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Consonance and Dissonance of Simultaneous Trichords in Western Music: A Listening Experiment to Test Models of Harmonicity and Roughness

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Abstract

Previous empirical studies have suggested that the perceived consonance/dissonance (C/D) of a musical chord depends on its psychoacoustic smoothness (lack of roughness), spectral harmonicity (perceptual fusion), and/or musical familiarity. We tested the dependence of C/D on smoothness and harmonicity in a hearing experiment that included all 19 possible trichords in musical pitch-class set theory. In each trial, a listener heard a chord (duration: 300 or 500 ms) and rated its C/D on an 11-point scale. Each trichord was presented 10 times: 4 times constructed from octave-complex tones (OCTs, sounding like an electronic organ) and 6 times from natural piano tones. Each OCT chord was presented in 4 different transpositions. The piano chords were in close or open position, and root position or 1st/2nd inversion (2 levels of spacing x 3 levels of inversion = 6 levels of voicing). We found no main effect of timbre (OCT versus piano) and no interaction between trichord and timbre. Results correlated closely with predictions of simple models of roughness and harmonicity. The roughness model performed better, and the predictions correlated with each other. A combined model was not superior to roughness alone. The results were consistent with a multifactorial model of the C/D of a musical chord, the main factors being roughness, harmonicity, and familiarity.

Keywords: music, chord, consonance, dissonance, roughness, harmonicity

Introduction

The structure of Western music depends on the consonance and dissonance (C/D) of its sonorities (Christensen, 2006; Parncutt & Hair, 2011), but perception of C/D and theories about it changed from one historical period to the next (Tenney, 1988). Before the advent of written polyphony in the 12th century (Bevilacqua, 2016), Western music was melodic and rhythmic, and C/D was understood to refer to successive tones (melodic C/D). Later and today, C/D more often referred to simultaneous tone combinations (harmonic C/D).

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Intervals that are considered consonant when heard melodically are not necessarily consonant when heard harmonically. An octave or perfect 5th interval is consonant in both cases, but the major 2nd interval is melodically consonant (it is the most common interval between successive tones in melody; Vos & Troost, 1989) and harmonically dissonant (due to psychoacoustic roughness; Huron, 1994).

The C/D of a chord of three or more simultaneous tones in music theory is not a simple combination of the C/D of the intervals between the chord tones. For example, a minor triad (e.g., CE♭G) may sound (or be considered) less consonant than a major triad (CEG), even though the intervals among the tones are the same (in both cases, there are three intervals: perfect 5th, major 3rd, minor 3rd). Similarly, a major-minor 7th chord (Mm7) or dominant 7th (e.g., G7, GBDF) may seem more consonant than a major 7th chord (e.g., Gmaj7, GBDF♯), although only the former contains a dissonant tritone interval (BF). In both cases, the difference (major versus minor, MmT versus maj7) may be due to *harmonicity*: the degree to which the chord's spectrum resembles a harmonic series. A major triad is more similar to a harmonic series than a minor triad, and a major-minor 7th chord is more similar to a harmonic series than a major 7th chord.

Modelling Roughness and Harmonicity

In a previous study (Parncutt et al., 2019), we showed that the prevalence of trichords in a musical score database of Western polyphony from the 13th to the 19th century correlated with predictions of simple models of roughness and harmonicity. Assuming that chord prevalence depends primarily on C/D, and taking into account historical knowledge about how new chords were introduced to musical vocabulary, we concluded that C/D in polyphonic Western music depends mainly on three factors: roughness, harmonicity, and familiarity. That was consistent with an independent finding, based on a reanalysis of published consonance perception data and modelling of a music notation database (Harrison & Pearce, 2020), in which simultaneous consonance was found to depend on interference (roughness), periodicity/harmonicity, and cultural familiarity.

The theory of psychoacoustic roughness was developed by Helmholtz (1863), and the basic ideas were repeatedly confirmed in empirical 20th-century studies (e.g., Plomp & Levelt, 1965). Musical sounds are subjectively rough when their amplitude envelope changes rapidly and periodically. Fast amplitude modulation (or beating) is caused by partials (pure-tone components) that are nearby in frequency (lying within the same auditory filter or critical band). They reinforce each other when in phase, and cancel when out of phase, producing beating. The sound is rough if the beats are too fast to hear separately (20 Hz) but not too fast that the sensation disappears (above about 300 Hz; Terhardt, 1968). The magnitude of the roughness sensation produced by two partials depends on carrier (or mean) frequency, rate of modulation (the difference between the two frequencies), and modulation depth.

Several quantitative models of roughness have been proposed (e.g., Aures, 1985; Bigand et al., 1996; Daniel & Weber, 1997; Hutchinson & Knopoff, 1978; Pressnitzer et al., 2000), but none is widely accepted or has proven useful in music theory and analysis. Roughness can nevertheless explain the rank order of dissonance of harmonic (simultaneous) intervals between musical tones. According to Malmberg (1918), and in broad agreement with mainstream music theory, those intervals are perfect octave, perfect 5th, major 6th, major 3rd, perfect 4th, minor 6th, minor 3rd, tritone, minor 7th, major 2nd, major 7th, and minor 2nd (in order of increasing roughness). Consonant intervals (those near the start of this list) tend to have partials that exactly coincide when the interval between the two fundamental frequencies is tuned to a simple ratio of small integers. For example, in the interval of a perfect 5th, the 3rd harmonic of the lower tone coincides with the 2nd harmonic of the higher tone. There is also a tendency for larger intervals to be more consonant than smaller intervals, because their fundamentals are further apart.

Another theory of C/D that was developed in the late 19th century is Stumpf's (1883) theory of perceptual fusion or unitary hearing (Schneider, 1997). Stumpf's empirical data suggested that a complex spectrum, including that of a musical chord, was more likely to be perceived as a fused whole if it was more similar to the harmonic series. Whereas Stumpf's theory corresponds to what we are calling harmonicity, he opposed naturalistic explanations, preferring to focus on cultural and phenomenological issues.

Existing theories and models of roughness and harmonicity may not explain all variations in the C/D of harmonic intervals. From a music-theoretic perspective, the harmonic perfect 4th and minor 6th intervals are more dissonant than might be expected from roughness alone. Many medieval music theorists regarded the harmonic perfect 4th as especially dissonant despite its simple frequency ratio, and in Renaissance and classical harmony, perfect 4th intervals between the bass voice and any other part are regarded as dissonances that require resolution (Naishtat, 1996; Thomson, 1996). Terhardt's (1974) theory of chord roots offers a possible explanation. In the harmonic perfect 4th and minor 6th intervals, the higher tone may be perceptually more salient than the lower – comparable with the root of a chord (Parncutt, 1988). For other intervals, the lower tone tends to be more salient, which is perceived as more normal or consonant. The perceived harmonicity of a musical sound may be enhanced if the fundamental of the implied harmonic series corresponds to the bass voice.

Some empirical tests of C/D did not refer directly to roughness or harmonicity. Roberts (1986) empirically established a rank order of consonance for four triads (major, minor, diminished, augmented), and demonstrated that root positions are more consonant than inversions, and that chords tuned to (familiar) 12-EDO are perceived to be more consonant than chords tuned to (unfamiliar) Just or Pythagorean tuning. Other studies have concluded that C/D is based on either harmonicity (McDermott et al., 2010) or roughness (Plomp & Levelt, 1965), but not both.

Eberlein (1994, p. 34) argued that roughness was more important than harmonicity (fusion). McDermott et al. (2010) argued the opposite – that C/D was mainly determined by harmonicity – but their conclusion may have been a consequence of their limited range of the stimuli.

Music-Theoretic Foundations

Many ideas in music theory, including many ideas about C/D, are octave-generalized: they do not depend on the octave register in which the tones are played, but are understood to be independent of register. A *pitch class* is an octave-generalized pitch, or a pitch whose octave register is not specified. For example, there are seven tones on the modern piano keyboard that are called “G” and hence belong to the pitch class “G”. There are twelve pitch classes, one for each tone in the chromatic scale.

Western tonal music is based on *triadic* chord progressions. A *triad* is traditionally regarded as a chord comprising three pitch classes that is created by stacking 3rd intervals, and the tones in a triad are called root, 3rd, and 5th. For example, the root of a C-major triad is C, the 3rd is E, and the 5th is G. That is true regardless of the octave registers in which the tones appear. If the lowest tone is not C, the chord is *inverted*.

Western tonality in the “common-practice period” (roughly 1650-1900) was based on four triads: major, minor, diminished, and augmented (Randel, 2003). Stacking a major 3rd [4 semitones] below a minor 3rd [3] creates a major triad (in semitones relative to the root: [047]), and reversing the order of intervals creates a minor triad [037]. Stacking two minor 3rds [3] creates a diminished triad [036]; two major 3rds [4], an augmented triad [048]. Music theory textbooks often imply that these four triads are the most common tone chords in Western tonal music. In fact, the most common simultaneities of three pitch classes in mainstream Western tonal music are major, minor, suspended [057], and diminished, in that order; the augmented triad is remarkably rare (Parncutt et al., 2019).

In the key or tonality of C major, which is defined by the scale CDEFGAB of which C is the tonic or main reference pitch, there are seven diatonic triads: C major (comprising tones CEG), D minor (DFA), E minor (EGB), F major (FAC), G major (GBD), A minor (ACE), and B diminished (BDF). In the key of C (harmonic) minor, which is defined by the scale CDE♭FGA♭B, the 7 diatonic triads are C minor (CE♭G), D diminished (DFA♭), E♭ major (E♭GB♭) (not augmented, as the scale implies), F minor (FA♭C), G major (GBD), A♭ major (A♭CE♭), and B diminished (BDF). Diatonic chord progressions are confined to these chords, but even harmonically simple music in the keys of C major or minor often includes mixtures of chords from both keys and/or non-diatonic triads such as D major (DF♯A) or B♭

major (B♭DF). The same applies to any major or minor key (*modal mixture*: Beach & Schubert, 1998).

Pitch-class set theory enumerates all possible pitch class combinations (regarded as mathematical sets) that are available for composition, within given restrictions. For example, it lists all possible combinations of three pitch classes, or *trichords,* that can be created in the chromatic scale (cf. Straus, 2005). The number of trichords can be reduced by grouping together those that are *transpositions* of each other. For example, D major [269] is a transposition of C major [047], both being major triads. After eliminating transpositions, 19 possible trichords remain, four of which are the triads listed above (major, minor, diminished, augmented). The number of possible trichords can be reduced from 19 to 12 by grouping together those that are intervallic inversions of each other; for example, the major and minor triads ([047] and [037]) can be regarded as different versions of the same *pitch class set*. In the following, we will regard major and minor triads as different trichords.

Whereas all 19 trichords appear regularly in tonal music, the most consonant ones appear more often than the most dissonant ones (Parncutt et al., 2019). The most dissonant trichord, and least often used, is the chromatic cluster [012] (e.g., C-C♯-D); it occurs occasionally in classical music when a chromatic appoggiatura rises by a semitone to the root of a major-minor 7th chord (for example, when B is played simultaneously with a C7 chord, and the B resolves to C).

Ratio Theory

In ancient Greece, the Pythagoreans conceived of musical intervals as ratios of string lengths on a monochord (Caleon & Subramaniam, 2007), and regarded all musical intervals as combinations of octaves (1:2), perfect 5ths (2:3), and perfect 4ths (3:4). Adding intervals meant multiplying ratios. For example, a major 3rd interval was regarded as the sum of four perfect 5ths minus two octaves, with a ratio of $(3/2)^4/2^2 = 81/64$. That is 4.08 semitones relative to today's chromatic scale on a modern keyboard, which has twelve equal divisions of the octave (12-EDO) relative to the logarithm of frequency. Two millennia later, in the 16th century, music theorists such as Gioseffo Zarlino assigned a ratio of 4:5 to the major 3rd interval (in *Just* or *pure* tuning or intonation), which is 3.86 semitones. After the emergence of the mathematical theory of spectral analysis in the 17th century, and culminating with Fourier in the 18th, musical intervals were regarded not as length ratios, but (mathematically equivalently) as *frequency ratios* (Rasch, 2002), raising questions about how the human brain might perceive such ratios.

Jean-Philippe Rameau (1750) and other 18th-century music theorists assigned numerical ratios to musical triads. In *Just* tuning/intonation, the major triad was 4:5:6 and the minor was 10:12:15 (note that 10:12 = 5:6, the Just minor 3rd, and 12:15 = 4:5, the Just major 3rd). The diminished triad could be regarded as 5:6:7 if the mistuning of the upper tone was ignored, which in this case is about 1/3 semitone flat relative to a diminished triad in 12-EDO. But if the diminished triad was regarded as a stack of two equal Just minor 3rds (5:6) – a solution that corresponded more closely to musical practice – the ratio became more complex: 25:30:36. The augmented triad was 16:20:25, again assuming two stacked Just intervals. All these ratios could also be rendered in In Pythagorean tuning, but with larger numbers. For example, a Pythagorean major triad was 64:81:128.

In historic music theory, number ratios were part of aesthetic theory, and seemed to explain the beauty of various arts including architecture (Christensen, 2004). Even today, if we wish to explain the important role of the major triad in Western music, we cannot fail to notice that it occurs naturally among the 4th, 5th, and 6th harmonics of a voiced speech sound or musical instrument tone, or in inversion among the 3rd, 4th, and 5th.

But results of empirical studies on intonation have repeatedly contradicted the idea that musical intervals *are*, or *correspond to*, simple number ratios. Ratio theory predicts that most intervals come in two different variants (Just and Pythagorean), but there is no evidence in musical composition or practice for such a separation. If the ratio of the diminished triad is 5:6:7, we might predict that chord to be more consonant than the minor triad, whose ratio has larger numbers (10:12:15), but that is clearly not the case. In a modern empirical-psychological or cultural-studies approach, musical triads, like all other familiar musical pitch combinations, are cultural products, and no triad is more or less "natural" than any other. Instead, triads vary in C/D.

Chord roots pose additional theoretical problems. The root is the tone after which the chord is usually named: the root of C major (CEG) is C, and the root of F half-diminished (FA♭C♭E♭) is F. The root is also the tone that is usually voiced in the bass, and it seems to act as a reference pitch relative to which the other tones are heard. Ratio theory can explain why the root of a C major triad or C major-minor 7th is C, but it cannot easily account for the roots of chords that do not correspond clearly to a harmonic series.

In an attempt to solve this problem, several 19th-century music theorists developed an approach called *harmonic dualism* (Snyder, 1980). They treated major and minor triads as equal but opposite, while assuming that musical intervals correspond to simple integer ratios. Riemann and Rameau independently entertained the idea that musical tones have *undertones* or subharmonics from which the minor triad could be derived, by analogy to the derivation of the major triad from overtones or harmonics. Both eventually realized that undertones do not physically exist and abandoned the idea. The failure of harmonic dualism to explain the minor triad was symptomatic of a broader failure of mathematical ratio theory to explain musical intervals and chords. If the ratio model is regarded as a research paradigm, the observations that contradict it can be regarded as anomalies that are sufficient to overthrow it (cf. Kuhn & Hawkins, 1963).

Harmonic dualism treated major and minor triads differently, and the minor triad as a special case. But a scientific model of the root and C/D of musical chords should treat all possible chords equally. It should be possible to input any chord to a quantitative model, and to test and improve the model's predictions regarding C/D and possible roots by comparing predictions with empirical estimates of C/D or prevalence.

Our Approach

To empirically test the relative importance of harmonicity and roughness for the C/D of a typical sample of Western musical chords, we included all 19 possible trichords in our experiment. The chords were diverse, ranging from low to high harmonicity and from low to high roughness. Alongside major and minor triads, which have high harmonicity and low roughness, some chords in our sample had high harmonicity and high roughness (e.g., [024]), some had low harmonicity and low roughness (e.g., [048]), and others had low harmonicity and high roughness (e.g., [012]).

To include musically representative variation in timbre, while at the same time checking whether results depended on timbre, each chord was constructed from either natural piano tones or octave-complex tones (OCTs) that sound like an electronic organ. An OCT has partials spaced at octave intervals across the audible spectrum, so it represents a pitch class (i.e., pitch regardless of octave register). Constructing chords from OCTs allows octave-generalized music theory to be tested by eliminating confounds based on pitch distance or pitch register from the experimental design.

Hypotheses

In a previous study (Parncutt & Radovanovic, in press), we predicted and confirmed that the prevalence of a trichord as it occurs in musical scores (how often it happens, averaged over different styles and periods) correlates with its consonance, as predicted using psychoacoustic models of harmonicity and roughness. Put simply, composers tend to use sounds that they like, and there is a general tendency to prefer sounds that are more harmonic and less rough. In the present study, we predicted that the consonance of a chord, as judged by a modern listener in a hearing experiment, would correlate with its predicted consonance according to the same models of harmonicity and roughness.

More specifically, we hypothesized that the historical prevalence of *unprepared* trichords (trichords with simultaneous tone onsets in musical scores), as investigated by Parncutt and Radovanovic (in press), would correlate more strongly with perceived consonance in our experiment than the historical prevalence of *prepared* trichords (trichords in which one or more tone onsets are anticipated and held over from a previous sonority). Two possible reasons arise. First, the historical prevalence of the unprepared chords may be distributed more normally, with unprepared chords being more dissonant than prepared chords. Second, participants in our experiment heard only unprepared chords, so we expected a closer connection to prevalence distributions of unprepared chords in the literature.

Another prediction was that perceived consonance in our experiment would correlate more strongly with roughness than harmonicity. Roughness tends to be more perceptually salient for chords presented out of context than for chords embedded in a chord progression (Wright & Bregman, 1987), consistent with conventions of voice-leading in musical composition, in which dissonant tones are *resolved* by stepwise motion from dissonance to consonance (Forte, 1974, p. 125). Given findings in the literature about the importance of harmonicity for C/D, we expected that a linear combination of harmonicity and roughness would improve the correlation between predictions and our findings relative to roughness alone.

We hypothesized that piano chords would be rated more consonant than OCT chords, because the sound of piano chords dies away quickly, reducing the chance that roughness will be perceived. We also hypothesized that piano chords would be rated more consonant when sounded in open position (with larger intervals between adjacent tones) than in close position. That is because larger intervals tend to produce less roughness than smaller intervals. For the purpose of this hypothesis, we assumed that the open and close voicings that we chose for the study were equally familiar to our participants from their passive musical experience.

We expected an interaction between trichord and inversion, such that consonance would depend on inversion in a different way for each trichord. Although there is a general tendency for triads to appear more often in root position than inversion, the relationship depends on the chord: the diminished triad, for example, happens more often in inversion than conventional root position (Forte, 1974, p. 66). Parncutt and Radovanovic (in press, Figure 5) found in their sample of jazz arrangements that minor triads (in semitones relative to the root: [037]) happen more often in first inversion (as [049]) than major triads; diminished triads ([036] appear similarly often in all three inversions; and suspended-4th triads [057] often appear in inversion as [027].

Method

Open Practices Statement

The primary data from the consonance ratings, the predictors, and the prevalence data from Parncutt et al. (2019), as well as analysis files, are available via the Open Science Framework (osf.io/me7ju). We report all data exclusions and all relevant measures and manipulations in the study.

Sample

The sample consisted of 21 participants (9 female, 12 male). They had a wide range of experience regularly playing or performing a musical instrument, from 0 to 32 years with a mean of 15.3 (SD = 10.5). Some participants received 10 euros for their participation; others were research colleagues and participated without compensation. To ensure that each participant had correctly understood the experimental instructions, we checked that her or his ratings correlated with the mean ratings of all participants. A post hoc power analysis using G*Power 3.1.9.7 (Faul et al., 2007) with $N = 21$ and $\alpha = .017$ yielded an estimated power of $1\text{-}\beta = 1.000$ for the main effect of trichord across timbre conditions.

Material

Chords were created digitally in Audacity software by adding OCTs or piano tones with simultaneous onsets. OCTs were created by adding pure tones with equal sound pressure level (SPL) across the audible range (from C0 = 16.35 Hz to C10 = 16744 Hz); octaves were exactly 2:1 and phase relations were not randomized. Piano tones were uncompressed .WAV files of tones played at moderate dynamic level on a Steinway Model B grand piano, downloaded on 22 October 2021 from https://ivyaudio.com/Piano-in-162. The SPL of the OCT chords was informally adjusted for subjective equal loudness with the piano chords; after this adjustment, SPL was higher for piano chords than for OCT chords just after onset, after which it decayed rapidly. The duration of piano chords was informally adjusted for equal subjective duration with OCT chords; after adjustment, piano chords had duration of 500 ms and OCT chords of 300 ms. The difference was due to the dying out of the piano sound, whereas the OCT sound remained constant.

Design

The experiment comprised 190 trials that were presented in a different random order for each participant. The main independent variable was Trichord, with 19 levels corresponding to 19 Tn-sets of cardinality 3 (Rahn, 1980). In semitones relative to a lower reference pitch, these are 012, 013, 023, 014, 034, 015, 045, 016, 056, 024, 025, 035, 026, 046, 027, 036, 037, 047, and 048. In tonal music theory, the last five in this list are called suspended, diminished, minor, major, and augmented respectively.

Each trichord was presented 10 times: six from Piano tones and four from OCTs. In that sense, Timbre can be regarded as a second independent variable with two unequal levels, Piano and OCT. Within the OCT chords, there were 4 levels (Transpositions), in which the chord's reference pitch class (the "0" in 012, 013 etc.) was transposed to C, E♭, F♯, or A. Within the Piano chords, there were 6 levels, within which there were two independent variables: Spacing (2 levels) and Inversion (3 levels). Spacing was either close or open; to convert close to open, the middle tone in the chord was transposed up an octave. The three levels of Inversion were root position, first inversion, and second inversion. The mean pitch of the three tones in each Piano trial was D4 (294 Hz) plus or minus a quartertone, relative to a logarithmic scale of frequency. For example, the major triad in close root position was B♭4D4F4; in open root position, F♯3C♯4A♯4.

Procedure

At the beginning of the experiment, participants gave their consent to participate and to the processing of their data and were asked to use headphones throughout. They were asked to understand consonance as “how well the tones go together in a musical sense” and dissonance as the opposite. In each trial, they heard a chord and rated its C/D on a continuous 11-point scale from very dissonant on the left (0) to very consonant on the right (10) by mouse click. Ten of the 21 participants did the experiment in our lab using PsychoPy (Peirce et al., 2019); the other 11 participated remotely using Pavlovia (pavlovia.org), due to the COVID pandemic. The experiment typically took about 10 minutes.

Modelling

We compared the data against the same 18 predictors for the C/D of a trichord as were used in Parncutt et al. (2019): 6 for roughness, 7 for harmonicity, 3 for unevenness, and 2 for diatonicity, as follows.

Roughness

The first roughness predictor was the number of semitones in the pc-set. It involved multiplying the interval vector of the trichord by the interval vector <100000> (cf. Forte, 1973). The first position in an interval vector represents intervals of one semitone, the second represents intervals of two semitones, and so on. For example, trichord CEF has an interval vector of <100110>: it contains one semitone (EF), one interval of 4 semitones (CE), and one of 5 (CF). Multiplying the interval vector of this trichord by <100000> yields 1x1 + 0x0 + 0x0 + 0x1 + 0x1 + 0x0 = 1. In other words, the trichord contains one semitone interval. The second predictor was the sum of the number of semitones and the number of tritones, calculated using the vector <100001>. The third also considered the number of whole tones (<110001>). The fourth weighted the semitone interval higher than the whole tone and tritone (<310001>). The fifth weighted the tritone higher than the whole tone, but lower than the semitone (<310002>). The sixth considered all six interval classes, weighting them relative to each other as proposed by Huron (1994): <−1.428, −0.582, +0.594, +0.386, +1.240, −0.453>.

Harmonicity

The first harmonicity predictor was the number of perfect 4th intervals in the chord (<000010>). The second also considered the number of major 3rd intervals, but gave more weight to the 4ths because they occur lower in the harmonic series (<000120>). The third included the minor 3rd and again attempted a reasonable relative weighting of the three intervals (<001240>). The fourth included the major 2nd interval (which occurs in the harmonic series between harmonics 8 and 9): <012360>. The last three harmonicity predictors were calculated using the chord-root model of Parncutt (1988), with root-support intervals P8, P5, M3, m7 and M2 and corresponding root-support weights 10, 5, 3, 2, and 1. The fifth harmonicity predictor was the *weight* of the most salient pitch class in the chord according to that model, the sixth was *salience* of the most salient pitch class, and the seventh was the *pitch ambiguity* of the chord.

Unevenness

The first unevenness predictor was the largest interval between any two of the chord's pitch classes on the pitch class circle in semitones. For example, in a major triad, the biggest interval is 5 semitones, occurring between the chord's fifth and its (higher) root. The second predictor was the difference between the largest and smallest interval in the chord on the pitch class circle; for a major triad, that was 5 - 3 = 2. The third predictor was the standard deviation of the three intervals between adjacent tones.

Diatonicity

The first diatonicity predictor had the value 1 if the chord was diatonic in any major scale and 0 if it was not. The second was the number of times the chord occurs in the same diatonic scale. For example, a major triad appears in a C-major scale at three positions: C, F and G.

Statistical Analysis

Due to the high correlations between the predictors, we refrained from fitting our data using a linear regression model. To check whether the difference between the two correlation coefficients was significant, we used an online calculator (Lee & Preacher, 2013; Steiger, 1980). In a similar way, we compared the historical prevalence of unprepared and prepared trichords from Parncutt et al. (2019) with perceived consonance in our experiment.

We performed two repeated measures analyses of variances (ANOVA) to examine whether different Trichords were perceived to have different consonance or dissonance within each Timbre condition: once for OCT chords only (19 Trichords x 4 Transpositions) and once for Piano Tone chords only (19 Trichords x 2 Spacings x 3 Inversions). After calculating the mean of the consonance ratings for each trichord across the manipulations in each condition, we used another repeated measures ANOVA to test our hypothesis that participants would rate chords in the Piano Tone condition more consonant than the OCT condition.

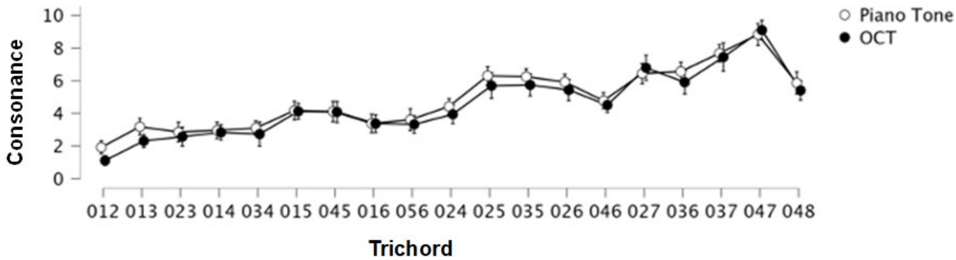
For all ANOVAs, we tested the sphericity of the data using the Mauchly test. If the sphericity assumption was violated, we adjusted degrees of freedom according to Greenhouse-Geisser. We set $\alpha < .017$ to account for multiple testing (three ANOVAs) according to Bonferroni.

Results

The mean perceived consonance for each chord in the Piano Tone and OCT conditions is shown in Figure 1. There was a highly significant main effect of Trichord, $F(3.754, 360) = 94.724$, $p < .001$, $\eta_p^2 = .826$, but no main effect of Timbre, $F(1, 20) = 1.371$, $p = .255$, $\eta_p^2 = .064$. There was no interaction between Timbre and Trichord, $F(7.622, 360) = 1.123$, $p = .351$, $\eta_p^2 = .053$.

Figure 1

Mean Consonance Ratings of the OCT and Piano Tone Conditions



Note. The error bars are 95% confidence intervals.

Correlation with Historical Prevalence of Trichords

We predicted that the historical prevalence of a trichord would correlate with its perceived consonance. We also expected that perceived consonance would correlate more strongly with the prevalence of unprepared trichords than prepared trichords. Table 1 shows an excerpt of the correlations. As expected, the historical prevalence of both the unprepared trichords (Pearson's *r* unprepared) and the prepared trichords (Pearson's *r* prepared) correlated highly with perceived consonance in our experiment. But when comparing the correlation coefficients for unprepared and prepared chords from Parncutt et al. (2019) and their respective perceived consonance from the present research according to the procedure suggested by Steiger (1980), we found mixed results. Against our hypothesis, the perceived consonance correlated significantly more strongly with the prevalence of prepared than unprepared chords for the 17th, 18th, and 19th centuries. For the 13th, 14th, 15th, and 16th centuries, there were no significant differences. We set $\alpha = .004$ to account for multiple correlations and .007 for the multiple comparisons with Fisher's *z'* (Steiger, 1980). The full table including 95% confidence intervals for each correlation can be found on OSF.

Table 1

Correlation of Historical Prevalence and Perceived Consonance of Trichords

Century	Pearson's *r* unprepared	*p* unprepared	Pearson's *r* prepared	*p* prepared	Fisher's *z'*	*p* Fisher's *z'*
13th	.739	< .001	.702	< .001	0.310	.756
14th	.712	< .001	.816	< .001	-1.585	.113
15th	.677	.001	.754	< .001	-1.956	.051
16th	.671	.002	.738	< .001	-2.073	.038
17th	.666	.002	.768	< .001	-2.732	.006
18th	.685	.001	.774	< .001	-3.321	< .001
19th	.686	.001	.738	< .001	-2.804	.005

Correlation with Predicted Harmonicity and Roughness

We expected the mean perceived consonance for each trichord to correlate more strongly with roughness than harmonicity. To examine that hypothesis, we correlated the predictors of Parncutt et al. (2019) with our empirical data. The table of all predictors as well as the full table of correlations adapted from Parncutt et al. (2019) can be found on OSF.

Table 2 shows how each predictor correlated with the mean consonance rating for each trichord. The most successful harmonicity model was Harmonicity4 (<012360>), according to which harmonicity is the sum of the number of major 2nd intervals in the trichord, the number of minor 3rds (multiplied by 2), the number of major 3rds (x3), and the number of perfect 4ths (x6). The most successful roughness model was Roughness4 (<310001>), the sum of the number of minor 2nds (x3), the number of major 2nds, and the number of tritones. We disregarded the high correlation for Unevenness2, because that predictor correlated strongly with Roughness4 ($r = .846$). The correlations for diatonicity were relatively weak.

Table 2

Correlation of Predictors and Mean Consonance Ratings across 19 Trichords

Model	Vector	Pearson's *r*
Roughness1	100000	-.817
Roughness2	100001	-.647
Roughness3	110001	-.642
Roughness4	310001	-.898
Roughness5	310002	-.858
Roughness6	Huron	-.894
Harmonicity1	000010	.464
Harmonicity2	000120	.513
Harmonicity3	001240	.683
Harmonicity4	012360	.724
Harmonicity5	-	.579
Harmonicity6	-	.229
Harmonicity7	-	-.479
Unevenness1	-	-.774
Unevenness2	-	-.836
Unevenness3	-	-.832
Diatonicity1	-	.595
Diatonicity1	-	.567

Table 3 shows how Roughness4 and Harmonicity4 correlated with consonance ratings of trichords in more detail. We set $\alpha = .017$ to account for multiple correlations. As shown in the table, these two predictors correlate with each other, making them unsuitable for fitting the data using a linear regression model (Cohen, 1988; Poole & O'Farrell, 1971).

Table 3

Correlation of Harmonicity4 and Roughness4 with Perceived Consonance

Variable		Empirical Means	Harmonicity4 <012360>
Empirical	Pearson	-	
Means	Correlation	-	
Harmonicity4	*r*	.724	-
<012360>	*p*	< .001	-
	Upper 95% CI	.910	-
	Lower 95% CI	.422	-
Roughness4	*r*	-.898	-.673
<310001>	*p*	< .001	.002
	Upper 95% CI	-.852	-.374
	Lower 95% CI	-.956	-.862

Note. The 95% confidence intervals (CI) are based on 1000 bootstraps.

Comparing the correlation coefficients for Harmonicity4 and Roughness4 from Table 3 according to Steiger (1980), we found a significant difference (Fisher's z' = -5.789, $p < .001$). That is consistent with our hypothesis that, in this kind of experiment, perceived consonance correlates more strongly with roughness than harmonicity.

Consonance of Chords from Octave-Complex versus Piano Tones

A comparison of the mean consonance ratings of each chord in the OCT and Piano Tone conditions using a repeated measures two-way ANOVA showed no significant difference between the two conditions, $F(1, 20) = 1.371$, $p = .255$, $\eta_p^2 = .064$. Looking more closely at the levels within the OCT condition, the data showed no significant main effect of Transposition, $F(2.056, 60) = 2.843$, $p = .068$, $\eta_p^2 = .124$, and no significant interaction between Transposition and Trichord, $F(54, 1080) = 1.087$, $p = .314$, $\eta_p^2 = .052$. The significant main effect of Trichord could also be observed in the OCT condition alone, $F(5.211, 360) = 55.200$, $p < .001$, $\eta_p^2 = .734$.

Looking at possible effects within the Piano Tone condition, we did not find any significant main effects of Spacing or Inversion (see Table 4) – contrary to expectations that open positions would be less dissonant than close positions, and inversions would be more dissonant than root positions. There was an interaction between Inversion and Trichord that we had expected based on preferences for specific inversions in music. There was also an interaction between Spacing and Trichord, and a three-way interaction between Inversion, Spacing and Trichord, as shown in Figure 2. These data did not, however, conform to expectations based on music theory. In particular, root positions of major and minor triads (047 and 037) were not consistently more consonant than first inversions, and first inversions were not consistently more consonant than second inversions. Nor was the root position of the diminished triad 036 consistently less consonant than its inversions, as we had expected.

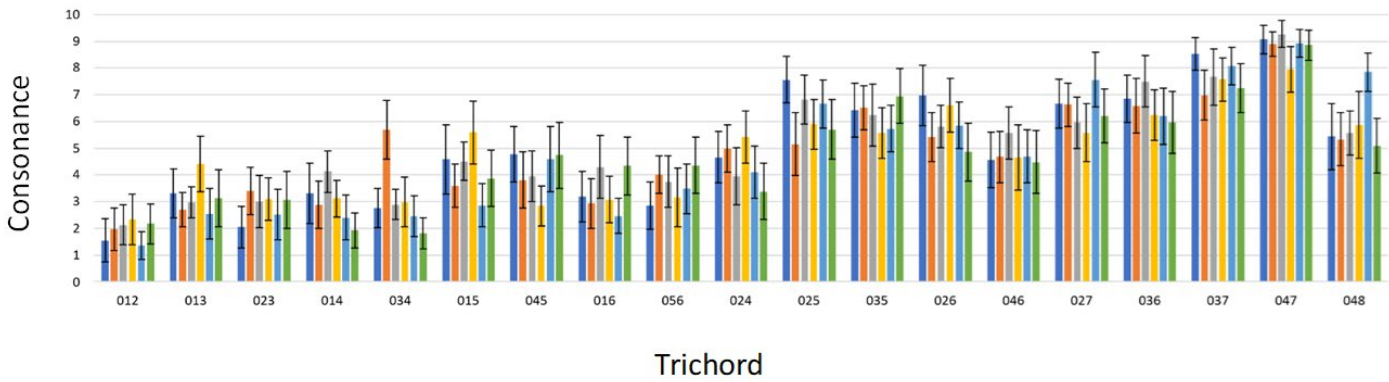
Table 4

Perceived Consonance in the Piano Tone Condition

Cases	*df*	*F*	*p*	η^2p
Inversion	2.000; 40.000	2.975	.062	.129
Spacing	1.000; 20.000	1.833	.191	.084
Trichord	5.053; 360.000	63.722	< .001	.761
Inversion ✻ Spacing	2.000; 40.000	0.104	.901	.005
Inversion ✻ Trichord	36.000; 720.000	3.563	< .001	.151
Spacing ✻ Trichord	18.000; 360.000	4.029	< .001	.168
Inversion ✻ Spacing ✻ Trichord	36.000; 720.000	3.529	< .001	.150

Figure 2

The Three-Way Interaction between Inversion, Spacing, and Trichord for Piano Chords



Note. For each trichord, the 1st column (dark blue) is for root position and close position (e.g., B♭D4F4), the 2nd (orange) for root position and open position (e.g., F♯3C♯4A♯4), the 3rd (grey) for first inversion close (e.g., B♭3D♭4G♭4), the 4th (yellow) for first inversion open (e.g., F♯3D4A4), the 5th for second inversion close (e.g., A3D4F♯4), and the 6th for second inversion open (e.g., F3D4B♭4). The error bars are 95% confidence intervals.

Discussion

Our study confirms that the C/D of trichords in Western tonal music depends on either roughness or harmonicity or both. It also suggests that the C/D of isolated sonorities depends more on roughness than on harmonicity. Taken by itself, our study does not necessarily demonstrate that the C/D of trichords depends on *both* roughness and harmonicity. That conclusion may nevertheless be drawn from an overview of other empirical and theoretical literature, in which the C/D of trichords has repeatedly been shown to depend on either roughness or harmonicity or both.

Although our consonance ratings correlated more strongly with roughness than harmonicity, we do not see that as evidence that roughness is *generally* more important than harmonicity for the harmonic vocabulary of Western tonal music, in which chords are not heard in isolation but in relation to preceding and following chords. In a musical context, the listener's attention may be attracted away from roughness and toward voice-leading. That could explain the resolution effect in music theory, whereby harmonic dissonance is resolved by stepwise resolution to harmonic consonance. In medieval counterpoint, for example, a major 6th interval was often resolved by stepwise contrary motion to an octave. In later tonal harmony, a triad with a suspended 4th was typically resolved by a falling step from 4th to 3rd, and a dissonant 7th on a dominant chord was resolved by falling step to a 3rd above the tonic.

The high negative correlation between our predictors for roughness and harmonicity can be explained theoretically. When a human with normal hearing hears a typical harmonic complex tone in speech or music, roughly ten harmonic partials are aurally relevant; that is, they contribute to the perception of the tone's overall pitch, loudness, and timbre (Terhardt et al., 1982). In a cognitive approach, harmonicity involves a template-matching process, where the template represents memory for typical harmonic spectral-pitch patterns. In that process, the lower partials play a more important role than the higher ones, because lower partials are more often clearly audible in typical voiced speech sounds.

Harmonics get closer together with higher harmonic number – not relative to absolute frequency in Hz, but relative to the logarithm of frequency (or musical pitch, or the piano keyboard) as well as critical bandwidth (the effective bandwidth of auditory filters of the inner ear). The harmonicity model, therefore, predicts that smaller musical intervals in the range of 1 to 5 semitones have lower harmonicity than larger intervals. That is almost the exact opposite of roughness: in the range of 1 to 5 semitones, the smaller the interval, the greater the roughness (e.g., Huron, 1994). Again, the reason has to do with critical bandwidth. Put another way, the reason why harmonicity only involves about ten harmonic partials is because higher adjacent partials (e.g., harmonics 10 and 11) fall well within one critical band and cannot be resolved. Pairs of partials are perceived as rough for the same reason.

Apart from intervals of 1 to 5 semitones, the only remaining interval class is 6 semitones (the tritone). That interval is dissonant according to both the roughness and the harmonicity model. It is relatively high in roughness because the 2nd harmonic of the upper tone lies a semitone away from the 3rd harmonic of the lower tone, and relatively low in harmonicity because it lies between relatively high harmonics (5 and 7, or 7 and 10), one of which is often masked in the voiced sounds of everyday speech.

There is a long tradition in music theory of linking the C/D of a musical interval to its ratio. Intervals with more complex ratios (however defined or operationalized) are understood to be more dissonant. Our study adds to existing evidence that the traditional ratio-based approach to musical C/D is incorrect or misleading. Musical intervals are not ratios, nor do they (or should they) correspond exactly to ratios. They are culturally learned pitch distances, and their C/D depends on the ability of the inner ear to separate nearby frequencies (*roughness*, reflecting its inability to do that for small intervals) and the ability of the auditory cortex to recognize either periodicity in the time domain or (equivalently) *harmonicity* in the frequency domain.

C/D also depends on musical familiarity, such that more common sounds are perceived to be more consonant. The cultural contingencies within which music develops influences C/D. Our experiment did not investigate this factor; instead, we assumed it from the literature. Since more common sounds tend to be more harmonic and less rough, it is difficult to establish whether the C/D of a specific sound, as perceived by a specific person on a specific occasion, is mainly or primarily due to "nature" (roughness or harmonicity) or "nurture" (familiarity). Since all three components of C/D – roughness, harmonicity, and familiarity – correlate with each other, it is difficult to compare their relative importance.

Future work could investigate individual differences. Participants may respond differently depending on cultural background (e.g., Indian versus Western) or, within Western culture, depending on musical experience. Among musicians, there may be differences depending on the instrument. However, it is difficult to formulate specific hypotheses in advance. We can hardly test whether harmonicity is more important for one group and roughness for another if the two measures correlate with each other, but the two predictors could perhaps be separated by manipulation of the spectrum.

Our results on the dependence of consonance on spacing and inversion were inconclusive (but see Parncutt & Radovanovic, in press). Whereas the interactions were significant, we were unable to account for the details. To address this question, a design with more statistical power would be necessary.

Conclusion

Our findings were consistent with our initial expectation, based on the existing empirical literature, that the perceived consonance of familiar musical chords depends on three main factors: smoothness (lack of psychoacoustic roughness), harmonicity, and familiarity. We did not test the effect of familiarity, and we were unable to confirm from our data that *both* smoothness and harmonicity play a role, due to the correlation between the corresponding predictors. Instead, a comparison of correlation coefficients suggested that smoothness played a more important role than harmonicity in this kind of experiment. It is possible that the relationship between smoothness and harmonicity is different when sounds are heard in a musical context. Results also correlated with the prevalence of simultaneities in a historical music database; contrary to our expectation the correlation was stronger if the tone onsets in the musical score were not simultaneous ("prepared" chords) for music composed in the 17th-19th centuries.

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