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IS THERE A GRIČ ATTRACTOR?

Postoji li grički atraktor?

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Abstract — In this study the time series of monthly averages of temperature, cloudiness and solar radiation are analyzed to investigate the chaotic properties of the local climate, and the possibility of the estimation of an minimum number of independent variables necessary to model the time evolution of the underlying climate system. The analysis of observations is based on a statistical procedure, called "correlation algorithm", leading to the measures of dimensionality or degrees of freedom which control the underlying dynamics. The results show that there is no indication of the existence of a low-dimensional attractor of the Zagreb-Grič time series.

Within a climate system, it is possible to isolate a three-variable subsystem, controlling the large amplitudes of the local climate. These are: temperature, cloudiness and solar radiation. The existence of autocorrelation in the time series, due to solar radiation, has enabled the detection of a low-dimensional attractor. When the seasonal cycle has been theoretically filtered out of the series, the correlation disapped, as well as the low-dimensional climate subsystem.

Key words: climate attractor, climate system

Sažetak — U radu su analizirani vremenski nizovi mjesečnih srednjaka temperature, naoblake i Sunčevog zračenja opservatorija Zagreb-Grič u razdoblju 188-1995, s ciljem ispitivanja njihovih kaotičnih osobina, i mogućnosti određivanja minimalnog broja neovisnih varijabli potrebnih za opis vremenske evolucije pripadnog klimatskog sistema. Analiza se zasniva na statističkoj metodi poznatoj kao korelacijski algoritam, i daje mogućnost računanja mjera dimenzionalnosti odnosno broja stupnjeva slobode pripadnog dinamičkog sistema. Rezultati primjene metode pokazuju da nije moguće doći do korelacijske dimenzije atraktora, odnosno da niskodimenzionalni grički atraktor ne postoji.

Unutar klimatskog sistema može se izdvojiti podsistem, karakteriziran sa samo tri varijable koje opisuju utjecaje velikih amplituda djelovanja: temperaturom, naoblakom i Sunčevim zračenjem. Postojanje klimatskog podsistema unutar nizova mjesečnih srednjaka uzrokovano je autokorelacijom zbog prevladavajućeg utjecaja Sunčeva zračenja. Eliminacija tog utjecaja ima za posljedicu iščezavanje autokorelacije, odnosno niskodimenzionalnog atraktora lokalnog klimatskog podsistema.

Ključne riječi: klimatski atraktor, klimatski sistem

1. INTRODUCTION

Those trying to explain the world we live in have always hoped that, in the realm of complexity and irregularity observed in nature, simplicity would be found behind everything, and unpredictable events would finally become predictable.

In recent decades, the theoretical understanding of some complex dynamic systems has undergone a rapid development. By dynamic system, we mean a system whose evolution from some initial state can be described by a set of rules. These rules may be conveniently expressed by sets of differential equations. Dynamic systems also include systems where the exact present state only approximately determines a near future state; but there is no assurance that distant future states will be even approximately determined. This extended definition covers many real physical systems, whose behavior commonly involves at least some randomness or uncertainty. Among these systems a prominent one is the atmosphere plus its immediate surroundings.

Low-order models, and, indeed, many larger models, are examples of dynamic systems. The theory of dynamic systems, or the qualitative theory of ordinary differential equations, dates back at least to Poincare, but it has experienced a surge of interest in the past decade. A dynamic system may consist of any finite number of equations, but, for obvious reasons, the most thoroughly examined systems have been small ones. Low-order models of the atmosphere and other fluid systems have provided pure mathematics with many of their specific examples. The best example is the system created by Lorenz (1963), a three-coefficient simplification of a Galerkin approximation originally developed for studying the Rayleigh-Benard convection, which has been cited many times during the last decade.

Dynamic systems are commonly described in geometrical terms. The dependent variables are treated as coordinates in a multidimensional space. A particular state then becomes a point in space. As the state varies in accordance with the equations, it traces out a trajectory, or orbit. An exact periodic solution becomes a closed curve, while a steady-state solution becomes a fixed point, which is treated as a special type of closed orbit. Two distinct orbits cannot intersect, although they can approach each other asymptotically.

Over the last decades scientists from many disciplines have accepted a new way of looking at complex dynamic systems: *chaos* theory. A chaotic system is one that exhibits sensitive dependence on the initial conditions, i.e. where the approximate present state is insufficient to determine approximate states in the distant future, whether or not the exact present state determines the future. The solutions of equations describing dynamic system are extremely dependent on the initial conditions; near trajectories diverge exponentially, and therefore long-range predictability is lost. The numerical values of any variable at equally widely spaced intervals, e.g., once-a-year observations of temperature at a single whether station, will have the appearance of random numbers.

A primary motivation for studying chaos is this: Given an observation of irregular behavior, is there a simple explanation that can account for it? And if so, how simple? There is a growing consensus that a useful understanding of the physical world will require more than finally uncovering the fundamental laws of physics. Simple systems, which obey simple laws, can nonetheless exhibit exotic and unexpected behavior. Nature is filled with surprises that turn out to be direct consequences of Newton's laws.

Of particular interest are bounded systems, where each orbit eventually enters a fixed region of space and subsequently remains there. For such systems, a basic subset of space is the *attractor*. Given any particular orbit, there are certain points which the orbit will be repeatedly approaching, arbitrarily closely, at regular or irregular intervals; these are the limit or attracting points for that orbit. A point having a greater-than-zero probability of being an attracting point for a randomly selected orbit lies in the attractor. An attractor may consist of a stable fixed point, a stable closed curve, or something more complicated. The orbit passing through any point of the attractor lies entirely in the attractor. Orbits not contained in the attractor are likely to approach it asymptotically; exceptions include unstable fixed points and unstable closed orbits. When the solution of a system is determined by numerical integration, it is usually safe to assume that, after an initial transient segment has been discarded, the solution describes a part of the attractor as closely as a the round-off error will permit.

In a lot of cases we have no a priori knowledge about the equations describing the system, at least not complete knowledge. An experimentalist, confronted with a physical system, measures at regular and discrete intervals of time the value of a state variable (e.g. temperature) and records the time series: $x(t_0)$, $x(t_1)$, $x(t_2)$, ..., with $x(t_i) \in R$ and $t_i = t_0 + i\Delta t$. The question is, how to, by using the only information available, the $x(t)$ time series, reach the same information available from the knowledge of equations describing the dynamics of the system. Recently developed methods have made it possible to obtain some measures of the dimensionality or degrees of freedom which control the observed turbulent flows by analysing of observations (Grassberger and Procaccia, 1983, 1984). But these methods of data analysis also provide an objective test for the quality of the model solutions because they give a quantitative measure for its time evolution.

A review of the theory of the *statistical* properties of dynamic systems can be found in Eckmann and Ruelle (1985), and Abarbanel et al. (1993). We also refer to a great book by Lorenz (1993), which provides a clear account of chaos and fractals, and the application of the science of chaos to meteorology.

In this paper, the time series of temperature, cloudiness and solar radiation are analyzed to investigate the chaotic properties of the local climate, and the possibility of an estimation of the minimum number of independent variables necessary to model the time evolution of the underlying climate system. At the time when we started this study the possibility of the existence of low-dimensional attractors in the weather and climate systems was still widely accepted. Some explanations (e.g., Fraedrich, 1986), reasoning the appearence of low-dimensional attractors, seemed to be acceptable. During the last several years great progress has been done in understanding of reasons accounting for such results. We have followed this progress in our investigation, taking into account the specific purpose of this study and the characteristics of the data used. Special attention has been paid to the investigation of the influence of seasonal data variations on the results. In Section 2 of this paper the methodological background and a short review of the results presented up to day are described. Section 3 presents the application to the Zagreb-Grič Observatory data. A conclusion is given in Section 4.

2. THE METHODOLOGY FOR ESTIMATING THE DIMENSIONS AN ATTRACTOR

2.1. Basic concepts

Consider a system whose state may be described by n variables $X_1, \ldots, X_n$. Let the system be governed by the set of equations

$$\frac{dX_i}{dt} = F_i(X_1, \ldots, X_n) \qquad i=1, \ldots, n, \qquad (1)$$

where time t is the single independent variable, and the functions F_i possess continuous first partial derivatives. Such a system may be studied by means of *phase space* - an n-dimensional Euclidean space Γ whose coordinates are $X_1, \ldots, X_n$. Each *point* in phase space represents a possible instantaneous state of the system. A state varying in accordance with (1) is represented by a moving *particle* in phase space, traveling along a trajectory in phase space. If the system exhibits an attractor, all trajectories initiated from different initial conditions will eventually converge and stay on a submanifold of the total available space-*attractor*.

Deterministically developed systems have attractors characterized by an integer dimension that is equal to the topological dimension of the submanifold in the state space. Trajectories converging on them do not diverge, but stay at a constant distance from each other. When the attracting submanifold is not topological, it is called a 'fractal' set and is characterized by a dimension that is not an integer (Mandelbrot, 1983). The corresponding attractors are called *'strange'* attractors. An important property of these attractors is the divergence of initially nearby trajectories. The dimensionality of an attractor, whether fractal or not, indicates the minimum number of variables present in the evolution of the system. Therefore, the determination of the dimension of an attractor sets a number of constraints that should be satisfied by a model used to predict the evolution of the system.

2.2. Delay-Time Embedding

In the case where an exact mathematical formulation of the system is not available, the state space can be replaced by the phase space which can be produced using a single record of some observable variable from that system. If this system is the atmosphere then the observable variable could be, for instance, temperature, pressure, or geopotential.

The analyses of observations are based on statistical procedures which lead to measures of the dimensionality or degrees of freedom which control the underlying dynamics (Grassberger and Procaccia, 1983, 1984).

The measurement $x(t)$ represents a projection $\pi: R^n \rightarrow R$ from the full state vector $X(t) \in R^n$

The partial differential equations describing the dynamics of the system can be transformed to a set of n time dependent ordinary differential equations. The resulting set of ordinary differential equations (1) defines the time development of n expansion coefficients X_i. Thus the phase space containing the time evolution of the underlying process is spanned by the n different variables X_i, i

$=1, ..., n$ of the dynamic system. Portraits of the time evolution of the system are formed by trajectories in this n-dimensional phase space. The time evolution may be described by a vector $\xi(t)$:

$$\xi(t) = [X_1(t), ..., X_n(t)], \qquad (2)$$

whose components define the position of the trajectory in the phase space. System (1) can be reduced to a single highly nonlinear differential equation for one of the variables X_i (t), say $X(t)$, if all others are eliminated by differentiation. This leads to an n th order differential equation

$$X^{(n)} = F[X, X', ..., X^{(n-1)}], \qquad (3)$$

which is equivalent to a set of n equations describing the time evolution, $X(t)$, plus its n-1 derivatives $X(t), X'(t), ..., X^{(n-1)}(t)$:

$$X(t) = [X(t), X'(t), ..., X^{(n-1)}(t)]. \qquad (4)$$

The initial value problem posed by the single state variable $X(t)$ and its n-1 successive derivatives starts the time evolution (3) or (4) which appears in the same n-dimensional phase space of n coordinates (i.e., the time series plus its $(n$-$1)$ derivatives). This is represented by the vector components (4) which define the position of the trajectory of the time evolution. Adding further derivatives (e.g., $X^{(n)}(t)$) to the vector (4) is superfluous, because it does not produce more independent information.

If a manifold within the original n-dimensional phase space $X_1, ..., X_n$ (1), (2) is considered, it can be described in the new phase space $X, X', X''...$ spanned by a single variable and its derivatives (3), (4). It should be noted that the dimensionality of the new phase space may be smaller than that of the original phase space. This is an embedding theorem which is valid for almost all smooth dynamic systems. The theorem implies that d-dimensional manifolds (described by the dynamic system (1) and evolving in the n-dimensional phase space with coordinates X_i, i=1,..., n) can be embedded into a (m=2D+1)-dimensional space (e.g. spanned by the variable $X(t)$ and its successive derivatives which define the embedded dynamics). Takens and Mane have provided often-cited proofs that this procedure does (almost always) reconstruct the original state space of a dynamic system, as long as the embedding dimension $m > 2D+1$, where D is the fractal dimension of the underlying attractor (Theiler, 1990). As long as $m > D$, the reconstructed set will almost always have the same dimension as the attractor (Eckmann and Ruelle, 1985).

Thus, for deriving the dimension of attractors from single state variables it is sufficient to embed them into an m-dimensional space spanned by the time series and its m-1 derivatives:

$$X(t) = [X(t), X'(t), ..., X^{(m-1)}(t)], \qquad (5)$$

i.e. it is not necessary to know the original phase space (or independent state variables) and its dimension n as long as m is chosen large enough.

Instead of the continuous variable $X(t)$ and its derivatives, $X^{(m-1)}(t)$, a discrete time series $X(t)$ and its shifts by (m-1) time lags (m-1)τ may be considered to identify structures in the time evolution of the single state variable. Packard et al. (1980) devised a delay scheme to reconstruct the state space by embedding the time series into a higher-dimensional space. From the time-delayed values of the scalar time series, a vector $\hat{X} \in R^m$ is created:

$$\hat{X}(t) = \{X(t), X(t-\tau), ..., X[t-(m-1)\tau]\}, \qquad (6)$$

where the time delay τ and the embedding dimension m are parameters of the embedding procedure. Here $\hat{X}(t)$ represents a more comprehensive description of the state of the system at time t than does $x(t)$ and can be thought of as a map of $\pi^{(m)}: R^n \to R^m$ from the full state $X(t)$ to the reconstructed state $\hat{X}(t)$.

The time-delay embedding makes it possible for one to analyze the self-organizing behavior of a complex dynamic system without knowing the full state at any given time. Although almost any delay time τ and embedding dimension $m > D$ will work in principle (with unlimited, infinitely precise data), it is nontrivial to choose the embedding parameters in an optimal way. In general, one wants τ to be not too much less than, and (m-1)τ not too much greater than some characteristic decorrelation time. The linear autocorrelation time is one such characteristic (Theiler, 1990).

2.3. Correlation Dimension

The most popular way to compute an attractor dimension is to use the correlation algorithm, which estimates dimension based on the statistics of pairwise distances. The box counting algorithm (Mandelbrot, 1983) and the correlation algorithm are both in the class of fixed size algorithms because they are based on the scaling of mass with size for fixed-size balls (or grids). Grassberger and Procaccia (1983, 1984) have suggested to calculate a direct arithmetic average of the pointwise mass function, which gives what they call a correlation integral. The procedure consists of counting pairs of points $X(t_i)$, $X(t_j)$ on a geometrical object which are a distance r_{ij} apart:

$$r_{ij} = |X(t_i) - X(t_j)| \tag{7}$$

The number $N(r)$ of such pairs, whose distance is smaller than the prescribed threshold, $r_{ij} < r$, is formally determined by

$$N(r) = \sum_{i,j\neq i}^{N} \Theta\left(r - \left|X_i - X_j\right|\right) \tag{8}$$

where Θ is the Heaviside function with $\Theta(a)=0$ or 1, if $a>0$ or <0; N is the total number of points. This leads to a cumulative distribution function, which is normalized by the total number of N^2 pairs:

$$(1/N^2)\,N(r). \tag{9}$$

It describes how the number of pairs grows with the increasing threshold distance r. For $N\rightarrow\infty$, the growth rate changing with the dimension d is determined by the cumulative distribution function

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} N(r) \sim r^D \tag{10}$$

Now the dimension of a geometrical object, say an attractor in phase space, can be determined by the cumulative frequency distribution $C(r)$ of distances of pairs of points which are situated on the time trajectory of the dynamic system; the slope of the distribution, i.e. $lnC(r)$ versus lnr leads to the dimension

$$D = \frac{ln\ C(r)}{ln\ r} \tag{11}$$

Not only is this a particularly elegant formulation but it has the substantial advantage that the function $C(r)$ is approximated even for r as small as the minimum interpoint distance

Typically, having chosen τ, one performs a dimension analysis for increasing values of m and looks for a plateau in the plot of D versus m. The scaling exponent D is the correlation dimension for that m. In practice, we increase the value of m and check for a *saturation value D* (i.e. a further increase of m does not affect D), which will be an estimation of the correlation dimension of the attractor.

2.5. Application to weather and climate variables

The Grassberger-Procaccia (GP) method, or correlation algorithm for estimating the dimension of attractors has been the most popular method applied to time series of various variables representing the weather and climate system during the last decade. The first to apply this method to climate data were Nicolis and Nicolis (1984). They applied the correlation dimension method to the time series describing the isotope record of deep-sea cores, and their results indicated the existence of the fractal dimension D=3.1. They have suggested that a model involving four variables could already provide a description of the salient features of the climate system. Based on the same method, Fraedrich (1986) evaluated the dimensions of different weather and climate single variable time series to estimate possible low-dimensional attractors. Various weather and climate variables (local pressure, relative sunshine duration, zonal wave amplitudes, and δ^{18}O-record of the Meteor core 13519 in the tropical Atlantic) were analyzed, and for all of them a low dimensionality (between three and five) was found. These results might account for the required four independent variables, necessary to model the corresponding system dynamics.

In the following years, the correlation algorithm was used in a number of papers applied to different meteorological time series. The quantities selected for analysis included (besides the above mentioned ones) upper-level geopotential heights (Essex et al, 1987, Keppenne and Nicolis, 1989), low-level vertical velocity components (Tsonis and Elsner, 1988), rainfall intensities (Sharifi et al, 1990), cyclone positions (Fraedrich and Leslie, 1989), tree-ring widths (Grassberger, 1986). All these findings raised hopes that suitably constructed models with relatively few variables might recapture the dynamics of the weather and climate. Reported values of attractor dimension typically fall between three and eight. Because the atmosphere is so complex, these values have seemed surprisingly low, and doubts as to their appropriateness were expressed even by the originators of the method (Grassberger, 1986, 1987, Procaccia, 1988). Procaccia (1988) stressed the importance of using a sufficient amount of data to provide meaningful calculations of a dimension. Grassberger (1986) tested the results obtained by Nicolis and Nicolis (1984) and his results did not give any hint of a finite-dimensional attractor. This was a warning that spuriously small dimension estimates can be obtained from using too few, too finely sampled and too highly smoothed data. For example, Grassberger (1987) found the results obtained by Fraedich (1986) wrong due to including in the sum (8) pairs whose time separation is less than the correlation time.

Practical implementations of the Grassberger-Procaccia method were stressed by Theiler (1990), Tsonis et al. (1993) and Grassberger et al (1991). Here are the most important facts, found responsible for the appearence of low-dimensional attractors in the above meteorological studies:

1. In the sum over pairs (X_i, X_k), the diagonal terms i=k should of course not be included. Where this seems to have been done (Fraedrich, 1986), the results should be taken with great care.

2. But leaving out the diagonal terms is in general not sufficient. We have to remember that the correlation sum should reflect the clustering of points in phase space due to purely geometrical effects, not due to dynamic correlation. Thus, all pairs should be discarded whose distance in time is not much larger than the correlation time t_{correl}, as pointed out in Theiler (1986). It seems that a number of spuriously small observed attractor dimensions are obsolete due to a neglect of this. A popular alternative to excluding just pairs with $|t_i - t_k| < t_{correl}$ is to take only a subset of the time sequence where the delay $|t_n - t_{n-1}|$ between successive points is $> t_{correl}$. While this does, indeed, eliminate the above problem, it has the drawback of substantial reducing the statistics, and is thus not recommended when the time sequence is extremely long.

3. The time sequence has, of course, to be long enough to sample the attractor reasonably, and the system has to be stationary. This remark is related to the preceding remark: if the attractor is not yet sampled enough, *all* points in the time sequence are dynamicly correlated. The neglect of this obvious requirement is probably the most common reason why many authors found low dimensional chaos in real-world phenomena. The small dimensions found in these analyses are dimensions of individual trajectories, but not of invariant measures.

4. Small measured correlation dimensions can be misleading in systems with strong intermittency, or with two different dynamics acting at different times. Take, for instance, wind speed measurements near the coast with strong winds during the day and weak winds during the night and morning. Most pairs (i, k) with $(v_i \approx v_k)$ will come from night or morning measurements, representing a phase space region which on the global "attractor" scale is essentially a single point. A small measured dimension would then result trivially. Such a measurement was done by Tsonis and Elsner (1988). Though the night data were left out in this analysis, the morning and evening data were still retained. They account most likely for the effect observed.

4. The minimal length of the time series needed for a dimension estimate has been much discussed (Nerenberg and Essex, 1990, Theiler, 1990). It is often difficult to obtain long time sequences since it is hard to keep the system stationary over long times. In some cases it is possible to repeat the measurement. But, unfortunately, this is not the case in the meteorology.

5. The main problem in the optimal choice of the embedding appears to be not so much the optimal embedding but the optimal choice of metric in the embedding space. The most obvious arbitrariness here concerns the choice between the Euclidean and the maximum norm. The maximum norm was first proposed by Takens , and has some advantages over the Euclidean norm (Grassberger et al., 1991).

Lorenz (1991) proposed another explanation for the seemingly low estimates of correlation dimension. It does not say that most real-data studies are meaningless, but that they need to be reinterpreted. Lorenz (1991) showed that if the variable selected for analysis is strongly coupled to only a few variables of the system, the estimated value of D, if N is only moderately large, will be considerably smaller than the dimension as determined by other standard methods, such as the Kaplan-Yorke (1979) conjecture. As suggested by Tsonis and Elsner (1989), the atmosphere may be viewed as a loosely coupled set of lower-dimensional subsystems, and the correlation algorithm, as practiced, attempts to measure the dimension of a subsystem.

3. APPLICATION OF THE CORRELATION ALGORITHM TO THE ZAGREB-GRIČ LONG TERM METEOROLOGICAL MEASUREMENTS

3.1. Data

The method for the estimation of a correlation dimension, as formulated by Grassberger and

Procaccia (1983) has been applied to the long-term meteorological measurements of temperature, cloudiness and solar radiation, carried out at the Zagreb-Grič Observatory over a period of 130 years. The measurements on the Zagreb hill Grič (φ=45.82°, λ=16.03°), northwest Croatia, began in the year 1862 and until December 1995 a series of 1572 monthly mean values was completed. The climatic data represent a homogeneous series, as the location of the Observatory has remained unchanged, and the instruments inside the Observatory have been only slightly and insignificantly changed (Penzar et al., 1992a). Details of the used time series for the period 1862—1865 are presented in Fig. 1. The most obvious characteristic is the seasonal cycle, most expressed in the temperature time series. The monthly mean temperatures have been calculated from the daily means, and the daily means according to the formula: $(t_7+t_{14}+2t_{21})/4$. The data on the quantity of cloudiness are given in tenths, measured visually. Global solar radiation is expressed in monthly amounts.

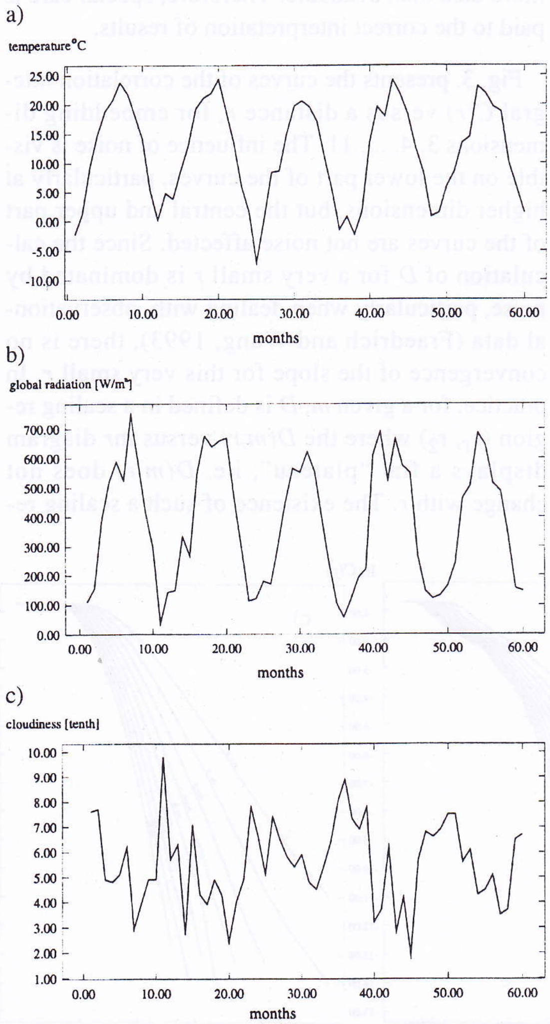


Figure 1. The time series of monthly mean values of temperature (a), cloudiness (b), and global solar radiation (c) (details for the period 1862—1865).

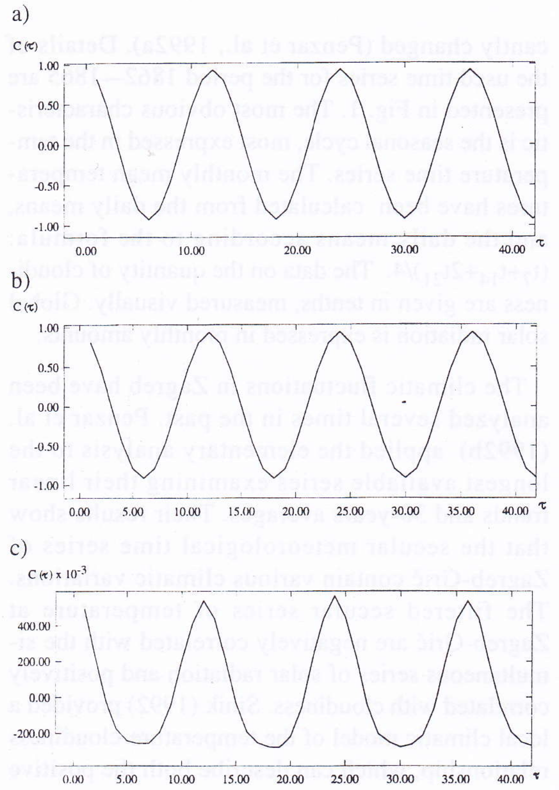
Slika 1. Vremenski nizovi mjesečnih srednjaka temperature (a), naoblake (b), i globalnog Sunčeva zračenja (c) (detalj za razdoblje 1862—1865).

The climatic fluctuations in Zagreb have been analyzed several times in the past. Penzar et al. (1992b) applied the elementary analysis to the longest available series examining their linear trends and 30-years averages. Their results show that the secular meteorological time series of Zagreb-Grič contain various climatic variations. The filtered secular series of temperature at Zagreb-Grič are negatively correlated with the simultaneous series of solar radiation and positively correlated with cloudiness. Šinik (1992) provided a local climatic model of the temperature-cloudiness relationship, which can describe both the positive and negative impacts of cloud radiative forcing upon surface temperature. Theoretically generated temperature series prove the prevailing greenhouse effect of clouds upon recent climatic variations in temperature at the Zagreb-Grič station.

We are specially interested in the comparison of the results obtained in Šinik (1992) with the results of the statistical methods used in this paper. Therefore, the temperature time series is the most important in our discussion.

3.2. Application of the Grassberger-Procaccia algorithm to the Zagreb-Grič time series

The basic problem in the application of the correlation algorithm to the time series of monthly averages is their shortness and high autocorrelation. The autocorrelation function is presented in Fig. 2. It shows a high positive correlation between the months within the same season, and a negative correlation between the summer and winter months. Because of its periodic shape, it is not possible to construct independent coordinates of the points on the attractor. The decorrelation time, used for the choice of the time delay τ, may be defined as the lag time at which the autocorrelation falls below a threshold value. This threshold value is not uniquely defined, and in general it depends on the problem in hand and the assumptions about the data set. In meteorology, this threshold value is commonly



Slika 2. Funkcija autokorelacije vremenskih nizova mjesečnih srednjaka temperature (a), naoblake (b), i globalnog Sunčevog zračenja (c)

Figure 2. The autocorrelation functions of the monthly mean temperature (a), cloudiness (b), and global solar radiation (c) time series.

defined as $1/e$ (Tsonis and Elsner, 1988). Because the autocorrelation function of this particular data set is periodic and it is not possible to select a threshold value, we have chosen $\tau=3$, a value for which the autocorrelation function reaches 0 value for the first time.

In the calculation of sum (8) a constraint $|t_i - t_j| < \tau$ has been used (as described in Section 2.5), i.e. only pairs that are separated by a time interval greater than the decorrelation time are included. Too small number of data N has been showed as most dangerous for the invalidation of the procedure. In the present case we are not able to ensure more data than available. Therefore, special care is paid to the correct interpretation of results.

Fig. 3. presents the curves of the correlation integral $C(r)$ versus a distance r, for embedding dimensions 3, 4, ..., 11. The influence of noise is visible on the lower part of the curves, particularly at higher dimensions, but the central and upper part of the curves are not noise affected. Since the calculation of D for a very small r is dominated by noise, particularly when dealing with observational data (Fraedrich and Wang, 1993), there is no convergence of the slope for this very small r. In practice, for a given m, D is defined in a scaling region (r_1, r_2) where the $D(m,r)$ versus $\ln r$ diagram displays a flat "plateau", i.e. $D(m,r)$ does not change with r. The existence of such a scaling re-

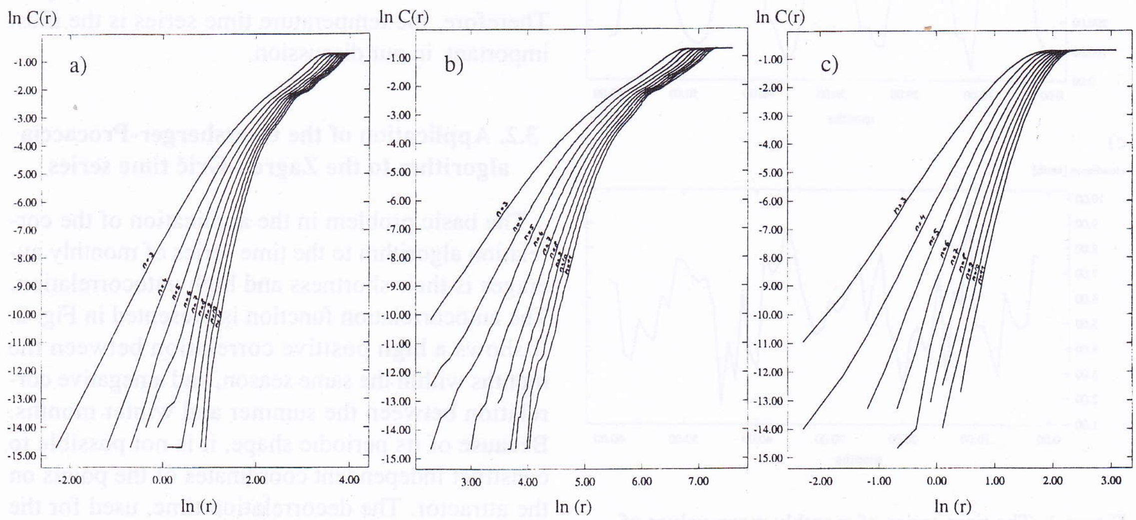


Figure 3. Curves of ln $C(r)$ versus ln r for embedding dimensions 3, 4, ...,11, obtained by applying the Grassberger-Procaccia algorithm to the time series of monthly average temperature, cloudiness, and monthly amounts of solar radiation at Zagreb-Grič.

Slika 3. Krivulje ln $C(r)$ u ovisnosti o ln r za dimenzije faznog prostora 3, 4,...,11, dobivene primjenom Grassberger-Procaccia algoritma na vremenske nizove mjesečnih srednjaka temperature, naoblake, i mjesečnih suma globalnog Sunčevog zračenja na opservatoriju Zagreb-Grič.

gion on the correlation integral curve is the basic constraint for the plausible fit of the straight line to a plot of $lnC(r)$ versus lnr. An erroneous fit of the straight line can give spurious values of D. This fact was discussed in detail by Essex et al. (1987), and emphasized again by Fraedrich and Wang (1993). In most studies reporting about attractors of low dimension there has been no indication of clear-cut scaling regions displayed in the form of a slope versus distance diagram.

Fig. 4a. displays a $D(m,r)$ - ln$r$ diagram, derived from the correlation integral curves for the temperature time series (Fig. 3a.). The scaling region is

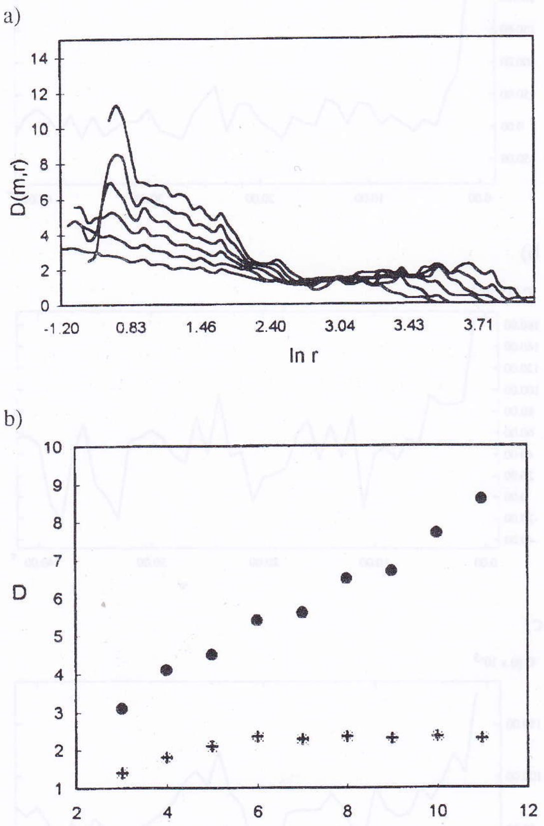


Figure 4. Dimension D as a function of the distance ln r, for different embedding dimensions m (a), and dimension D as a function of the embedding dimensions m (b), for the time series of the monthly average temperature at Zagreb-Grič.

Slika 4. Dimenzija D u ovisnosti o udaljenosti ln r, za različite dimenzije faznog prostora m (a), i dimenzija D u ovisnosti o dimeziji faznog prostora m (b), za vremenski niz mjesečnih srednjaka temperature na opservatoriju Zagreb-Grič.

indicated by the flat plateau, approximately between the lnr values 2.4 - 3.4. In Fig. 3a. this region of lnr values corresponds to the first wavy part of correlation integral $C(r)$ curves. The wavy shape of the correlation integral curves for a temperature and a radiation (less expressed) in their upper part is the influence of an autocorrelation function. We have estimated values of D separately for the first wavy part of the temperature correlation integral curves, and for their lower region. A dependence of the correlation dimension D on different embedding dimensions m for both the upper (wavy part) and lower part of the temperature correlation integral is presented in Fig. 4b. The crosses correspond to the first wavy part of the $C(r)$ curves, i.e. the flat plateau indicated in Fig. 4a.

A steady increase in the correlation dimension D with embedding dimension m (presented in figure by circles) would show that a low-dimensional attractor of the Zagreb local climate does not exist. But, there is a clear indication of the existence of a low-dimensional attractor coming from the upper part of the same temperature correlation integral (presented in the figure by crosses). Its dimension has a value between 2 and 3. The same applies to the solar radiation correlation integral (Fig. 3b).

Here, we refer again to Eckmann and Ruelle (1985) and Lorenz (1991). Theoretically, the statistical dimension of the system can be obtained indifferently from x or any other physical variable of the system. But, as noted by Eckmann and Ruelle, experimental uncertainties change this situation. At the level of accuracy of an experiment, some degrees of freedom may effectively be driven by others and, having small amplitudes, pass unnoticed. Therefore, as shown by Lorenz (1991), different selected variables can yield different estimates of D. To see how this situation can arise, let us consider a *product* dynamic system **I**×**II**. Take an observable $x=x_1+x_2$, depending on the subsystems **I** and **II**, respectively, and let the amplitude r_1 of the signal x_1 be much smaller than that of x_2. In the range r < r_1 we have statistical information on the complete system **I**×**II**, giving an informational dimension α. In the range r >> r_1 we have statistical information only on system **II**, giving an informational dimension α' (Eckmann and Ruelle, 1985).

In the present study, a low correlation dimension between 2 and 3 would indicate the existence of a underlying climatic subspace with 3 strongly coupled variables. In the climate system the air temperature is correlated most strongly with the incoming solar radiation. According to the results obtained by Šinik (1992), cloudiness is the third variable coupled in the local climate subsystem. Šinik evaluated the "cloudy" model of the Zagreb temperature regime (mentioned in Section 3.1.), which describes the prevailing influence of solar radiation, temperature, and cloudiness on the local climate system.

3.3. Filtering out the solar influence

The variation of solar radiation intensity, which is the consequence of the Earth's evolution around the Sun, determines the basic climatic characteristics of the climate of any latitude. The great influence of solar radiation makes it impossible to detect other, less important, influences in the time series of monthly averages. Seasonal variations of the monthly mean temperature (also of solar radiation and cloudiness) are responsible for a strong, periodic autocorrelation. We account the existence of the three-variable climate subsystem to the correlation influence. If we excluded the seasonal regime of the monthly averages, the wavy shape of the correlation integral should disappear, the tem-

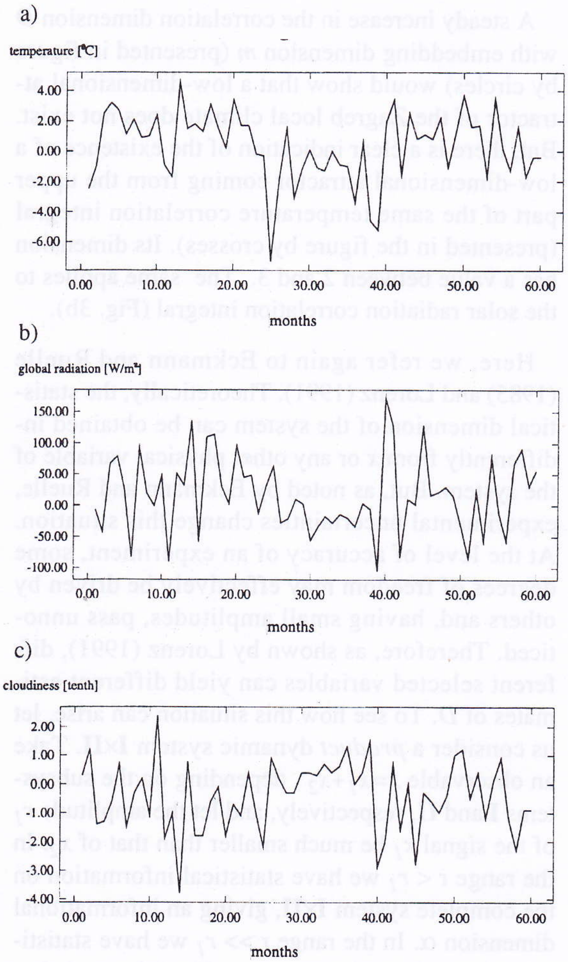


Figure 5. Filtered time series of the monthly mean values of temperature (a), cloudiness (b), and global solar radiation (c) time series (details for the period 1862—1865).

Slika 5. Filtrirani vremenski nizovi mjesečnih srednjaka temperature (a), naoblake (b), i globalnog Sunčeva zračenja (c) (detalj za razdoblje 1862.—1865).

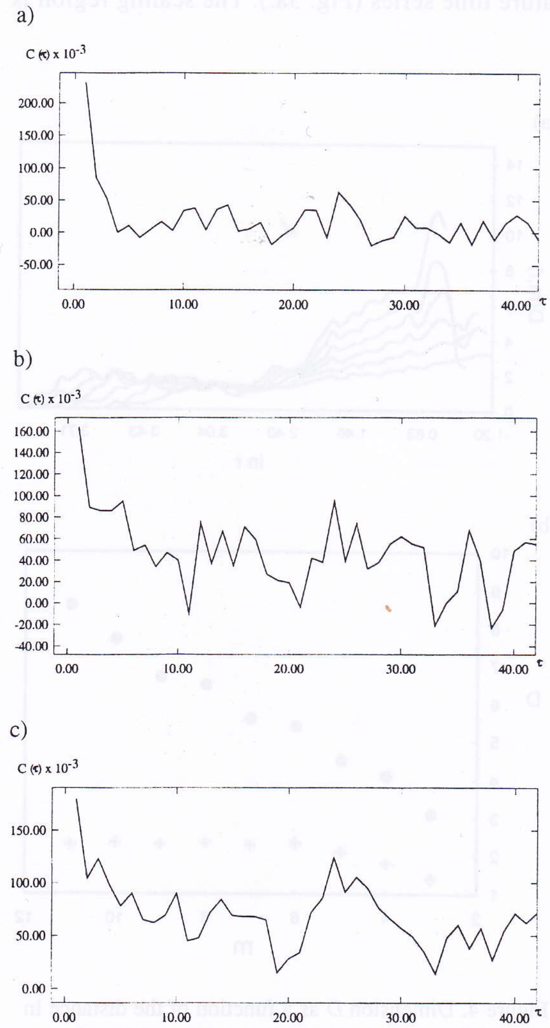


Figure 6. The autocorrelation functions of the filtered monthly mean temperature (a), cloudiness (b), and global solar radiation (c) time series.

Slika 6. Funkcija autokorelacije filtriranih vremenskih nizova mjesečnih srednjaka temperature (a), naoblake (b), i globalnog Sunčeva zračenja (c) .

perature and cloudiness time series would not be any more strongly coupled with solar radiation, and the detection of the low-dimensional subsystem would not be possible.

This fact can be tested by a hypothetical series with the influence of the seasonal cycles filtered out, theoretically, similarly to the procedure applied to the large series of daily temperatures by Zeng et al. (1992). The series were evaluated by subtracting from each monthly mean value its long term average value, computed for the whole period 1862-1995. The series obtained have values oscillating around zero, and are presented in Fig. 5. Autocorrelation in the new series vanishes (Fig. 6), which allows the application of the correlation algorithm with τ=3 to construct linearly independent coordinates of points in the embedding space. The small magnitude of the number of data N now becomes a dominant problem in the application of the Grassberger-Procaccia procedure. The correlation

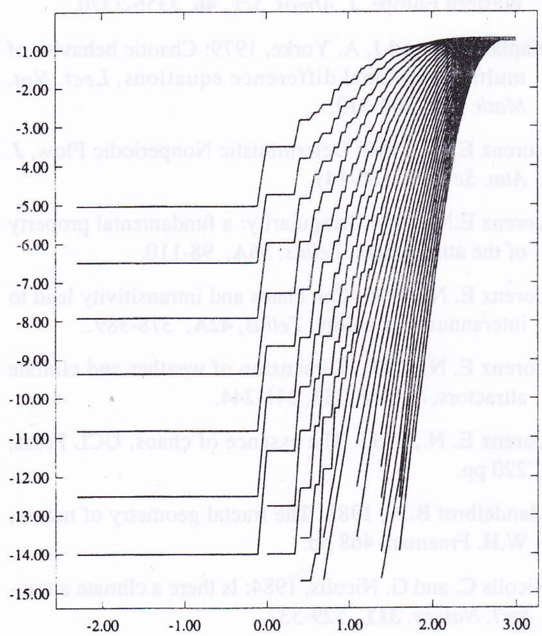


Figure 7. Curves of ln $C(r)$ versus ln r for embedding dimensions 3, 4, ...,11, obtained by applying the Grassberger-Procaccia algorithm to the filtered time series of monthly average temperature, at Zagreb-Grič.

Slika 7. Krivulje ln $C(r)$ u ovisnosti o ln r za dimenzije faznog prostora 3, 4,...,11, dobivene primjenom Grassberger-Procaccia algoritma na filtrirani vremenski niz mjesečnih srednjaka temperature na opservatoriju Zagreb-Grič.

integral ln$C(r)$ versus lnr diagram (presented only for the temperature time series in Fig. 7) proves that the curves $C(r)$ have lost their wavy parts, and that the noise spreads over the greatest part of the curves, making an estimation of D impossible.

4. CONCLUSION

While instability is a fundamental concept in meteorology (Lorenz, 1984), it is not obvious that strange attractors have important applications. Nevertheless, as recommended by the ECMWF Workshop (1988), the evaluation of the dimensionality of atmospheric attractors may have an impact on the number of elements needed in a Monte Carlo ensemble forecast of the extended range.

Following new developments and concepts developed for the evaluation of nonlinear dynamic systems, we have evaluated the chaotic properties of the atmosphere from the time series of temperature, cloudiness and solar radiation measured at the Zagreb-Grič Observatory. The application of correlation algorithm, as developed by Grassberger and Procaccia (1983, 1984), to the time series of the monthly averages of the Zagreb-Grič Observatory did not enable us to obtain the small value of correlation dimension, which would indicate the existence of low-dimensional underlying attractors.

This is in agreement with the results of similar studies elsewhere, which have also shown that, due to the limitations of the GP algorithm for a limited number of data, the most, if not all, of the previous estimates of low-dimensional attractors in the atmosphere are unreliable.

When a dynamic system is an atmospheric model, the points on the attractor represent those states which are compatible with the climate. Although all scales of atmospheric motion are interconnected, it is standard practice in investigating the dynamics of a particular type of system to disregard the presence of systems of larger or smaller scale. Such an approach can yield only partial explanations, but attempts to deal with all scales at once often yield nothing at all (Lorenz, 1984).

The data we deal with in weather and climate are coarse, and small scale processes are absent. These large scale coarse data are likely to obey their own closed dynamics. The low-dimensional subspaces in the climate system may be characterized by low-dimensional attractors which describe large-amplitude influences of just a few governing variables. The existence of autocorrelations in the time series of temperature and solar radiation caused the appearance of wavy deformations on the upper part of the integral correlation curves ln$C(r)$. This has enabled the detection of a low-dimensional attractor, which owes its existence to the seasonal variations of the monthly mean temperature (or radiation).

This result is in agreement with previous studies of local climate, which have detected temperature, cloudiness and solar radiation as a three-variable system controlling the large amplitude of the local climate (Šinik, 1992). When the seasonal cycle is theoretically filtered out of the series, the autocorrelations disappear, as well as the low-dimensional attractor.

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