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APPLICATION OF THE QUEUING THEORY IN THE PLANNING OF OPTIMAL NUMBER OF SERVERS (RAMPS) IN CLOSED PARKING SYSTEMS

ABSTRACT

The principal objective of this scientific paper is to learn how to efficiently organise traffic areas and especially the size of parking capacities and hence how to ensure a quality parking service to local population and tourists as a component of the overall offer in urban and tourist destinations and how to ensure a return of investments in a reasonable period to parties investing in the parking capacity. What is the optimal capacity and how to calculate it in the best possible way by connecting parking supply and demand? This paper presents the application of the queuing theory to the planning of the optimal number of servers (ramps) in closed parking systems, since parking area can be defined as a queuing system. The illustrated model has been tested on the example of the "Delta" parking area in the City of Rijeka and the particular value of the model is its universal application. This approach has shown that by using the queuing theory, the optimal number of servers (ramps) in closed parking systems can be determined.

Keywords: *optimal number of servers (ramps), queuing theory, queuing system, planning of parking area capacity "Delta" parking area in the City of Rijeka*

1. INTRODUCTION

The demand for parking services is not constant, but it varies from lowest to highest. The span between the lowest and the highest demand and the dynamism of changes are fundamental factors which influence the required size of parking area capacities and the financial effect of parking.

The parking activity, whose objective is to "produce" parking service, is faced with primary difficulties as early as in the stage of planning: how to ensure parking service in the period of increased demand; the dimensioning of the optimal parking area capacity should be performed based on what demand; what to do with the surplus of parking area capacity when the demand is reduced; which percentage of under capacity and over capacity of the parking area is acceptable; how to balance the use of parking area capacity in conditions of fluctuation with the process of long term increase of demand and of capacity?

Besides, unlike the means of transportation which, in a case of variation larger than the planned circulation, can be rented, sold or purchased and therefore, temporarily or permanently, adjust capacity to the demand, for parking areas in the function of parking defined by location and purpose, and whose invested resources and lifetime are long term, this

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is not possible. For that reason, when constructing parking locations, it is necessary to also take into account a long term forecast of parking area requirements and demands (particularly important for tourist destinations), a forecast of cyclical oscillations (arrivals of tourists in tourist season requires a larger number of parking areas as against the rest of the year) and the possibility of extensions of parking capacities according to requirements.

Since arrivals of vehicles and length of parking time can be taken as a random (stochastic) variable and the empirical distribution of these variables then approximised with adequate theoretical distributions, for parking areas it is therefore possible to apply the analytical approach, i.e. to use formulas set out by the queuing theory in order to calculate the parking functioning ratio.

The objective of this paper is to demonstrate that by applying the queuing theory the optimal number of servers (ramps) in closed parking areas can be defined and that by applying the laid out model it is possible to make adequate business decisions regarding the planning and development of parking capacities.

In the national and international scientific and professional literature the queuing theory applied to the problematics of parking has not yet been consistently researched and presented to the public, but it has only been partially analysed and treated. In recent time, parking or still traffic, as referred to by some authors, is mentioned by some Croatian authors [5, 6] as a growing problem which will eventually paralise traffic in cities and tourist centres. The issue of the modelling of parking systems is significantly less represented; parking models have been analysed by G. Lubурić in 2005 [5] in his PhD dissertation entitled "*Model rješavanja problema parkiranja u gradskim središtima*" (Model for resolving the parking problem in urban centres), whilst in 1991 in the article published in the magazine "*Promet i prostor*" (Traffic and space) Ž Kerkez presents a model of dimensioning of the optimal size of parking area [3], but neither the former nor the latter paper apply the queuing theory.

2. FUNCTION OF MODEL IN THE PLANNING OF PARKING AREA CAPACITIES

Parking area is a complex system consisting of adequate system components and of their inter-dependence; it is therefore necessary prior to analysing and planning of parking capacities to define the model of parking.

The use of model in the analysis of a system has multiple advantages:

1. model enables analysing and experimenting with complex situations, not possible with a real system,
2. model enables to consider in advance most of the problem's relevant elements and to determine the inter-dependence of these elements,
3. experimenting on the model, which is an approximation of the real system, is shorter in time and cheaper than experimenting on the real model, all the more because in practice experimenting with the real system is not always possible,
4. model enables the subsequent insertion of the parameters' changes and the making of adequate business decisions for the system observed over a time.

Depending on the criterion, various types of models can be observed: according to function, structure, degree of randomness, time dependence and degree of quantification.

With relation to the object of research, this paper gives particular importance to stochastic (probabilistic) models having random model variables with known probability distributions.

The methods, i.e. procedures used to resolve the models are: analytical, simulational and heuristic methods.

Analytical methods are based on mathematical analysis procedures which produce solutions for an optimal functioning of the system. The solution of the model is given in the form of a formula into which various values of input variables and parameters are inserted, depending on the problem content [15].

Simulation is the method which uses trials (experiments) which produce variants and based on the set criteria the variant most likely to be closest to the optimal variant is then determined.

Analytical methods and the simulation method, due to their advantages and disadvantages, complement each other. In researches of traffic and parking systems it is advisable to apply the analytical method in those cases in which it is possible to set a mathematical model for the parking area under observation; or as an addition to the research use the method of simulation to analyse certain sections of more complex models, or when it is not possible to put up a mathematical model, simulational modelling is then applied.

In this paper, since the object of research are parking areas, models of the queuing theory used for the modelling of the queuing process have been selected.

The queuing theory is one of the methods of operational researches studying the problems of waiting lines whose task is to serve randomly arrived units or requests for a service. The queuing theory uses mathematical models to determine inter-dependence among the arrival of units, their waiting to be served, their serving and finally the exit of units from the system.

Basic terms of the queuing theory are the following:

1. customers (service users, clients),
2. servers (service places, locations providing service or performing processing),
3. queue (waiting line, amassing).

Considering the elements used to describe a process of the waiting line, the necessity to denote the type of the queuing problem has arisen. For this purpose, Kendall's notation has been accepted

$$v / w / x / y / z \tag{1}$$

where the letters stand for:

v - distribution of the arrival of units into the system,
w - distribution of time of the service of units,
x - number of servers,
y - capacity of the queuing system,
z - discipline of queue.

A large number of types of queuing problems arise in practice because the six elements used to describe the queuing process can be presented in a large number of variations.

Among all the possible queuing systems, the most common in practice are systems with waiting. These are queuing problems in which customers wait to be served if their number exceeds the number of the servers or wait for service places (servers) if the number of customers is lower than the number of servers. Among queuing systems with waiting, particularly important for parking areas are waiting lines with finite waiting time, i.e. limited number of places in the waiting line.

Basic parameters used for the calculation of adequate indices of the queuing system functioning are the intensity of arrival flow (λ) and intensity of servicing (μ) which in practical examples are determined based on data obtained by statistical observation or evaluation depending on the objective and task of the research.

The application of the queuing theory is shown in section 4 of the present paper.

3. STATISTICAL ANALYSIS OF THE INTENSITY OF VEHICLE ARRIVAL AND OF THE PARKING TIME LENGTH

The arrival of vehicles into parking areas has large oscillations during the year, month, day and hour. It is therefore difficult to pre-determine the number of vehicles arriving to or leaving a certain parking area on a certain day. However, for the planning of a parking area capacity it is useful to determine whether there is a certain regularity in the arrival of vehicles, i.e. in the number of vehicles and in the time of the arrival of these vehicles into the parking area.

If the data about the number of vehicles by days and months are compared, no relation between the day under observation and the following days could be noticed. This conclusion can be verified using the correlation method, i.e. a statistical method verifying the existence and form of relation between two or more observed phenomena. Considering the fact that a high number of value pairs has been set, we have opted for the correlation for grouped elements [10].

For this purpose a correlation table is drawn up in which the denominations of groups of one characteristic are put in the table's row heading (number of vehicles of the following day – phenomenon X), and the denominations of groups of another characteristic in the column heading of the same table (number of vehicles of previous day – phenomenon Y).

In order to determine the intensity of the relation between the number of vehicles of the following day and the number of vehicles of the previous day, it is necessary to calculate the correlation coefficient according to the formula:

$$r = \frac{\sum f_{x_i y_j} \cdot X_i Y_j - N\overline{X}\,\overline{Y}}{N\sigma_x \sigma_y} \quad , \tag{2}$$

where:

N – total number of days
σ_x – standard deviation for phenomenon X (number of vehicles of following day)
σ_y – standard deviation for phenomenon Y (number of vehicles of previous day).

From the obtained value of the correlation coefficient, for which $0 < r < 1$, the dependence between the observed variables can be concluded. If r has a small value, then there is no significant dependence in the order of daily arrivals of vehicles which then means that the arrivals of vehicles can be observed as if they were independent, random in statistical sense and that the number of the vehicles arriving into closed parking area can be taken as a random variable.

Analogously a statistical analysis is carried out of the parking time length and the conclusion is drawn up about significant or random dependence between the parking time length of the following and previous day.

If the arrivals of vehicles into the parking area as well as the length of parking time are random variables, it is necessary to determine the types of intensity of these variables, i.e. to verify if these variables act according to the rules of certain theoretical distributions.

Traffic entities (further on vehicles) arrive to the service place at random time moments. In most cases these time moments are mutually independent. The notion of the arrival of vehicle can be equated with the event which is realised at the entry into the queuing system. We can therefore talk about incoming flow of event.

The arrivals of units into the system can by deterministic and in these cases it is sufficient to know the value of λ (average number of units arriving into the system in a unit of time) or t_{dol} (time period between two consecutive arrivals). However, in practice it is much more common for arrivals of units to be random (stochastic), in which case besides λ and t_{dol} it is necessary to know their probability functions.

In real terms, an unlimited number of various distributions of arrivals is possible. In theory, they are described in distributions best suited to actual situation, but such distributions are after all approximations of actual processes.

Distributions of arrivals are most commonly classified by type of arrivals:

- regularly distributed arrivals,
- completely random arrivals (exponential distribution of arrivals),
- arrivals distributed according to Erlang distribution of order *k*,
- generally independently distributed arrivals.

Completely random arrivals (marked with *M*) refer to the units arriving into the system at random order with average time interval between two consecutive arrivals $\bar{t}_{dol}$ or with average number of units λ. As these are random arrivals, it is necessary to define the probability function. The distribution of the number of arrived units can be described using the Poisson density of distribution:

$$P_n = \frac{(\lambda t)^n}{n!} e^{-\lambda t};\ t \geq 0, n = 1,2,... \ , \tag{3}$$

and the distribution of time intervals (with *k* consecutive arrivals) with the Erlang density of distribution:

$$E_k(t) = \frac{1}{(k-1)!} \lambda^k t^{k-1} e^{-\lambda t} \quad ; \quad t \geq 0 \quad , \quad k = 1,2,... \tag{4}$$

When for the distribution of time intervals between two consecutive arrivals *k* = 1, the Erlang distribution changes to the exponential density of distribution:

$$Exp(t) = \lambda e^{-\lambda t} \quad ; \quad t \geq 0 \quad . \tag{5}$$

The servicing time is expressed in the number of time units necessary for the servicing of one unit, i.e. for the performing of a certain service. This ratio expresses the throughput of the server.

The servicing time can be analysed in a similar manner as the arrival of units, but there is a substantial difference between these two processes. The arrivals of units are distributed within the entire time interval under observation, whilst the servicing is defined only when there are units inside the system; if there is no unit, the server is unused.

The characteristics of the intensity of vehicles' arrival flow and of the time of servicing of vehicles in parking areas are the following:

▪ Stationarity is the property which shows random hesitation about the mean value. This property can also be accepted for parking systems so the intensity of vehicles' arrival flow does not depend on the time, but it is a constant value and represents the average number of vehicles arrived in a unit of time (λ).

▪ Arrivals of vehicles into the parking area are events which are consecutive one to another in moments randomly distributed in the interval under observation and they represent the incoming flow. Analogously the outgoing flow can be defined as well, i.e. the exiting of vehicles from the parking.

▪ Flows of events on parking areas are non-uniform (non-homogeneous) event flows because the demands for servicing vary according the type and the internal structure. It has been expected considering the task of the parking areas, their functioning within the traffic chain and technology of work in parking areas. However, in this paper we accept the hypothesis about the homogeneous flow considering the fact that passenger vehicles prevail in the total number.

- In parking areas the event flows are mostly non-regular (random) because the demands for servicing do not appear according to pre-determined order.
- In relation to the time of arrival, it can be accepted that event flows in parking areas are ordinary, which means that there is a very small probability that two vehicles will appear at the same time with the same demand, and it is accepted that vehicles enter into the parking area one after the other.
- The arrival of vehicles over a certain period does not depend on the number of vehicles previously arrived into the parking area. That is why we say that the arrivals of vehicles are flows without consequence. This flow property makes sense only if the vehicles arrive from more directions, not only one, which is the commonest case on parking areas.

From the aforementioned properties (stationarity, ordinarity, flow without consequence) it is deducible that flows in parking areas are simple random flows and because of that parking areas can be analysed as a mass service system. However, it can be seen in practice that incoming and outgoing event flows do not have all of these properties; the authors [2, 12] studying the queuing theory recommend not to reject the hypothesis of a simple event flow, but to implement certain generalisations that do not influence substantially the accuracy of the obtained results.

If the number of vehicles and the length of service period are random variables, it is not possible to pre-determine their values, but it is possible to pre-determine their probability distributions.

In order to be able to calculate the probability of the realisation of the random variable representing the number of arrival of vehicles and the number of serviced vehicles, it is necessary to carry out the following: gather the data about the entering, carry out a statystical analysis of these data and to verify the matching of the empirical distribution with the selected theoretical distributions.

The grouping of data results in empirical distributions about the vehicles, the number of entry ramps and all the other information related to the production of the parking service. Basic statistical parameters are calculated for these distributions, like: arithmetic mean, standard deviation, range of variation, measures of asymmetry and roundness. If samples are in question, then it is necessary to apply the sampling method, especially interval evaluations of certain parameters and the verification of the set up hypotheses. The relation between certain variables are tested using the correlation method, linear or non-linear.

Based on the parameters calculated for the assigned empirical distribution, the procedure is continued by calculating theoretical frequencies for the selected theoretical distribution and eventually the coincision between the empirical and the theoretical distribution is tested.

The determination of the type of distribution according to which the arrivals of vehicles and the service time behave, is done by statistical tests, i.e. by testing the hypothesis about the coincision of the assumed theoretical probability distribution with the empirical probability distribution. This paper has shows the χ^2 - test.

When it is determined that the arrival of vehicles and the service time behave according to some theoretical distribution, then it is possible to apply the queuing theory for calculating the index of functioning of the assigned system depending on the distribution of arrival of vehicles and the distribution of the length of service time.

If empirical distributions of the observed variables (arrivals of vehicles and service time) cannot be adapted to any of the theoretical distributions, then, as agreed by most authors [8], it is not possible to use the analytical approach, but it is necessary to apply simulation.

4. APPLICATION OF THE QUEUING THEORY IN THE PLANNING OF PARKING CAPACITIES

The problematics of parking in urban, tourist and other centres requires an interdisciplinary approach which takes into consideration all aspects of transportation needs, preservation of urban area, environment protection and economic plausibility of possible solutions. This paper presents the application of the queuing theory in the function of optimal dimensioning of parking areas using the example of the "Delta" parking area in the City of Rijeka. The analysis of the parking area as a queuing system and the making of business decisions related to the development of parking capacities can be supported by the shown model presenting the parking area.

4.1. PARKING AREA AS A QUEUING SYSTEM

Since the arrival of the vehicles into the parking area is irregular, statistical analysis has confirmed that the arrivals of vehicles and the length of service time can be observed as random variables which can be approximised with adequate theoretical distributions. This further on means that parking areas can be analysed as mass service systems. This paper takes into consideration "closed parking systems"[1] which represent parking locations into which all entering and exiting points are equipped with certain types of ramps (arm barriers), where when entering from the incoming terminal, the driver takes the parking ticket and enters into the parking area and the payment is made in toll booths.

Parking area represents a queuing system with the following structure: customers are vehicles forming (or not) a waiting line (depending on the current situation) in order to be served (parked) in a parking section and after the service has been completed (certain length of parking time), they exit the system.

According to the characteristics of the intensity of vehicles' arrival flow and the service time listed in section 3 of the present paper, ununiformity in use of parking capacity can be deducted; if the number of vehicles arrived into the parking area is greater than the number of vehicles the existing parking capacities can serve in a unit of time, then vehicles are lined up in waiting lines, and in reverse case vehicles do not wait, but however parking capacities are not fully exploited.

Defining the optimal number of parking spaces requires the taking into consideration of all the factors influencing the work of the parking area. The optimal number of parking spaces is the one providing satisfactory level of service to the user, and at the same time has good economic effects, i.e. small number of unserved vehicles and a large number of occupied parking spaces.

For the selected system of incoming terminals into the parking area the arrivals intensity flow λ represents the average number of vehicles arriving into the parking area in a unit of time (hour, day, year etc.) under observation.

For the selected system of incoming terminals into the parking area, the intensity of servicing μ represents also the average number of vehicles which can be served in a unit of time (hour, day, year etc.) under observation. The intensity of servicing is the reciprocal value of the average time necessary for the vehicle to enter the parking and the length of service time ($\mu = 1/\bar{t}_{usl}$).

Service time ($\bar{t}_{usl}$) is expressed in the number of units of time necessary for the servicing of one vehicle, i.e. for the entry of vehicles into the parking area. This data is used to express the throughhpput of the server. The average time needed for the vehicle to enter into the parking area is calculated as the arithmetic mean of the total time (sum of all waiting times at entry ramps) needed for the vehicles to enter into the parking area and that time consists of: time needed for the driver to stop at the entry ramp, to press the button or to read the bar code ticket, the time to lift the ramp and the time the driver needs to proceed to the parking space after the ramp has been lifted.

The relation between the intensity of the flow of arrivals and the intensity of the flow of servicing of vehicles is the degree of load of entry ramp $\rho=\lambda/\mu$, i.e. the coefficient of utilisation of the parking area $\rho_s=\lambda/S\mu$, where *S* stands for the number of entry ramps.

If the entry ramps are occupied, the vehicle waits in line until served. Customers are vehicles, and services are performed by servers – entry ramps. Considering the fact that the length of the waiting line is limited by the length of space in which vehicles can halt while waiting to enter the parking area, the parking area is defined as a multi-server queuing system with finite waiting line length.

From the point of view of the queuing theory, the following can be concluded for incoming/outgoing parking area terminals:

- considering that the arrival flow of vehicles is not an integral part of the system, the parking area is an open system,
- considering that more waiting lines are formed at the entry point into the parking area, we can talk about a multi-server queuing system,
- arrivals of vehicles into the parking area are distributed according to certain theoretical distributions (in this paper the distribution is Poisson's),
- service time is also distributed according to certain theoretical distributions (in this paper the distribution is exponential),
- servicing of vehicles is done according to the FIFO method (first-come-first-served),
- considering the fact that outside the entry ramps there is a certain number of space for vehicles waiting to enter the parking area, we can talk about a waiting line with limited length.

Based on the starting parameters and the characteristics of a concrete mass service system, adequate indices are calculated. These are values which express the functioning of a mass service system. According to the obtained dana, adequate conclusions are made for various number of entry ramps.

4.2. DETERMINATING PARKING CAPACITIES

Experience shows that, although the parking area has sufficient number of entry ramps, large traffic congestion creates each day at the entry into the parking. When the parking area fills up, entry ramps automatically prevent new vehicles from entering into the parking area, i.e. the drivers trying to enter are signalled that the parking area is complete and this initiates the creation of a line of vehicles trying to enter into the parking area.

A dilemma arises from the question whether the entry ramp or the parking space is a service place. Considering the fact that there are relatively poor experiences presented in national and international literature, in this paper the authors define entry ramps as service place and they base their analysis upon them; based on the obtained results they have calculated the number of parking spaces and therefore defined the required parking area capacity. It can therefore be deducted that, when the optimal size of the parking area is being defined, it is not sufficient to take into account only the entry ramps, but it is necessary to take also into account the number of parking spaces because the increase of the number of entry ramps does not mean an increase of parking area capacity.

The parking area capacity is expressed in the number of parking spaces, i.e. the number of vehicles which use the parking service. It is necessary to distinguish the statistical from the dynamic parking area capacity. Statistical capacity is expressed in the number of vehicles which can be parked at the same time, and the dynamical in the total number of vehicles counted in a unit of time.

The optimal number of entering points, i.e. ramps, according to experts in garage facilities and closed parking areas construction, amounts to one entering point per 250 parking spaces. This is the statistical parking area capacity.

The dynamical capacity is calculated taking into account the number of vehicles entering into the parking area in a day, then the average parking time length and the total working time of the parking area, so the number of the required parking spaces can be calculated using the formula

$$\Sigma_{ZM} = \lambda \times t_0 / T \tag{6}$$

where:

□$_{PM}$ – average number of occupied parking spaces during the day,

λ – average number of vehicles during the day he entered the parking lot,

t – average duration of parking for one car (hours),

T – total daily work time parking (hours).

It is important to underline that once obtained optimal solution for a problem is not optimal forever and that the change in any of the mentioned elements influences a smaller or greater change of the optimal solution. Having obtained the optimal solution does not mean that there will be no waiting line, but it means that we expect this line to be the shortest possible, depending on the set out optimisation criteria: waiting time (W_Q), number of vehicles in the waiting line (L_Q), probability of cancellation (P_{otk}) or costs emerged due to waiting in line and the non-occupance of service spaces, or similar.

Since the waiting time for the vehicles waiting to enter into the parking area is not paid to the driver, the criteria for optimality for the parking area should be the probability of cancellation (P_{otk}) because if this is high, the drivers will opt for another parking area (the service system will not be congested because *m* spaces have been set in the waiting line, but many will be cancelled). Imagine for example P_{otk} = 0.33 which means that the probability is that every third vehicle will be cancelled, which is problematic and disturbing! So, according to experience and practice as well as the peculiarities of each parking location, it is necessary to evaluate the maximum tolerable value of P_{otk} and then to determine the optimal solution which this criterion meets.

Planning the dimensioning of parking area capacity by average value is an important segment in the dimensioning of the optimal parking capacity which cannot be avoided, but however the time unit can be shortened, especially if it is a seasonal phenomenon (oscillations according to months, days, hours). However, even if the planning of service places would be based on the peak load (take one day with the highest number of vehicles in the year) it does not mean that there would be no waiting lines, because the arrivals of vehicles and the parking time length are stochastic variables (discussed in section 3 of the present paper).

All of the above is the consequence of the fact that traffic (parking) service cannot be stocked like, for example a commodity and because of that it is very difficult to obtain the ideal situation. Planning capacity in traffic is one of the most difficult problems with which under-capacity or over-capacity attempts to be avoided, i.e. it is attempted to reach the maximum utilisation. One of the solutions to the problem is the implementation of a varying number of service places (ramps). But with parking areas this is not rational, however a more favourable and acceptable solution is the implementation of a system of guidance and information about free parking spaces on other locations.

4.3. Dimensioning of optimal capacity of parking area illustrated by the example of the "Delta" parking area in the City of Rijeka

The model of defining the optimal size of parking area shown in sections 3, 4.1 and 4.2 of this paper has been applied to the planning of optimal capacity of the "Delta" parking area in the City of Rijeka.

Table 1 shows the number of vehicles arrived into the "Delta" parking area in 2005.

In order to determine the intensity of relation between the number of the vehicle of the following day and the number of vehicles of the previous day, the correlation coefficient has been calculated according to (1) from section 3 of this paper.

The obtained value of the correlation coefficient $r = 0.04$ shows that there is no significant dependence in the sequence of daily arrivals of vehicles and that the hypothesis of the number of vehicles arriving into closed parking areas as a random variable can be accepted.

By doing the statistical analysis of the parking time length, we would arrive to analogous conclusion because the parking time length is also a random variable.

After such a conclusion, it can be examined whether the arrivals of vehicles and the parking time length, being random variables, behave according to some theoretical distributions.

In order to verify the aforementioned hypotheses, figure 1 shows the number of vehicles arrived into the "Delta" parking area in 2004 and 2005.

Table 1. **Number of vehicles arrived into the "Delta" parking area in 2005 by days and months**

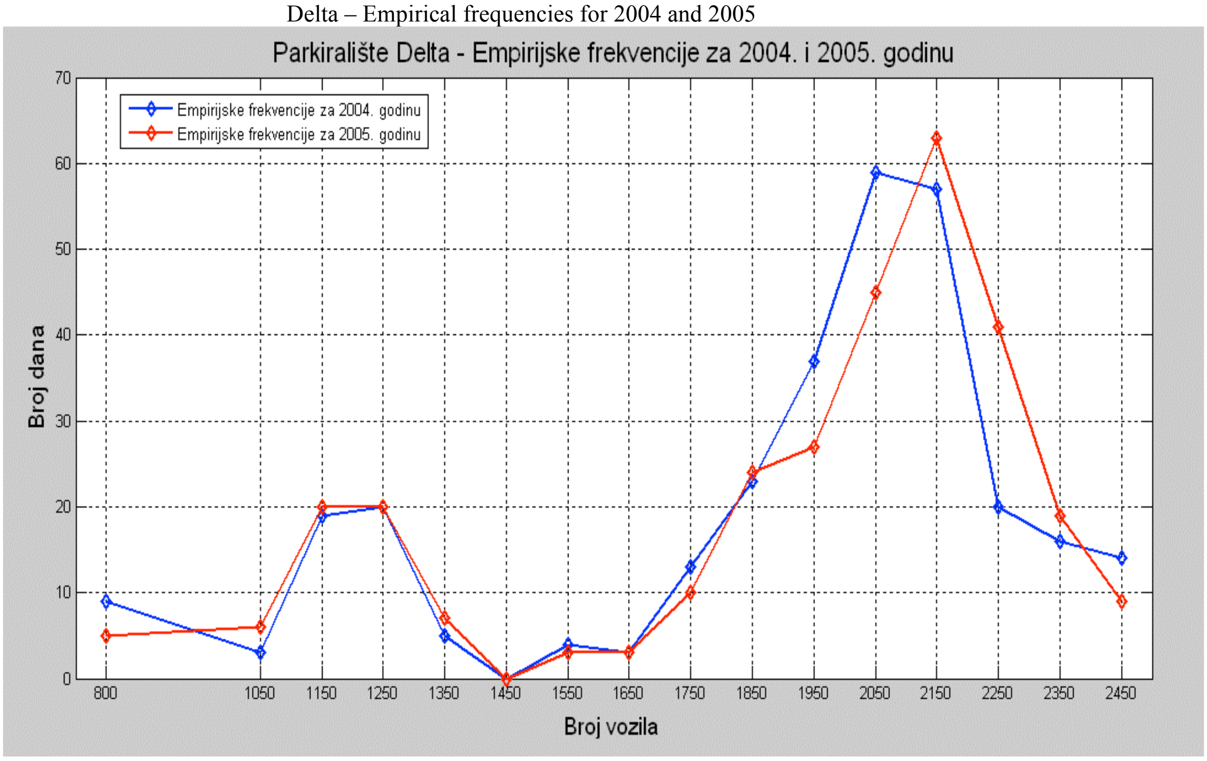
Month / Day	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Total
1.	-	1187	1859	2196	-	2189	2030	1949	2082	1189	-	1930	**16611**
2.	1158	1206	1787	1198	2279	2142	1144	1955	2260	-	2104	2093	**19326**
3.	1589	1681	1794	-	2121	2151	-	2217	1192	2130	2181	1015	**18071**
4.	1753	1824	1909	2070	2122	1225	2022	2407	-	2040	2071	-	**19443**
5.	1205	1159	1176	2012	1995	-	1837	-	2228	1874	1222	2243	**16951**
6.	-	-	-	2039	2198	2229	2116	1302	2143	1862	-	2127	**16016**
7.	1986	2174	2124	2009	1226	1829	2168	-	2040	2202	1871	2078	**21707**
8.	-	1858	1993	2098	-	2112	2220	2434	2181	-	2024	2200	**19120**
9.	1788	1825	2040	1119	2160	2037	1333	2080	2134	-	2105	2170	**20791**
10.	1821	1961	1996	-	1982	2253	-	2290	1290	2210	1998	1071	**18872**
11.	2006	2196	1980	1830	2157	1162	2305	2167	-	2119	2216	-	**20138**
12.	1119	1204	1186	2151	2280	-	2338	2440	2262	2046	1216	2005	**20247**
13.	2156	-	-	2064	2153	2238	2214	1289	2408	2050	-	2132	**18704**
14.	2259	2381	2094	2133	819	2083	1914	-	2255	2167	2201	2049	**22355**
15.	-	1884	2180	2282	-	2087	1984	-	2301	1103	2051	2165	**18037**
16.	1899	1957	2188	1266	2222	2159	1221	2442	2420	-	2099	2029	**21902**
17.	2203	2001	2258	-	2101	2126	-	2388	1399	2057	2115	1205	**19853**
18.	1877	1937	2414	2269	1980	1155	2191	1949	-	2059	2182	-	**20013**
19.	1824	1220	1339	1955	2159	-	2302	2021	2499	1956	1052	2351	**20678**
20.	1920	-	-	1873	2231	2159	1922	1233	2321	1918	-	2361	**17938**
21.	2125	1547	2196	2168	1312	2075	1859	-	2314	1853	2192	2230	**21871**
22.	1187	1608	1889	2387	-	-	2111	2479	2241	1076	2011	2270	**19259**
23.	-	1576	2141	1251	2268	2012	1277	2334	2361	-	1381	2326	**18927**
24.	1927	1846	2212	-	2322	1965	-	2228	1230	2182	2001	1109	**19022**
25.	1000	2082	1730	1945	1990	-	2122	2042	-	2069	1716	-	**16696**
26.	1080	1174	1198	2259	-	-	1785	2371	2259	2098	974	-	**15198**
27.	1223	-	-	2171	2299	1958	1798	1388	2235	2099	-	2257	**17428**
28.	1124	2137	-	2100	1183	1875	1645	-	2179	2214	2390	2053	**18900**
29.	617	-	2234	2329	-	1762	1792	2268	1867	1291	1864	1894	**17918**
30.	-	-	2157	1272	2131	2025	1025	2187	2195	-	2154	2060	**17206**
31.	1287	-	2217	-	2315	-	-	2103	-	2170	-	714	**10806**
Total	**40133**	**41625**	**50291**	**50446**	**50005**	**47008**	**48675**	**51963**	**54296**	**48034**	**47391**	**50137**	**580004**

Note: Sign "─" denotes holidays, i.e. days (Sunday and national holidays) when parking fee was not charged or months with less than 31 days.

Source: Statistical data of the "Delta" database (processed by authors)

Fig 1. **Dynamics of arrivals of vehicles into the "Delta" parking area in 2004 and 2005**

Delta – Empirical frequencies for 2004 and 2005



↑ Number of days → Number of vehicles

From figure 1 it can be noted that the number of days per number of vehicles acts very ununiform, which only corroborates the previous conclusion that the arrival of vehicles is a random variable and that the difference between the two years under observation is not significant.

However, figure 1 shows that regardless of the year, the number of days can be divided into two intervals: the first interval comprises the days with 600 to 1600 vehicles per day and the second interval are days with 1600 to 2500 vehicles per day.

It can therefore be concluded that the average number of vehicles per day will significantly vary in regard to the interval: for the first interval $\overline{X}_1 = 1{,}178$ vehicles per day and for the second 2,089 vehicles per day.

The mentioned fact points to the need for a statistical analysis of each interval distinctly, however, from the practice point of view, this causes particular problems when planning of parking area capacities is concerned. How to plan the required number of parking spaces? If we take the average number per year or the number of vehicles in the first and in the second interval, the parking area will be unutilised or on the other hand the number of parking spaces will be unsufficient in certain days of the year.

Based on the completed statistical analysis, it follows that it is essential to verify whether the assigned distribution follows the rules of a theoretical distribution, i.e. it is essential to compare the coincision of the empirical distribution and the selected theoretical distribution. The comparison has been done by applying the χ^2- test, and among the theoretical distributions, the normal and the Poisson distributions have been selected.

The calculation of theoretical frequencies depends on the selected theoretical distribution. For the Poisson distribution $f_{t_i} = N \bullet P(x)$, however, the number of vehicles is a relatively large number (around 2,000) so the calculation of theoretical frequencies has been rendered very difficult, even with the use of a computer. Statistical literature mentions that the tabulation of values for n > 20 with the Poisson distribution is not practical.

Since the Poisson distribution when $n \to \infty$ becomes less asyimmetrical and aspires towards a normal curve, in these cases $P(\lambda)$ can be approximised with normal distribution, which has been done in figure 2.

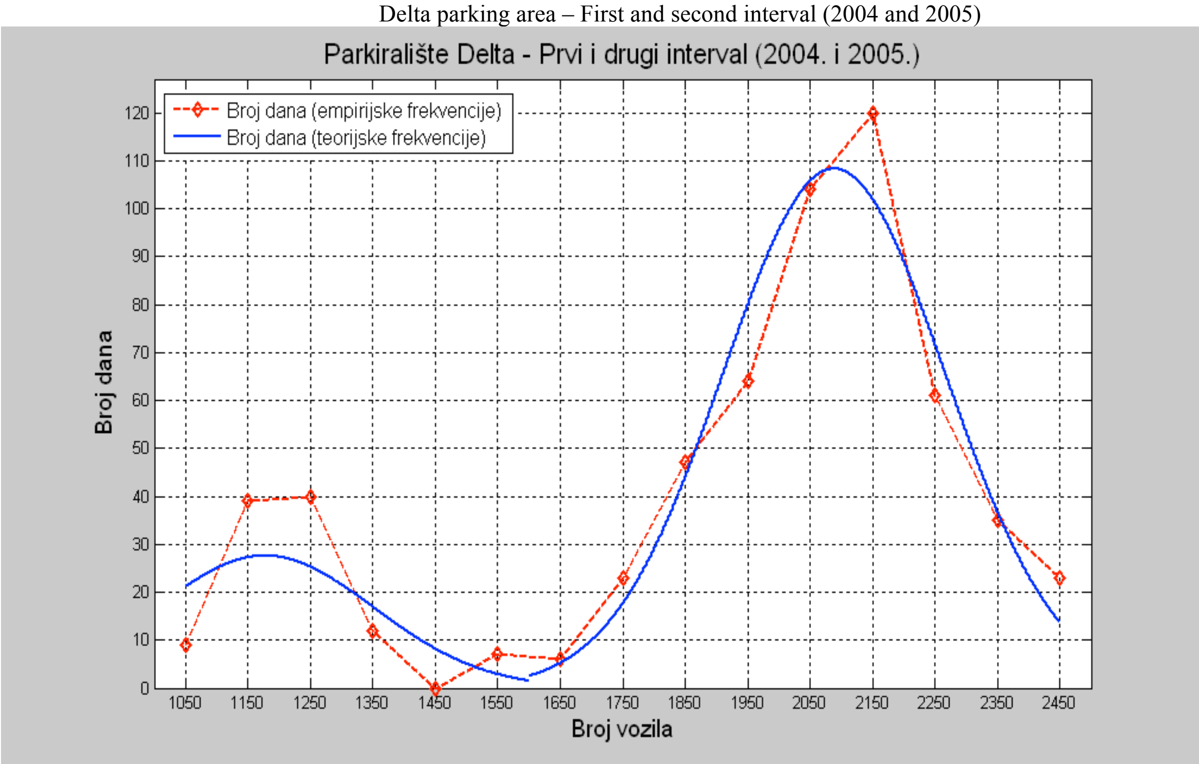
The comparison of empirical data with the normal distribution has been done distinctly for the 1st and the 2nd interval. Figure 2 shows evidently that the days in the 2nd interval adjust well to the normal distribution, but in the 1st interval this is not the case. This conclusion is corroborated by the values χ^2:

1st interval: $\chi^2 = 37.116$ $\chi^2_{0..99} = 11.341$; significant difference

2nd interval: $\chi^2 = 16.140$ $\chi^2_{0..99} = 16.812$; statistically not significant difference.

Fig 2. **Comparison of empirical and theoretical frequencies for the "Delta" parking area for 2004 and 2005**

Delta parking area – First and second interval (2004 and 2005)



↑ Number of days → Number of vehicles

In these cases, when empirical distributions cannot be reduced to some theoretical distributions, in theory it is advisable to apply either simulation or use of hypothesised distribution, at least as approximation of the real problem. The following authors support approximation:

- S. Vukadinović [12] states that the capacity and other characteristics of the service process depend relatively little on the form of distribution, and more on the average value (parameter λ),
- D. Gross [2] indicates that for the statistical queuing models the hypothesis that the intensity of flow of arrivals and the intensity of the serviceing follow the Poisson distribution is most often accepted.

Based on the mentioned facts, we accept the hypothesis that the distribution of days per number of vehicles on the parking behave according to the Poisson distribution.

On the basis of the above results, it follows that when defining the optimal parking area capacity the use of the queuing theory is justified.

The number of spaces in the waiting line: total length of space appointed to the waiting of the vehicles in order to be able to enter into the parking area is 80 m; if the average length of a vehicle in the waiting line is 5 m, it follows that the maximum of 16 automobiles can be present on the reserved space in one moment, i.e. $m = 16$.

Therefore, the observed servicing process is classified as a queuing problem with finite number of vehicles in the waiting line, $M/M/S/16$. Every next vehicle (17^{th} one) in the waiting line will be cancelled from the waiting line because the line of vehicles would otherwise continue on the roads intended for the circulation of motor vehicles.

The intensity of the vehicles' arrival flow: the calculation will use the average number of vehicles arriving daily into the parking area in 2005; λ = 1,921 vehicles per day (with 14-hour working time and 302 days a year because the rest of the days are holidays and the parking fee is not charged) or the average of 137 vehicles/hour, i.e. 302 vehicles/hour in peak hours and maximum load of the parking area.

Intensity of servicing: the intensity of servicing (μ) is obtained in the calculation as a reciprocal value of the average servicing time ($\bar{t}_{usl}$ = arithmetic mean of the servicing time); if the servicing time represents the time necessary for the driver (parking area customer) to stop its vehicle in front of the entry terminal, to take the parking ticket and to enter into the parking area and it amounts to an average of 15 s, then $\bar{t}_{usl}$ = 15 s = 0.0041666 hours, and the intensity of servicing

$$\mu = 1/\bar{t}_{usl} = 240 \text{ vehicles/hour.} \tag{7}$$

It is evident that in peak hours more vehicles arrive in a unit of time in relation to the possibility of their servicing with only one entry ramp. Based on the definition of basic parameters on the "Delta" parking area as a servicing system with finite length of the queuing line ($M/M/S/16$), we arrive to $\rho = \lambda / \mu = 302/240 = 1.25833$, and with adequate formulas [14] indices of parking area functioning have been calculated.

Based on the conducted analysis of the functioning of entry ramps on the "Delta" parking area, as a mass service system with finite length of waiting line, it is deductible that the increase of the number of entry ramps influences the increase/decrease of indices values of the parking system. Two variants have been analysed: variant A, when the average number of serviced vehicles in a day with two entry ramps is taken into account, and variant B when the maximum number of serviced vehicles in the peak hour of the occupancy (for example from 8 am to 9 am) with one, i.e. two ramps is taken into account.

All the obtained indices for the average number of vehicles, served during the day, point out that only one ramp is necessary, but the one capable of servicing all the arrived vehicles. Therefore more attention will be paid here to variant B where it is evident that a large problem exists when a high number of vehicles in peak hours want to enter, but also exit the parking system. The question arises whether the reason for the creation of waiting lines in front of the entrance points into the parking system is the insufficient number of parking spaces within the system or lower/insufficient number of service places, in this case entry ramps.

Analysing the indices for variant B and the maximal number of vehicles the following arises:

- in the system with only one entry ramp and the maximum number of vehicles in peak hour, the ramp load degree (ρ) is higher than one which imposes the conclusion that there will be the amassment of vehicles at the entering points into the parking area which will eventually result in the impossibility of a normal functioning of the system and in a "large" probability of cancellation (P_{otk}). The system with 2 entries has ρ_s smaller than one, and therefore meets the basic requirement of the customer (vehicle) being served eventually,

- probability that there is no vehicle in the queuing system (P_0), i.e. that the capacity of the service place is inused is very small for the the system with 1 entering point, whilst the system with 2 entering points has a significantly higher probability of 22.76%,
- probability that the vehicle entering into the system will not be serviced, i.e. that it will be cancelled, in the system with 1 incoming terminal is 20.86%, whilst for the system with 2 incoming terminals it is minor,
- probability that the vehicle entering into the system will be serviced (servicing probability) in the system with 1 incoming terminal amounts to 79.14%, whilst for the system with 2 entries it aspires to be 100%,
- average number of vehicles in the waiting line for the system with 1 incoming terminal is 12 vehicles, whilst for those with 2 entries in average no vehicles need to wait in line to enter the parking area,
- average number of vehicles currently serviced is 1 – it is the vehicle entering the parking area,
- average waiting time in line for the system with 1 incoming terminal is 148 seconds (2.5 minutes) and 10 seconds for the system with 2 incoming terminals,
- average servicing time for the system with 1 incoming terminal is 11.88 seconds and 15 seconds for the system with 2 incoming terminals,
- average time inside the servicing system for the system with 1 incoming terminal is 160 seconds (2.664 minutes) and 24 seconds for the system with 2 incoming terminals.

Based on the presented it can be easily concluded that having only one entry ramp worsens considerably the quality of the servicing of vehicles with the possibility of cancellation in the system, whilst values for cases when two entry ramps are put on the entering point improves considerably the servicing quality. It can be concluded that two entry ramps is the optimal number considering the intensity of arrival of vehicles into the parking area and their servicing time at the entry of the vehicles.

If we take into consideration the fact that there is an average of 1,915 vehicles arriving daily into the "Delta" parking area and that the average length of parking is 2 hours[2] and the opening hours are 14 a day, the formula (6) shows us that the required number of parking spaces is 274. Since the actual number of parking spaces is around 500[3], the question still arises why there are long lines at the entry. Since 40% of the parking area is occupied by privileged parking ticket holders (residents and companies) we arrive to the real reason of the lack of parking space.[4] This category of users, leaving their vehicles for longer periods, is the cause of the problem.

It is therefore necessary to calculate the required number of parking spaces on the "Delta" parking area in relation to the number of vehicles entering into the parking area during the day, average time of stay in the parking and the total working hours of the parking. If the calculation takes into account the hypothesis about the mentioned percentage of occupancy of the parking area by privileged parking ticket holders (N) and the fact that λ = 1,915 and the average stay time t = 2 hours and the working hours T = 14 hours, by inserting these into the formula for the number of parking spaces (8) we arrive to the required number of parking spaces in the "Delta" parking area of 602 spaces.

$$\sum EM = \sum PM - \frac{N \times t_1}{T} \tag{8}$$

All of the above outlined leads to the conclusion that lines of vehicles forming at the entry into the mentioned parking area are not caused by poor organisation of entry ramps or their insufficient capacity, but by the fact that the City of Rijeka chronically lacks parking space, i.e. there is a disproportion between the supply and the demand for parking space.

5. CONCLUSION

The arrival of vehicles into parking areas has large oscillations during the year, month, day and hour. It is therefore difficult to pre-determine the number of vehicles arriving to or leaving a certain parking area on a certain day. However, for the planning of a parking area capacity it is useful to determine whether there is a certain regularity in the arrival of vehicles, i.e. in the number of vehicles and in the time of the arrival of these vehicles into the parking area.

Since arrivals of vehicles and length of time of their servicing can be taken as random (stochastic) variables and the empirical distribution of these variables then approximised with adequate theoretical distributions, for parking areas it is therefore possible to apply the analytical approach, i.e. to use formulas set out by the queuing theory in order to calculate the parking functioning ratio.

A dilemma arises from the question whether the entry ramp or the parking space is a service place. Considering the fact that there are relatively poor experiences presented in national and international literature, in this paper the authors define entry ramps as service place and they base their analysis upon them; based on the obtained results they have calculated the number of parking spaces and therefore defined the required parking area capacity. It can therefore be deducted that, when the optimal size of the parking area is being defined, it is not sufficient to take into account only the entry ramps, but it is necessary to take also into account the number of parking spaces because the increase of the number of entry ramps does not mean an increase of parking area capacity.

The objective of this paper is to demonstrate that by applying the queuing theory the optimal number of servers (ramps) and the required capacity (number of parking spaces) in closed parking areas can be defined. After all, the verification of the set out model of planning of optimal capacity of parking area capacity upon the actual "Delta" parking area in the City of Rijeka has shown the indisputable applicability of the results of a scientific research to actual parking area capacities. A particular merit of the model is its universal applicability because the presented methodology can be applied to any other closed parking area, i.e. parking area with ramps in current or future, changed conditions.

REFERENTIAL NOTES

[1]"Open parking system" refers to those parking areas in which the service is charged via parking machines and/or mobile phones, through the so-called *m*-parking service.

[2]Average stay on the parking (not including customers with privileged pre-paid tickets) is calculated in the following way: total income of the analysed day/number of charged parking tickets (for example 3.000,00 kuna (income of the analysed day)/475 (number of charged parking tickets) = 6.32 kuna (average income per charged parking ticket). Since parking fee of the "Delta" parking area is 4.00 kuna per hour, i.e. 8.00 kuna for two hours, we calculate that the average time of stay is 2 hours.

[3]Actual number of parking spaces on the "Delta" parking area is 458, but in author's experience, this parking area can easily accommodate around 500 vehicles.

[4]Data based on the experience of one of the authors and documentation of the company Rijeka promet.

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PRIMJENA TEORIJE REDOVA ČEKANJA U PLANIRANJU OPTIMALNOG BROJA USLUŽNIH MJESTA (RAMPI) NA ZATVORENIM PARKIRALIŠNIM SUSTAVIMA

SAŽETAK

Temeljni cilj ove znanstvene rasprave je kako učinkovito organizirati prometne prostore i napose veličinu parkirališnih kapaciteta čime se domicilnom stanovništvu, ali i turistima omogućava kvalitetna parkirališna usluga kao sastavnica cjelokupne ponude gradskih i turističkih destinacija, a investitoru u parkirališni kapacitet povrat investicije u primjerenom roku. Koji je to optimalan kapacitet i kako ga izračunati na najbolji mogući način povezujući parkirališnu ponudu i potražnju ? U ovom je radu prikazana primjena teorije redova čekanja u planiranju optimalnog broja uslužnih mjesta (rampi) na zatvorenim parkirališnim sustavima, budući da se parkiralište može definirati kao sustav opsluživanja. Prikazani je model testiran na primjeru parkirališta „Delta“ u Gradu Rijeci, a posebna je vrijednost modela univerzalna primjenjivost. Takvim pristupom dokazano je da se primjenom teorije redova čekanja može utvrditi optimalan broj uslužnih mjesta (rampi) na zatvorenim parkirališnim sustavima.

Ključne riječi: *optimalan broj uslužnih mjesta (rampi), teorija redova čekanja, sustav opsluživanja, planiranje parkirališnih kapaciteta, parkiralište "Delta" u Gradu Rijeci*