_d \Delta r} + \frac{W}{2rk_d} + \frac{\phi_{i,j-1,n} - \phi_{i,j,n}}{(\Delta z)^2} - \frac{\phi_{i,j,n} - \phi_{i,j+1,n}}{(\Delta z)^2}\right] \tag{18}$$

7. Upward heat absorption

$$H_{i,j,n+1} = H_{i,j,n} + A\left[\frac{\phi_{i-1,j,n} - \phi_{i,j,n}}{(\Delta r)^2} + \frac{\phi_{i,j,n} - \phi_{i+1,j,n}}{(\Delta r)^2} + \frac{\phi_{i-1,j,n} - \phi_{i+1,j,n}}{2r\Delta r} + \frac{\phi_{i,j+1,n} - \phi_{i,j,n}}{(\Delta z)^2} - \frac{W}{k_d \Delta z}\right] \tag{19}$$

8. Downward heat absorption

$$H_{i,j,n+1} = H_{i,j,n} + A\left[\frac{\phi_{i-1,j,n} - \phi_{i,j,n}}{(\Delta r)^2} + \frac{\phi_{i,j,n} - \phi_{i-1,j,n}}{(\Delta r)^2} + \frac{\phi_{i-1,j,n} - \phi_{i+1,j,n}}{2r\Delta r} + \frac{\phi_{i,j-1,n} - \phi_{i,j,n}}{(\Delta z)^2} + \frac{W}{k_d \Delta z}\right] \tag{20}$$

9. Heat absorption at the lower corner

$$H_{i,j,n+1} = H_{i,j,n} + A\left[\frac{\phi_{i+1,j,n} - \phi_{i,j,n}}{(\Delta r)^2} - \frac{W}{k_d \Delta r} + \frac{W}{2rk_d} + \frac{\phi_{i,j-1,n} - \phi_{i,j,n}}{(\Delta z)^2} + \frac{W}{k_d \Delta z}\right] \tag{21}$$

10. Heat absorption at the upper corner

$$H_{i,j,n+1} = H_{i,j,n} + A\left[\frac{\phi_{i+1,j,n} - \phi_{i,j,n}}{(\Delta r)^2} - \frac{W}{k_d \Delta r} + \right.$$
$$\left. + \frac{W}{2rk_d} + \frac{\phi_{i,j+1,n} - \phi_{i,j,n}}{(\Delta z)^2} - \frac{W}{k_d \Delta z}\right] \quad (22)$$

The following are the equations used for the calculation of the quantity of heat radiation W / (J/m^{2}s) from the ingot, mould, and anti-piping refractory surfaces into the atmosphere [9]:

$$W_M = \eta\left(\vartheta_M - \vartheta a\right)^{5/4} +$$
$$+ \sigma E_M\left[\left(\vartheta_M + 273\right)^4 - \left(\vartheta_a + 273\right)^4\right] \quad (23)$$

1. Heat radiation from the top surface

$$\eta = 3{,}2564 \cdot 10^{-3}$$

2. Heat radiation from the side

$$\eta = 2{,}5586 \cdot 10^{-3}$$

3. Heat radiation from the bottom surface

$$\eta = 1{,}7445 \cdot 10^{-3}$$

Heat transfer at the boundary in the period between the pouring of molten steel and the air gap formation

When the ingot contacts with the mould or anti-piping refractory, the heat transfer at the boundary is taken place by conduction, and the relations shown by the following equations are obtained.

$$H_{Si,j,n+1} = H_{Si,j,n} + C_{pS}\left(\vartheta_{Si,j,n+1} - \vartheta_{Si,j,n}\right) \quad (24)$$

$$H_{Mi,j,n+1} = H_{Mi,j,n} + C_{pM}\left(\vartheta_{Mi,j,n+1} - \vartheta_{Mi,j,n}\right) \quad (25)$$

$$H_{Bi,j,n+1} = H_{Bi,j,n} + C_{pB}\left(\vartheta_{Bi,j,n+1} - \vartheta_{Bi,j,n}\right) \quad (26)$$

Heat transfer after the air gap formation

When an air gap exists between the ingot and the mould or the anti-piping refractory, heat transfer between them is supposed to be made by radiation. In this case, the heat flux W between the two of them is shown by Equations (27) to (29), respectively.

$$W_{S\to M} = \sigma\frac{E_S E_M}{E_S + E_M - E_S E_M} \times$$
$$\times\left[\left(\vartheta_S + 273\right)^4 - \left(\vartheta_M + 273\right)^4\right] \quad (27)$$

$$W_{S\to B} = \sigma\frac{E_S E_B}{E_S + E_B - E_S E_B} \times$$
$$\times\left[\left(\vartheta_S + 273\right)^4 - \left(\vartheta_B + 273\right)^4\right] \quad (28)$$

$$W_{B\to M} = \sigma\frac{E_B E_M}{E_B + E_M - E_B E_M} \times$$
$$\times\left[\left(\vartheta_B + 273\right)^4 - \left(\vartheta_M + 273\right)^4\right] \quad (29)$$

RESULTS AND DISCUSSION

In calculating the freezing process of the ingot in the chilled mould the space step is $\Delta r = \Delta z = 10$ cm, and the time step is $\Delta t = 2$ min. The pouring temperature is 1580 °C, and the temperature of the atmosphere is 50 °C. The position of the isosolidus and the isoliquidus in different points of ingot is shown in Figure 2. The temperatures of solidus and liquidus are taken from the equilibrium diagram Fe-C [10], and they are 1478,4 °C and 1516,5 °C.

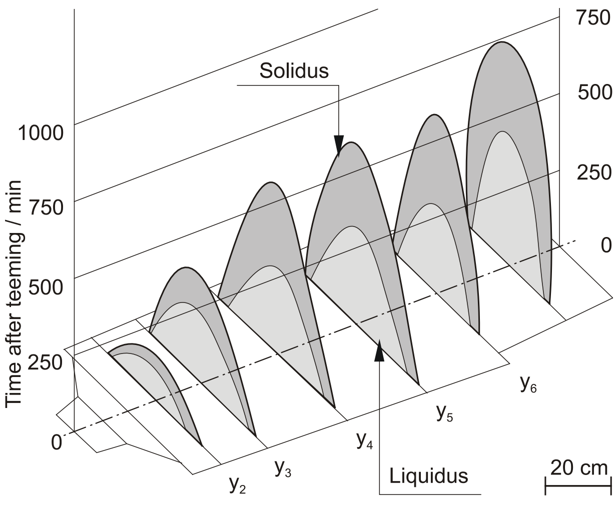

Figure 2. **Positions of isosolidus and isoliquidus in the steel ingot**
Slika 2. **Položaj izosolidusa i izolikvidusa u čeličnom ingotu**

In this figure, the typical positions in the ingot are represented by the dotted lines and solid lines. The dotted lines refer to the period immediately after the pouring until the beginning of solidification, and the solid lines to the period after the beginning of solidification until its termination. On the basis of the results thus obtained, the relation between the freezing time and the freezing thickness in any direction at any position in the ingot can readily be known.

Such relations also serve to find out the freezing rate, i.e. the velocity of the advancing solidus line. From Figure 2. it can be concluded that the solidification time of the ingot in the chilled mould is 12,5 hr. From the analysis of results, it was found that the freezing time was short on the bottom, but long on the middle and top of the ingot. It is thought these phenomena are owing to the following reasons that a large quantity of heat escapes from the bottom surface of the mould, by conduction, and a small quantity of heat gradually escapes from the surface of middle and top of the ingot to the mould by radiation.

CONCLUSION

Three-dimensional mathematical model of the solidification of steel ingot in a chilled mould has been formulated and investigated in the paper. The thermo physical data are temperature dependent, what gives nonlinearity to the model. The model is solved by means of the finite difference the method of enthalpy. The solidification algorithm is programmed in ASCII FORTRAN and solved on the SPERRY 1106 computer. The freezing time was short on the bottom, but long on the middle and top of the ingot. On the basis of the model it can be theoretically estimated freezing rate in any position of the ingot. Optimum stripping time for 20 t steel ingot is 12,5 hr.

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Abbreviations

a - temperature conductivity
c_p - specific heat at constant pressure
E - emissivity
$H_{i,j,n}$ - heat content per unit weight of the heat conductive substance at point (i, j) and the time t
ΔH_f - latent heat of fusion
k - thermal conductivity
r - space coordinate
t - time
W - quantity of heat radiation
z - space coordinate
ϑ - temperature
η - surface convection constant
σ - Stefan-Boltzmann's constant

Index

S - ingot
M - mould
B - refractory
d - at basic temperature ϑ_d
a - atmosphere
i - point in the direction of the r - coordinate
j - point in the direction of the z - coordinate
n - point in the time t