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Stability analysis and periodic optimal control problems for impulsive periodic system

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Abstract. In this paper, stability analysis and periodic optimal control problems for a class of an impulsive periodic system in Banach spaces is considered. Combining exponential stabilizability and impulsive evolution operators, we present the existence of periodic optimal controls without assuming the semigroup is compact or exponentially stable. Finally, an example is given for demonstration.

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1. Introduction

It is well-known that periodic motion is a very important and special phenomenon not only in natural science but also in social science such as climate, food supplement, insecticide population, sustainable development. Periodic systems with applications on finite dimensional spaces have been extensively studied. Impulsive periodic systems on finite dimensional spaces are considered and some important results are obtained (see [5, 9]).

Since the end of the last century, many authors including us have paied great attention to impulsive systems on infinite dimensional spaces. Particulary, Ahmed et al. investigated optimal control problems for systems governed by an impulsive system (see [2, 3, 4]). We also gave a series of results for the first order (second order) semilinear impulsive systems, integral-differential impulsive system, strongly nonlinear impulsive systems and their optimal control problems (see for instance [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24] and the references therein).

Recently, we have begun to investigate the impulsive periodic system on infinite dimensional spaces. The suitable impulsive evolution operator corresponding to a homogenous impulsive periodic system was introduced and its properties (boundedness, periodicity, compactness and exponential stability) were given. Some results including the existence of the periodic PC-mild solutions and the alternative theorem, criteria of Massera type, asymptotical stability and robustness by perturbation for a linear impulsive periodic system were established. For a semilinear impulsive periodic system, some fixed point theorems such as the Banach fixed point theorem,

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Horn's fixed point theorem and Leary-Schauder fixed point theorem were applied to obtain the existence of the periodic PC-mild solutions, respectively. In order to do it, we had to construct a suitable *Poincaré* operator, discuss its properties and derive some generalized Gronwall inequalities for the estimate of the PC-mild solutions (see [10, 11, 12]).

To our knowledge, optimal control problems arising in systems governed by an impulsive periodic system on infinite dimensional spaces have not been extensively investigated. Herein, we study the following optimal control problem (P1):

$$\text{Minimize } L(x,u)\text{: } L(x,u)=\int_0^{T_0}(g(x(t))+h(u(t)))\,dt, \tag{1}$$

subject to the impulsive periodic boundary problem

$$\begin{cases} \dot{x}(t)=Ax(t)+Bu(t), & t\in[0,T_0]\backslash\widetilde{D},\\ \Delta x(\tau_k)=C_kx(\tau_k), & k=1,2,\cdots,\delta,\\ x(0)=x(T_0), & u\in L^2(0,T_0;U), \end{cases} \tag{2}$$

on real Hilbert spaces H and U, where $\Delta x(\tau_k)=x(\tau_k^+)-x(\tau_k^-)$, $\tau_{k+\delta}=\tau_k+T_0$, $\widetilde{D}=\{\tau_1,\tau_2,\cdots,\tau_\delta\}\subset(0,T_0)$, T_0 is a fixed positive number and $\delta\in\mathbb{N}$ denoted the number of impulsive points between 0 and T_0. The operator A is the infinitesimal generator of a C_0-semigroup $\{T(t), t\geq 0\}$ on H. Operator $B\in \pounds_b(U,H)$ and $C_{k+\delta}=C_k\in H$. x denotes the T_0-periodic PC-mild solution of system (2) corresponding to the control $u\in L^2([0,T_0];U)$. The functions g: $H\to\mathbb{R}$ and h: $U\to\bar{\mathbb{R}}=]-\infty,+\infty]$. In this paper, using exponential stabilizability and discussing the impulsive evolution operators, without compactness and exponential stability of a semigroup generated by original principle operator A, we present the existence of periodic optimal controls for problem (P1) under weaker assumptions.

In order to study the impulsive periodic system on infinite dimensional spaces, we constructed the impulsive periodic evolution operator $\{S(\cdot,\cdot)\}$ associated with A and $\{C_k;\tau_k\}_{k=1}^{\infty}$ which is very important in the sequel. It can be seen from the discussion on the linear impulsive periodic system, the invertibility of $[I-S(T_0,0)]$ is the key of the existence of a PC-mild solution of system (2) (see [13]). For the invertibility of $[I-S(T_0,0)]$, compactness or exponential stability of $\{T(t),t\geq 0\}$ generated by A is needed. By virtue of the concept of exponential stabilizability, which is introduced by Barbu in [6] to weaken the assumptions on the existence of periodic PC-mild solutions, we replace the problem (P1) by problem (P2):

$$\text{Minimize } \widetilde{L}(x,v)\text{: } \widetilde{L}(x,v)=\int_0^{T_0}(g(x(t))+h(v(t)+Fx(t)))\,dt, \tag{3}$$

subject to

$$\begin{cases} \dot{x}(t)=A_Fx(t)+Bv(t), & t\in[0,T_0]\backslash\widetilde{D},\\ \Delta x(\tau_k)=C_kx(\tau_k), & k=1,2,\cdots,\delta,\\ x(0)=x(T_0), & v\in L^2(0,T_0;U), \end{cases} \tag{4}$$

where $A_F=A+BF$, $F\in\pounds_b(H,U)$ such that A_F generates an exponentially stable semigroup. Discussing the impulsive evolution operator $\{S_F(\cdot,\cdot)\}$ associated with operator A_F and $\{C_k;\tau_k\}_{k=1}^{\infty}$ and giving some sufficient conditions for invertibility of $[I-S_F(T_0,0)]$, we prove that every periodic PC-mild solution of (2) is a periodic PC-mild solution of (4) with $v=u-Fx$ and vice versa. Therefore, the equivalence between problem (P1) and problem (P2) is shown. Utilizing some techniques of semigroup theory and functional analysis, we present the existence of periodic optimal controls for problem (P2), which implies the existence of solutions for problem (P1).

The main result of this paper is the existence of optimal control for problem (P1) (given by Theorem 5). However, the novelty of this paper over other related results in literature lies in the fact that the invertibility of $[I-S(T_0,0)]$ is replaced by a weaker condition. In addition, some sufficient conditions for invertibility of $[I-S_F(T_0,0)]$ are presented. This will extend the scope of application such as the case of a wave equation with impulse.

This paper is organized as follows. In Section 2, impulsive evolution operator $\{S_F(\cdot,\cdot)\}$ and its exponential stability are studied and some sufficient conditions guaranteeing $[I-S_F(T_0,0)]^{-1}\in \pounds_b(H)$ are given. Section 3 is devoted to the equivalence of (P1) and (P2). In Section 4, the existence of optimal periodic arcs for (P2) is presented. Hence, the existence of optimal controls for (P1) is obtained. At last, two examples are given to demonstrate the applicability of our results.

2. Impulsive evolution operator and exponential stability

Let H be a Hilbert space. $\pounds(H)$ denotes the space of linear operators in H; $\pounds_b(H)$ denotes the space of bounded linear operators in H. $\pounds_b(H)$ is the Hilbert space with the usual supremum norm. Define $\widetilde{D}=\{\tau_1,\cdots,\tau_\delta\}\subset[0,T_0]$. We introduce $PC([0,T_0];H)\equiv\{x:[0,T_0]\to H\mid x$ is continuous at $t\in[0,T_0]\backslash\widetilde{D}$, x is continuous from left and has right-hand limits at $t\in\widetilde{D}\}$ and $PC^1([0,T_0];H)\equiv\{x\in PC([0,T_0];H)\mid \dot{x}\in PC([0,T_0];H\}$. Set

$$\|x\|_{PC}=\max\left\{\sup_{t\in[0,T_0]}\|x(t+0)\|,\ \sup_{t\in[0,T_0]}\|x(t-0)\|\right\},\ \|x\|_{PC^1}=\|x\|_{PC}+\|\dot{x}\|_{PC}.$$

Then $(PC([0,T_0];H),\|\cdot\|_{PC})$ $((PC^1([0,T_0];H),\|\cdot\|_{PC^1}))$ is a Hilbert space.

The basic hypotheses are the following:

Assumption [H1]:

[H1.1]: A is the infinitesimal generator of a C_0-semigroup $\{T(t),t\geq 0\}$ in H.

[H1.2]: There exists δ such that $\tau_{k+\delta}=\tau_k+T_0$.

[H1.3]: For each $k\in\mathbb{Z}_0^+$, $C_k\in\pounds_b(X)$ and $C_{k+\delta}=C_k$.

Under the assumption [H1], we consider the Cauchy problem

$$\begin{cases}\dot{x}\,(t)=Ax(t), & t\in[0,T_0]\backslash\widetilde{D},\\ \Delta x(\tau_k)=C_kx(\tau_k), & k=1,2,\cdots,\delta,\\ x(0)=x_0.\end{cases}\tag{5}$$

For Cauchy problem (5), if $x_0\in D(A)$ and $D(A)$ is an invariant subspace of C_k, using Theorem 5.2.2, ([1], p.144), step by step, one can verify that Cauchy problem (5) has a unique classical solution $x \in PC^1([0,T_0];H)$ represented by $x(t) = S(t,0)x_0$ where

$$S(\cdot,\cdot)\colon\ \Delta = \{(t,\theta) \in [0,T_0] \times [0,T_0] \mid 0 \le \theta \le t \le T_0\} \longrightarrow \pounds(H)$$

given by

$$S(t,\theta) = \begin{cases} T(t-\theta), & \tau_{k-1} \le \theta \le t \le \tau_k, \\ T(t-\tau_k^+)(I+C_k)T(\tau_k-\theta), & d\tau_{k-1} \le \theta < \tau_k < t \le \tau_{k+1}, \\ T(t-\tau_k^+)\Big[\prod_{\theta<\tau_j<t}(I+C_j)T(\tau_j-\tau_{j-1}^+)\Big](I+C_i)T(\tau_i-\theta), & \\ \qquad\qquad \tau_{i-1} \le \theta < \tau_i \le \cdots < \tau_k < t \le \tau_{k+1}. & \end{cases} \tag{6}$$

Definition 1. *The operator $\{S(t,\theta),(t,\theta)\in\Delta\}$ given by (6) is called the impulsive evolution operator associated with operator A and $\{C_k;\tau_k\}_{k=1}^{\infty}$.*

Lemma 1. *Impulsive evolution operator $\{S(t,\theta),(t,\theta)\in\Delta\}$ has the following properties:*

(1) For $0 \le \theta \le t \le T_0$, there exists a constant $M_{T_0} > 0$ such that

$$\sup_{0\le\theta\le t\le T_0} \|S(t,\theta)\| \le M_{T_0}.$$

(2) For $0 \le \theta < r < t \le T_0$, $r \ne \tau_k$, $S(t,\theta) = S(t,r)S(r,\theta)$.
(3) For $0 \le \theta \le t \le T_0$ and $N \in \mathbb{Z}_0^+$, $S(t+NT_0,\theta+NT_0) = S(t,\theta)$.
(4) For $0 \le t \le T_0$ and $N \in \mathbb{Z}_0^+$, $S(NT_0+t,0) = S(t,0)\,[S(T_0,0)]^N$.
(5) For $0 \le \theta < t$, there exits $M \ge 1$, $\omega \in \mathbb{R}$ such that

$$\|S(t,\theta)\| \le M\exp\Big\{\omega(t-\theta) + \sum_{\theta\le\tau_k<t} \ln(M\|I+C_k\|)\Big\}.$$

It is well known that if there exist constants $M_0 \ge 0$ and $\omega_0 > 0$ such that the semigroup $\{T(t), t\ge 0\}$ generated by A satisfies $\|T(t)\| \le M_0 e^{-\omega_0 t}$, $t>0$, the semigroup $\{T(t), t \ge 0\}$ is said to be exponentially stable. In general, a semigroup may not be exponentially stable.

Let $B \in \pounds_b(U,H)$. A pair (A,B) is said to be exponentially stabilizable, if there exists $F \in \pounds_b(H,U)$ such that $A_F = A + BF$ generates an exponentially stable C_0-semigroup $\{T_F(t), t\ge 0\}$, that is, there exist $K_F \ge 0$ and $\nu_F > 0$ such that

$$\|T_F(t)\| \le K_F e^{-\nu_F t},\ t>0.$$

Remark 1. *By Theorem 5.4 of [1], the following inequality*

$$\int_0^\infty \|T_F(t)\xi\|^p dt < \infty,\ \ \textit{for every } \xi \in X,\ t>0,\ 1\le p<\infty \tag{7}$$

implies that the exponential stability of $\{T_F(t), t\ge 0\}$.

Impulsive evolution operator $S(\cdot,\cdot)$ plays an important role in the sequel. Here, we need to discuss the exponential stability and exponential stabilizability of the impulsive evolution operator.

Definition 2. *$\{S(t,\theta), t \geq \theta \geq 0\}$ is called exponential stability if there exist $K \geq 0$ and $\nu > 0$ such that*

$$\|S(t,\theta)\| \leq Ke^{-\nu(t-\theta)},\ t > \theta \geq 0.$$

Consider the Cauchy problem

$$\begin{cases} \dot{x}(t) = (A+BF)x(t),\ t \in [0,T_0]\backslash\widetilde{D}, \\ \Delta x(\tau_k) = C_k x(\tau_k), \quad k = 1,2,\cdots,\delta, \\ x(0) = x_0. \end{cases} \tag{8}$$

The impulsive evolution operator $S_F(\cdot,\cdot)$: $\Delta = \{(t,\theta) \in [0,T_0] \times [0,T_0] \mid 0 \leq \theta \leq t \leq T_0\} \longrightarrow \pounds(H)$ associated with operator $A_F = A + BF$ and $\{C_k;\tau_k\}_{k=1}^{\infty}$ can be given by

$$S_F(t,\theta) = \begin{cases} T_F(t-\theta), & \tau_{k-1} \leq \theta \leq t \leq \tau_k, \\ T_F(t-\tau_k^+)(I+C_k)T_F(\tau_k-\theta), & \tau_{k-1} \leq \theta < \tau_k < t \leq \tau_{k+1}, \\ T_F(t-\tau_k^+)\Big[\prod_{\theta<\tau_j<t}(I+C_j)T_F(\tau_j-\tau_{j-1}^+)\Big](I+C_i)T_F(\tau_i-\theta), & \\ \qquad \tau_{i-1} \leq \theta < \tau_i \leq \cdots < \tau_k < t \leq \tau_{k+1}. & \end{cases} \tag{9}$$

It is not difficult to verify that $\{S_F(t,\theta),(t,\theta) \in \Delta\}$ also satisfies similar properties in Lemma 1.

Assumption [H2]: The pair (A,B) is exponentially stabilizable.

Under the assumptions [H1] and [H2], we give some sufficient conditions guaranteeing exponential stability of $\{S_F(\cdot,\cdot)\}$.

Lemma 2. *Assumptions [H1] and [H2] hold. There exists $0 < \lambda < \nu_F$ such that*

$$\left(\prod_{k=1}^{\delta} K_F\|I+C_k\|\right)e^{-\lambda T_0} < 1.$$

Then $\{S_F(t,\theta), t \geq \theta \geq 0\}$ is exponentially stable.

Proof. Without loss of generality, for $\tau_{i-1} \leq \theta < \tau_i \leq \cdots < \tau_k < t \leq \tau_{k+1}$, we obtain

$$\begin{aligned} \|S_F(t,\theta)\| &= \left\|T_F(t-\tau_k^+)\Big[\prod_{\theta<\tau_j<t}(I+C_j)T_F(\tau_j-\tau_{j-1}^+)\Big](I+C_i)T_F(\tau_i-\theta)\right\| \\ &\leq K_F e^{-(\nu_F-\lambda)(t-\theta)}\left[\left(\prod_{\theta<\tau_k<t} K_F\|I+C_k\|\right)e^{-\lambda(t-\theta)}\right],\ t > \theta \geq 0 \end{aligned}$$

Suppose $t \in (nT_0, (n+1)T_0]$, let $b = \max_{s\in[0,T_0]} \left\{\prod_{0\le\tau_k<s} K_F\|I+C_k\|\right\}$, then

$$\begin{aligned}\left(\prod_{\theta<\tau_k<t} K_F\|I+C_k\|\right)e^{-\lambda(t-\theta)} &\le \left(\prod_{0\le\tau_k<nT_0} K_F\|I+C_k\|\right)e^{-\lambda nT_0} \\ &\quad\times\left(\prod_{nT_0\le\tau_k<t} K_F\|I+C_k\|\right)e^{-\lambda(t-nT_0)}e^{\lambda\theta} \\ &\le \left[\left(\prod_{k=1}^{\delta} K_F\|I+C_k\|\right)e^{-\lambda T_0}\right]^n be^{\lambda\theta} \ \le \ be^{\lambda\theta}.\end{aligned}$$

Let $K = K_F be^{\lambda\theta} > 0$, $\nu = \nu_F - \lambda > 0$, then $\|S_F(t,\theta)\| \le Ke^{-\nu(t-\theta)}$, $t > \theta \ge 0$. □

Lemma 3. *Assumptions [H1] and [H2] hold. Suppose*

$$0 < \mu_1 = \inf_{k=1,2,\cdots,\delta}(\tau_k - \tau_{k-1}) \le \sup_{k=1,2,\cdots,\delta}(\tau_k - \tau_{k-1}) = \mu_2 < \infty. \tag{10}$$

If there exists $\gamma > 0$ *such that*

$$-\nu_F + \frac{1}{\mu}\ln(K_F\|I+C_k\|) \le -\gamma < 0,\ k = 1,2,\cdots,\delta, \tag{11}$$

where

$$\mu = \begin{cases}\mu_1,\ \gamma-\nu_F<0,\\ \mu_2,\ \gamma-\nu_F\ge 0\end{cases}$$

then $\{S_F(t,\theta), t\ge\theta\ge 0\}$ *is exponentially stable.*

Proof. It comes from (11) that

$$\sum_{\theta\le\tau_k<t}\ln(K_F\|I+C_k\|) \le -\sum_{\theta\le\tau_k<t}\mu(\gamma-\nu_F) = -\mu(\gamma-\nu_F)N(\theta,t)$$

where $N(\theta,t)$ denotes the number of impulsive points in $[\theta,t)$.

For $\tau_{i-1} \le \theta < \tau_i \le \cdots < \tau_k < t \le \tau_{k+1}$, by (10), we obtain the following inequality

$$(N(\theta,t)-1)\mu_1 \le t-\theta \le (N(\theta,t)+1)\mu_2.$$

This implies

$$\frac{1}{\mu_2}(t-\theta)-1 \le N(\theta,t) \le \frac{1}{\mu_1}(t-\theta)+1.$$

Then

$$\begin{aligned}-\mu(\gamma-\nu_F)N(\theta,t) &\le \begin{cases}-\mu_1(\gamma-\nu_F)\left[\frac{1}{\mu_1}(t-\theta)+1\right]\\ = -(\gamma-\nu_F)(t-\theta)-\mu_1(\gamma-\nu_F),\ \gamma-\nu_F<0.\\ -\mu_2(\gamma-\nu_F)\left[\frac{1}{\mu_2}(t-\theta)-1\right]\\ = -(\gamma-\nu_F)(t-\theta)+\mu_2(\gamma-\nu_F),\ \gamma-\nu_F\ge 0.\end{cases}\\ &= -(\gamma-\nu_F)(t-\theta)+\mu|\gamma-\nu_F|.\end{aligned}$$

Thus,

$$-\nu_F(t-\theta)+\sum_{\theta\le\tau_k<t}\ln(K_F\|I+C_k\|)\le-\gamma(t-\theta)+\mu|\gamma-\nu_F|.$$

Similar to (5) of Lemma 1, one can obtain

$$\|S_F(t,\theta)\|\le K_F\exp\left\{\nu_F(t-\theta)+\sum_{\theta\le\tau_k<t}\ln(K_F\|I+C_k\|)\right\}\le K_Fe^{\mu|\gamma-\nu_F|}e^{-\gamma(t-\theta)}.$$

Let $K=K_Fe^{\mu|\gamma-\nu_F|}>0$, $\nu=\gamma>0$, $\|S_F(t,\theta)\|\le Ke^{-\nu(t-\theta)}$, $t>\theta\ge 0$. □

Corollary 1. *Let assumption [H1] and (10) hold. There exist $M\ge 1$, $\omega\in\mathbb{R}$ such that $\|T_F(t)\|\le Me^{(\omega+\|BF\|)t}$, $t\ge 0$. If there exists $\gamma>0$ such that*

$$(\omega+\|BF\|)+\frac{1}{\mu}\ln(M\|I+C_k\|)\le-\gamma<0,\ k=1,2,\cdots,\delta. \tag{12}$$

where

$$\mu=\begin{cases}\mu_1,\ \gamma+\omega+\|BF\|<0,\\ \mu_2,\ \gamma+\omega+\|BF\|\ge 0.\end{cases}$$

Then $\{S_F(t,\theta),t>\theta\ge 0\}$ is exponentially stable.

Now some sufficient conditions for the existence of inversion of $[I-S_F(T_0,0)]$ can be given.

Theorem 1. *Under the assumptions of Lemma 2, the operator $I-S_F(T_0,0)$ is inverse and $[I-S_F(T_0,0)]^{-1}\in\pounds_b(H)$.*

Proof. Consider the operator $Q=\sum_{n=0}^{\infty}[S_F(T_0,0)]^n$. Under the assumptions of Lemma 2, $\{S_F(\cdot,\cdot)\}$ is exponentially stable. It comes from the periodicity of $\{S_F(\cdot,\cdot)\}$ that

$$\big\|[S_F(T_0,0)]^n\big\|\le\big\|S_F(nT_0,0)\big\|\le Ke^{-\nu nT_0}\to 0,\text{ as } n\to\infty.$$

Thus, we obtain

$$\|Q\|\le\sum_{n=0}^{\infty}\big\|[S_F(T_0,0)]^n\big\|\le\sum_{n=0}^{\infty}Ke^{-\nu nT_0}.$$

Obviously, the series $\sum_{n=0}^{\infty}Ke^{-\nu nT_0}$ is convergent, thus operator $Q\in\pounds_b(H)$. From $[I-S_F(T_0,0)]Q=Q[I-S_F(T_0,0)]=I$, we have $Q=[I-S_F(T_0,0)]^{-1}\in\pounds_b(H)$. □

Theorem 2. *Under the assumptions of Lemma 3, the operator $I-S_F(T_0,0)$ is inverse and $[I-S_F(T_0,0)]^{-1}\in\pounds_b(H)$.*

Further, we give a little bit stronger condition which will guarantee exponential stability of $\{S_F(\cdot,\cdot)\}$. However, it is more easy to demonstrate.

Corollary 2. *Assumptions [H1] and [H2] hold. If*

$$\nu_F>\frac{\sum_{k=1}^{\delta}\ln\|I+C_k\|+(\delta+1)\ln K_F}{T_0}, \tag{13}$$

then the impulsive evolution operator $S_F(nT_0,0)$ is strongly convergent to zero at infinity (i.e., $S_F(nT_0,0)\to 0$ as $n\to\infty$). Further, the operator $I-S_F(T_0,0)$ is inverse and $[I-S_F(T_0,0)]^{-1}\in\pounds_b(H)$.

Remark 2. *If* $\|S_F(T_0,0)\| = L_F < 1$*, then* $S_F(nT_0,0) \to 0$ *as* $n \to \infty$ *and the operator* $I - S_F(T_0,0)$ *is inverse and* $[I - S_F(T_0,0)]^{-1} \in \pounds_b(H)$.

3. Optimal control problem of an impulsive periodic system

We study the following optimal control problem (P1).

$$\text{(P1): Minimize } L(x,u)\text{: } L(x,u) = \int_0^{T_0} (g(x(t)) + h(u(t)))\,dt \tag{14}$$

subject to

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), & t \in [0,T_0]\backslash\widetilde{D}, \quad x \in PC([0,T_0];H), \\ \Delta x(\tau_k) = C_k x(\tau_k), & k = 1,2,\cdots,\delta, \\ x(0) = x(T_0), & u \in L^2(0,T_0;U). \end{cases} \tag{15}$$

Definition 3. *A function* $x \in PC([0,T_0];H)$ *is said to be a* T_0*-periodic* PC*-mild solution of the controlled impulsive periodic system (15) if* x *satisfies*

$$x(t) = S(t,0)x(0) + \int_0^t S(t,\theta)Bu(\theta)d\theta, \ \textit{for } t \in [0,T_0] \ \textit{and } x(0) = x(T_0). \tag{16}$$

If system (15) has a T_0-periodic PC-mild solution corresponding to u, $(x,u) \in PC([0,T_0];H) \times L^2(0,T_0;U)$ is said to be an admissible pair. Set

$$U_{ad} = \{(x,u) \mid (x,u) \text{ is admissible}\}$$

which is called an admissible set. Problem (P1) can be rewritten as:

Find $(x^*,u^*) \in U_{ad}$ such that

$$L(x^*,u^*) \le L(x,u) \text{ for all } (x,u) \in U_{ad}.$$

In fact, if the condition

$$[I - S(T_0,0)]^{-1} \in \pounds_b(H) \tag{17}$$

is satisfied, then for every $u \in L^2(0,T_0;U)$ the T_0-periodic PC-mild solution of system (15) can be given by

$$x(t) = S(t,0)x_0 + \int_0^t S(t,\theta)Bu(\theta)d\theta, \text{ for all } t \in [0,T_0]$$

where

$$x_0 = [I - S(T_0,0)]^{-1} \int_0^{T_0} S(T_0,\theta)Bu(\theta)d\theta.$$

If the condition (17) fails, then system (15) has no solutions for every $u \in L^2(0,T_0;U)$.

Under assumptions [H1] and [H2], we can write system (15) formally in the form

$$\begin{cases} \dot{x}(t) = A_F x(t) + B(u(t) - Fx(t)), & t \in [0,T_0]\backslash\widetilde{D}, \quad x \in PC([0,T_0];H), \\ \Delta x(\tau_k) = C_k x(\tau_k), & k = 1,2,\cdots,\delta, \\ x(0) = x(T_0), & u \in L^2(0,T_0;U), \end{cases} \tag{18}$$

and substitute $u - Fx = v$ so $u = v + Fx$.

Therefore, we led to the problem (P2):

$$\text{Minimize } \widetilde{L}(x,v)\text{: } \widetilde{L}(x,v) = \int_0^{T_0} \left(g(x(t)) + h(v(t) + Fx(t))\right) dt, \tag{19}$$

subject to

$$\begin{cases} \dot{x}(t) = A_F x(t) + Bv(t), & t \in [0,T_0]\backslash\widetilde{D},\ x \in PC([0,T_0];H), \\ \Delta x(\tau_k) = C_k x(\tau_k), & k = 1,2,\cdots,\delta, \\ x(0) = x(T_0), & v \in L^2(0,T_0;U). \end{cases} \tag{20}$$

It can be seen from the proof of Theorem 1, if $\{S_F(\cdot,\cdot)\}$ is exponentially stable, then $[I - S_F(T_0,0)]^{-1}$ exists and $[I - S_F(T_0,0)]^{-1} \in \pounds_b(H)$. Set

$$x(0) = [I - S_F(T_0,0)]^{-1} \int_0^{T_0} S_F(T_0,\theta)Bv(\theta)d\theta,$$

then $x \in PC([0,T_0];H)$ given by

$$x(t) = S_F(t,0)x(0) + \int_0^t S_F(t,\theta)Bv(\theta)d\theta$$

is the periodic PC-mild solution of (20)

By Theorem 1 and Theorem 2, we have the following existence result.

Theorem 3. *For every $v \in L^2(0,T_0;U)$, impulsive periodic system (20) has a unique periodic PC-mild solution provided that assumptions of Lemma 1 or assumptions of Lemma 2 are satisfied.*

In order to show that the equivalence of problem (P1) and problem (P2), we have to prove that every PC-mild solution of (15) is a PC-mild solution of (20) with $v = u - Fx$ and vice versa. It is not obvious for a PC-mild solution. Here is the equivalence.

Theorem 4. *Under the assumptions [H1] and [H2], if $\{S_F(\cdot,\cdot)\}$ is exponentially stable, then every PC-mild solution of (15) is a PC-mild solution of (20) with $v = u - Fx$ and vice versa. Therefore, the problem (P1) is equivalent to problem (P2).*

Proof. It is obvious that every strong solution of system (15) is a strong solution of system (20). We prove only that (16) implies

$$x(t) = S_F(t,0)x(0) + \int_0^t S_F(t,\theta)Bv(\theta)d\theta \tag{21}$$

and

$$x(0) = [I - S_F(T_0,0)]^{-1} \int_0^{T_0} S_F(T_0,\theta)Bv(\theta)d\theta, \tag{22}$$

as the inverse statement will have the same proof. Therefore, let x satisfy (16) and denote the Yosida approximation of A by A_λ. Let x_λ be the strong solution of

$$\begin{cases} \dot{x}_\lambda(t) = A_\lambda x_\lambda(t) + Bu(t), & t \in [0, T_0]\backslash\widetilde{D}, \quad x_\lambda \in PC([0, T_0]; H), \\ \Delta x_\lambda(\tau_k) = C_k x_\lambda(\tau_k), & k = 1, 2, \cdots, \delta, \\ x_\lambda(0) = x(0), & u \in L^2(0, T_0; U). \end{cases} \tag{23}$$

Taking into account that

$$T_\lambda(t)x(0) \to T(t)x(0) \text{ as } \lambda \to 0, \text{ uniformly in } t \in [0, T_0],$$

it follows that for each $t \in [0, T_0]$ but fixed,

$$S_\lambda(t,\theta)x(0) \to S(t,\theta)x(0) \text{ as } \lambda \to 0, \text{ uniformly in } \theta \in [0, t],$$

where the operator $\{S_\lambda(t,\theta), (t,\theta) \in \Delta\}$ is the impulsive evolution operator associated with A_λ and $\{C_k; \tau_k\}_{k=1}^{\infty}$.

In fact, for $\tau_{k-1} \leq \theta \leq t \leq \tau_k$,

$$S_\lambda(t,\theta)x(0) = T_\lambda(t-\theta)x(0) \to T(t-\theta)x(0) = S(t,\theta)x(0) \text{ as } \lambda \to 0,$$

uniformly in $\theta \in [0, t]$.

For $\tau_{k-1} \leq \theta < \tau_k < t \leq \tau_{k+1}$, $S_\lambda(t,\theta)x(0) = T_\lambda(t-\tau_k^+)(I + C_k)T_\lambda(\tau_k - \theta)x(0)$.

Since $T_\lambda(\tau_k - \theta)x(0) \to T(\tau_k - \theta)x(0)$ as $\lambda \to 0$, uniformly in $\theta \in [0, \tau_k]$,

$$(I + C_k)T_\lambda(\tau_k - \theta)x(0) \to (I + C_k)T(\tau_k - \theta)x(0) \text{ as } \lambda \to 0, \text{ uniformly in } \theta \in [0, \tau_k].$$

Further,

$$S_\lambda(t,\theta)x(0) \to S(t,\theta)x(0) \text{ as } \lambda \to 0, \text{ uniformly in } \theta \in [0, t],$$

For $\tau_{i-1} \leq \theta < \tau_i \leq \cdots < \tau_k < t \leq \tau_{k+1}$, step by step,

$$\begin{aligned} &\left[\prod_{\theta<\tau_j<t} (I + C_j)T_\lambda(\tau_j - \tau_{j-1}^+)\right](I + C_i)T_\lambda(\tau_i - \theta)x(0) \\ \to &\left[\prod_{\theta<\tau_j<t} (I + C_j)T(\tau_j - \tau_{j-1}^+)\right](I + C_i)T(\tau_i - \theta)x(0) \end{aligned}$$

as $\lambda \to 0$, uniformly in $\theta \in [0, \tau_k]$. Of course, we have

$$S_\lambda(t,\theta)x(0) \to S(t,\theta)x(0) \text{ as } \lambda \to 0, \text{ uniformly in } \theta \in [0, t].$$

On the other hand, define $q^\lambda(\theta) = S_\lambda(t,\theta)Bu(\theta) - S(t,\theta)Bu(\theta)$, then

$$\|q^\lambda(\theta)\| = \|(S_\lambda(t,\theta) - S(t,\theta))Bu(\theta)\| \leq 2M_{T_0}\|B\|\|u\|_{L^2(U;H)} \in L^1(0, T_0; H).$$

Since $q^\lambda(\theta) \to 0$ a.e. $\theta \in [0, t]$ as $\lambda \to 0$, by virtue of Majorized Convergence Theorem, we obtain

$$\int_0^t q^\lambda(\theta)d\theta \to 0 \text{ as } \lambda \to 0.$$

This implies that $x_\lambda \to x$ in $PC([0,T_0];H)$ as $\lambda \to 0$.

However, (23) can be written as

$$\begin{cases} \dot{x}_\lambda(t) = (A_\lambda + BF)x_\lambda(t) + Bv_\lambda(t), & t \in [0,T_0]\backslash \widetilde{D}, x_\lambda \in PC([0,T_0];H), \\ \Delta x_\lambda(\tau_k) = C_k x_\lambda(\tau_k), & k = 1,2,\cdots,\delta, \\ x_\lambda(0) = x(0), & u \in L^2(0,T_0;U), \end{cases} \tag{24}$$

with $v_\lambda = u - Fx_\lambda$.

Similarly, one can obtain that x_λ in (24) is also convergent to the solution of (21) with $v = u - Fx$.

At the same time, it is easy to see that $U_{ad} \neq \emptyset$ and problem (P1) is equivalent to problem (P2). □

4. Existence of optimal controls

In this section, we present the existence of optimal controls for problem (P1) which is the main result of this paper.

We make the following assumptions:

[H3]: The function $h : U \to \bar{\mathbb{R}}$ is convex and lower semicontinuous; $\mathrm{Int}D(h) \neq \emptyset$, where $D(h) = \{u \in U; h(u) < +\infty\}$. Moreover, h: $U \to [0,+\infty)$ has the growth properties below:

$$\lim_{\|u\|_U \to \infty} \frac{h(u)}{\|u\|_U} = +\infty. \tag{25}$$

[H4]: The function $g : H \to \mathbb{R}$ is convex and lower semicontinuous. For arbitrary $x \in H$,

$$\varpi\|x\| + \mathcal{C} \leq g(x) < +\infty, \tag{26}$$

for some $\varpi > 0$ and $\mathcal{C} \geq 0$.

Theorem 5. *In addition to assumptions of Theorem 4, assumptions [H3] and [H4] hold. Then problem (P1) has at least one optimal control pair* (x^*, u^*).

Proof. By virtue of Theorem 4, it is sufficient to show the existence of optimal controls for Problem (P2). Set

$$\inf\left\{\widetilde{L}(x,v) \mid \widetilde{L}(x,v), \text{ over all } (x,v) \text{ as in } (21)\right\} = d.$$

If $d = +\infty$, there is nothing to prove. By assumptions [H3] and [H4], we know $d \geq 0$.

Let (x_n, v_n) with $x_n \in PC([0,T_0];H)$ and $v_n \in L^2(0,T_0;U)$ be a minimizing sequence for problem (P2). This means

$$d \leq \int_0^{T_0} (g(x_n(t)) + h(v_n(t) + Fx_n(t)))\,dt \leq d + \frac{1}{n}, \quad n = 1,2,\cdots. \tag{27}$$

Set

$$u_n(t) = v_n(t) + Fx_n(t). \tag{28}$$

It is obvious that (27) implies that

$$\int_0^{T_0} h(u_n(t))dt \le d+1. \tag{29}$$

Let E be any measurable subset of $[0,T_0]$ and $\sigma > 0$. Clearly, $E = E_1 \cup E_2$ with $E_1 = E \cap \{t; \|u_n(t)\|_U < \sigma\}$ and $E_2 = E \cap \{t; \|u_n(t)\|_U \ge \sigma\}$.

It can be seen from assumption [H3] that there exists $\phi(\cdot)$ such that

$$h(u) \ge \phi(\sigma)\|u\|_U, \qquad \text{for all} \qquad \|u\|_U \ge \sigma, \tag{30}$$

where $\lim_{\sigma\to\infty}\phi(\sigma) = +\infty$.

By a standard argument, we have

$$\begin{aligned}\int_E \|u_n(t)\|_U dt &= \int_{E_1} \|u_n(t)\|_U dt + \int_{E_2} \|u_n(t)\|_U dt \\ &\le \sigma m(E_1) + \frac{1}{\phi(\sigma)}\int_0^{T_0} h(u_n(t))dt \le \sigma m(E) + \frac{d+1}{\phi(\sigma)}. \end{aligned}\tag{31}$$

This implies that the set $\{u_n\}$ is uniformly integrable on $[0,T_0]$. In view of the Dunford-Petties Theorem, (31) implies that $\{u_n\}$ is sequentially weakly compact in $L^1(0,T_0;U)$. Say $u_n \to u^*$ weakly in $L^1(0,T_0;U)$.

Moreover, (26) and (27) imply

$$\int_0^{T_0} \|x_n(t)\|dt \le \frac{1}{\varpi}\int_0^{T_0} \left(g(x_n(t)) + h(u_n(t))\right)dt \le \frac{d+1}{\varpi}. \tag{32}$$

Taking into account that the pair (x_n, v_n) satisfies

$$x_n(t) = S_F(t,0)x_n(0) + \int_0^t S_F(t,\theta)Bv_n(\theta)d\theta \tag{33}$$

and

$$x_n(0) = [I - S_F(T_0,0)]^{-1}\int_0^{T_0} S_F(T_0,\theta)Bv_n(\theta)d\theta. \tag{34}$$

By elementary computation, there exists a constant $M > 0$ such that

$$\|x_n(t)\| \le M, \qquad \text{for all} \qquad t \in [0,T_0].$$

i.e., $\{x_n\}$ is bounded in Banach space $(L^1(0,T_0;H))^* = L^\infty(0,T_0;H)$. By Alaoglu Theorem, we have $x_n \to x^*$ weakly star convergent in $L^\infty(0,T_0;H)$.

Set $v_n = u_n - Fx_n$ and $F \in \pounds_b(H,U)$, then

$$v_n \to u^* - Fx^* = v^* \text{ weakly in } L^1(0,T_0;U).$$

There exists a function $\widetilde{x}(\cdot) : [0,T_0] \to H$ such that

$$\widetilde{x}(t) = S_F(t,0)\widetilde{x}(0) + \int_0^t S_F(t,\theta)Bv^*(\theta)d\theta,$$

with

$$\widetilde{x}(0) = [I - S_F(T_0, 0)]^{-1} \int_0^{T_0} S_F(T_0, \theta) B v^*(\theta) d\theta.$$

Clearly,

$$x_n(t) \to \widetilde{x}(t) \text{ weakly convergent in } H, \text{ for each } t \in [0, T_0].$$

One can verify $x_n \to \widetilde{x}$ weakly convergent in $L^1(0, T_0; H)$. This implies that $\widetilde{x} = x^*$. Hence x^* is the T_0-period PC-mild solution of system (15) corresponding to the control $v \in L^2(0, T; U)$ given by

$$x^*(t) = S_F(t, 0) x^*(0) + \int_0^t S_F(t, \theta) B v^*(\theta) d\theta \tag{35}$$

with

$$x^*(0) = [I - S_F(T_0, 0)]^{-1} \int_0^{T_0} S_F(T_0, \theta) B v^*(\theta) d\theta. \tag{36}$$

Letting $n \to \infty$ in (27), using assumptions [H3] and [H4] again, by Theorem 2.1 of [7], we can obtain

$$\begin{aligned} d &= \lim_{n\to\infty} \int_0^{T_0} \left(g(x_n(t)) + h(v_n(t) + F x_n(t))\right) dt \\ &\geq \int_0^{T_0} \left(g(x^*(t)) + h(v^*(t) + F x^*(t))\right) dt \geq d. \end{aligned}$$

Thus, we can conclude that $d = \widetilde{L}(x^*, v^*)$.

Let $u^* = v^* + F x^*$, $(x^*, u^*) \in U_{ad}$ is the optimal pair for problem (P1). □

5. An example

As an application of Theorem 5, we study the following problem:

Minimize

$$\int_Q \big(g_0(y, x(y,t)) + g_1(y, x_t(y,t)) + g_2(y, \nabla x(y,t))\big) dy dt + \int_Q h_0(u(y,t)) dy dt \tag{37}$$

subject to

$$u \in L^2(Q),\ Q = \Omega \times (0, T_0),\ x \in PC([0, T_0];\ H_0^1(\Omega)),\ x_t \in PC([0, T_0]; L^2(Q)),$$

satisfying the wave equation with impulse:

$$\begin{cases} x_{tt}(y,t) - \Delta x(y,t) = u(y,t), \quad y \in \Omega, \quad t \in [0, T_0] \backslash \widetilde{D}, \\ \Delta x(y, \tau_k) = B_1 x(y, \tau_k), \\ \Delta x_t(y, \tau_k) = B_2 x_t(y, \tau_k), \quad y \in \Omega, \quad k = 1, 2, \cdots, \delta, \\ x = 0, \qquad\qquad \text{in} \quad \Sigma = \partial\Omega \times (0, T_0), \\ x(y, 0) = x(y, T_0); \quad x_t(y, 0) = x_t(y, T_0), \quad y \in \Omega. \end{cases} \tag{38}$$

Here $\Omega \subseteq \mathbb{R}^n$ is a bounded domain with a smooth boundary $\partial\Omega$. The functions g_0: $\mathbb{R} \to \mathbb{R}$, g_1: $\mathbb{R} \to \mathbb{R}$, and g_2: $\mathbb{R}^n \to \mathbb{R}$ are supposed to satisfy the following hypotheses:

(J_1) $g_0 = g_0(y,x)$, $g_1 = g(y,z)$, and $g_2 = g(y,w)$, are continuous and convex in x, z, w (respectively) and measurable in y. There exist some $\alpha_i \in L^\infty(\Omega)$, $\beta_i \in L^1(\Omega)$, $i = 0, 1$, $\alpha_2 \in L^\infty(\Omega;\mathbb{R})$, and $\beta_2 \in L^1(\Omega;\mathbb{R})$ such that

$$\begin{aligned} g_0(y,x) &\geq \alpha_0(y)(x) + \beta_0(y) \quad \text{a.e.} \quad y \in \Omega, \quad x \in \mathbb{R}, \\ g_1(y,z) &\geq \alpha_1(y)(z) + \beta_1(y) \quad \text{a.e.} \quad y \in \Omega, \quad z \in \mathbb{R}, \\ g_2(y,w) &\geq (\alpha_2(y), w) + \beta_2(y) \quad \text{a.e.} \quad y \in \Omega, \quad w \in \mathbb{R}^n. \end{aligned}$$

and for some $c > 0$,

$$\begin{aligned} g_0(y,x) &\leq c(1 + \|x\|^2) \quad \text{a.e.} \quad y \in \Omega, \quad x \in \mathbb{R}, \\ g_1(y,z) &\leq c(1 + \|z\|^2) \quad \text{a.e.} \quad y \in \Omega, \quad z \in \mathbb{R}, \\ g_2(y,w) &\leq c(1 + \|w\|^2) \quad \text{a.e.} \quad y \in \Omega, \quad w \in \mathbb{R}^n. \end{aligned}$$

The function h_0: $\mathbb{R} \to \bar{\mathbb{R}}$. Denote h: $L^2(\Omega) \to \bar{\mathbb{R}}$ given by $h(q) = \int_\Omega h_0(q(y))dy$. h is supposed to satisfy the following hypotheses:

(J_2) h is a convex lower semicontinuous function with $\mathrm{Int}D(h) \neq \emptyset$.

An example of such h is

$$h(u) = \begin{cases} h^0(u), & u \in U_0, \\ 0, & \text{otherwise}, \end{cases} \tag{39}$$

where h^0: $L^2(\Omega) \to \mathbb{R}$ is a continuous convex function and $U_0 \subset L^2(\Omega)$ is a closed convex subset with nonempty interior.

Let $H = H_0^1(\Omega) \times L^2(\Omega)$, $U = L^2(\Omega)$, and denote

$$\mathfrak{X}(t)(y) = \begin{pmatrix} x(y,t) \\ x_t(y,t) \end{pmatrix}, \mathfrak{A} = \begin{pmatrix} 0 & I \\ \Delta & 0 \end{pmatrix}, \mathfrak{B} = \begin{pmatrix} 0 \\ I \end{pmatrix}, \mathfrak{C} = \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix}, \mathfrak{U}(t)(y) = u(y,t),$$

$$g(X) = \int_\Omega \big(g_0(y,x) + g_1(y,z) + g_2(y,\nabla x)\big)dy, \quad X = (x,z) \in H.$$

then system (38) can be written by

$$\begin{cases} \dot{\mathfrak{X}} = \mathfrak{A}\mathfrak{X} + \mathfrak{B}\mathfrak{U}(t), & t \in [0,T_0]\backslash\widetilde{D}, \\ \Delta\mathfrak{X}(\tau_k) = \mathfrak{C}\mathfrak{X}(\tau_k), & k = 1, 2, \cdots, \delta, \\ \mathfrak{X}(0) = \mathfrak{X}(T_0). \end{cases} \tag{40}$$

with the cost function

$$\int_0^{T_0} \big(g(\mathfrak{X}(t)) + h(\mathfrak{U}(t))\big)dt.$$

The pair $(\mathfrak{A}, \mathfrak{B})$ is exponentially stabilizable (as the controlled wave equation is null controllable in finite time (see [8])). By Lemma 2, when

$$\lambda > \frac{\delta \ln(\sum_{i=1}^2 \|I + B_i\|)}{T_0},$$

one can obtain that $\{\mathfrak{S}_F(t,\theta), t > \theta \geq 0\}$ is exponentially stable.

Note that

$$g_0(y,z) + g_2(y,w) \geq \varpi\|w\|^2 + \|z\| + \psi(y), \text{ a.e. in } \Omega,\ y \in \Omega,\ z \in \mathbb{R},\ w \in \mathbb{R}^n,$$

with some $\varpi > 0$ and $\psi \in L^1(\Omega)$. If h satisfies (25) then, by Theorem 5, problem (37) has at least one solution (x^*, u^*).

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