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What is a Genuine Mathematical Explanation in Empirical Science?*

Abstract
The question presented in the title of this paper was posed by Daniele Molinini almost a decade ago, with the contention that the answer to it had not been clearly articulated until that point. The heightened prevalence of the term genuine mathematical explanation can be traced back to Alan Baker's paper published approximately two decades ago. Regrettably, even today, after two decades, we can hardly say that there has been a significant advancement in the comprehension of the conceptual nuances of this term. Baker's Enhanced Indispensability Argument, in contrast to the Quine-Putnam Indispensability Argument, is founded on the significance of mathematical explanation within the realm of science, specifically emphasizing the role of what is termed as genuine mathematical explanation. This concept is cited by authors advocating for Platonist perspectives, as well as by those who maintain nominalist viewpoints. We will scrutinize the interpretations of this term offered by three authors, endeavoring to identify commonalities, with the hope that our analysis may contribute modestly to the crystallization of its meaning, aligning as closely as feasible with intuition.

Keywords
philosophy of mathematics, mathematical Platonism, enhanced indispensability argument, genuine mathematical explanation

1. Introduction

The question presented in the title of this text was posed by Daniele Molinini almost a decade ago, with the contention that the answer to it had not been

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clearly articulated until that point.[1] The heightened prevalence of the term *genuine mathematical explanation* (hereafter referred to as GME) can be traced back to Alan Baker's paper published approximately two decades ago.[2] Regrettably, even today, after two decades, we can hardly say that there has been a significant advancement in the comprehension of the conceptual nuances of this term. Various scholars interpret it divergently, underscoring a recurring theme within philosophical discourse wherein concepts lacking universal acceptance and explicit definition breed ambiguity, thereby hindering clarity across theories that incorporate them. This circumstance is not uncommon in philosophy. Of particular concern to the adherents of Platonism in mathematics is the pivotal role of GME in contemporary platonic arguments, with its persistence of ambiguity presenting a vexing issue.

The claim regarding the existence of mathematical objects constitutes a fundamental thesis of mathematical Platonism, engendering contentious debates between Platonists and nominalists in the field of the philosophy of mathematics. In recent discourse, Platonism draws, among other considerations, upon the Quine-Putnam indispensability argument (IA) and Baker's Enhanced Indispensability Argument (EIA). The latter argument, in contrast to the former, is founded on the significance of mathematical explanation within the realm of science, specifically emphasizing the role of what is termed as genuine mathematical explanation. This concept is cited by authors advocating for Platonist perspectives,[3] as well as by those who maintain nominalist viewpoints.[4] It appears to us, as we endeavor to demonstrate, that in prior literature, this term has been utilized ambiguously, lacking precision and varying from author to author, collectively rendering discussions in which this concept remains unclear. Consequently, the Platonist endeavor suffers particular detriment, given the significant role GME plays within it. We will scrutinize the interpretations of this term offered by three authors, endeavoring to identify commonalities, with the hope that our analysis may contribute modestly to the crystallization of its meaning, aligning as closely as feasible with intuition. We will identify the areas that we believe represent the main doubts concerning the understanding of GME, critically review the achievements of the three selected authors, and provide some potential semantic solutions that would be as close as possible to the intuition that scientific terminology and methodology imply.

2. Baker's Genuineness as an Introduction to Indispensability

Most authors discussing GME do not feel the need to explicitly define the term but rather employ it as self-evident and elementary, with sporadic attempts at characterization. Before Baker formulated the EIA to achieve Platonist objectives, he also employed other tools such as inference to the best explanation (IBE). He arrives at such conclusions only after assigning the status of GME,[5] to a specific procedure, that this approach ensures sufficient conditions for the application of IBE. However, does a mathematical explanation in science qualify as *genuine*? It is genuine when the mathematical component of a scientific explanation is explanatory *in its own right*.[6] Baker elucidates this linguistic construction through meticulously analyzed examples, highlighting three necessary conditions that should be met for an explanation of a phenomenon employing mathematical facts to be considered a *genuine* explanation:

1) The application of mathematical tools exhibits an external character, that is: it does not involve the mathematical explanation of mathematical problems;

2) The phenomenon in question must be in need of explanation;

3) The phenomenon must have been identified independently of the putative explanation (otherwise it is more like a prediction).[7]

For an original explanation to be considered GME, the mathematical component of the explanation must be explanatory in its own right, rather than functioning as a descriptive or calculational framework for the overall explanation.[8] In other words, mathematical elements must contribute to the explanatory power of explanations. Mathematical entities are not merely expected to play a "static" role of quantification in explanations, as is typically assumed as regards the indispensable role of mathematical objects in IA. Rather, they should play a "dynamic" role, whereby the use of mathematical objects critically enhances the explanatory power of the entire explanation. In this sense, the concept of "genuineness" serves as the chronological and essential precursor to Baker's concept of the indispensable explanatory role of mathematics utilized in formulating EIA.[9] Baker illustrates his understanding of GME by recognizing situations in which mathematical explanation constitutes a part or component of the overall explanation of a physical phenomenon.[10]

Such determination of origin naturally raises several additional questions. Firstly, considering arguments suggesting that EIA is largely built on the concept of GME,[11] and knowing that the indispensable explanatory role of mathematics, as a crucial component of EIA, implies a unique and irreplaceable function of "selected" mathematical objects and theories in explanation, the

1 More precisely, the question is: "What is a genuine mathematical explanation in empirical science and on what basis do we consider it as such?". See: Daniele Molinini, "Evidence, explanation, and enhanced indispensability", *Synthese* 193 (2016) 2, pp. 403–422, here p. 417, doi: https://doi.org/10.1007/s11229-014-0494-2.

2 Alan Baker, "Are there genuine mathematical explanations of physical phenomena?", *Mind* 114 (2005) 454, pp. 223–238, doi: https://doi.org/10.1093/mind/fzi223.

3 Ibid.

4 Mary Leng, "Models, structures, and the explanatory role of mathematics in empirical science", *Synthese* 199 (2021), pp. 10415–10440, doi: https://doi.org/10.1007/s11229-021-03253-x; Robert Knowles, Juha Saatsi, "Mathematics and Explanatory Generality: Nothing but Cognitive Salience", *Erkenntnis* 86 (2021), pp. 1119–1137, doi: https://doi.org/10.1007/s10670-019-00146-x.

5 A. Baker, "Are there genuine mathematical explanations of physical phenomena?", p. 236.

6 Ibid., p. 223. In the literature, we also encounter the phrase *ipso facto*.

7 Ibid., pp. 233–34.

8 Ibid., p. 234.

9 Alan Baker, "Mathematical explanation in science", *British Journal of Philosophy of Science* 60 (2009) 3, pp. 611–633.

10 In A. Baker, "Are there genuine mathematical explanations of physical phenomena?", p. 235, we encounter arguments aimed at securing the status of GME for the explanation of the *periodical cicadas* case.

11 On the identification of GME with explanations containing indispensable mathematical objects see D. Molinini, "Evidence, explanation, and enhanced indispensability", p. 407; Silvia Jonas, "Mathematical Indispensability and Arguments from Design", *Philosophia* 49 (2021) 5, pp. 2085–2102, here p. 2088, doi: https://doi.org/10.1007/s11406-021-00356-0; Markus Pantsar, "Objectivity in Mathematics, Without Mathematical Objects", *Philosophia Mathematica* 29 (2021) 3, pp. 318–352, here p. 327, doi: https://doi.org/10.1093/philmat/nkab010.

question arises as to whether a similar situation exists with GME? Secondly, is the component or partial (non)participation of mathematical explanation in the overall explanation of a phenomenon discretely determinable through two states, that is, whether it can be recognized as either the state *exists*, or the state *does not exist*. If so, what tools and mechanisms should be used for such recognition?[12] If an explanation is GME, can one discuss a gradation of cases based on the degree of involvement of mathematical explanation in the overall explanation, and how small measures could be registered in this sense?

According to Baker's definition of GME, the response to the first question is negative. Specifically, within the parameters outlined for GME, Baker does not stipulate the requirement of exclusivity or indispensability of the mathematical explanatory tool utilized in a particular case. For an explanation to qualify as GME, according to Baker's criteria, it is necessary and sufficient for the mathematical component of the explanation to form a part or component of the overall explanation of the phenomenon. Thus, the elementary principle derived from number theory, stating that a prime number possesses a minimal number of divisors, is employed in answering the question: why do periodical cicadas have prime periods? Because prime numbers minimize their frequency of intersection with other period lengths. If we were to exclude the mathematical explanatory component in this instance, the entire explanation of the phenomenon would be rendered worthless and incomplete. However, the proposed formulation of GME does not rule out the possibility that there could exist another explanation for a specific empirical phenomenon, of which a crucial part would be an alternative mathematical explanation. In other words, if there were a mathematical explanation, for example, from the field of mathematical analysis or topology, which, when combined with general empirical laws, provided a different acceptable explanation, then according to Baker, we would consider it GME.

Finding answers to the second set of questions appears more challenging because it is not entirely clear whether we have precise criteria for distinguishing the static – descriptive and quantificational roles of mathematics from the dynamic – explanatory role in explaining empirical phenomena. In extremely simplified examples, such as when we use numbers solely to express the measure of a specific physical quantity, it may be easy to determine the presence or absence of a mathematical explanation for explaining the empirical phenomenon. However, such clarity may not be evident in all cases. For instance, the fact that the mass of three electrons equals three times the mass of one electron may seem so trivial to us that it might obscure the underlying mathematical assertion.[13]

$$1 + 1 + 1 = 3 \tag{1}$$

This fact is elementary and self-evident to the extent that it diminishes its standing as an exclusively mathematical truth, employed for explicating an empirical phenomenon. Does this render it merely a trivial quantificational utilization of mathematics devoid of any explanatory function? Similarly, would the same principle apply in a scenario where we utilize the assertion that

$$2 + 3 = 5 \tag{2}$$

to explain some other physical fact? If the answer were affirmative, then we could proceed with similar examples and questions. Let us now assume that, for explaining the empirical phenomenon *P*, more complex mathematical objects and assertions are used. Let these, for example, be the following two statements:

$$\sum_{k=1}^{n} F_k^2 = F_n F_{n+1} \quad (3)$$

where F_i are Fibonacci numbers,[14] and

$$e^{i\pi} + 1 = 0 \text{ [15]} \quad (4)$$

Can we also discuss here a static, quantificational, non-explanatory use of mathematics to explain phenomenon *P*? If the answer were affirmative, then we might become concerned about what distinguishes the explanatory capacity possessed by the facts of number theory which leads to one explanation of periodical cicadas,[16] from the explanatory capacity of the facts given in statements (3) and (4), which enabled the explanation of phenomenon *P*. Moreover, arriving at facts (3) and (4) does not require any simpler mathematical reasoning than that which was used within number theory for the case of cicadas. Perhaps an objection to the previous analysis could be that the explanatory power of mathematics does not depend solely on the *complexity* of the mathematical assertions and theories under consideration, but rather on the *role* of those assertions and theories in explaining the phenomenon. However, when discussing the general phenomenon *P*, we cannot ascertain that role until we learn all the specific details regarding *P*. Nevertheless, even in cases where we are fully acquainted with all the details of the application of mathematical facts, such as in the example of calculating the mass of electrons, arriving at a definitive conclusion about the existence of the explanatory role of mathematics is not easy. The role of assertion (1) concerning the determination of the mass of multiple electrons is unquestionable, yet has it not the degree of *complexity* of the mathematical assertion itself prompted us to contemplate whether there is any explanatory role of mathematics in that case at all?

If we were to allow that the application of assertions (3) and (4) is not static but rather dynamic and explanatory, then there arises a question as to what

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"... one needs a further argument to see how the mathematical explanans can *in principle* have any explanatory relevance for an explanandum that is purely physical, free of any traces of mathematical vocabulary." – Sorin loan Bangu, "Inference to the Best Explanation and Mathematical Realism", *Synthese* 160 (2008) 1, pp. 13–20.

13
A similar example is used in Markus Pantsar, "Mathematical Explanations and Mathematical Applications", in: Bharath Sriraman (eds.), *Handbook of the Mathematics of the Arts and Sciences*, Springer, Cham 2021, pp. 2587–2602, here p. 2588.

14
Recursive definition of the n^{th} Fibonacci number F_n is $F_n = F_{n-1} + F_{n-2}$, where is $F_1 = F_2 = 1$. For further details, see, for example, Thomas Koshy, *Fibonacci and Lucas Numbers with Applications*, vol. 1, Wiley, New Jersey 2017, p. 87.

15
For more on the so-called Euler's identity, see, for example, Robin Wilson, *Euler's Pioneering Equation. The Most Beautiful Theorem in Mathematics*, Oxford University Press, New York 2018, here pp. 137–138.

16
This concerns a case of a specific type of North American cicadas (*Magicicada*) whose lifespan is expressed solely in prime numbers, either 13 or 17. In explaining this phenomenon, elementary facts from number theory (prime numbers have fewer divisors than composite numbers) as well as some assumptions from biology have been used. For further details, refer to A. Baker, "Are there genuine mathematical explanations of physical phenomena?".

the essential difference is between the use of assertions (1) and (2) on the one hand and assertions (3) and (4) on the other hand? What confers explanatory power upon the latter two examples while the former two do not possess such a quality? In each of the four instances, assertions involving *sums* are encountered. Admittedly, these sums are derived through disparate mathematical endeavors; nonetheless, revisiting this matter would inevitably reignite discourse on the significance of the intricacies of mathematical principles concerning the discourse on explanatory capacity.

The previous consideration reveals how challenging it is to delineate situations in which we could speak of the use of the *explanatory power of mathematics* in explaining physical phenomena from situations in which this is not the case. This fact not only undermines the endeavors to craft a cogent definition of GME but also comprises the basic elements of distinguishing between IA and EIA. Indeed, the pivotal discrepancy between these arguments hinges precisely on the role of mathematics, whether explanatory or not, in explaining empirical phenomena. Nonetheless, it appears that, in broad terms, a demarcation can be made between the explanatory role of mathematics and one that is not predicated on the question of whether the utilization of mathematical tools is confined solely to quantification and nomenclature or encompasses the employment of mathematical assertions irrespective of their degree of complexity. In other words, we contend that it is impossible to utilize a tool in the form of a mathematical statement, no matter how elementary, without such employment possessing some measure of explanatory power. The mere utilization of assertion *T* to explicate phenomenon *P* implies that the content of assertion *T* contributes to explaining certain aspects of *P*, and indicates that the mathematical rationale behind *T*, whether through proof or axiomatic foundation, concurrently constitutes a component of the comprehensive explanation for *P*. Thus, if any of the assertions (1), (2), (3), or (4) were to feature explaining phenomenon *P*, that role would necessarily be explanatory.

3. Molinini's Pragmatism

Although based on Baker's definitions, we cannot explicitly discern the identification between mathematical objects in GME and those for which Baker will assert are explanatorily indispensable in EIA, implicitly, this identification is grounded in the use of GME as a precursor to the construction of EIA, and intuitively, it is supported by the linguistic construction in *its own right*. It appears to us that we are justified in believing that explanatory indispensability, as a key characteristic of EIA, emerges within an environment fostered by the existence of GME. In holding this belief, we are not alone.

> "His (Baker's) idea is that wherever we have a genuine mathematical explanation of a phenomenon covered by a scientific theory *T*, and this explanation essentially depends on some mathematical entities, we have to regard these mathematical entities as explanatorily indispensable and therefore we have to be committed to the existence of these entities *via* EIA."[17]

Molinini contends that Baker nearly equates mathematical objects that appear in 'indispensable explanatory role' in empirical science with mathematical objects that play a genuine explanatory role in our best science.[18] More precisely, from the latter quote, among other things, it would follow that if we have a genuine mathematical explanation in which mathematical objects play a crucial role, then those objects would also have the status of indispensable objects. Therefore, according to the above reasoning, a genuine explanatory role implies an indispensable explanatory role. However, let's take a step back. Based on the above, we still do not know when we will say that a mathematical explanation is a genuine. Molinini astutely observes that Baker would need to allocate more space to clarifying the concept of GME to make clear the status of genuineness he attributes to specific scientific examples. In this regard, what does Molinini propose? He suggests that the final definition of GME should reflect the intuition upon which scientists rely in crafting their own explanations, with this intuition being subject to influences tied to the specificity of the particular scientific community, scientific practice, and historical circumstances. Such a conception of GME leads to a strong pragmatist view, an ultimate relativization of GME, and its overly broad definition, according to which any mathematical explanation of empirical phenomena based on a consistently structured formal context would be deemed genuine.[19]

> "Mathematics explains […] when it permits to reason in a very 'natural' way on the empirical fact we are investigating. For the researchers belonging to the logical relativity project, logical reasoning is the natural way to reason about relativity and explain relativistic phenomena, whereas for other scientists geometrical reasoning is the natural way to explain the Lorentz contraction.[20]

In other words, the facts of the theory of relativity can be genuinely explained through logical means using Zermelo-Fraenkel set theory, but also through geometric means using the Minkowski geometry. Neither option has priority in terms of genuineness. The choice of approach in explanation depends solely on how "natural" a particular approach is for the specific researchers or community to which they belong. The situation is similar when it comes to Euler's theorem in rigid body kinematics.[21] Whether one uses group theory or geometric tools to explain this empirical phenomenon is entirely irrelevant when it comes to the status of genuineness. The relevant details are only formal correctness and the sense of "naturalness" in the choice of approach in explanation, i.e., how much a particular mathematical tool "fits" into the hands of a specific research community or group.

What are the consequences of such a pragmatic stance? Firstly, the idea that "naturalness" should be the primary criterion in selecting a mathematical explanation for a specific empirical phenomenon leads to excessive freedom that threatens to abolish any criteria. It appears neither meaningful nor beneficial for the Platonist project to rely exclusively on the sense of "naturalness" within a research group for the formal status of genuineness.[22] Theoretically

17
D. Molinini, "Evidence, explanation, and enhanced indispensability", p. 407.

18
Ibid., p. 408.

19
A loose understanding of genuineness allows Molinini to attribute the status of GME to every option/mathematical explanation that explains *P*. For further details, see D. Molinini, "Evidence, explanation, and enhanced indispensability", pp. 419–421.

20
Ibid. p. 421.

21
For more on the previous examples, refer to D. Molinini, "Evidence, explanation, and enhanced indispensability".

22
At this point, it is essential to provide a couple of additional explanations. First, to discuss the originality, and thus the uniqueness, of the chosen mathematical explanation, an objective criterion is necessary to define such originality. The notion of originality in a scientific explanation would be contradictory if it were based solely on the pragmatic foundations of a particular scientific community. In that scenario, we would end up discussing

speaking, following the proposed approach, for phenomenon P, we could have an arbitrary number of mathematical explanations $E_1, E_2, ..., E_n$ ($n \in N$) that are "natural" for research groups $G_1, G_2, ..., G_k$ ($k \in N$ and $k \geq n$) addressing phenomenon P, with each E_i being a GME. In this context, within the set $E_P = \{E_i, 1 \leq i \leq n\}$ we would not be able to discern any hierarchy. There would be no ordering relation defined in relation to the degree of genuineness, based on which we could say that E_P is a linear order or a partial order. We would not be able to determine its cardinality either, since we can never know if all possibilities of finding new mathematical explanations for phenomenon P have been exhausted. Therefore, if we were to ultimately explain what the GME of phenomenon P is, we could not do so in any way other than by enumerating the known elements of the E_P, with the caveat that all elements of this set may never be known. Indeed, considering the development of existing and the emergence of new mathematical theories and their applications, it is clear that the question of definitively determining the set E_P for any given phenomenon P will always remain open. Such an approach does not seem to yield an outcome that aligns with the expected intuition of the concept of genuineness.

Secondly, the pragmatic approach does not promise a rosy future regarding the role of EIA in supporting the Platonist project. If we accept pragmatism as a criterion for selecting and identifying GMEs of empirical phenomenon P, as shown above, we will end up in a pluralistic environment, possibly obtaining multiple identified solutions without knowing whether this is a finite set of possible outcomes. On the other hand, if we read Baker as proposed by Molinini, understanding that the mathematical entities that are part of GME and are crucial for the overall explanation of the phenomenon have already secured themselves the status of explanatory indispensability, we arrive at incompatible positions. Mathematical objects as part of one GME E_1 of phenomenon P are clearly dispensable because they are interchangeable with mathematical objects that are part of an alternative GME E_2 of phenomenon P, but at the same time, they would be indispensable if we rely on Molinini's understanding of Baker. Both Baker and Molinini allow some kind of pluralism as regards genuineness: the former by not imposing exclusivity conditions when defining GME, and the latter by not preventing exclusivity with the same concept through a pragmatic position.

4. Leng's Structuralism

Mary Leng, a proponent of nominalist views, provides an insight into GME within the context of her own theory. From her perspective, mathematical explanations of empirical phenomena are possible due to the structural properties of mathematical theories, whose instantiations correspond to concrete phenomena in the physical world. The explication of the empirical domain through abstract mathematical constructs is facilitated through the utilization of mathematical models, serving as the initial stage in the process of generalizing physical reality. Acting as a conduit between abstract mathematical constructs and tangible physical occurrences, models function as a pivotal link. Some mathematical explanations of physical phenomena are indeed possible due to the structural properties of mathematical theories. Structural explanations explain a phenomenon by showing it to have been an inevitable consequence of the structural features instantiated in the physical system under consideration.[23] In other words, mathematical structuralism is a generalized representation of the structural properties of the physical world, and it is due to this that mathematical explanations of empirical phenomena become possible. Leng refrains from explicitly defining the concept of originality, presumably considering it intuitively self-evident, and asserts that structural mathematical explanations of empirical phenomena can be classified as *genuine* explanations for two cumulative reasons.. The first is that such explanations

"…. provide answers to the kind of 'why' questions that we ask in demanding explanations of phenomena. Why do the cicadas have the period length they have? Because the sequence of successive years is an instance of a natural number structure; the sequence of years in which the cicadas appear is an instance of an arithmetic progression within that structure; in any natural number structure, arithmetic progressions with prime differences between terms will overlap minimally with other progressions; and non-overlapping periods are advantageous."[24]

Another reason Leng invokes is her belief that structural explanations are, in fact, what are known as distinctively mathematical explanations.[25] Regarding the standpoint that distinctiveness[26] implies genuineness, she is not alone:

"An account of distinctively mathematical explanations aims to fit scientific practice by deeming to be explanatory only what would (if true) constitute genuine scientific explanations […]."[27]

Perhaps a somewhat more tangible and, for our purposes, more useful description of distinctiveness is obtained with the following paragraph:

an arbitrary number of original explanations, which would not make sense. Proponents of the pragmatic perspective might use the following concrete example as an argument. We might, for instance, choose different coordinate systems to determine the coordinates of a single mass point situated in space. Even if we restrict ourselves to the Cartesian coordinate system, there are infinitely many possible origins and consequently infinitely many possible coordinates for that single mass point. Which point we should select as the origin is a pragmatic question, depending on various factors. Indeed, in this case, the number of starting points and corresponding coordinate systems used to position a specific mass point would be infinite, yet each of those constructions for positioning the mass point would still be a coordinate system. Furthermore, between any two such coordinate systems, an isomorphic mapping can be established that enables us to identify them. In other words, we can identify all these coordinate systems. On the other hand, such identification is not feasible in the case of different mathematical explanations of physical phenomena. Second, why would the freedom of the pragmatic approach undermine the Platonist project? If we allow that one physical phenomenon can be originally explained with several different mathematical explanations, that is, by using various mathematical tools, then none of those tools would be indispensable; that is, they could be replaced by other tools. This would imply that EIA cannot be applied in this specific situation, meaning that the existential status of mathematical entities, which in the specific case serve as explanatory tools, cannot be guaranteed by utilizing EIA. I thank an anonymous reader for highlighting these important points.

23
M. Leng, "Models, structures, and the explanatory role of mathematics in empirical science", p. 10421.

24
Ibid., p. 10428.

25
Ibid.

26
Regarding *distinctive explanation*, a debate has emerged in the literature over the past few years, making it difficult to speak of a unique and explicit definition of this concept. See Carl F. Craver, Mark Povich, "The directionality of distinctively mathematical explanations", *Studies in History and Philosophy of Science* 63 (2017), pp. 31–38; Marc Lange, "A reply to Craver and Povich on the directionality of distinctively mathematical explanations", *Studies in the History of Philosophy of Science* 67 (2018), pp. 85–88, doi: https://doi.org/10.1016/j.shpsa.2018.01.002.

27
Marc Lange, "What makes a scientific explanation distinctively mathematical?", *British Journal of Philosophy of Science* 64 (2013) 3, pp. 485–511, here p. 508, doi: https://doi.org/10.1093/bjps/axs012.

"Distinctively mathematical explanations are "non-causal" […] they do not work by supplying information about a given event's causal history or, more broadly, about the world's network of causal relations. A distinctively mathematical explanation works instead […] roughly by showing how the fact to be explained could not have been otherwise […]."[28]

According to Leng, a structural mathematical explanation of a phenomenon would also be the GME because, among other reasons, it satisfies two conditions: the methodological orientation of the explanation towards finding reasons for the occurrence of the phenomenon (C_1) and the necessity of arriving at a unique *explanandum* (description of the phenomenon in question) regardless of any causality (C_2). Viewed in this way, the class of GME would be a subclass of the class of structural mathematical explanations. Moreover, to identify GME, it would suffice for a mathematical explanation, not necessarily structural, to satisfy the two aforementioned conditions. We say "not necessarily structural" because Leng isolates only the two aforementioned conditions from all the conditions she believes structuralism should imply. In this way, we implicitly obtain a new framework for defining GME. What is the nature of the conditions that create this framework? Let us first consider the circumstances relating to the operational recognition of GME, the verifiability of conditions C_1 and C_2, i.e., the steps that should be taken to examine the satisfaction of such conditions in a specific case. It appears that examining condition C_1 should be feasible without major difficulties because any explanation, including a mathematical one, should inherently provide an answer to the question of *why* the phenomenon occurs. Alternatively, if we attempt a counterexample, let us imagine a mathematical (or any other) *explanation* of a phenomenon that does not provide an answer to *why* the phenomenon occurs. Such an attempt, or rather a caricature of a mathematical explanation, can be found in the literature.

"Why are all planetary orbits elliptical (approximately)? Because each planetary orbit is (approximately) the locus of points for which the sum of the distances from two fixed points is a constant, and that locus is (as a matter of mathematical fact) an ellipse."[29]

Undoubtedly, in this example, we have a question concerning the shape of orbits and a sort of response that utilizes mathematical apparatus related to the definition of an ellipse. However, the response provided would be more appropriate for questions such as "Why could we say that planetary orbits have elliptical shapes?" or "What gives us the right to claim that planetary orbits are elliptical?" The question from the last cited paragraph aims to search for the reasons that *conditioned* the formation of the shapes of planetary orbits, so the provided response cannot be considered an appropriate explanation. The response given above does not explain *why* planets move along the paths they do, what the reason for such movement is, how such a type of movement originated, etc. Therefore, by nature, any explanation, including a mathematical one, to be considered an explanation at all, should offer reasons based on which we would obtain an answer to the question of *why*, concerning the specific phenomenon. Otherwise, we would have difficulty considering such a procedure as an explanation.

When it comes to verifying the satisfaction of the C_2 condition, circumstances appear to be not as simple as in the case of the C_1 condition. Specifically, the condition of absence of causality in the explanation may be operationally verifiable, but it is not certain that the same holds for the remaining part of C_2. Indeed, it is not entirely clear how a procedure could be implemented to demonstrate "how the fact to be explained could not have been otherwise". In other words, it is unclear how we would show in all cases that the explanation uniquely traces the *explanandum*, leaving no possibility for it to be otherwise. Even in the case of a purely mathematical explanation whose methodology is devoid of the probabilistic assumptions abundant in science, we could not speak of a strictly deterministic relationship between the explanation and the unique *explanandum* (consequence). Indeed, from the fact that *p* implies (explains) *q*, it does not follow that p implies (explains) *exclusively q*. Moreover, considering hybrid forms of explanation which are a blend of mathematical facts and scientific facts or assumptions, the situation becomes significantly more complex. For instance, the explanation in the cicadas case of is based on certain probabilities and scientific assumptions.[30] In such circumstances, we cannot assert that the current explanation guarantees the fact/*explanandum* being attempted to be explained/inferred. In other words, in this case, based on the given explanation, we lack a formal-theoretical guarantee that the event which is supposed to be explained could not have been different.

What outline has Leng's bipartite framework provided for GME? GME offers a response to the question of *why* regarding the explanation of a phenomenon, it is free from causal procedures and is intended to ensure the uniqueness and necessity of the *explanandum*. However, does this definition effectively guarantee the uniqueness of GME? In other words, does the presence of a GME explanation for phenomenon *P* preclude the possibility of the existence of another GME_1 explanation that would also elucidate *P*? Based on the proposed framework, we cannot assert the existence of exclusivity. Indeed, Leng's definition neither formally nor theoretically excludes the possibility of the existence of GME_1 and GME_2 phenomena explaining *P*. Furthermore, we cannot state that there exists a natural number *n* such that we can claim the existence of the set $E = \{GME_i; i = 1, ..., n\}$, comprising all genuine mathematical explanations of phenomenon *P*. Namely, considering the constant evolution of mathematics, the emergence of new mathematical and scientific theories, as well as the polymorphic approach to problem-solving and explaining phenomena which becomes increasingly rich and complex, we can never be certain that the current repertoire of explanations of phenomenon *P* available to the scientific community is finite. All such explanations should merely satisfy the criteria of not involving a causal procedure, addressing the question of *why*, and being sufficiently formal-methodological and explanatorily robust to convince us that phenomenon *P* is inevitable as a consequence of the explanation.

5. Common Characteristics of Three Approaches, Intuition, and Problems

The common thread among all three presented perspectives is the stance that the GME of phenomenon *P* need not be unique. In other words, none of the

28
Marc Lange, *Because without cause. Non-causal explanations in science and mathematics*, Oxford University Press, Oxford 2016, pp. 5–6.

29
M. Lange, "What makes a scientific explanation distinctively mathematical?", p. 508.

30
In this explanation, assumptions from evolutionary theory are utilized, such as the idea that species adaptations dictate their survival. Additionally, the assumption is made that the predators of cicadas have life cycles of fixed durations. See A. Baker, "Are there genuine mathematical explanations of physical phenomena?".

three authors presuppose the uniqueness of GME. Each of them implicitly or explicitly allows for the existence of multiple GMEs *for a given phenomenon P*. Furthermore, it has been demonstrated that each of the three perspectives, theoretically speaking, permits the existence of an arbitrary number of GMEs. Even if such circumstances seem natural to us, it is not clear how such a concept of GME could be the closest ancestor of the notion of indispensability, as suggested by Molinini.

Do our intuition and linguistic-meaning experience provide support for this road? It does not seem to us that this is the expected interpretation of the concept of genuineness concerning mathematical explanations in science. If we consult dictionaries regarding the meaning of the term *genuine*, we will see that they synonymously refer us, among other things, to terms such as authentic, original, and it also connotes definite origin from a source.[31] It seems that such terms are incomparably closer to a singular rather than pluralistic understanding of the entities to which they refer. Similarly, it is unclear how we could deem multiple mathematical explanations as genuine, authentic, or original. The genuineness of explanation *E* for phenomenon *P* should precisely mean that it is explanatory *in its own right*, *ipso facto*, making it *uniquely* original, irreplaceable, and therefore an indispensable procedure for understanding *P*. This means that no other explanation should have the explanatory capacity that *E* possesses.[32] It seems that such understanding is dictated by intuition regarding GME. This concept of GME would not exclude the possibility of the existence of different mathematical explanations, but we would not attribute the characteristic of genuineness to them. They would not have the explanatory capacity and strength that GME possesses. An alternative explanation could replace GME in explaining phenomenon *P*, which means that GME would be substitutable, and dispensable, i.e., the mathematical objects featured in GME would not be indispensable. Therefore, contrary to Molinini's viewpoint, we believe that at a general level, the existence of a conceptual difference between *genuineness* and *indispensability* is expected. The GME of the phenomenon *P* does not necessarily have to be the only and indispensable mathematical methodological procedure by which *P* is explained. GME does not exclude the possibility of other explanations, but ones that would not be genuine. Following this general consideration and an attempt to connect the intuition about GME with uniqueness, we encounter a challenging practical problem: finding criteria based on which, in the case of the existence of multiple mathematical explanations of the phenomenon *P*, we could highlight only one as genuine. Additionally, such a criterion should be applicable even in cases where there is currently only one mathematical explanation for a particular phenomenon,[33] and we would like to determine whether it qualifies as GME.

How to propose a criterion for recognizing GME of phenomenon *P*? One solution might be based on the degree of basicness of the mathematical theory that constitutes the main part of the mathematical explanation of phenomenon *P*. As is known, among some mathematical theories, it is possible to establish a certain hierarchy in terms of basicness. In this sense, on the set of all mathematical theories, we can define a binary relation ρ = "is basic for" as follows:

$$T_1 \, \rho \, T_2 \text{ if and only if } T_2 \text{ uses the results of } T_1.$$

For example, set theory and Euclidean geometry are commonly regarded as basic mathematical theories on which the statements of many other "derived" theories are built. In this sense, we could assign the status of GME to the explanation, if there are multiple, whose mathematical part belongs to a theory of higher degree of basicness. Thus, such a criterion would be based on the formal "age" of the theory used in the mathematical part of the explanation. The weaknesses of this solution are evident from the unpleasant properties of the relation ρ. Indeed, since the possibility that for two different theories T_1 and T_2 each of them uses the results of the other is not excluded, i.e., if both $T_1 \rho\, T_2$ and $T_2 \rho\, T_1$ hold, we do not have a guarantee that the relation ρ is antisymmetric, thus, we do not have a guarantee that it is a partial order relation. Also, the situation where neither of the two theories T_1 and T_2 uses the results of the other is not excluded, i.e., that these theories are incomparable with respect to the relation ρ. In other words, the set of all mathematical theories as well as some of its subsets with relation ρ will not be a linear order.[34] Due to such circumstances, it is not unexpected that the relation ρ will not be useful in some situations, i.e., there will be sets of theories where we will not be able to find the one that is more basic than the others.

Another criterion that might guide us in the search for mathematical genuineness could be based on the naturalness of the connection between empirical phenomena and the mathematical theory/structure used in the explanation. For example, in the case of the *Seven Bridges of Königsberg problem*,[35] it seems that using graph theory or topology would be expected before using mathematical analysis or number theory. Similarly, in the case of the periodical cicadas, it appears more natural to seek an explanation using number theory rather than *Lobachevsky geometry* or partial differential equations. Unfortunately, there are obvious problems associated with such a criterion. The naturalness of the connection between an empirical problem and a mathematical structure is an informal characteristic that is difficult to precisely express, which is why deciding on greater or lesser naturalness can easily lead to problems. Furthermore, the fact that we speak of various forms of morphisms between different mathematical structures, thanks to which parts of these structures are almost completely identified with each other, does not give us the right to declare any of them less or more natural for explaining a

31
See, for example, Henry Watson Fowler, *A Dictionary of Modern English Usage*, Oxford University Press, Oxford 2009, p. 214, or an online dictionary https://www.collinsdictionary.com/dictionary/english/genuine (accessed on 30 April 2025).

32
Perhaps it is not entirely clear why all the different (but "equivalent") mathematical explanations could not be authentic and original. As previously explained in footnote 22, in the example with coordinate systems, the existence of isomorphisms between certain mathematical structures allows us to identify such structures, as well as the mathematical explanations that utilize them. Therefore, in such cases, there is virtually no doubt about authenticity. Such doubt exists only in instances where a physical phenomenon can be explained by various mathematical explanations, with no proof that these explanations are equivalent. Such is the case of the explanation of the fact of the theory of relativity, which was mentioned earlier.

33
We say *currently* because at any given point in time, we cannot know what mathematical explanations the future development of mathematical theories may bring for a particular phenomenon.

34
For more on the elementary properties of relations, see, for example, Ivo Duntsch, Günther Gediga, *Sets, Relations, Functions*, Methodos Publishers, Bangor 2000.

35
The problem, famously posed by Euler in the 18th century, asks whether the seven bridges of the city of Königsberg over the river Pregel can all be traversed in a single trip without doubling back, with the additional requirement that the trip ends in the same place it began.

phenomenon.[36] Finally, let us note the problem that arises in relation to both proposed criteria. They attempt to choose from multiple explanations the one that would be "more" genuine than the others, but not the *only* genuine one.

Although mentioned in the literature over the past couple of decades, GME has rarely been thoroughly analyzed as a concept, whether implicitly or explicitly. Among other things, we aimed to demonstrate that there is no consensus within the scientific community that uses it regarding its meaning. By examining the perspectives of three authors who have discussed the concept, we attempted to analyze them, highlight their weaknesses, and implications, and notice their common characteristics. The diversity of each of these perspectives is evidence of the existence of ambiguity and uncertainties regarding this concept. To modestly contribute to sharpening the meaning of GME, we proposed a definition that differs from all three presented here, but which we believe is closest to the generally accepted intuition. We believe that emphasizing the characteristic of uniqueness could be crucial in the path toward a final definition of the concept, even at the cost of the problems such a definition might bring. Additionally, in this way, the position of EIA, whose conceptual apparatus represents a natural heritage of GME, would be stronger and conceptually more consistent.

Vladimir Drekalović

Što je pravo matematičko objašnjenje u empirijskoj znanosti?

Sažetak

Pitanje iz naslova ovoga članka postavio je Daniele Molinini prije gotovo deset godina, uz tvrdnju da do tada na njega nije bio jasno artikuliran odgovor. Povećana učestalost pojma autentično matematičko objašnjenje može se pratiti unatrag do rada Alana Bakera, objavljenog prije otprilike dvadeset godina. Nažalost i danas, dva desetljeća kasnije, teško možemo tvrditi da je došlo do značajnijeg napretka u razumijevanju pojmovnih nijansi toga izraza. Bakerov pojačani argument neizostavnosti, za razliku od Quine–Putnamova argumenta neizostavnosti, temelji se na važnosti matematičkog objašnjenja u znanosti, s posebnim naglaskom na ono što se naziva pravim matematičkim objašnjenjem. Ovaj se pojam navodi kako u tekstovima autora koji zagovaraju platonistička stajališta, tako i onih koji zastupaju nominalističke pozicije. Razmotrit ćemo tumačenja toga izraza kod trojice autora, nastojeći uočiti zajedničke elemente, s nadom da će naša analiza makar skromno pridonijeti kristalizaciji njegova značenja, u mjeri u kojoj je to moguće u skladu s intuicijom.

Ključne riječi

filozofija matematike, matematički platonizam, pojačani argument neizostavnosti, pravo matematičko objašnjenje

36 At this point, it makes sense to ask a specific question. Namely, if a certain mathematical structure or theory offers technically simpler solutions and a more technically elegant explanation of a phenomenon, why should this not give us the right to declare it to be more natural (as in fact happens)? For example, sometimes the use of spherical coordinates leads to incomparably easier equations, and hence such a system is usually labeled as being more natural for explaining that particular phenomenon. It seems, however, that such a point of view would mean equating the meanings of the terms *natural* and *simple*, which is undoubtedly not the case. Indeed, the greater naturalness of an explanation in bringing about another explanation cannot be derived only from the greater simplicity of one of the explanations. As is known from practice, an explanation can be considered more natural for reasons that have nothing to do with its complexity. For example, *naturalness* can be related to the traditional methodology that a specific scientific community cultivates according to certain scientific approaches; it can be related to the fertility, productivity, and amount of scientific consequences caused by a specific scientific explanation, etc.

Vladimir Drekalović

Was ist eine genuine mathematische Erklärung in der empirischen Wissenschaft?

Zusammenfassung

Die im Titel dieses Textes aufgeworfene Frage wurde von Daniele Molinini vor nahezu einem Jahrzehnt gestellt – mit der Feststellung, dass die Antwort darauf bis zu diesem Zeitpunkt nicht eindeutig artikuliert worden war. Die zunehmende Verbreitung des Begriffs „genuine mathematische Erklärung“ lässt sich auf Alan Bakers Artikel zurückführen, der vor annähernd zwei Jahrzehnten veröffentlicht wurde. Bedauerlicherweise lässt sich selbst heute, nach zwei Jahrzehnten, kaum behaupten, dass es eine substanzielle Weiterentwicklung in der Auffassung der begrifflichen Nuancen dieser Notion gegeben hat. Bakers verstärktes Unentbehrlichkeitsargument gründet sich – im Gegensatz zum Quine-Putnam-Unentbehrlichkeitsargument – auf den Stellenwert der mathematischen Erklärung im Bereich der Wissenschaft und stellt dabei namentlich die Rolle der sogenannten „genuinen mathematischen Erklärung“ in den Fokus. Dieser Begriff wird sowohl von Autoren angeführt, die platonistische Perspektiven befürworten, als auch von denen, die nominalistische Standpunkte vertreten. Wir werden die Interpretationen dieses Begriffs, die von drei Autoren vorgebracht wurden, eingehend prüfen und dabei bestrebt sein, gemeinsame Merkmale herauszuarbeiten – in der Hoffnung, dass unsere Analyse einen bescheidenen Beitrag zur Kristallisierung seiner Bedeutung leisten kann und sich hierbei so nah wie möglich an die Intuition anlehnt.

Schlüsselwörter

Philosophie der Mathematik, mathematischer Platonismus, verstärktes Unentbehrlichkeitsargument, genuine mathematische Erklärung

Vladimir Drekalović

Qu’est-ce qu’une explication mathématique véritable dans les sciences empiriques ?

Résumé

La question posée dans le titre de ce texte a été soulevée par Daniele Molinini il y a près d’une décennie, avec l’observation que, jusqu’alors, aucune réponse claire n’y avait été apportée. La prévalence accrue du terme « explication mathématique véritable » remonte à l’article d’Alan Baker publié il y a environ deux décennies. Malheureusement, même aujourd’hui, après vingt ans, il est difficile de dire qu’il y a eu une avancée significative dans la compréhension des nuances conceptuelles de ce terme. L’argument de l’indispensabilité renforcée de Baker, par opposition à l’argument de l’indispensabilité de Quine et Putnam, repose sur l’importance de l’explication mathématique dans le domaine scientifique, en mettant particulièrement en évidence le rôle de ce qui est désigné par le terme « explication mathématique véritable ». Ce concept est cité aussi bien par des auteurs défendant des perspectives platonistes que par ceux qui soutiennent des points de vue nominalistes. Nous examinerons les interprétations de ce terme proposées par trois auteurs, cherchant à identifier des points communs, dans l’espoir que notre analyse contribue modestement à la clarification de son sens, en s’alignant autant que possible avec l’intuition.

Mots-clés

philosophie des mathématiques, platonisme mathématique, argument de l’indispensabilité renforcée, explication mathématique véritable