Odgojne znanosti
Vol. 12, br. 2, 2010, str. 413-426
Izvorni znanstveni članak

DUAL INTERPRETATIONS IN PRIMARY EDUCATION MATHEMATICS AS ASPECT OF CRITICAL THINKING OF STUDENTS

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Summary - *The effectiveness of learning increases significantly if the teacher teaches pupils different ways of thinking and reasoning. Critical thinking reaches high levels when during the teaching process the pupils are trained to acquire knowledge, seeing it from different points of view. In our study the dual treatment was applied in dual interpretation of concepts, dual solutions of exercises, dual formulations and solutions of mathematical problems. The presented experience gives primary education teachers a new alternative for teaching mathematics, meanwhile the pupils are given the opportunity to develop critical thinking through encouragement to see things in dual point of view. This experience experimented in mathematics, can be extended to other subjects, returning to work method for teachers. It is possible that dual treatment becomes a method of reasoning and critical thinking for pupils starting from the lower classes of primary education.*

Key words*: teaching and learning in primary education; critical thinking; dual treatment of concepts, exercises and problems.*

Introduction

Dualism is a philosophical doctrine which states that the world and all phenomena have their source in two initial independent bases denying one another. In general things exist in relation to their opposites. Even today in stating that there can not mean "victory" without meaning "loss", "good" without "bad" etc. While in the field of mathematics, the meaning of "big" can not exist without the meaning of "small", "wide" without the meaning of "tall", "maximum" without "minimum" etc. According to Gao (1999, p. xi) mathematics has its roots in dualism

A lot of researchers have studied mathematics' dual nature (see, for example, Artstein-Avidan & Milman, 2007; Gao, 1999; Gray & Tall, 1994; Jasterbov, 2001; Jasterbov, Menshikova & Epifanova, 2006). The researchers Erdnyev and Erdnyev (1996), Gray and Tall (1994), Jasterbov (2001), Jasterbov, Menshikova and Epifanova (2006), emphasize that the dual nature of mathematics should be taken into account also during its study as school subject. This was one of the reasons that the object of our study was the possibility to include the duality in the process of learning mathematics.

The second reason of the study was to give our experience for the possibility of including elements of the dual treatment in primary education mathematics. Different authors give their experience on the possibility of involvement of dual treatments in higher mathematics classes, so the authors Jasterbov, Menshikova, Epifanova (2006), etc give their experience for dual treatments in the high school mathematics, meanwhile the authors Gao (1999), Jasterbov (2001), Kërënxhi (2009a), etc for university mathematics. One such experience we haven't found in the primary education, that's why this has been the field of our studies since 2007, (see Gjoci & Kërënxhi, 2009a, 2009b).

Thirdly, we think that the experience of our study, for the possibility of incorporating elements of dual treatment in primary education mathematics, will be another experience for the teachers to the education of students of active learning and critical thinking. "... the teaching activity is linked with the creation of the pupil's personality, meaning the formation of their personal abilities, and as a result the students have some active positions" (Kërënxhi & Gjoci, 2008, p. 174). According to Temple, Crawfrord, Saul, Mathews and Makinster (2006) when we say that the student actively fit we understand that this student is curious, asks questions, discovers the latest news and uses his knowledge to solve problems. However, to achieve critical thinking, we should attach the practice of students to see issues from different perspectives, the ability to explore nuances and implications of ideas and maintaining a position based on reason to the range mentioned above. If students are taught to learn actively, seeing issues under dual point of view they will be prepared for the day when school will end and when will have to continue to learn in life itself.

After several years of study and experimentation on some classes of primary education, we present our experience on dual treatments in primary education mathematics and recommend it to teachers, hoping that our ideas will help teachers in their noble job and students' help at the same time encourage the development of critical thinking.

The aim and objectives of the research

Research in the field of learning has shown the importance for student's compilation, implementation and analysis of ideas (Wittrock, 1991). To reach

the stage of critical thinking, active learning practices should add that the issues looked at different points of view (Temple, et. al., 2006). These perspectives on the deal through dual interpretation of concepts, exercises and problems in mathematics aim to improve the teaching in primary education.

"The aims to merge the interaction in the lesson need the development of new model in the process of teaching, as well as in the process of learning …" (Gjoci, 2008, p. 121). The aim of this study was that through collaboration with primary teachers from some schools to choose a whole number of concepts, exercises and mathematical problems which allowed dual treatment. The selected models to be used in classes of mathematics to develop to the pupils an important aspect of critical thinking and exactly the ability to see issues in dual point of view.

The study would be achieved if we fulfilled the following objectives:

- Awareness of primary school teachers for the possibility of dual treatment of some concepts and problems of the mathematics text.
- Training of teachers for the selection in mathematics text of those concepts, exercises and problems that accepted dual treatment.
- Training of teachers for dual interpretation of concepts, dual formulation of problems and treatment of solutions in dual point of view, where the possibility existed.
- Awareness of the teachers so that the above abilities they could gradually pass to their pupils.
- Education of desire to the teachers to teach pupils so that they could see things in dual point of view.

Research hypothesis

After selection and study of literature of the problem that was interesting to us we set the main hypothesis: development of critical thinking through dual treatment, teachers and students should be taught to see the given knowledge (meanings, exercises, problems) in dual point of view.

Starting from this hypothesis below are the problems that will constitute our study:

1. The level selection of dual point of view inclusion in primary education mathematics.

2. The selection of the most effective methods that will enable this point of view in primary education.

3. The monitoring of the effectiveness in incorporating of dual point of view in primary education mathematics.

Sample, methods and the research instruments

The study was conducted during 2009-2010, in 10 classes of primary education of 9-years schools in the city of Elbasan. This city is located in central area of Albania. To the study were subservient 10 teachers and 342 pupils. Thus were distributed in 10 classes: in two year 1-s, in two year 2-s, in two year 3-s, in two year 4-s and in two year 5-s. These teachers had prior experience in their work with experimental classes because during their practice of education they had been working for experimentation of teaching techniques. During this study teachers were trained by taking some theoretical knowledge and guidance for the inclusion of dual treatment in primary education mathematics.

This study uses descriptive and analytical methods. During this study combined methods were used. The research instruments were questionnaires and conversations with teachers. The results have emerged from surveys, observations, tests, data of which are been deferred to the statistical process.

Research dynamics

The study passed in the following steps:

1) The possibility of dual treatment of mathematics in primary education

"… thinking in general is directed to what is unknown. By nature man is curious, keen to learn about everything. So we thought that exactly this trend, which can be encouraged during learning of mathematics, must combine to the ability to learn the theory and mathematics exercises, viewing it in dual point of view" (Kërënxhi, 2009a, p.102). This is possible thanks to the dual nature of mathematics. "There are great possibilities to make dual content of "common ties" simple, a method of thinking of pupils, by the lower classes, recalling that from the small rises glorious" (Erdnyev & Erdnyev, 1996, p.169).

But how could the dual point of view be included in teaching of mathematics? Which dual treatments would be the focus of our study? Did the dual point of view exist in exercises? Could the problems accept dual formulation? At what class in primary education can dual treatment start? These issues and others constitute our questions in this step of the study.

2) The study of curricula, the conversations with teachers, the creation of the work group

We emphasize from the beginning that the strategies that will be recommended from us will not change the content of curricula, but only the way it will be treated to the pupils. So the main question that we wanted to ask ourselves during the study of curricular program was: how much do the cur-

ricula fulfill the conditions about the possibility of the fulfillment of pupil's dual thinking?

By the carefully studying of texts: Mathematics 1, (Dedej et. al., 2009a), Mathematics 2, (Dedej et. al., 2009b), Mathematics 3, (Dedej et. al., 2009c), Mathematics 4, (Dedej et. al., 2009d), Mathematics 5, (Star et. al., 2009) we concluded that they contained examples, for which teachers could make interpretations and dual formulations. But meanwhile they had also some disadvantages of introducing such models, which they had been included in the text would have given the better opportunities for dual interpretations for teachers and pupils.

After that we worked with some primary school teachers and invited them one by one to give us their opinions. With each of them we talked about our ideas about the possibility of involvement of dual treatments in mathematics learning. We concluded that teachers appreciated our idea and didn't hesitate to give us their opinions. They expressed willingness to cooperate with us as our ideas were valued as interesting. Then we created the working group with 10 teachers. All of them were experienced teachers and some of them had been previously involved in research and experimentation. We divide this group into two groups of five teachers each. The first group was formed by teachers who gave lessons from first grade to fifth, who would work under the dual treatment models. These teachers were given training in the theory of duality in mathematics. The group would decide in which classes the dual treatments would be done and how it would be done. Teachers of the second group, who also taught from first grade to fifth, would continue normal classes and would serve us as a comparative group to evaluate the achieved results by the first group.

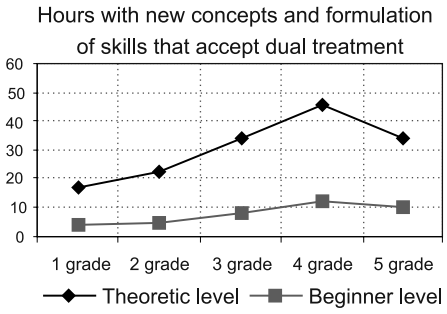
3) The results of questionnaire, first conclusions

We surveyed just teachers of the first group, so teachers who would experiment with us. We surveyed them to tell us in details the topics of lesson where they thought to implement the duality, the concepts that could be interpreted from them in dual point of view, the models of exercises in which they could make dual interpretation and problems that could be treated by them in dual form. We also asked them to explain to us in details how they thought they would go about these treatments. After we got out from questionnaires the data of teachers about the number of hours where they thought to apply dual treatments, we compared them with the indicators determined by us on the grounds of what we thought the outcome would be from the involvement of dual treatment in lessons. We had got out our indicators from the study that we had done to the curricula. These indicators in the table below we call "theoretical".

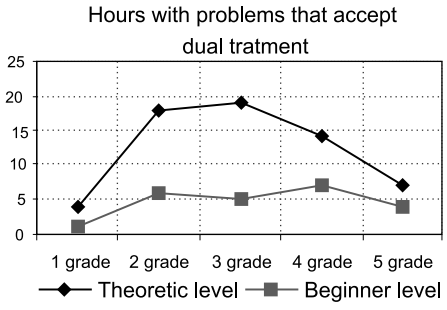
Table 1. The distribution of data about the possibility of dual treatment in teaching hours at the beginning of experiment

Grades	The kind of information	Topic hours of study			Hours with new concepts and formation of skills			Hours that have problems		
		The number of hours	From them accept dual treatment		The number of hours	From them accept dual treatment		The total number of hours	From them accept dual treatment	
			Nr	%		Nr	%		Nr	%
1 Grade	Theoretic	175	77	44	34	17	50	19	4	21
1 Grade	From teachers	175	13	7	34	4	12	19	1	5
2 Grade	Theoretic	175	75	43	46	22	48	60	18	30
2 Grade	From teachers	175	9	5	46	5	11	60	6	10
3 Grade	Theoretic	175	68	39	60	34	57	53	19	36
3 Grade	From teachers	175	11	6	60	8	13	53	5	9
4 Grade	Theoretic	140	62	44	87	46	53	32	14	44
4 Grade	From teachers	140	22	16	87	12	14	32	7	22
5 Grade	Theoretic	140	48	34	81	34	42	29	7	24
5 Grade	From teachers	140	12	9	81	10	12	29	4	14

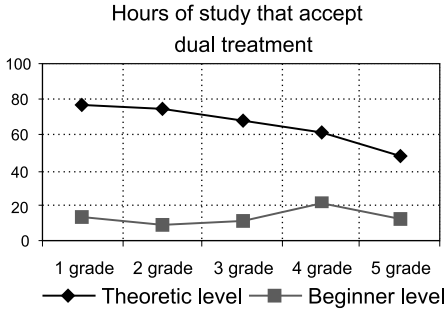
Graph 1



Graph 2



Graph 3



The graphs distribution of data about the possibility of dual treatment (at the beginning of the experiment)

To understand better the difference between data, we showed them in graphics 1, 2, 3. In fact graphics should include only points, but we have joined the points with lines so that these graphics can be clearer.

The comparison of the data showed that there existed differences between our study and the opinions of teachers about the possibility of including dual elements in mathematics classes. This meant two things: our indicators were high, or teachers had difficulty in defining those concepts, exercises and problems that accepted dual treatment. Therefore, our efforts should aim that the indicators of teachers would approach as much as possible to our indicators.

4) Some findings from lesson observations

The surveys were done in two main directions with the aim:

- To understand how teachers operate generally in mathematics teaching. What were the difficulties they faced during dual treatments? How did they encourage questions and class discussions? How did they evaluate the pupil's learning?
- To evaluate the level of pupil understanding and learning habits. How did pupils understand what they learned or heard? Were they original in thoughts or did they generally tend to repeat ideas learnt from the lesson? Could they implement them in other topics, in new situations? Did they work well with friends? Could they carry out discussions and listen to each-other? Could they contribute to their ideas to understand new meanings?

Despite discussions and surveys that we had done with the teachers, our observations helped us to understand better the deficiencies of teachers' understanding of the duality. Although we were already trying to clarify the teachers about the meanings and dual interpretations in teaching of mathematics, our explanations were not complete because we concluded some misunderstandings. For example:

1. From the observation in the 3rd grade the lesson 5.3, of the teacher I.T. we learnt that the teacher understood the dual treatment of the problem only as the solution in two different ways. From the studying of the curriculum it was a well known fact that the program of the third class required: "To solve problems it is recommended the use of schemes and solutions in different ways. Pupils should be formed to see things trained from different points of view" (Dedej et. al., 2000, p. 6). We discussed with the teachers about the requirements of the program and came to the conclusion that since the problem accepted nor only dual solution but also dual formulation, we could now include its dual formulation in learning.

2. From the observation in the fourth grade, the lesson 5.8, of the teacher B.K. we noted that she accepted as dual the opposite problem of given prob-

lem. Without underestimating the importance of the opposite problem we discussed with the teacher the fact that the opposite problem of a given problem is not the dual problem, because in the dual problem, the solution doesn't change but only the formulation does. So when we say dual problem we consider a new formulation of the problem that can be given by the teacher or the pupil. The dual problem we can give in connection with the initial given problem and in connection with its opposite problem (for more see Gjoci & Këršnxhi, 2009a, p. 457-461).

5) Results and discussions

We felt especially the positive results in observations made during recent months of the school year. During the observed lessons we noticed a significant improvement in teaching, optimism and security. But most importantly, these elements were seen not only in teachers but also in students. Let us stop in two examples that we can mention.

The teacher of the third grade I.T. at the topic 16.10, after the pupils solved two problems, asked them the questions:

a) Which of the problems can be solved in another way? If you can, show the other solution.

b) Which of the problems can we formulate in a different way? If you can, show the other formulation.

The teacher of the fourth grade B.K., who in the first observation accepted as dual problem of a given problem its opposite problem, in the lesson 7.8, managed, in collaboration with pupils, to create an example that included the problem, its opposite problem and two their dual problems. Pupils gave the solutions of these problems.

Another aspect that we were interested in learning was how much did pupils benefit from all this work, so how much did this dual way of learning mathematics influence them. To obtain answers on this issue we prepared several tests with exercises which we would do to the parallel classes and we would compare the answers. During the testing that we did to the two year 5-s parallel classes in the lesson 14.8 we gave pupils a problem with these questions: Can you build a scheme for solving problem? Give a different formulation for this problem to be resolved with the same operations and build scheme for its solution. After that each class was divided into 4 groups and we waited pupils' written responses. The pupils of experimental class gave the correct answer, meanwhile the pupils of non-experimental class gave correct answers only for one model.

In first two parallel classes we gave the exercises: with the numbers 2, 7, 5 write as much equalities and inequalities as you can. Do you see any connection between the inequalities $5>2$ and $2<5$? By comparing the answers given

by the pupils of two classes it was noticed that the pupils of experimental class gave more complete answers than the pupils of non-experimental class. So for example, to the pupils of non-experimental class we find equalities such as 2+5=7 (99%), 5+2=7 (72%), but generally the equalities 7=2+5 (43%), 7=5+2 (21%) are missing. These equalities a note we find without deficiencies to the pupils of experimental class (87%). For given inequalities pupils of experimental class answer almost all that is the same inequality but read in two different directions (83%), while from the pupils of non experimental class answer partly only 24% of pupils.

6) Analysis of findings

At the end of the school year the teachers of first group were surveyed again to give us information for teaching topics where they had implemented duality, for hours of learning concepts and skills training where teachers or pupils have made dual interpretation, as well as the hours with problems treated in the dual form. These data of last phase of experimentation was compared with previous data are shown in Table 2.

From the table 2 we can take some results that are collected in the table 3.

Table 2. The continuance of data of dual treatments at the end of the experimentation

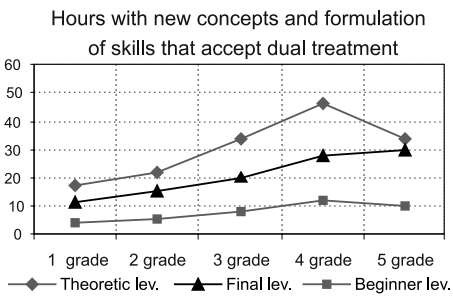
		Topics of teaching hours			Hours with new concepts and formation of skills			Hours that has problems		
		The total number of hours	From them accept dual treatment		The total number of hours	From them accept dual treatment		The total number of hours	From them accept dual treatment	
Grades	The kind of information		Nr	%		Nr	%		Nr	%
1 grade	Theoretic	175	77	44	34	17	50	19	4	21
	In the first phase	175	13	7	34	4	12	19	1	5
	In conclusion	175	54	31	34	11	32	19	4	21
2 grade	Theoretic	175	75	43	46	22	48	60	18	30
	In the first phase	175	9	5	46	5	11	60	6	10
	In conclusion	175	56	32	46	15	33	60	15	25
3 grade	Theoretic	175	68	39	60	34	57	53	19	36
	In the first phase	175	11	6	60	8	13	53	5	9
	In conclusion	175	42	24	60	20	33	53	15	28
4 grade	Theoretic	140	62	44	87	46	53	32	14	44
	In the first phase	140	22	16	87	12	14	32	7	22
	In conclusion	140	46	33	87	28	35	32	12	37
5 grade	Theoretic	140	48	34	81	34	42	29	7	24
	In the first phase	140	12	9	81	10	12	29	4	14
	In conclusion	140	40	29	81	30	37	29	6	21

Table 3. The continuance of dual treatment at the beginning and at the end of experiment

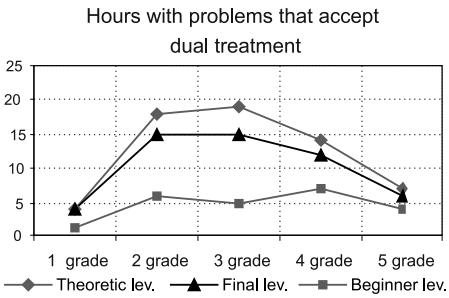
Kind of dual treatment	% of hours where was thought to apply dual treatment	% of hours where was applied dual treatment
In topics of lessons	8%	30%
In new concepts and formation of skills	13%	34%
In problems	12%	27%

To understand better values of the table 2, we build the graphs 4, 5, 6. The black line indicates the level of dual treatments implemented realized by teachers and pupils, the gray line indicates the level above her duality theory for the possibility of dual treatment, while the line below shows the planned level from the teachers in the first phase of the experiment for the possibility of the inclusion of dual treatment.

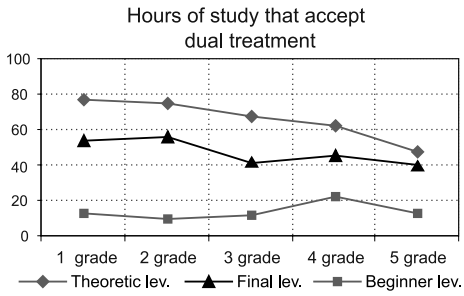
Graph 4



Graph 5



Graph 6



The graphic distribution of data about the dual treatment (at the beginning and in the end of the experiment)

Conclusion

When we started we were studying the opinion and this opinion was later reinforced also by experimental teachers. The inclusion of dual point of view in primary education mathematics will be considered completed only if we together with teachers will manage to include it in the process of teaching mathematics of primary education three categories the dual interpretation of mathematical concepts; the dual treatment of mathematical exercises; the dual formulation of mathematical problems and their solution's way. These will be completed through interpretation, treatments, dual formulations that primary education teachers will be doing during teaching, concepts, exercises and problems classes that were dual issue property or as we can say otherwise as the resulting duality. Meanwhile we can say that the study has reached its aim.

Considering teaching of mathematics as a process and not simply as a product (Gray & Tall, 1994) for us, the way of formation of mathematical knowledge took a very important role. In this context it was important that knowledge is formed and not be broadcast. In relation to the existing program, the program made some experimental changes in the ways of the elaboration of didactic material and performance, creating another space for critical thinking.

During the experimental program the logic formation of pupils received special attention. That's why priority was given to the cultivation and the development of the properties of mathematical objects and the laws, the creation of pupils' habits forming in claims and argument to make simple deductions. Considering that the "intuition" can't function in absence of knowledge and experience (Gjoci, 2007, p. 10), in the experimental program it was insisted also in the preparation of intuition to see things in dual point of view. We applied the saying: what you hear, you forget; what you see, you remember; what you do, you understand.

The proposed model of dual treatment executes a full treatment of the concept with a high degree of processing it through various interpretations of equivalent. In some cases, the need that base concepts had for a further processing was linked to the possibility that the program was created. In order to create a more complete imagination about the concepts that enabled the dual treatment among pupils, they were treated in such a context whenever they reappeared. We think that the dual treatment has great value in forming the pupils (in terms of giving concepts in low grades of primary education) than the persistence in giving precise definitions.

Problem's solutions has forming values, therefore in this issue we insisted on the view that learning is a process more than a result (Gray & Tall, 1994). But in the meantime the clear difference between the types of exercises aimed at the formation of skill and those aimed at logical development was maintained.

We support the idea that the activity of the teacher affects the process of teaching, which from its side affects the process of learning. "The dual nature of mathematical knowledge should be taken into account also during learning mathematics at school" (Gjoci & Kërënxhi, 2009a, p. 458). Critical thinking under the dual perspective for the teacher, among other means: to seeing the texts and themes of learning from dual points of view, creating dual ideas which are argued good and to illustrate them with examples of dual issues, giving arguments and to illustrate with concrete examples of dual treatments, before seeking treatment from pupils for a dual specific problem should have completely clear what response to expect from the pupil, making pupils think about the meaning of what they learned in classes and understand the change that brings a new dual situation; making pupils apply what they learnt during the solution of dual problems.

Indicators of experimental classes on the acquisition of concepts and themes that carried dual treatment, compared to non experimental classes were encouraging. The interest of different levels for teaching mathematics marked a step forward. As for the experimental program it covered, where practicable dual treatment of concepts and knowledge was included without causing increasing of teaching load. This does not mean that there were no problems during the experimentation. On the contrary at this stage they were numerous and complex issues. In the future a lot of attention needs to be paid to the details of the chosen route. To achieve better results, the started work requires deeper studies scientific as well as pedagogical ones.

Our current study shows that the inclusion of dual treatments in the process of learning of mathematics in primary education, significantly affects the rate of increasing the understanding of knowledge of this subject, leading students to the developing of critical thinking. We think that there are opportunities that the dual study is coming to be a method of thinking and reasoning to students starting from the lower classes of primary education. Active teaching methods that have basic dual treatments can be applied by the primary education teacher in other subjects. When studying mathematics, biology, literature and any other subject, students can be encouraged to think and learn actively and apply and extend beyond what they have learned through duality point of view.

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DVOJNA INTERPRETACIJA U NASTAVI MATEMATIKE PRIMARNOG OBRAZOVANJA KAO ASPEKT KRITIČKOG MIŠLJENJA UČENIKA

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Sažetak - *Uspješnost učenja osjetno se poboljšava ako učitelj uči učenike različitim načinima mišljenja i razmišljanja. Kritičko mišljenje doseže visoku razinu kada se tijekom procesa učenja učenike poučava stjecanje znanja promatranjem iz različitih perspektiva. U istraživanju su primijenjena dvojna interpretacija koncepata, dvojna rješenja zadataka, dvojne formulacije i rješenja matematičkih zadataka. Prikazana iskustva nude učiteljima primarnog obrazovanja alternativnu poučavanju matematike, dok se učenicima pruža prilika za razvoj kritičkog mišljenja poticanjem promatranja iz više perspektiva. Ovaj eksperiment proveden je u nastavi matematike ali je primjenjiv u drugim predmetima kao metoda rada učitelja. Moguće je da dvojni pristup postane metoda razvoja kritičkog mišljenja za učenike počevši od nižih razreda osnovne škole.*

Ključne riječi: *učenje i poučavanje u primarnom obrazovanju, kritičko mišljenje, dvojni pristup konceptima, vježbe i problemi*