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Application of Extended Kalman Filter for Road Condition Estimation

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Original scientific paper

High quality estimation of tire-road friction forces has important role in many automotive control systems like anti-lock brake systems (ABS), traction control systems etc. For this purpose an extended Kalman filter augmented with integral term has been employed. A procedure for selecting appropriate integral gain has been proposed. The proposed estimator has been compared to the well-known passivity based state estimator.

Key words: automotive applications, estimation techniques, Kalman filter, anti-lock brake system, software for measurement

1 INTRODUCTION

Recently, a great attention in automotive industry has been given to the active safety of the road vehicles. This includes anti-lock brake systems, traction control, vehicle dynamic systems, etc. Since the forces transferred from the tires to the road primarily determine vehicle motion, good information about them has significant importance in such systems. However, these forces are usually hard to measure, or sensors for them are too complex and expensive for implementation. Therefore, it is necessary to estimate friction forces from easily measurable signals like wheel angular velocity, car velocity, etc. Another problem arises from time variability of the tire-road friction model parameters, specifically of the parameter that represents road surface condition (dry, wet, snow, ice, etc.).

In [1] the passivity based state estimator based on dynamic LuGre friction model [2] has been proposed for the estimation of friction force and road condition parameter. Stability and convergence of this state estimator has been proven but only under assumption that there is no measurement noise. Such state estimator achieved excellent tracking performance. However, measurement noise is always present in real applications and it significantly degrades estimation accuracy.

In this paper, a different approach based on extended Kalman filter (EKF) is proposed. EKF is widely used in practical applications as a state estimator due to its simple design procedure. Additional integral term has been added to the original EKF in order to capture model parameter variations (PI EKF) [3, 4]. The main problem in designing of PI EKF is to obtain optimal integral gain which must be selected in a way that overall system remains stable, and that the dynamic response of the integral action is comparable to the fastest disturbance present. Higher values of integral gain imply faster response but also more noisy estimation and in extreme case instability. An optimization procedure is used for integral gain adaptation with respect to the level of the measurement noise.

2 PROBLEM DESCRIPTION

As mentioned in introduction information about friction forces between wheels and road has important role in car active safety systems. For the purpose of estimation of these forces simplified model of the contact between wheel and road has been used. This model is described by the following set of equations (Figure 1):

$$m\dot{v} = F + F_n \sigma_2 v_r \tag{1}$$

$$J\dot{\omega} = -rF + u_r - \sigma_\omega \omega \tag{2}$$

where:

- m – wheel mass, kg;
- J – moment of inertia of the wheel, kgm^2;
- u_r – torque acting on wheel axis, Nm;
- F – friction force between wheel and road, N;
- σ_ω – coefficient of viscous friction, Nm/s;
- $v_r = r\omega - v$ – relative velocity, m/s.

Friction force F in equations (1) and (2) can be regarded either as a static or a dynamic friction. In literature, numerous friction models can be found, which can be easily adapted to describe tire-road friction effects. Among them the most widely used models are Pacejka static model [5] (in literature al-

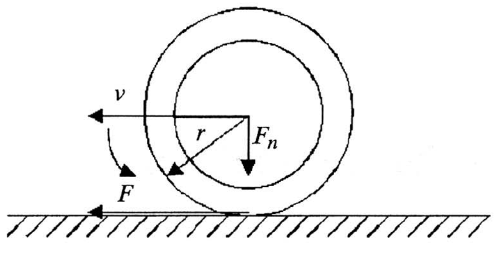

Fig. 1 One wheel friction model

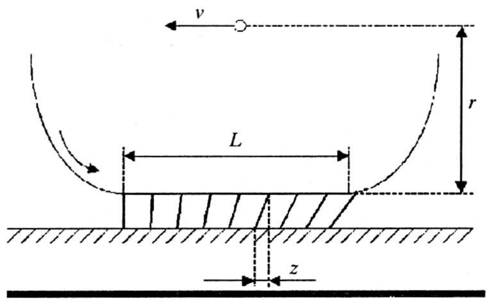

Fig. 2 Tire/road contact patch model

so known as a Pacejka magic formula) and LuGre dynamic model [2] given by the following set of equations:

$$F = F_n(\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r) \tag{3}$$

$$\dot{z} = v_r - \theta \frac{\sigma_0 |v_r|}{g(v_r)} z \tag{4}$$

$$g(v_r) = \mu_c + (\mu_s - \mu_c) e^{-\sqrt{|vr/vs|}} \tag{5}$$

where:

- z – internal friction state (average bristle deflection), m;
- σ_0 – rubber longitudinal lumped stiffnes, 1/m;
- σ_1 – rubber longitudinal lumped damping, s/m;
- σ_2 – viscous relative damping, Ns/m;
- L – contact patch length, m;
- μ_c – normalized Coulomb friction;
- μ_s – normalized static friction;
- v_s – Stribeck relative velocity, m/s;
- v_r – relative velocity $(r\omega - v)$, m/s;
- θ – road condition coefficient.

In order to eliminate a steady-state error that occurs in the LuGre tire model given by (3), (4) and (5), equation (4) is modified as follows [6]:

$$\dot{z} = v_r - \left[\theta \frac{\sigma_0 |v_r|}{g(v_r)} + \frac{\kappa}{L} r|\omega| \right] z, \tag{6}$$

where κ is a lumped model constant (usually between 1.2 and 1.4).

Tire/road friction model given by equations (3), (5) and (6) can be rewritten in matrix form:

$$\begin{bmatrix} \dot{v} \\ \dot{\omega} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{F_n}{m}\left[\left(\sigma_0 z + \sigma_1\left(r\omega - v - \left[\theta\frac{\sigma_0|r\omega - v|}{g(r\omega - v)} + \frac{\kappa}{L} r|\omega|\right] z\right)\right) + \frac{F_n}{m}\sigma_2 (r\omega - v)\right] \\ -\frac{rF_n}{J}\left(\sigma_0 z + \sigma_1\left(r\omega - v - \left[\theta\frac{\sigma_0|r\omega - v|}{g(r\omega - v)} + \frac{\kappa}{L} r|\omega|\right] z\right)\right) - \frac{\sigma_\omega}{J}\omega \\ v_r - \left[\theta\frac{\sigma_0|v_r|}{g(v_r)} + \frac{\kappa}{L} r|\omega|\right] z \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \\ 0 \end{bmatrix} u_r. \tag{7}$$

It is assumed that only wheel angular speed ω is measurable while car speed v, bristle deflection z, as well as road condition parameter θ have to be estimated. Thus measurement equation is:

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} v & \omega & z \end{bmatrix}^T. \tag{8}$$

3 DESIGN OF THE PI EKF ESTIMATOR

We assume that nonlinear system can be described by the following set of difference equations:

$$x(k+1) = f(x(k), u(k), v(k), \theta), \tag{9}$$

$$y(k) = h(x(k), w(k)), \tag{10}$$

where $w(k)$ and $v(k)$ represent the process and measurement noise, respectively. The assumption is that process and measurement noise are Gaussian stochastic variables with zero mean and Q and R variances, respectively. Nonlinear process given by equation (7) and (8) can be transformed to the form given by (9) and (10) applying common discretization procedure.

Kalman filter was originally developed for state estimation of linear systems. In order to apply it to nonlinear systems it is necessary to linearize system using Taylor series expansion around true states of nonlinear system. Since these states are not measurable, linearization should be made around the previous state estimation:

$$\begin{aligned} A_{[i,j]} &= \frac{\partial f_{[i]}}{\partial x_{[j]}}(\hat{x}, u, 0), \quad V_{[i,j]} = \frac{\partial f_{[i]}}{\partial v_{[j]}}(\hat{x}, u, 0), \\ H_{[i,j]} &= \frac{\partial h_{[i]}}{\partial x_{[j]}}(\hat{x}^-, 0), \quad W_{[i,j]} = \frac{\partial h_{[i]}}{\partial w_{[j]}}(\hat{x}^-, 0). \end{aligned} \tag{11}$$

After the linearization has been made, the same procedure can be applied as for linear systems, using the matrices given by equation (11), as follows [7]:

– measurement update

$$K(k) = P^{-}(k)H(k)^T(H(k)P^{-}(k)H(k)^T + \\ + W(k)R(k)W(k)^T)^{-1}, \quad (12)$$

$$\hat{x}(k) = \hat{x}^{-}(k) + K(k)(y(k) - h(\hat{x}^{-}(k), 0)), \quad (13)$$

$$P(k) = (I - K(k)H(k))P^{-}(k); \quad (14)$$

– time update

$$\hat{x}^{-}(k+1) = f(\hat{x}(k), u(k), 0), \quad (15)$$

$$P^{-}(k+1) = A(k)P(k)A(k)^T + V(k)Q(k)V(k)^T, \quad (16)$$

where:

$\hat{x}^{-}(k)$ – *a-priori* state estimates,

$\hat{x}(k)$ – *a-posteriori* state estimates,

$P^{-}(k)$ – *a-priori* covariance matrix,

$P(k)$ – *a-posteriori* covariance matrix,

$K(k)$ – Kalman gain.

PI EKF estimator for nonlinear systems, described by equations (7) and (8), is given by the following set of difference equations [3]:

$$\hat{x}^{-}(k+1) = f(\hat{x}(k), u(k), \hat{\theta}(k)), \quad (17)$$

$$\hat{x}(k+1) = \hat{x}^{-}(k+1) + K(k) \cdot (y(k+1) - \\ - h(\hat{x}^{-}(k+1), \hat{\theta}(k)), \quad (18)$$

$$\hat{\theta}(k+1) = \hat{\theta}(k) + K_I \cdot (y(k+1) - h(\hat{x}^{-}(k+1), \hat{\theta}(k)). \quad (19)$$

Its principle scheme is shown in Figure 3.

From equations (17)–(19) it can be seen that PI EKF differs from ordinary EKF only in an additional term for capturing variations in model parameter θ. Parameter θ is updated according to the difference between measured output signal and its estimation, while the dynamic of the parameter estimation is determined by the integral gain K_I. Large values of integral gain imply faster transient response but also can cause overshoots and oscillations around true value of the parameter particularly when considerable measurement noise is present. So the key problem in designing of PI EKF estimator is selection of the appropriate integral gain K_I, in order to achieve satisfactory behavior with respect to the level of the measurement noise. Optimal values of integral gain K_I were obtained by an offline optimization procedure based on the values of measurement noise variances. For noise variances in specified range, optimal value of K_I was obtained by minimizing of the ISE (Integral Square Error) criterion, using Matlab function *fmin*.

$$J = \int_0^T (\theta - \hat{\theta})^2 \mathrm{d}t \rightarrow \min. \quad (20)$$

4 SIMULATION RESULTS

The simulations were performed using the tire/road model given by equations (7) and (8). For the purpose of comparison passivity based state estimator has been designed for the same model according to the procedure given in [1]. The model parameters used in simulations are given in Table 1.

Table 1 **Model parameters used in simulations**

Parameter	Value	Parameter	Value
σ_0	40, N/m	r	0.25, m
σ_1	49487, Ns/m	m	5, kg
σ_2	0.0018, Ns/m	J	0.2344, kgm^2
μ_C	0.5	F_n	14, kgm^2/s^2
μ_s	0.9	κ	1.1
v_s	12.5, m/s	L	0.25, m

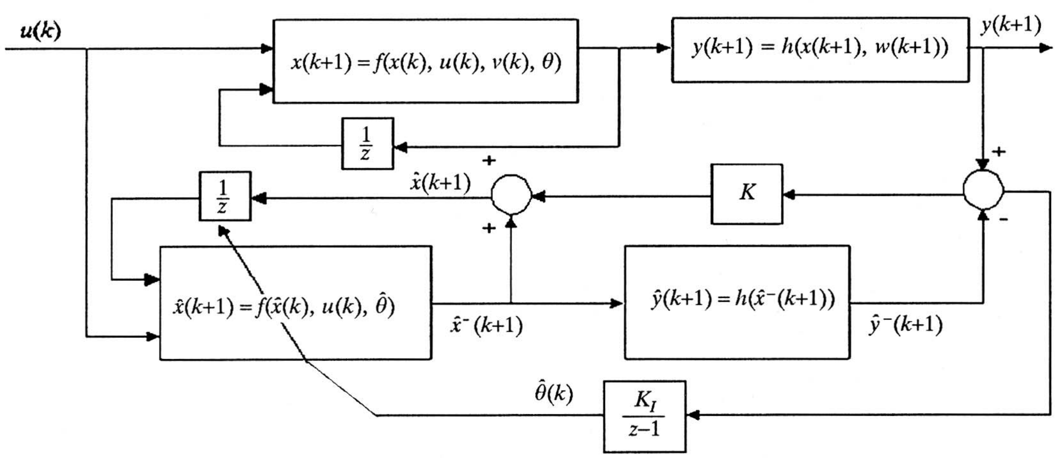

Fig. 3 PI EKF state estimator

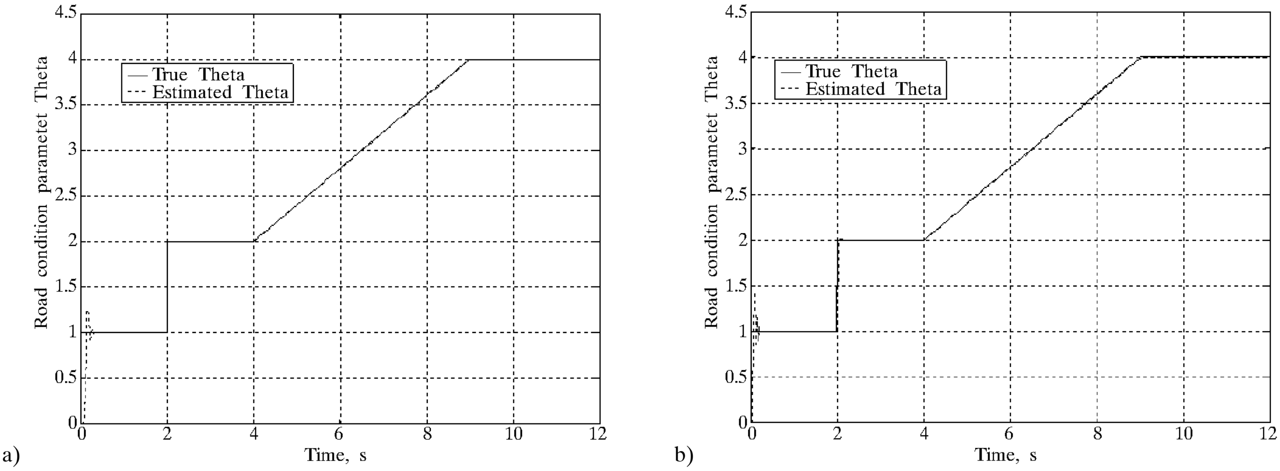

Fig. 4 Road condition parameter θ estimation without measurement noise: a) passivity based estimator, b) PI EKF estimator

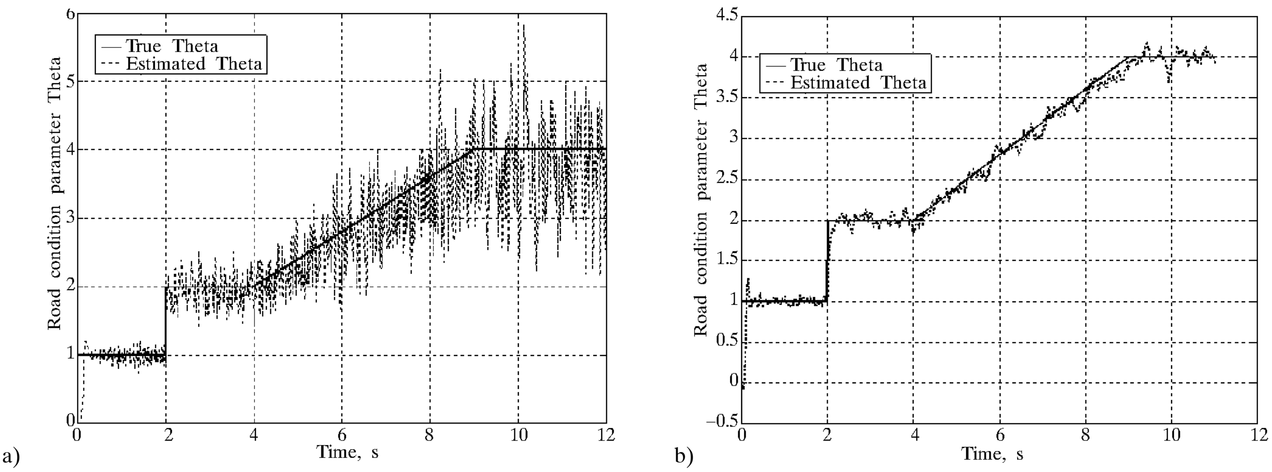

Fig. 5 Road condition parameter θ estimation with measurement noise of variance R = 0.1: a) passivity based estimator, b) PI Kalman estimator

The main goal of this work is design of the state estimator that is able to estimate variations in road conditions. Validation of the PI EKF estimator was done through simulations by varying the parameter θ in such a way that four different conditions of the road were simulated. Parameter θ in all experiments was changed as follows: for the first 2 seconds it represents dry asphalt, then sudden change in $t=2$ s to wet asphalt, and then smooth change from wet asphalt to snow in time period $t=4$ s to $t=9$ s, and after that it was constant representing snow conditions.

Two different simulations were carried out, one without measurement noise (Figure 4) and another with measurement noise of variance $R=0.1$ (Figure 5), for both passivity based and PI EKF state estimator. Optimal integral gain for PI EKF was picked-up from the curve shown in Figure 6, which was obtained by optimization procedure according to equation (20). From Figure 5 it can be seen that

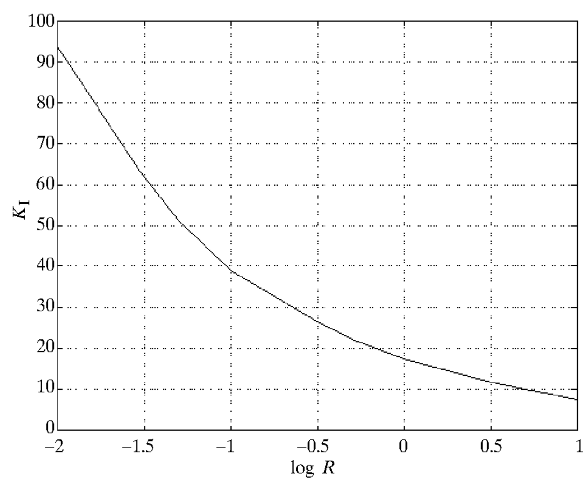

Fig. 6 Optimal integral gain K_I obtained by minimization of the ISE criterion (20)

PI EKF state estimator provides significantly less noisy estimation of the road condition parameter θ with satisfactory fast transient response, while for noiseless simulations (Figure 4) both state estimators produce similar results.

In order to check robustness of PI EKF estimator two additional simulations with the same noise variance ($R=0.1$) and integral gain K_I optimized for ten times greater and ten time smaller value of noise variance than one used in first simulations, have been performed. Figures 7, 8 and 9 show estimation of the state variables and parameter θ, obtained with PI EKF in the presence of measurement noise of variance $R=0.1$ and with K_I optimized for noise variances $R=0.1$, $R=1$ and $R=0.01$, respectively.

From Figure 7 it can be seen that estimation of the car velocity and wheel angular velocity are very accurate, while estimation of the bristle deflection z is noisy but satisfactory accurate and fast.

When integral gain K_I was optimized for ten times greater noise variance than its actual value was (Figure 8), the road condition coefficient estimation is less noisy but with somewhat slower response to sudden changes of road condition coefficient. Estimation of the bristle deflection is also less noisy. To the contrary for K_I optimized for ten times smaller noise variance than its actual value was (Figure 9), estimation of the road condition parameter as well as estimation of the bristle deflection are more noisy and less accurate. These behaviors were expected since road condition coefficient value was corrected according to the estimation error of the output signal.

Thus, it can be concluded that it is better to put somewhat smaller value of integral gain K_I than the optimal value, in order to make this estimator robust to noise variance level variations. Such an approach is closely related to the H_∞ estimator design i.e. the worst-case design.

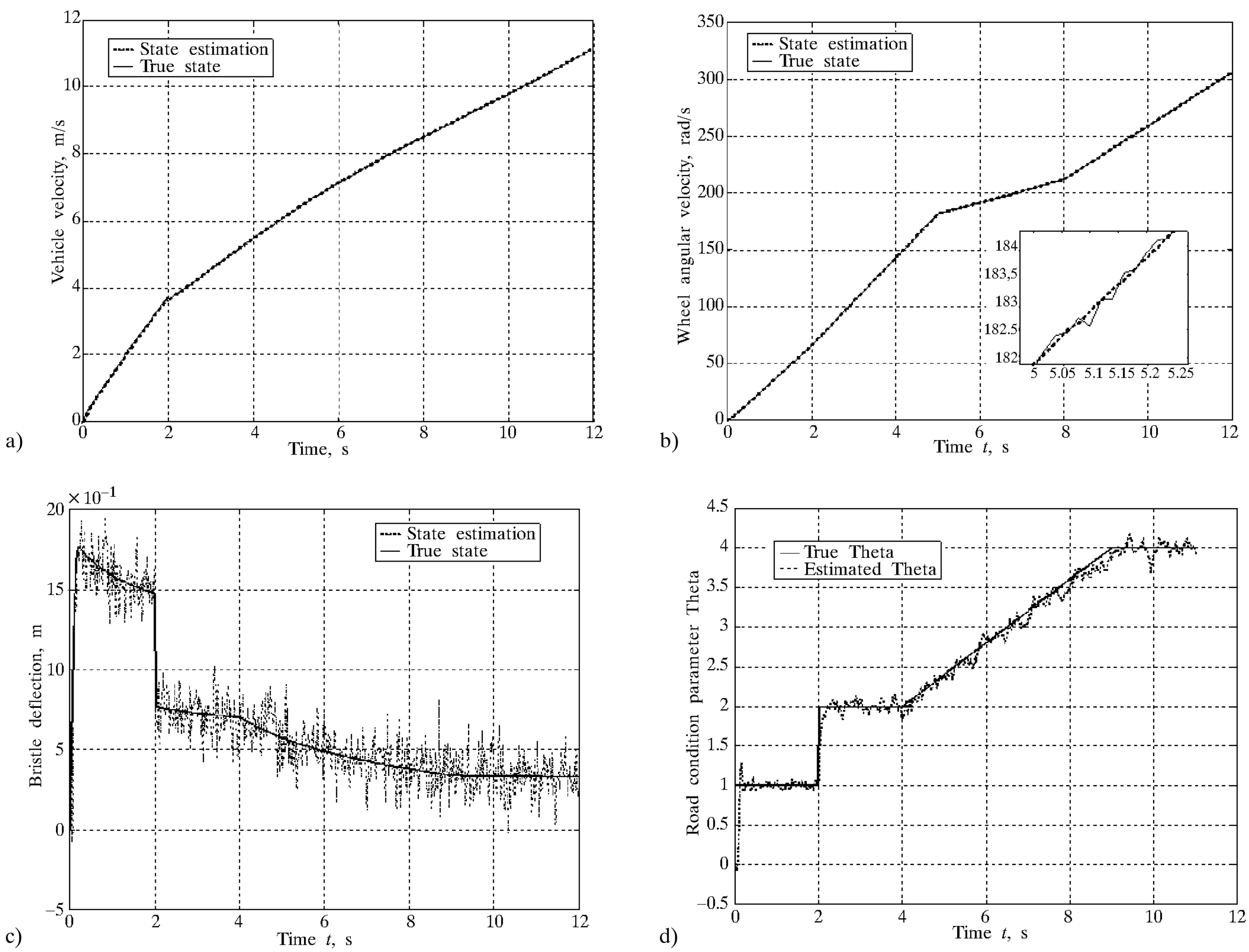

Fig. 7 Estimation of the a) vehicle velocity v, b) wheel angular velocity ω, c) bristle deflection and d) parameter θ obtained in the presence of measurement noise of variance R = 0.1 and integral gain optimized for noise variance R = 0.1 (optimal value of K_I)

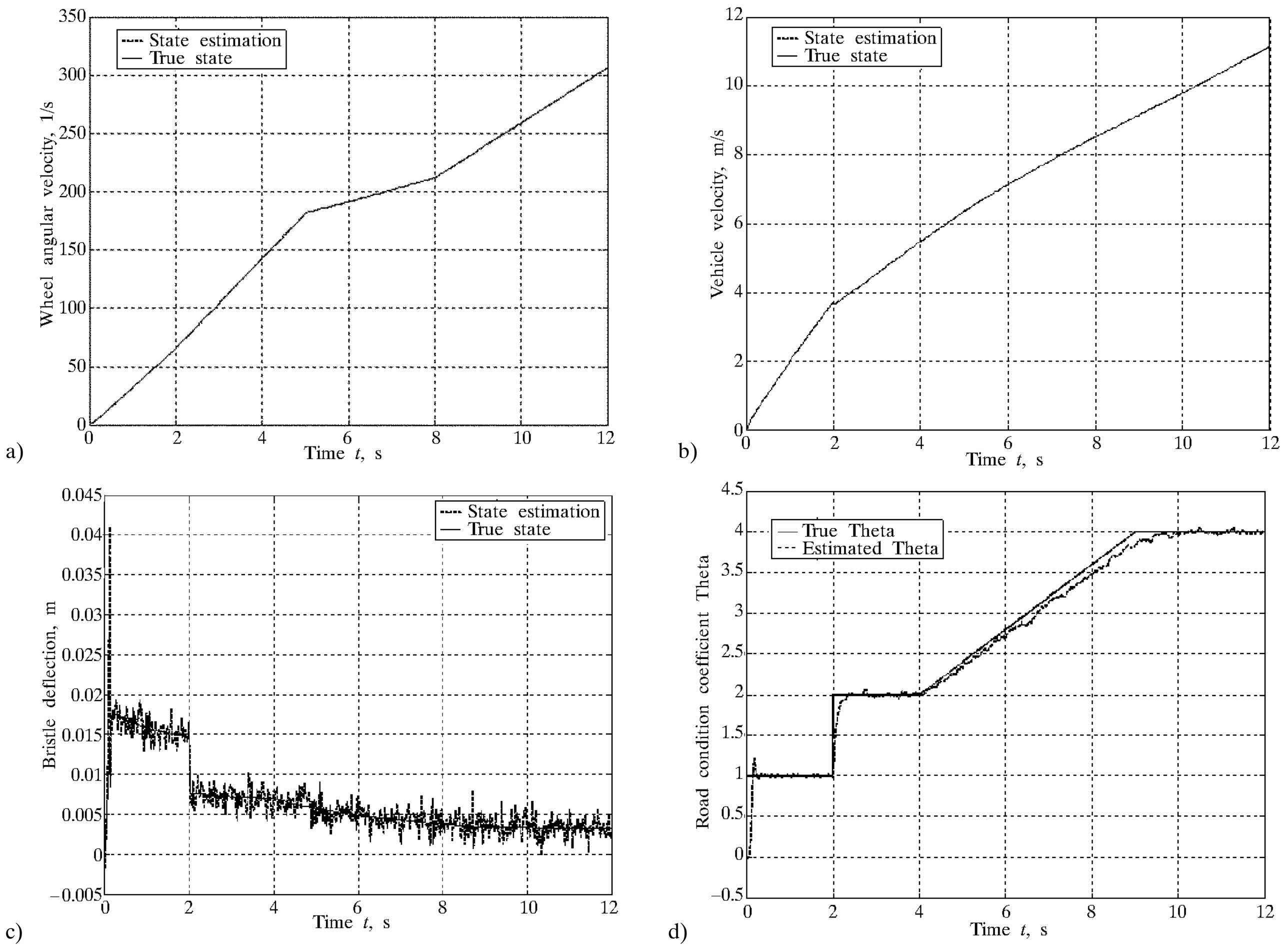

Fig. 8 Estimation of the a) vehicle velocity v, b) wheel angular velocity ω, c) bristle deflection z and d) parameter θ obtained in the presence of measurement noise with variance R = 0.1 and integral gain optimized for noise variance R = 1

5 CONCLUSION

In this paper extended Kalman state estimator augmented with integral term for road condition has been proposed. Comparison with passivity based state estimator has also been made. Simulation results show that PI Kalman estimator produces less noisy estimation of the road condition parameter with satisfactory fast response. It also produces very accurate and fast estimation of the state variables. Simulations without measurement noise suggest similar performances of both state estimators. Simulations with integral gain optimized for ten times greater and ten times smaller measurement noise variances than its actual value, pointed out a robustness of the proposed estimator for smaller noise variances, while greater noise variances degraded estimator performance.

6 ACKNOWLEDGEMENT

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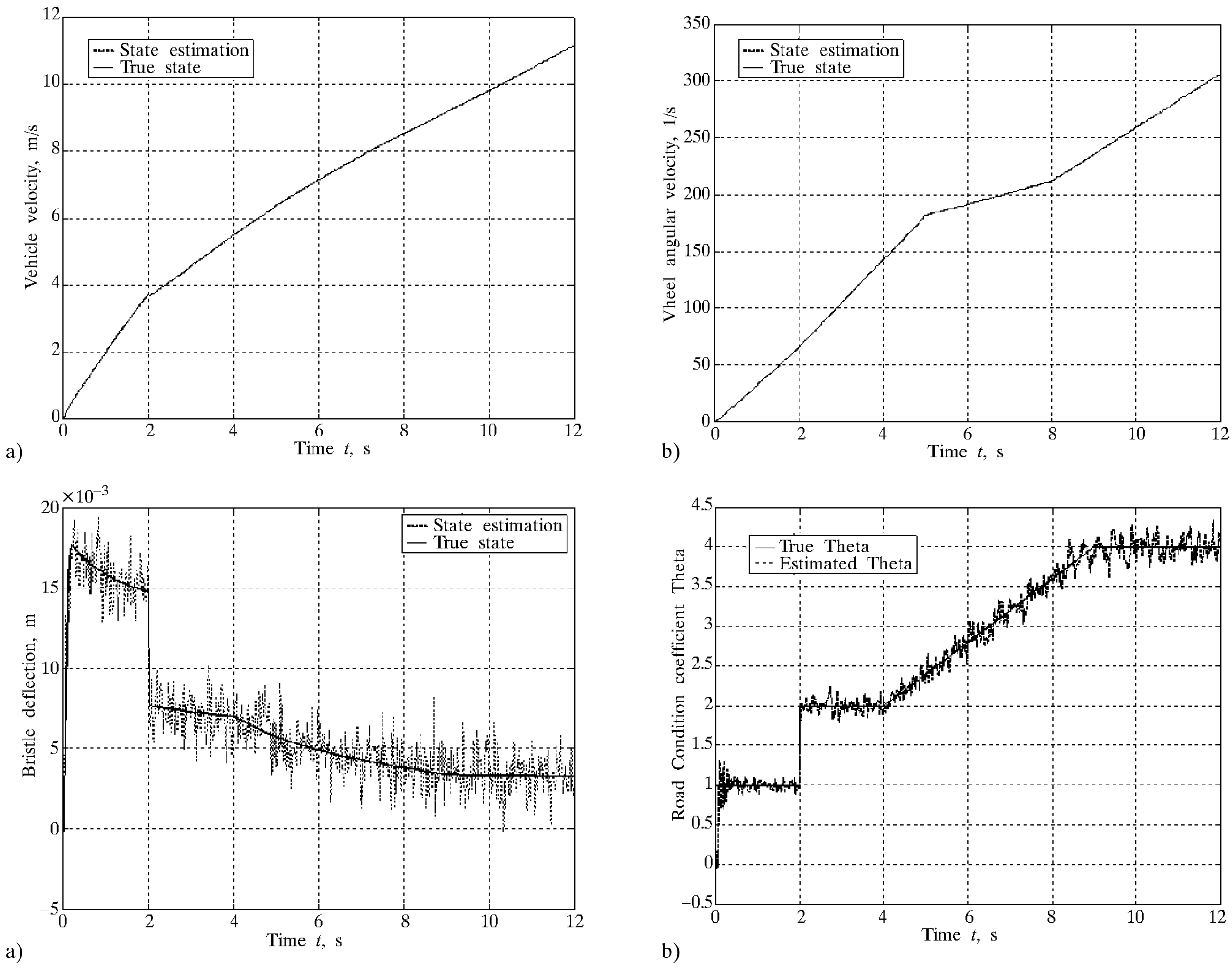

Fig. 9 Estimation of the a) vehicle velocity v, b) wheel angular velocity ω, c) bristle deflection z and d) parameter θ obtained in the presence of measurement noise with variance $R = 0.1$ and integral gain optimized for noise variance $R = 0.01$

Primjena proširenog Kalmanovog filtra za estimaciju stanja na podlozi. Kvalitetna estimacija sile trenja između automobilskog kotača i podloge ima veliki značaj u sustavima sigurnosti kod suvremenih automobila kao što su sustav kontrole proklizavanja (ABS), sustav upravljanja vučnom silom (TC) i sl. U svrhu estimacije sile trenja u ovom radu se koristi prošireni Kalmanov filtar kojem je dodan integralni član. Predložen je postupak odabira optimalnog pojačanja integralnog člana. Predloženi estimator je uspoređen s estimatorom stanja na podlozi zasnovanim na teoriji pasivnosti.

Ključne riječi: automobilske primjene, tehnike estimacije, Kalmanov filtar, sustav kontrole proklizavanja, mjerni softver

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