

Modelling of the recent crustal movements at the territory of Croatia, Slovenia and Bosnia and Herzegovina

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The paper presents the results of determining vertical displacements of the Earth's crust at the territory of Croatia, Slovenia, and Bosnia and Herzegovina on the basis of the data obtained in the process of establishing geometric levelling networks of the highest order of accuracy. These are the networks of the Austrian precise levelling, I. levelling of high accuracy and II. levelling of high accuracy that was established successively within the period of about 100 years (1874–1973). In accordance with the hypotheses made and the selected methodology for defining the displacements, a concrete numeric quantification of displacements has been obtained, the size and the direction of Earth's crust displacements have been determined and their characteristics and empirical parameters of behaviour indicated. It is shown that bench mark displacements are of significant amount as related to the measurement accuracy of height differences in levelling networks and that long-wave and short-wave component of their variation can be noticed in correlation with the positional distribution of bench marks. The displacements have been modelled by means of so called grid models created by the combination of the regression modelling and the minimal curvature surface modelling. Thus, on the basis of modelled vertical displacement values the speed of uniform vertical crustal movements has been determined. In spite of the fact that a very simple methodology of displacement and movement determination has been used, that is based on the absolute heights of identical bench marks positioned in various epochs – networks, the obtained results present an adequate starting point for the selection and application of more sophisticated methodologies in determining and modelling of displacements in the forthcoming research, i.e. the starting point that enables interdisciplinary explanation of geodynamic processes leading to the changes in the shape and geometry of the Earth's crust at the observed area.

Keywords: recent crust movements, vertical movements, bench marks, levelling networks, grid model

1. Introduction

Continuous monitoring of the position of discrete points on the Earth's physical surface over a certain period of time is an important data source for

the determination of recent vertical movements, and for the determination of appropriate kinematic parameters. Various types of geodetic points included into the state geodetic networks (triangulation networks, trilateration networks, levelling networks, GPS networks, etc.) materialised on the Earth's physical surface by means of adequate surface or underground construction stabilisation are considered appropriate for such purpose. They are positioned by applying geodetic methodologies of relative and absolute positioning in accordance with the stabilisation durability and the circumstances in which the surveys are repeated in various moments of time (epochs). Although the primary purpose of geodetic points is to realise state, i.e. regional, continental or global geodetic reference coordinate systems, it is obvious that the stabilisation durability, preservation in the course of time, availability of repeated measurements data in various epochs, positioning quality and positional distribution of points across the state territory or regions make them a suitable base for displacement determination. In this situation the displacements and movements offer the possibility of getting concrete, i.e. quantity related insight about how far the Earth's crust and its contact surface with the atmosphere (topographic surface) are subject to the influences of various endogenous and exogenous geodynamic forces, i.e. to what extent its shape and geometry are changing.

The most interesting are those geodetic points whose quality of construction stabilisation does not allow the appearance of its own movements in the course of time, but the displacements and movements can be regarded as representative for the Earth's crust. Along with the given precondition, the most interesting are by all means those geodetic points that are positioned using the geodetic measurements of the highest order of accuracy, having the quality of survey in various epochs adequately homogenous and obviously higher than the size of displacements. At the same time, especially useful are the points that make a sufficiently large and in terms of position adequately distributed base in the region of interest for the determination and modelling of displacements. Namely, the information about the displacements on a series of discrete points enables their generalisation to the total physical surface of the Earth by means of adequate mathematical models. The operating value of permanently stabilized geodetic points for the purpose of determining the displacements and movements is also significantly emphasized by the total number of geodetic resurveys, i.e. number of epochs that they have been undertaken during the course of time. All given elements are consequently closely connected not only with just scientific and professional context of planning and systematic performance of so called fundamental geodetic works at the state territory, but equally with historical, technological, organisational and other circumstances that are specific for certain country or countries, because the given circumstances actually defining the concrete results of the fundamental geodetic works.

After being concerned only and exclusively with the vertical component of the Earth's crust movements, and in accordance with historical and geodetic

heritage dating from the period of Austro-Hungarian Monarchy and Yugoslavia, there were the appropriate data for the territory of Croatia and the neighbouring Slovenia, and Bosnia and Herzegovina for the purpose of determining vertical displacements from three so called fundamental levelling (height) networks of geometric levelling of the highest order of accuracy. In the historical or time continuity, and in accordance with the principles and professional criteria of performing fundamental geodetic works, these networks served for the realisation of reference vertical coordinate systems of Austro-Hungarian Monarchy, Yugoslavia, and the Republic of Croatia after its becoming independent, Rožić (2001). These are the levelling networks of so called Austrian precise levelling – APN that were established in the time of Austro-Hungarian Monarchy, and the levelling networks of the so called I. levelling of high accuracy – INVT and the II. levelling of high accuracy – IINVT established in the time of former Yugoslavia.

Within the scope of the realisation of the mentioned levelling networks the construction stabilisation of a large number of geodetic height points – bench marks were made at the territory of Croatia, Slovenia, and Bosnia and Herzegovina that are mutually connected by means of measured height differences structured in levelling lines and levelling figures, i.e. levelling networks of firm geometric structure or configuration, using the methodology of relative height positioning by means of geometric levelling. In spite of some inconvenient circumstances, i.e. primarily destroyed part of initially stabilized bench marks and the need to stabilise completely new bench marks in the process of establishing younger levelling networks, it can be stated that there is a quite sufficient data material to be used for the determination of vertical displacements. Although a relatively small number of identical bench marks were consequently included into the all three levelling networks, there are a significant number of identical bench marks that are separately encompassed by the APN and INVT networks, and the bench marks that are separately encompassed by the INVT and IINVT networks. In spite of the fact that in both mentioned cases various bench marks were included into the networks evidently having different planar or ellipsoidal position, they were distributed along the same territory, i.e. the territory of Croatia, Slovenia, and Bosnia and Herzegovina. This fact results from not so much complete or exclusive incompatibility of levelling line routes, i.e. the incompatibility of geometric configuration of INVT network in comparison with the configuration of APN network (older network) and the IINVT network (younger network), but from the influence of the fact that the bench marks have not been preserved during time because of a series of different reasons (war destruction, reconstruction of roads and railroads, demolition of old houses or other construction objects etc.).

Since in about the last twenty years the original data of levelling measurements, as well as other relevant data regarding the levelling networks APN, INVT and IINVT have been collected and unified from the archives and various data sources, and have also been verified, translated from analogous into

digital format, analysed and professionally evaluated, they were made suitable to be used for the purpose of determining and modelling vertical displacements of the Earth's crust. Without any further detailed consideration of geodetic scientific and professional methodologies referring to relative and absolute bench mark height positioning, designing and realisation of levelling networks, as well as technical standards and professional norms applied in the realisation of the works on stabilisation of bench marks and network surveying, a series of useful data about their genesis, realisation, characteristics and original usage can be found in Rožić et al. (2006).

2. Data and methodology of the vertical displacements determination

Very simple methodology in determining vertical displacements of bench marks related to the usage of data obtained from levelling networks surveying is based on the comparison of absolute vertical positions of identical bench marks determined in various epochs, i.e. contained in different levelling networks. Namely, along with the definition and the realisation of the unique vertical reference coordinate system, for each bench mark included in specific network and referred to the belonging epoch, the values of absolute vertical position can be determined using appropriate processing of the measurement data for each levelling network separately. Positions are expressed with the belonging absolute height coordinate together with the indicators of the quality of the positioning made. If the height reference system had been realised and adequately spatially oriented in relation to the physical reality of the Earth's body, i.e. primarily in relation to the geoid (usually obtained by the mean sea level), the term of "height above mean sea level" or just height is colloquially used for absolute height coordinates of bench marks. The information about the height of the same bench marks in various epochs, and within the frame of the same reference height coordinate system enable the determination of its vertical displacements because the displacements of bench marks are directly equal to the differences in their heights.

Although the presented methodology and the concept are extremely simple, the practical realisation is unfortunately very complex. The reason lies first of all in the large amount of data collected by relative height measurement (geometric levelling), extensive numerical data processing and analysis of these data (identification and elimination of outliers and systematic measurement errors) and the application of adequate adjustment methodology for the purpose of determining unique values of bench mark heights (the elimination of random measurement errors). Additionally, the need of reliable identification of exclusively those bench marks that are really contained in various networks (epochs) and that are at the same time relevant for the determination of vertical displacements appears as a significant problem in the process. Their relevance results from the fact that the change of their vertical position

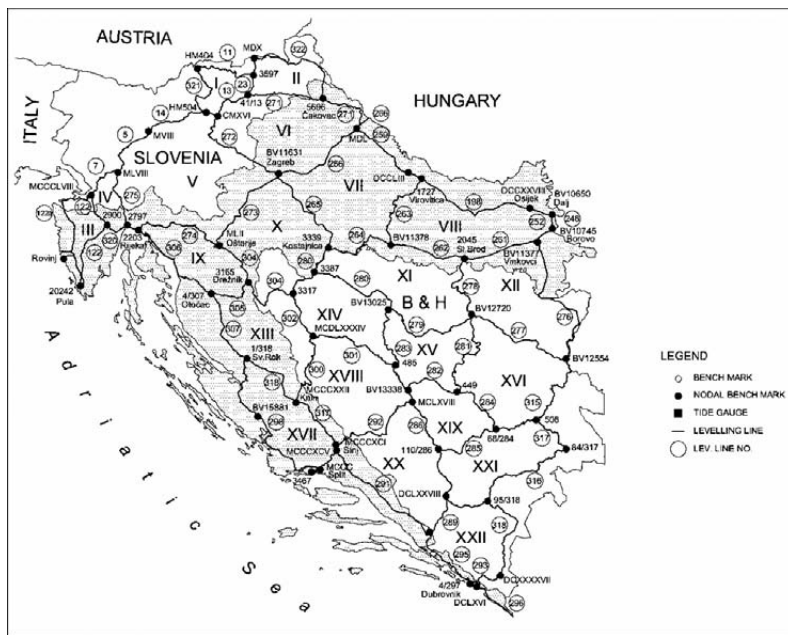


Figure 2. The network of the I. levelling of high accuracy – INVT.

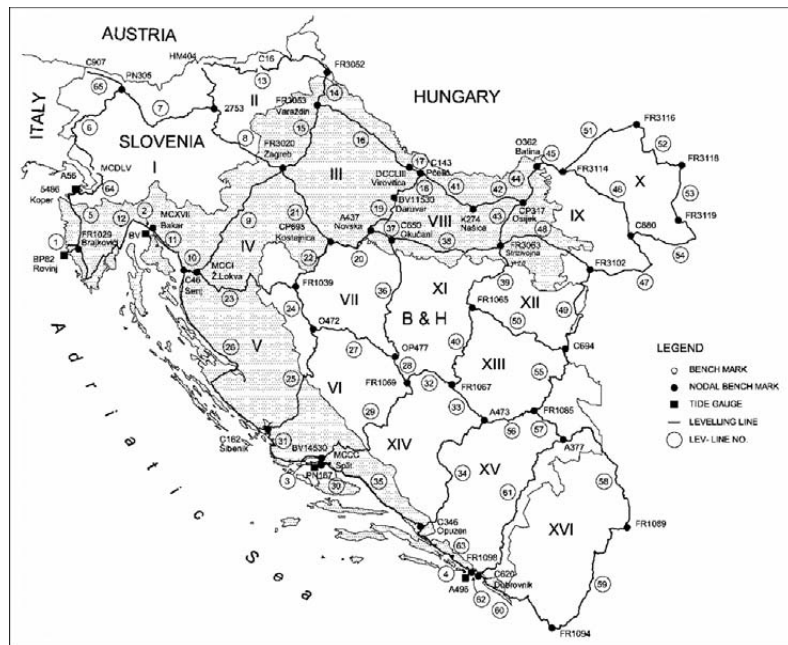


Figure 3. The network of the II. levelling of high accuracy – IINVT.

sation of any of the networks, there have been no gravimetric measurements made, the influence of non-parallel level surfaces of the gravity field has been eliminated by using normal gravity field.

The basic data about the surveying time dynamics of levelling networks APN, INVT and IINVT are given in the Table 1. It is obvious that the levelling network APN has been realized gradually and in a very long period of time, which is the consequence of objective organisational, economic and other circumstances in Austrian-Hungarian Monarchy having Croatia, Slovenia, and Bosnia and Herzegovina as their integral parts, and in the case of younger networks the realisation was essentially shorter. In the case of the network IINVT, as the youngest network, the time interval of network surveying is practically congruous with the theoretical ideal. It should be pointed out, that the length of time interval used for network surveying has by all means a direct influence on the determination of vertical movements because the heights of bench marks obtained by means of adjustment should be associated with explicit and unique time moment, i.e. epoch. In specific case, the surveying epoch of each of the levelling networks has been determined as the surveying moments mean value of each levelling line that constitute a single network. It is obvious that the time interval between the epochs of network surveying, i.e. the epochs of surveying the network APN and the network IINVT, assumes the amount of 78.3 years.

The explained procedure of determining the epochs of individual networks realising can certainly be subject to criticism since the time intervals of single networks surveying are very long, especially of the network APN, and somewhat more moderately of the network INVT. On the other hand, there is also a fact indicating that all surveying data, i.e. the surveying data referring to each single network are processed (adjustment) all together as homogeneous unit, i.e. regardless of various time moments of surveying its individual structural parts, i.e. individual levelling lines.

In order to compare the bench mark heights, independently determined from the data sets of each levelling network, it is necessary to meet a few elementary principles:

– The heights of the identical bench marks encompassed by the networks APN, INVT and IINVT, referring to various epochs (APN – 1. epoch, INVT – 2. epoch and IINVT – 3. epoch), must be determined in the same reference

Table 1. Time dynamics of surveying the levelling networks APN, INVT and IINVT.

Epoch	Levelling network	Network survey time interval (years)	Mean epoch of levelling lines survey (years)	Time interval (years)
1.	APN	1874–1909 (35)	1892.8	56.2
2.	INVT	1946–1963 (17)	1949.0	
3.	IINVT	1970–1973 (3)	1971.1	22.1

height coordinate system. Thereby, the definition and the realization of the height system includes the selection of adequate type of the heights in theoretical sense (orthometric heights, normal heights, normal-orthometric heights, dynamic heights, ellipsoidal heights, etc.), as well as the absolute orientation of coordinate system as related to the Earth's body (height datum), Hofmann-Wellenhof and Moritz (2005).

– The data of relative height measurement of single levelling networks belonging to various epochs and completely independent of each other should be processed and adjusted completely independently, but using the same methodology of data processing, i.e. by applying the same mathematical model and method of adjustment.

– All levelling networks should be the same or at least of similar geometric configuration, and should encompass approximately the same territory.

– The quality of levelling measurement and absolute vertical positioning of bench marks in all levelling networks, i.e. various measurement epochs should be homogeneous to the acceptable level.

– The same bench marks in various networks that show their own movements caused by low-quality construction stabilisation should be identified and excluded from the process of vertical movements' determination.

Respecting the given principles the data processing and adjustment of all levelling networks was done. Due to the lack of gravimetric measurements, being practical and at the same time pragmatic solution for the realisation of reference height coordinate system, the so called system of normal-orthometric heights was adopted. The spatial orientation of height coordinate system was made by adopting explicit level surface of gravity field as reference surface for the determination of heights, i.e. the surface of so called "zero absolute height". The spatial orientation of this surface was made on the location of the oldest Croatian tide gauge in Bakar ($\varphi = 45^\circ 18' 28''$ N, $\lambda = 14^\circ 32' 19''$ E), by determining and fixing the absolute height of so called reference bench mark of the height system above mean sea level. This is the bench mark BV that is connected with all three levelling networks. The height for this bench mark was determined to be 2.6601 m, corresponding to the mean sea level of the Adriatic Sea for the time epoch 1971.5 year derived from the data of continuous measurement of sea water level in the period of 18.6 years, Bilajbegović et al. (1986).

Separate and mutually independent adjustment of the networks APN, INVT and IINVT was made by applying the mathematical model of indirect measurements and the method of weighted least squares, Ghilani and Wolf (2006). On the basis of the network data adjustment unique values of heights of all bench marks were determined, as well as adequate indicators of measurement quality. The "a priori" measurements accuracy has been determined by means of reference probable errors u_F calculated from networks levelling figure misclosures and "a posteriori" accuracy by means of reference probable

Table 2. Measurements accuracy of the levelling networks.

Epoch	Network	u_F (mm/km)	u_o (mm/km)
1.	APN	± 3.20	± 3.27
2.	INVT	± 1.68	± 1.65
3.	IINVT	± 0.79	± 0.79

errors u_o calculated from the corrections of measurements obtained by adjustment, Table 2.

The data given in the Table 2, as well as the previous explanations referring to the networks APN, INVT and IINVT, speak in favour of the possibility to determine vertical displacements of bench marks on the basis of the heights obtained by means of adjustment. The heights of bench marks from all networks or time epochs have been determined in the same height reference coordinate system by applying the same type of heights and the same methodology of surveying data adjustment is used. The networks are located on the same territory and have to some extent quite congruent geometric configurations, although not completely identical. Since the methodology of geometric levelling is the most accurate geodetic classical terrestrial methodology of relative height positioning, the obtained results of heights and the quality of height positioning of bench marks can be regarded as the basis of adequate quality and homogeneity for the determination of bench mark vertical displacements, especially if the size and shape of the observed territory is taken into consideration.

Still, the analysis of the displacements calculated by means of bench marks heights, and especially their interpretation, should include also a certain amount of attention. First of all due to the fact that partly different mathematical models and the parameters of these models in the determination of the scale corrections of levelling staves and of the normal-orthometric correction of height differences in various epochs, i.e. networks, have been used. Since the mentioned corrections have been taken over directly in the original form from the original archive documentation, the influence of mutual diversity of the used models and parameters is reflected consequently on the bench mark heights obtained by means of adjustments, i.e. on bench mark displacements. One should also be careful regarding the fact that so called normal-orthometric heights are not completely appropriate for the determination of vertical displacements because of its theoretical characteristics. As well, the fact exists that geometric configurations of networks are mutually partly different. Namely, the differences in network geometric configurations cumulate the differences in the measurement errors propagation that reflect on the adjusted height values within the scope of the error elimination by means of the methods of least squares. The aggravating circumstances in the analysis and interpretation of displacements is by all means the previously described method of determining and associating the discrete time moments in which single net-

works were realized, i.e. the epochs, being especially emphasized in the case of APN network, and somewhat less the INVT network, because pretty long time periods spent in measuring single networks were focused on one explicit and unique time moment.

In spite of all given facts, the vertical displacements determined on the basis of bench mark heights can be, even if it is in the first approximation, a very indicative clue or indicator, especially if the displacements are significant in terms of their size and respectably larger than the unfavourable effects of previously stated influences. If the stated influences are e.g. at sub-millimetre and millimetre level (scale correction and normal-orthometric correction), i.e. if they partly reach even centimetre level (the influence of type of heights and the partial diversity of geometric configurations of networks), the displacements of bench marks at the decimetre level can be absolutely regarded as significant and usable. Especially if they are determined systematically and consistently, regardless of the simple methodology used for their determination. They can offer initial and adequately clear insight into the displacement sizes, the signs (the relative raising and sinking of the Earth's crust), positional distribution along the observed territory, empirical patterns of behaviour and other properties.

It does not hurt to point out the fact that the methodology of determining bench mark displacements by means of heights is very quick and efficient, because bench mark heights are just one of the several final results of levelling network adjustments, although they do not need to be the primary goal of these adjustments. Namely, in concrete case, the analysis, evaluation and homogenizing of levelling surveying data taken over from analogous original sources, the identification and elimination of measurement errors, the analysis of the presence of the remaining systematic influences, the determination of adjusted height difference values and the measurement accuracy indicators were the primary goal of measurement data processing, and the bench mark heights were a pragmatic and very useful result of this process. In other words, like the result at reach of hands after initial establishment and evaluation of original networks measurement data they can be used directly and purposefully without any further delay or complication. Of course, only providing that all networks are referring to the same height reference system, which was not difficult to achieve on the basis of the strategy and early planning of the execution of works.

3. Determination of bench mark vertical displacements

On the basis of extensive measurement data, adjustment data, bench mark position data represented by the ellipsoidal longitude λ and latitude φ (Bessel ellipsoid, the initial meridian "Greenwich") and other bench mark and levelling network data available from adequate registers and archive sources, the iden-

tical bench marks contained in the levelling network APN and INVT (1. and 2. epoch) were identified, the identical bench marks contained in the networks INVT and IINVT (2. and 3. epoch) and as well as the identical bench marks that are contained in all three networks (1., 2. and 3. epoch).

The identification of bench marks was very complex and demanding unlike expected and in spite of the fact that all necessary data had been transformed from analogous into digital form. Namely, the low-level of quality of original archive documentation that the data had been taken over from, the fact that some individual bench marks have had their numerical designations used in register and databases changed in the course of time, the fact that some individual bench marks were restabilised with the original numerical designations preserved in spite of significant vertical repositioning, the fact that sometimes completely different bench marks are inadequately marked with the same numerical designations, etc. had very unfavourable effects. The errors contained in the height difference measurement data were also noticed in some cases, as well as the errors of ellipsoidal bench mark positions, the unavailability of height measurement data was discovered to smaller extent that made the height positioning of some bench marks impossible, and the other various phenomena indicating or raising suspicions about the reliability of certain bench marks usage.

Along with the identification of bench marks included in various levelling networks that have indicated to be convenient for the determination of displacements, a great attention was paid to the identification and elimination of all those bench marks that showed a significant non representative values of presented vertical movements, i.e. the inconsistency of the displacement size as related to the vertical displacements of other bench marks in the local area on the one hand, and as related to the general trend of the change of displacement values on the other hand, i.e. so called local and global spatial outliers.

As the final result of the extensive data processing of levelling network data, of the identification of bench marks included in them and of the elimination of all bench marks that were found unreliable to be used, either because of geodetic professional reasons or because of occurring properties of so called global and local spatial outliers, definite data series of bench marks that can be used for concrete determination of vertical displacements at the territory of the Republic of Croatia, Slovenia, and Bosnia and Herzegovina were prepared.

In the levelling networks APN (1. epoch) and INVT (2. epoch) there were altogether 390 identical bench marks identified, out of which 49 bench marks are contained at the same time also in the levelling network IINVT (3. epoch), Fig. 4. The bench marks were originally stabilized and for the first time included into the survey of APN network, and in the course of time included into the survey of the INVT network in accordance with their preservation, Rožić (1999). The mentioned 49 bench marks are marked on Fig. 4 with a point within a checkbox. In the levelling networks INVT (2. epoch) and IINVT (3. epoch) there were altogether 1987 identical bench marks identified, Fig. 5.

Apart from the already mentioned 49 bench marks, all remaining bench marks were originally stabilized and for the first time included into the survey of the network INVT, and in accordance with their preservation in the network IINVT as well. It is obvious, that in younger networks INVT and IINVT, with a smaller time interval between them, there are a significantly larger number of identical bench marks, practically three times larger as related to the number of bench marks in the networks APN and INVT. Unfortunately, as a consequence of objective circumstances, only 49 identical bench marks are encompassed by all three networks, i.e. all three epochs. On the basis of the insight into Fig. 4 and Fig. 5, in spite of a small presentation scale, a specific pattern of positional distribution of bench marks across the territory of Croatia, Slovenia, and Bosnia and Herzegovina can be seen. This pattern is the consequence of design methodology and the realization of levelling networks of geometric levelling, and it is manifested in a characteristic bench mark line setup at relatively short distances, because the bench marks are placed usually at the distances of 0.3 to 0.5 km when stabilized. Namely, the bench marks positions are arranged along levelling lines as basic structural parts of the network. The pattern of positional distribution of bench marks at the observed

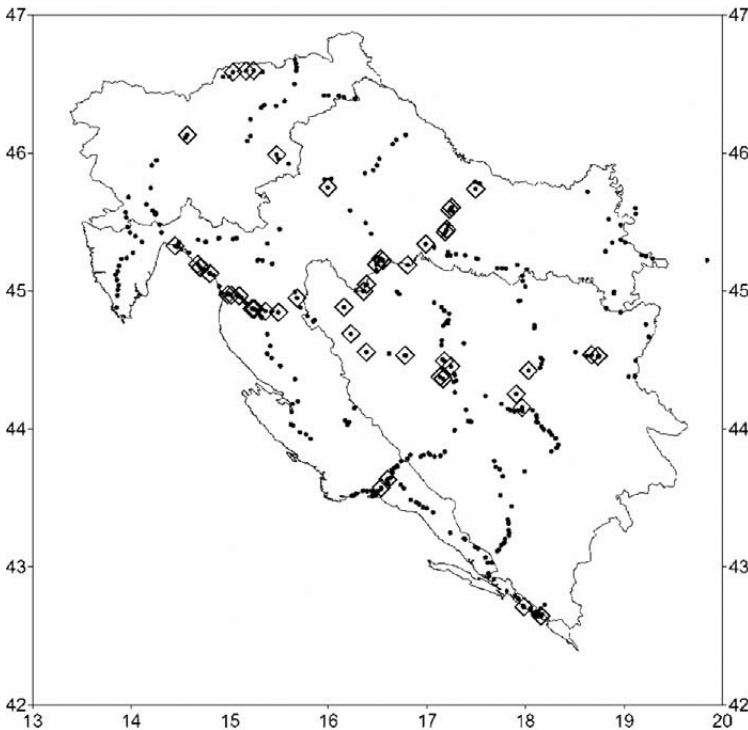


Figure 4. Identical bench marks encompassed by the levelling networks APN and INVT.

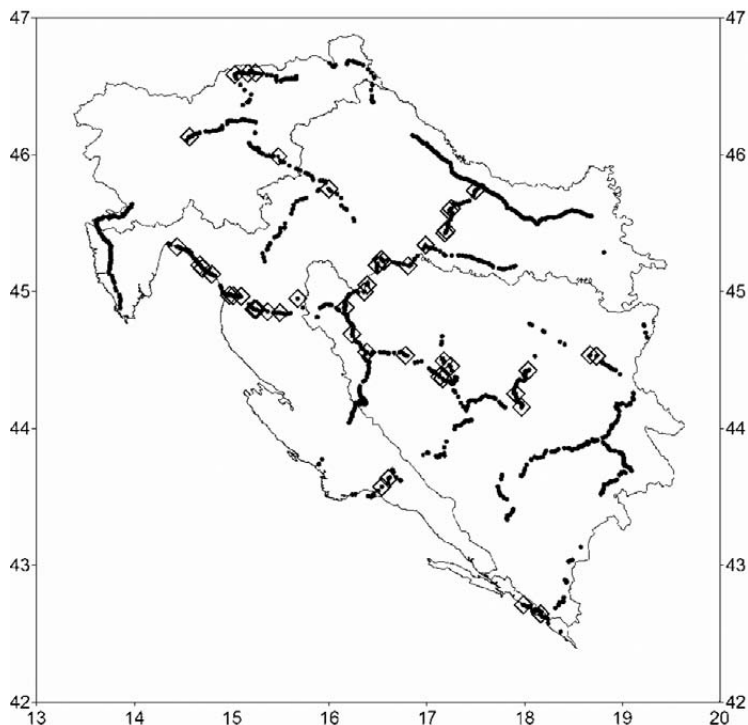


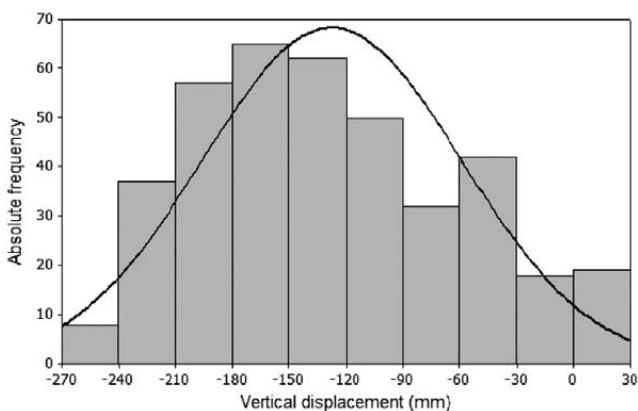
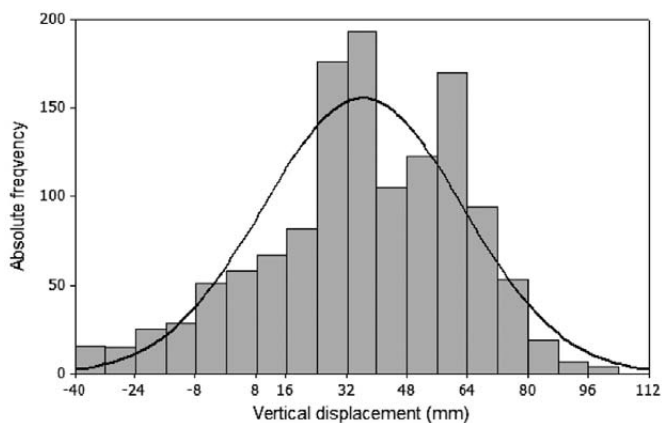
Figure 5. Identical bench marks encompassed by the levelling networks INVT and IINVT.

territory is not regular and homogeneous because their concentration along the levelling lines is in accordance with professional geodetic principles of network realisation, and with the level of bench mark preservation. As well the existence of larger areas in which there are no bench marks at all can be seen.

On the basis of obtained data about the heights H_{APN} , H_{INVT} and H_{IINVT} of the bench marks presented on the Fig. 4 and Fig. 5, vertical displacements ΔH were calculated. The sign of the displacements has been determined by subtracting the height of bench marks contained in older epoch from the heights of these bench marks corresponding to the younger epoch. Respectively, the vertical displacements show directly a relative change of vertical positions of bench marks, indicating raising or sinking, as related to the initial position determined in an older epoch. The basic indicators of empirical properties of the displacements, independently of the correlation with the ellipsoidal position and the positional distribution across the observed area, are given in the Table 3, and the histograms with the distribution of displacement absolute empirical frequencies and adjusted normal distribution curves on Fig. 6 and Fig. 7. The number of classes is in accordance with the amounts of standard deviations and the total number of displacements, i.e. bench marks included into individual data set.

Table 3. Statistical indicators of displacements.

	1. and 2. epoch	2. and 3. epoch
Vertical displacement	$\Delta H_{12} = H_{\text{INVT}} - H_{\text{APN}}$	$\Delta H_{23} = H_{\text{IINVT}} - H_{\text{INVT}}$
Time interval (years)	56.2	22.1
Number of bench marks	390	1287
Mean value (mm)	-127.4	36.2
Standard deviation (mm)	68.3	26.4
Minimum (mm)	-251.6	-39.1
Maximum (mm)	29.7	103.0
Dispersion range (mm)	281.3	142.1

**Figure 6.** Distribution of displacements (1. and 2. epoch).**Figure 7.** Distribution of displacements (2. and 3. epoch).

Statistical indicators of displacements determined from the 1. and 2. epoch, i.e. from the 2. and 3. epoch that are parallelly presented in the Table 3 clearly indicate that the vertical displacements in both cases are of significant amounts as related to the measuring accuracy of the networks, and that they reach a decimetre level. Displacements are distributed within the dispersion range without any larger discontinuities, i.e. they change the amount from minimal to maximal values rather continuously, Figs. 6 and 7. The dispersion range is more significantly expressed with the displacements determined from the 1. and 2. epochs, where it is approximately 28 cm, and with the displacements determined from the 2. and 3. epochs it is at the level of approximately 14 cm, i.e. practically of two times smaller amount. Such relation of dispersion range is to some extent logically related to the lengths of time intervals between single epochs, i.e. approximately 56 years between the 1. and 2. epochs, and about 22 years between the 2. and 3. epochs. It can be noticed that a significantly larger dispersion range was found in a longer time period between the epochs. Standard deviations of displacements indicating the dispersion size of single displacement values around the associated mean value is also significantly larger than with the value of movements determined from the 1. and 2. epoch. One should point out the fact that in the vertical displacements determined from the 1. and 2. epochs there are negative displacements prevailing (sinking of bench marks) that have extreme values of approximately 25 cm, and with the displacements determined from the 2. and 3. epochs there are positive movements prevailing (raising of bench marks) that have extreme values of about 10 cm. The given empirical pattern has been reflected in the displacement mean values, i.e. in the amount of about -13 cm in the first case and $+4$ cm in the second case. In spite of the fact that the histograms of displacements empirical distribution in both cases remind of normal distribution, the results of Person's test of normal distribution with the probability of 95% indicate that the distributions cannot be regarded as normal distributions.

Visual presentation of the amounts and directions of bench mark vertical displacements are presented on Fig. 8 and Fig. 9. The displacements are symbolically visualised with vertical streight lines (vectors) that are proportional to the displacement sizes, with the directions that are corresponding to raising or sinking of bench marks in the course of time. The figures present clearly the distribution of the direction and size of movements across the observed area, at the identical scale of presentation. On both figures one can notice completely specific patterns of global and local empirical displacement variation. In accordance with the positional distribution of bench marks and the amounts and directions of associated displacements it is quite clear, that there is a general trend present on the one hand, as well as a specific level of variation in local area of a each bench mark.

Without trying to interpret the displacements, taking the sources and causes of their nature and origin into consideration, it should be pointed out that in a larger part of the observed territory there is the significant raising of

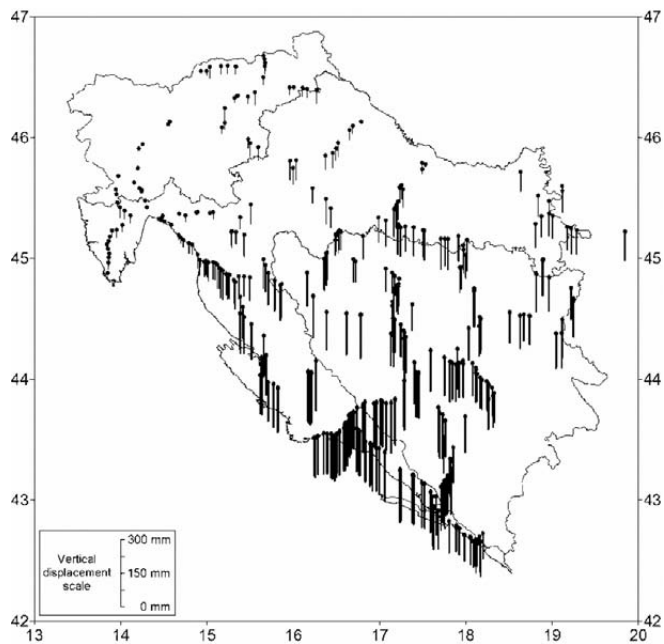


Figure 8. Vertical displacements – 1. and 2. epoch.

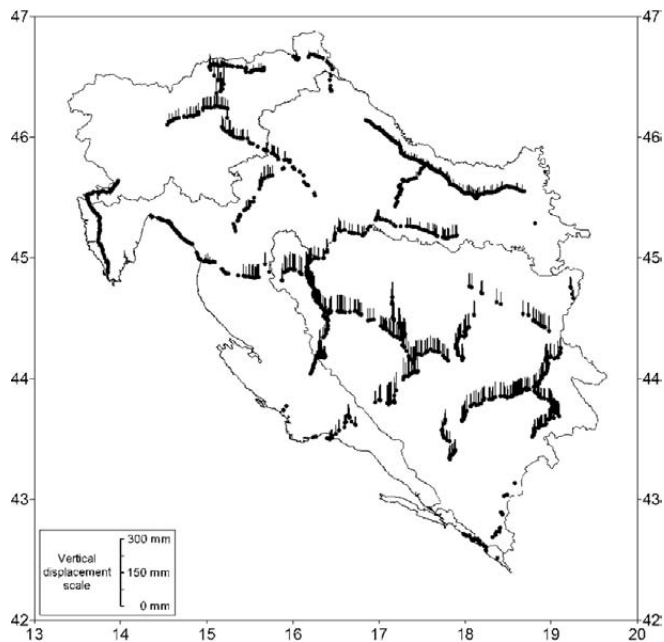


Figure 9. Vertical displacements – 2. and 3. epoch.

the Earth's crust between the 2. and 3. epoch present after very much noticeable and significant sinking between the 1. and 2. epoch. The mentioned pattern is quite intensive mostly at the territory of Bosnia and Herzegovina and southern Croatia, and it somewhat less noticeable at the territory of northern and north-western Croatia, and eastern Slovenia. On the other hand, after significant sinking of the Earth's crust at the territory of south-eastern Croatia between the 1. and 2. epoch there was no significant raising between the 2. and 3. epoch noticed, and the sinking and raising of the crust at the territory of eastern Croatia and Slovenia are rather small and moderate.

It is also interesting to point out that the sizes of bench mark displacements are pretty correlated with the lengths of time intervals between individual epochs. Much larger displacements have occurred in the period between the 1. and 2. epoch lasting 56 years than in the period between the 2. and 3. epoch lasting 22 years. It entails the hypothesis that along with the "expected" extrapolation retention of crust raising even after the 3. epoch it tends to return in its initial vertical position on a remarkable part of the observed territory, i.e. the position corresponding to the 1. epoch. Although this hypothesis is only of a speculative character, it is interesting, because it reminds of a cyclical pattern of the activity of geodynamic forces.

4. Vertical displacements modelling

Referring to the fact that the data on the displacements of bench marks between individual epochs are spatial data, i.e. the groups of arranged triplets $(\lambda, \varphi, \Delta H)$, where the vertical displacement can be interpreted as stochastic variable depending on the ellipsoidal position, it is possible to apply the mathematical modelling for the purpose of creating an appropriate mathematical displacement models. If it should be supposed for the sake of simplicity that the displacements are continuous variables, the mathematical model in terms of geometry assume the form of continuous spatial surface over observed region defining the size of displacements depending on argument of bench mark ellipsoidal position. The assumption of vertical displacements like continuous variables should be considered primarily in terms of fundamental property of created surface models, and it is a property of continuity over the observed area, i.e. the fact that the surface model does not contain any discontinuities. In the case of displacement modeling using the discontinuous surfaces the modeling process would certainly be considerably more demanding, though probably more consistent with physical reality. Specifically if it is evident that at the observed area there are some regional faults and recognizable long fault lines that can be assumed to lead to the height displacements discontinuity.

When selecting the method of modelling two essential elements should be taken into consideration. First of all, the total number and positional distribution of bench marks on the observed territory on the one hand and completely

specific empirical patterns of displacements variation on the other hand. The first element in both characteristic cases, i.e. in the case of the displacements between the 1. and 2. epoch, as well as in the case between the 2. and 3. epoch is not ideal because inhomogeneous and irregular bench mark positional distribution exists, Fig. 4 and Fig. 5. Beside specific pattern of line positional distribution of bench marks on a relatively short distance there are quite large areas without bench marks at all. Speaking in terms of modelling, the areas without bench marks can make the modelling remarkably more difficult and affect the reliability and quality of created models. The other element, in both given cases, indicates the fact that a certain level of empirical correlation can be noticed with respect to the positional distribution (sign and amount) of displacements that is globally shown in the trend pattern (long-wave variability of displacements) and the pattern of relatively moderate local variability (short-wave variability of displacements).

Taking all mentioned elements into consideration, the combination of regression modelling, Seber and Lee (1990), and of modelling with minimal curvature surface, Dewhurst (1990), is one of many various modelling methodologies that can be adequately taken into consideration and unify the effects of inhomogeneous positional distributions of bench marks across the observed area, as well as the patterns of their long-wave and short-wave variation. Namely, the spatial model surface that has a relatively moderate level of spatial indentedness encompassing adequately a long-wave component of displacement variation (trend) and retaining especially in extrapolation areas a relatively coherent and “logical” shape can be determined very efficiently by means of regression modelling. The residuals of regression modelling in which a short-wave component of displacement variation remain contained can be very efficiently modelled with the minimal curvature surface model, especially because of the properties that the model surface has got in larger interpolation and especially in extrapolation areas. In that case the displacement models developed by unifying the regression models and minimal curvature surface models assume the form of so called grid models, Collier (2002). The realization of such models in terms of data processing complexity does not represent a complex problem because the data sets are not too big, and because it is possible to use adequate specialised software tools having needed modelling functions and routines.

Without getting deeper into the theoretical base of every mentioned modelling method, the course of displacement modelling can be briefly described with the following steps:

- Creating of regression models in the form of explicitly defined functions of spatial surfaces $F_{\Delta H} = F(\lambda, \varphi)$ on the basis of the adjustment 3D data of bench mark displacements $(\lambda, \varphi, \Delta H)$ by applying the method of least squares.
- Modelling of the displacement residuals $(\lambda, \varphi, v_{\Delta H})$ obtained by means of regression modelling, $v_{\Delta H} = \Delta H - F(\lambda, \varphi)$, with the minimal curvature surface models, including the definition of adequate shape and size of the grid.

– Conversion of regression models from explicit function form into the form of a grid model, adopting at previously already defined grid.

– Unifying the grid models obtained by means of regression modelling and modelling with the minimal curvature surface into a final displacements grid model.

With regard to the fact that the selection of a really large number of completely different regression functions is at disposal, where functions include essentially different and large number of regression parameters, the appropriate software tool for automated application of a large number of diverse predefined regression functions on the same group of data was used. The regression model spatial surface showing the highest level of internal accuracy, with moderate trend and “logical” shape in extrapolation areas is adopted as final modelling result.

On the basis of regression modelling, using more than 200 predefined polynomial functions, based on displacement data determined from the 1. and 2. epoch the polynomial function with 11 regression parameters

$$\begin{aligned}
 F_{\Delta H_{12}} = & -1.776161 \cdot 10^5 - 3.520325 \cdot 10^4 \lambda^{-1} + 1.263565 \cdot 10^6 \lambda^{-2} - \\
 & - 2.241779 \cdot 10^7 \lambda^{-3} + 1.967622 \cdot 10^8 \lambda^{-4} - 6.839266 \cdot 10^8 \lambda^{-5} + \\
 & + 3.937434 \cdot 10^7 \varphi^{-1} - 3.484880 \cdot 10^9 \varphi^{-2} + 1.542686 \cdot 10^{11} \varphi^{-3} - \\
 & - 3.415839 \cdot 10^{12} \varphi^{-4} + 3.026566 \cdot 10^{11} \varphi^{-3}
 \end{aligned} \quad (1)$$

was identified as the most acceptable regression function describing the displacements long-wave variation (trend), and for the displacements determined from the 2. and 3. epoch the polynomial function of Taylor’s series with 10 parameters was identified

$$\begin{aligned}
 F_{\Delta H_{23}} = & -6.170430 \cdot 10^2 + 1.376517 \cdot 10^1 \lambda + 3.521808 \cdot 10^1 \varphi - \\
 & - 3.829052 \cdot 10^{-1} \lambda^2 - 7.108039 \cdot 10^{-1} \varphi^2 - 3.073657 \cdot 10^{-1} \lambda \cdot \varphi + \\
 & + 1.555355 \cdot 10^{-3} \lambda^3 + 5.078515 \cdot 10^{-3} \varphi^3 + 7.703290 \cdot 10^{-4} \lambda \cdot \varphi^2 + \\
 & + 6.651875 \cdot 10^{-3} \lambda^2 \cdot \varphi
 \end{aligned} \quad (2)$$

where the ellipsoidal longitudes λ and latitudes φ are directly expressed in degrees (minutes and seconds in decimal parts of the degrees) and the modelled displacement values in meters.

Characteristic indicators obtained by the regression modelling are presented in the Table 4, indicating that the internal quality of both regression models is very similar and that quite high level of model fitting to empirical data was achieved. Standard deviations defined by means of residuals are at the level of 1 to 2 centimetres, and the coefficients of the determination indicate that 80% to 90% of the displacements variations contained in empirical data are adequately explained by the created regression models.

On the basis of the regression functions given in the expressions (1) and (2) the visualisation of vertical displacement models by means of isolines, i.e. the

Table 4. Indicators of regression modelling.

Indicator	$F_{\Delta H_{12}}$	$F_{\Delta H_{23}}$
Number of bench marks	390	1287
Number of regression parameters	11	10
Sum of residuals (mm)	-9.74×10^{-5}	-4.48×10^{-8}
Average residual value (mm)	-2.50×10^{-7}	-3.48×10^{-11}
Sum of residual squares (mm ²)	198161.57	167046.85
Standard deviation (mm)	22.9	11.4
Coefficient of determination, R ²	0.89	0.81
Minimal residual (mm)	-54.4	-33.8
Maximal residual (mm)	51.1	51.1
Residual dispersion range (mm)	105.5	84.9

curves of the same displacement values has been made possible. The surface model in orthogonal projection corresponding to the displacements determined between the 1. and 2. epoch is presented on Fig. 10, and the surface model corresponding to the displacements determined between the 2. and 3.

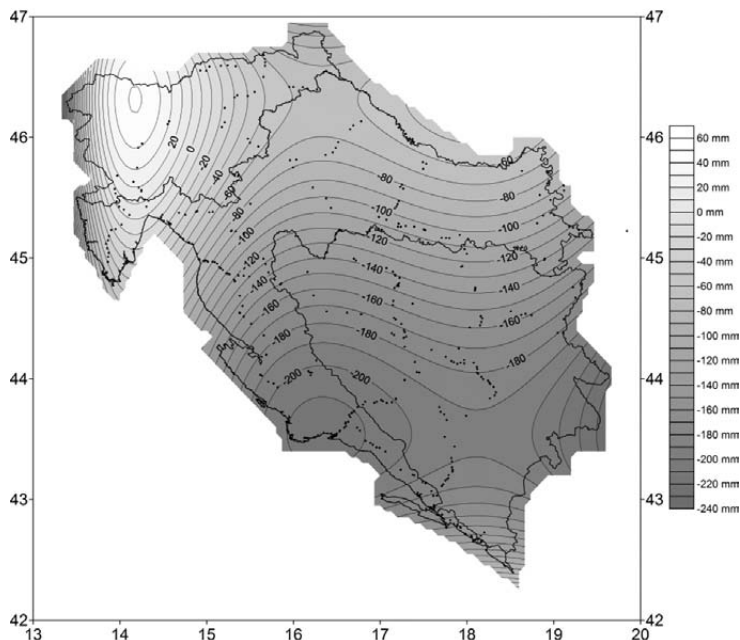


Figure 10. Displacement regression model – 1. and 2. epoch.

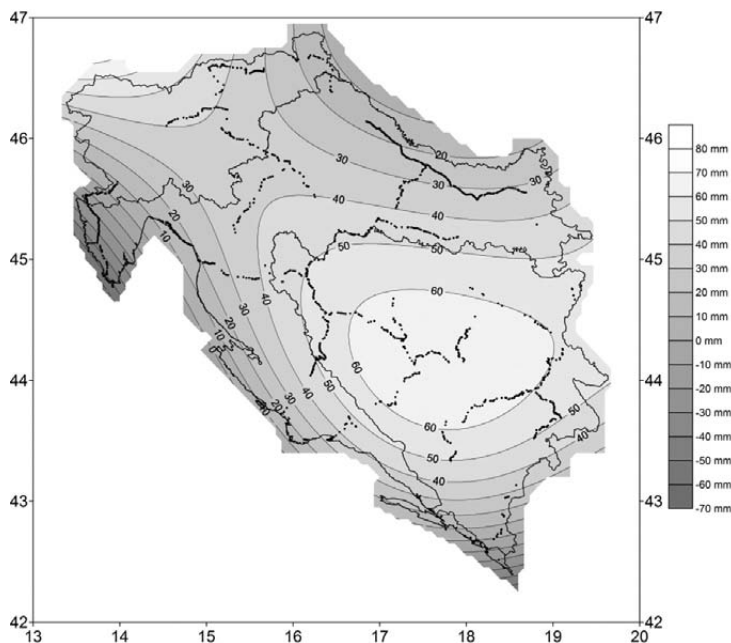


Figure 11. Displacement regression model – 2. and 3. epoch.

epoch is presented on Fig. 11. Both presentations give the position of bench marks included into regression modelling along with the isolines equidistance adjusted to the range of displacements dispersion, Table 4. It is obvious that the models show moderate vertical indentedness of surfaces without any emphasized extremes, along with good fitting to the empirical data, and also a relatively stable course in extrapolation areas. In other words, the regression model surfaces encompass very well long-wave displacements variability and represent the trend contained in empirical data in an acceptable level.

The displacement residuals obtained by means of regression modelling are unified with the ellipsoidal bench mark positions $(\lambda, \varphi, v_{\Delta H})$. The displacement residuals, in which the short-wave displacement variation is still contained, further on have been modelled by means of minimal curvature surfaces. As the presumption for modelling, in correlation with the total size and shape of the area encompassed as with the total number and positional distribution of bench marks, the adequate grid has been adopted. There has been a grid of rectangular shape defined with the origin at point $\lambda = 13^{\circ} 12' 0''$ E and $\varphi = 42^{\circ} 12' 0''$ N, with the longitude $\Delta\lambda = 6^{\circ} 40' 0''$ and the latitude spacing $\Delta\varphi = 4^{\circ} 48' 0''$ and with the size of grid cell of $4' \times 3'$. The grid contains 97 horizontal and 101 vertical lines, i.e. the total of 9797 grid nodes. Within the border of Croatia, Slovenia, and Bosnia and Herzegovina there are 4267 nodes contained, Fig. 12, and outside of the area there are the remaining 5530 nodes.

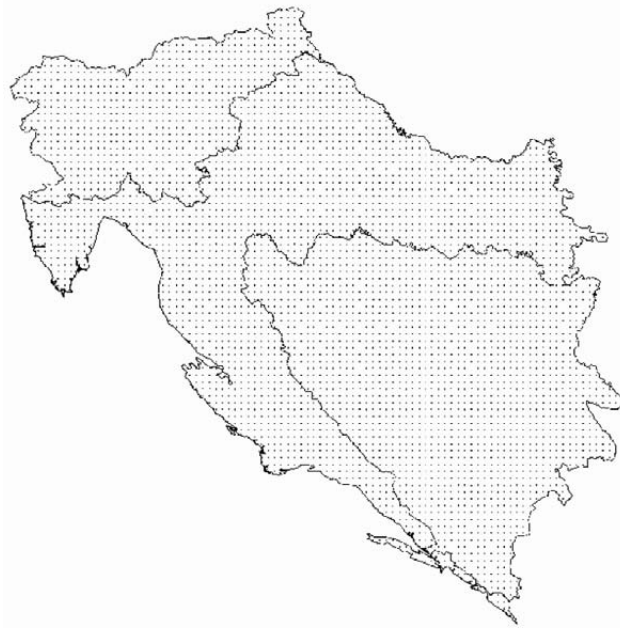


Figure 12. The grid visualization.

The indicators determined from the residuals obtained from the minimal curvature surface modelling are given in the Table 5, and the model surfaces on Fig. 13 and Fig. 14, with previously already used values of isoline equidistance. The internal accuracy of the created models given with standard deviations is mutually quite similar and on the millimetre level. The range of residual dispersions are also of clearly smaller amounts than the range of initial data being modelled, Table 4.

Table 5. Indicators of minimal curvature surfaces modelling.

Indicator	$v_{\Delta H_{12}}$	$v_{\Delta H_{23}}$
Bench mark number	390	1287
Sum of residuals (mm)	69.95	33.85
Average value of residuals (mm)	0.2	0.0
Sum of residual squares (mm ²)	18521.74	19682.91
Standard deviation (mm)	6.9	3.9
Minimal residual (mm)	-29.2	-21.4
Maximal residual (mm)	32.3	18.7
Residual dispersion range (mm)	61.5	40.1

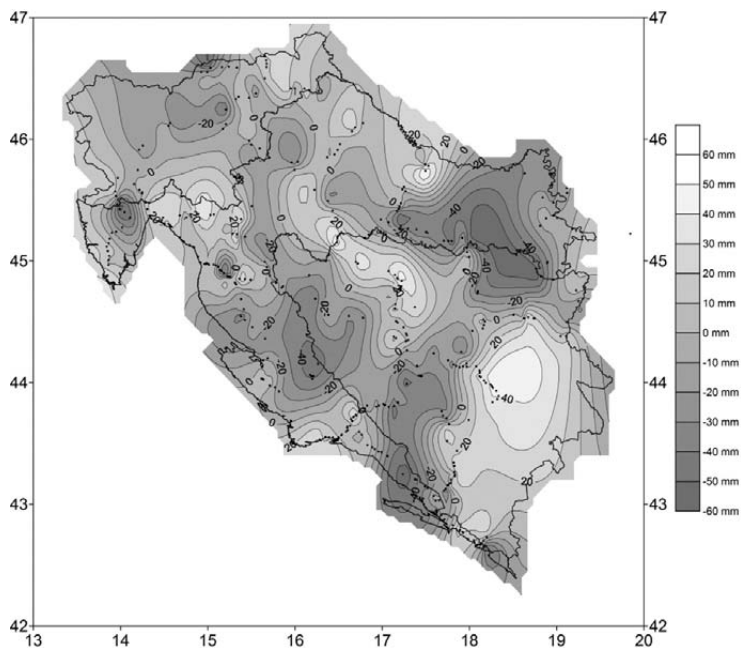


Figure 13. Minimal curvature surface displacement model – 1. and 2. epoch.

By unifying the regression models and the models obtained by means of minimal curvature surfaces the final models of displacements between single epochs have been created. The regression models have been first transformed into the form of a grid model adopting the same geometry size and shape of the grid that were used in modelling by means of minimal curvature surfaces. The conversion of regression models is very simple, because the associating grid models are created by calculating the displacement model values on the grid nodes using expression (1) and (2). Fig. 15 presents the final model of displacements between the 1. and 2. epoch and Fig. 16 the final model of displacements between the 2. and 3. epoch.

Fig. 17 presents the final model of vertical displacements between the 1. and 3. epoch. The model was created by grid addition operation of the grid models of displacements between the 1. and 2. epoch and between the 2. and 3. epoch, i.e. by means of direct summing up of modelled values on individual grid nodes. The mentioned grid model indicates the resultant of vertical displacements in the total time interval of approximately 78 years, and it was created in spite of the fact that there are only a small number of identical bench marks at disposal and with very inconvenient positional distribution, which are contained in all three epochs (49 bench marks).

Although these bench marks did not provide direct creation of the displacement model between the 1. and 3. epoch, displacements ΔH_{13} derived di-

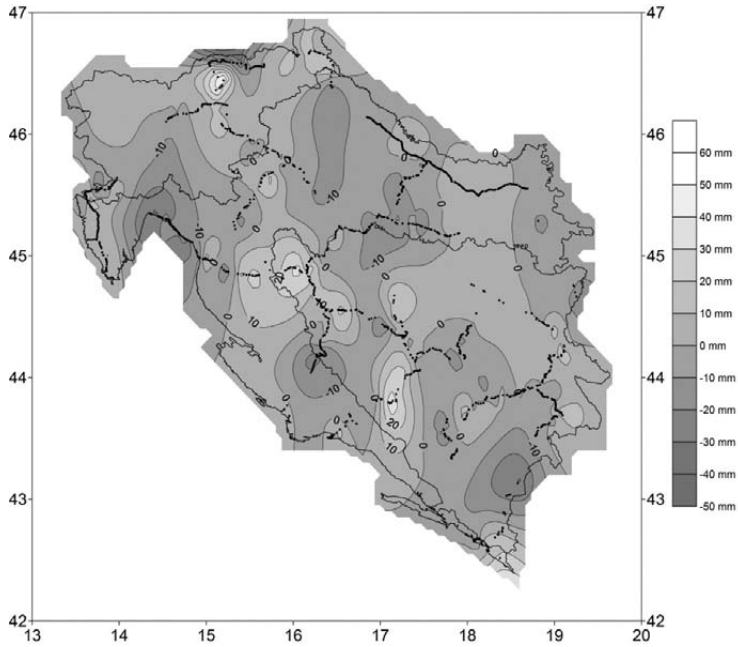


Figure 14. Minimal curvature surface displacement model – 2. and 3. epoch.

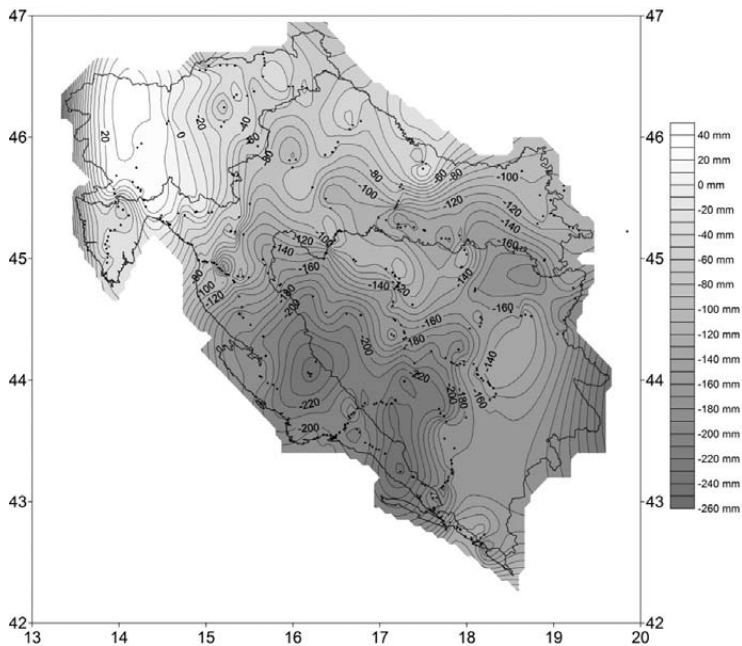


Figure 15. Final model of vertical displacements – 1. and 2. epoch.

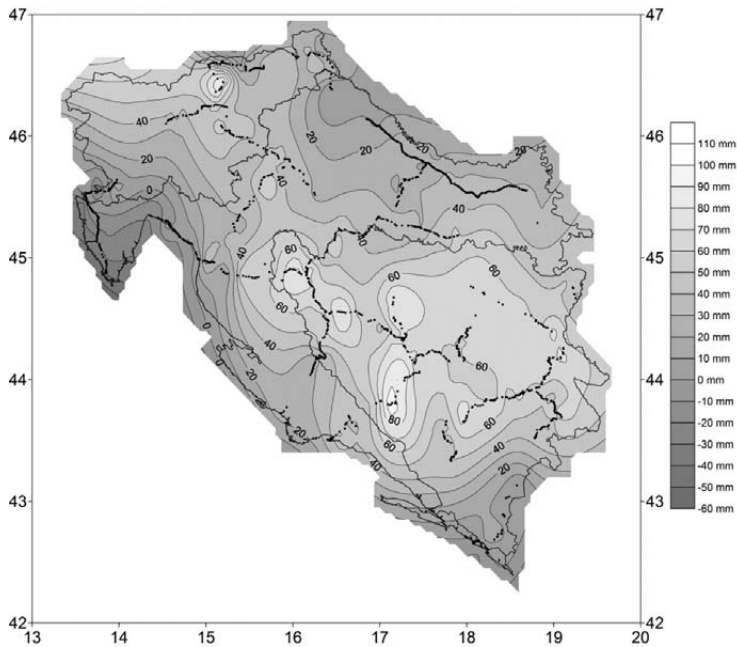


Figure 16. Final model of vertical displacements – 2. and 3. epoch.

rectly from their heights offer an independent insight into the so called external accuracy of this grid model. This accuracy is indicated by standard deviation calculated from the errors d_{13} obtained like differences between displacement values ΔH_{13} and the displacements determined from the belonging model. The indicators of error d_{13} empirical properties are presented in the Table 6. On the basis of the amount of standard deviation being 9.0 mm it can be stated

Table 6. Indicators – 1. and 3. epoch.

Indicator	d_{13}
Bench mark number	49
Sum of residuals (mm)	15.15
Average value of residuals (mm)	0.3
Sum of residual squares (mm ²)	3999.14
Standard discrepancy (mm)	9.0
Minimal residual (mm)	-29.6
Maximal residual (mm)	21.0
Residual dispersion range (mm)	50.7

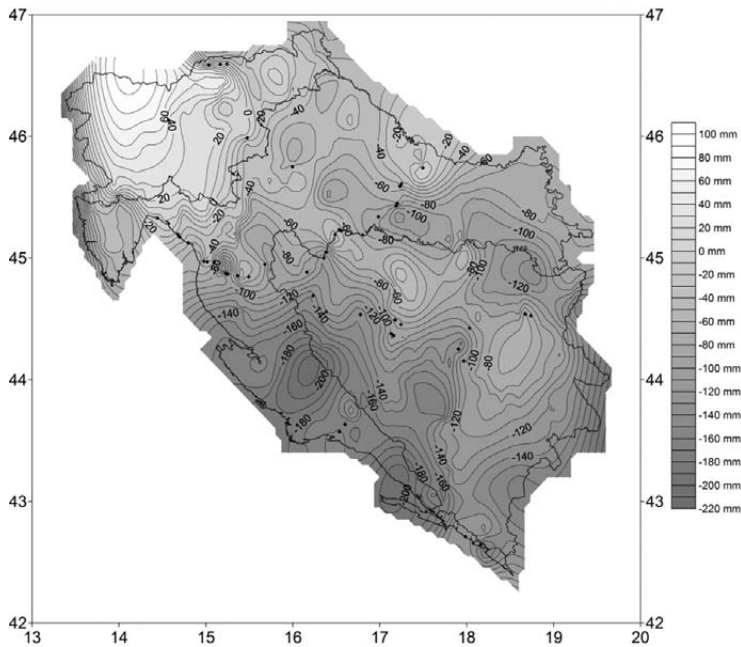


Figure 17. Final model of vertical displacements – 1. and 3. epoch.

that the model has got quite adequate external accuracy taking into account the size of the displacements and their range of dispersion.

All previously presented vertical displacement models realized by means of continuous spatial surfaces at the total observed area make the determination of vertical displacement for any point of known ellipsoidal position possible. The models are realized in the form of grid models for which the data of ellipsoidal position of grid nodes and associated modelled displacement values are contained in adequately formatted computer files. In such case, for any point of known ellipsoidal position differing from the position of grid nodes, the modelled value of the displacement is determined by applying bilinear interpolation out the displacement values on the nodes of the grid cell containing that point.

In spite of the simplicity of the methodology of determining and modelling the displacements, in spite of introducing certain completely theoretical presumptions and the specific selection of modelling methods, the presented models of displacements offer a clear insight into the resultant of geodynamic forces activity on to the Earth's crust and its surface along observed area. The models make it possible to find out whether there are or not some significant movements of the Earth's crust, and what is their size, how they are positionally distributed and what are their properties. The insight into their appearing and properties, especially because of the fact that the starting point for modelling is actually the result of truly long-lasting, complex, hard and de-

manding gathering, combining, harmonisation, organisation and analysing of data obtained from direct height survey of the levelling networks APN, INVT and IINVT, should be the starting point for the application of more complex and more sophisticated methodologies of modelling. These methodologies should not rely just on explicitly determined bench mark heights as starting quantities, but on the data about measured and corrected height differences, and in the process of modelling they should include more consistently the time components referring to the realisation of individual levelling lines that are contained in the networks, instead of introducing only one unique time moment of realization of networks that these levelling lines belong to.

5. Determination of the speed of vertical movements

On the basis of the models of vertical displacements the speed of uniform vertical movement of the Earth's crust connected with individual epochs can be easily determined and visualised. Namely, on the basis of the ratio of modelled displacement values, Fig. 15, Fig. 16 and Fig. 17, and the lengths of time intervals between individual epochs, Table 1, the speed of uniform movement expressed in millimetres per year (mm/year) were determined. Fig. 18, Fig. 19 and Fig. 20 present the speed of the movement of the Earth's crust between the 1. and 2. epoch, between the 2. and 3. epoch and finally between the 1. and 3. epoch. The speed is visualised with isolines, where the sign is also attached to the speed amount in order to make it obvious whether the crust is moving upwards (rising) or downwards, as related to the initial position.

Numerical indicators of the speed of uniform vertical movement relating to individual epochs are given in the Table 7, and they were determined on the basis of the grid node displacement values from the territory included into the models, Fig. 12. Maximal speed of the movement has assumed quite large amounts, somewhat larger than 4 mm/year.

Just as in the case of displacement models it should be pointed out that the interpretation of the speed of uniform vertical movements of the Earth's crust should be approached with adequate level of caution, taking into account all previously mentioned circumstances connected with the applied methodology of modelling. Still, even if in the first approximation, and as direct deriva-

Table 7. Indicators of the vertical movements speed on grid nodes.

Epoch	1. – 2.	2. – 3.	1. – 3.
Number of grid nodes	4267	4267	4267
Minimal speed (mm/year)	-4.5	-1.9	-2.8
Maximal speed (mm/year)	0.6	4.5	1.2
Mean speed value (mm/year)	-2.1	1.8	-1.0
Range of speed dispersion (mm/year)	5.1	6.4	4.0

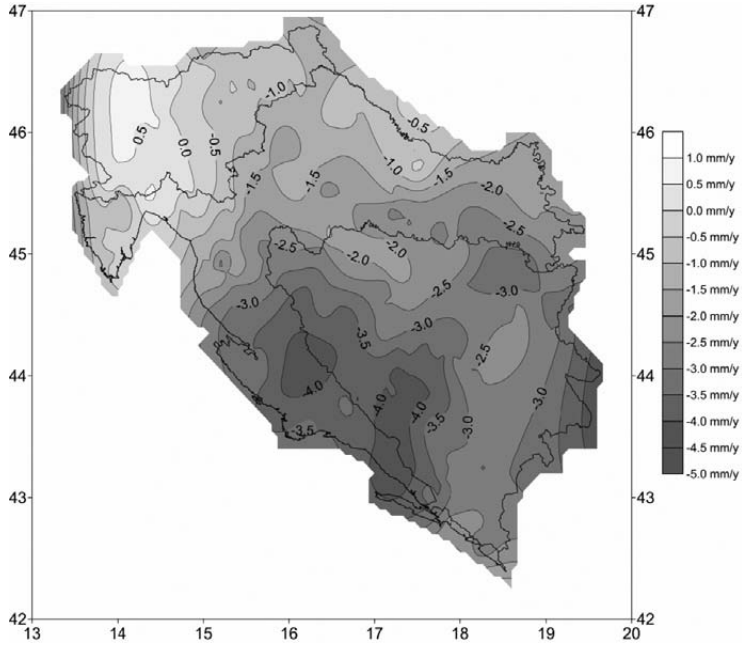


Figure 18. Vertical movement speed – 1. and 2. epoch.

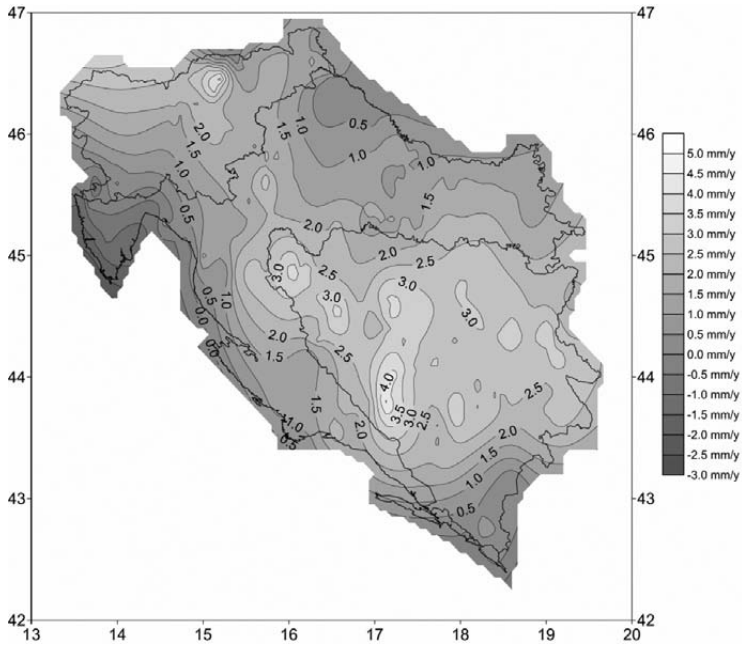


Figure 19. Vertical movement speed – 2. and 3. epoch.

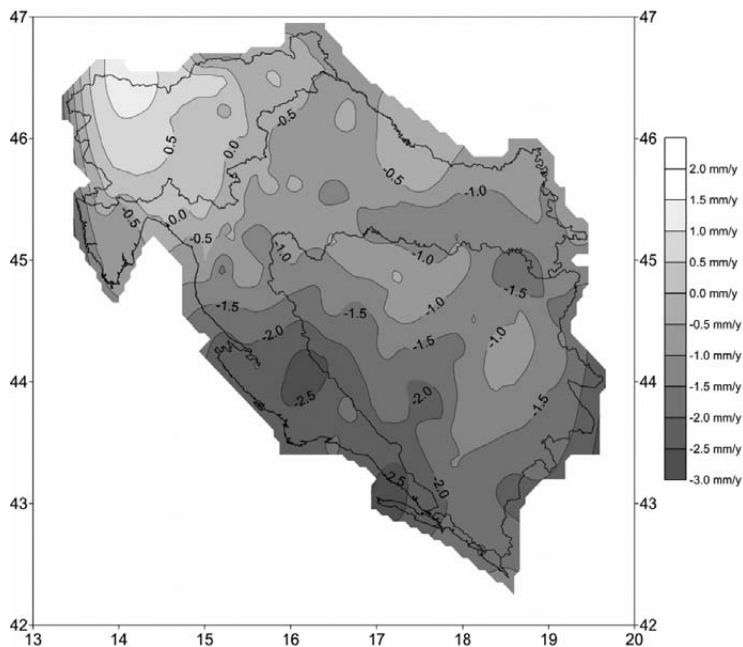


Figure 20. Vertical movement speed – 1. and 3. epoch.

tives of the displacement models, the speed of vertical movements at the observed territory gives at least an initial insight into the consequences of the systems of geodynamic forces which are changing the surface of the Earth's crust in a complex physical reality.

6. Conclusion

The presented detailed explanations, numerical results, models, indicators and their visualisation speak sufficiently for themselves and offer clear insight into the final results of the vertical displacements modelling process and the determination of the speed of uniform vertical movements of the Earth's crust at the territory of Croatia, Slovenia, and Bosnia and Herzegovina.

According to the information known to the authors, the vertical displacements, displacement models and the movements speed of Earth's crust have been determined for the first time systematically, completely and methodologically clearly, on the basis of the whole usable data obtained from the survey of all three height networks of the highest order of geometric levelling at the territory of Croatia, Slovenia, and Bosnia and Herzegovina, i.e. the networks APN, INVT and IINVT. The obtained results, taking into consideration the basic guidelines of the applied methodology, as well as all inconvenient circumstances, still offer a systematic and explicit insight into the amounts of dis-

placements and the movements' speed of the Earth's crust, their size and direction, as well as empirical and model properties.

The presented results can be the basis for the connection with other geoscientific disciplines (geodynamics, seismic, geology, hydrology, etc.) for the purpose of interpretation and detection of adequate correlations that can explain the sources, intensities, manners and patterns of the activities of various endogenous and exogenous force systems reflected on the changes of the shape and geometry of the Earth's crust. Due to the need to interpret the obtained data interdisciplinary, there are only those results presented in this work that are directly, i.e. rather strictly connected with scientific and professional field of geodesy, i.e. the principles and the methodology of height positioning and modelling of spatial data.

At any rate, the interpretation of the presented results should be approached with the adequate level of caution, taking into consideration the introduced theoretical presumptions and the pragmatism in the methodology of determining and modelling of data. The key element is the fact that the usage of quite simple methodology of determining the bench mark vertical displacements has occurred almost by itself, first of all, as consequence of a very complex long-lasting and very demanding work on collecting the original archived data about the levelling networks and their detailed organisation. In this context, i.e. in the context of organising, analysing and homogenization of the data sets, the adjustments of levelling networks as one of the unavoidable steps in this process have offered also the data about the heights of bench marks. Their direct, efficient and in terms of time quick usage has given, even if it was in the first approximation, the valuable insight into the direction and size of the displacements, their empirical patterns and properties. This information should be an adequate starting point for the continuation of the research of recent Earth's crust movements at observed region, for the purpose of applying more complex and demanding methodologies of displacement determining and modelling on the one hand, and equally for the purpose of their interdisciplinary interpretation on the other hand.

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SAŽETAK

Modeliranje recentnih pomaka Zemljine kore na teritoriju Hrvatske, Slovenije i Bosne i Hercegovine*Nevio Rožić, Ivan Razumović i Ivo Nazifovski*

U radu su izloženi rezultati određivanja visinskih pomaka zemljine kore na teritoriju Hrvatske, Slovenije i Bosne i Hercegovine na temelju podataka realizacije nivelmanskih mreža geometrijskih nivelmana najvišeg reda točnosti. To su mreže Austrijskog preciznog nivelmana, I. nivelmana visoke točnosti i II. nivelmana visoke točnosti, koje su sukcesivno realizirane unutar vremenskog raspona od približno 100 godina (1874–1973.). Sukladno postavljenim hipotezama i odabranoj metodologiji određivanja pomaka dobivena je konkretna numerička kvantifikacija pomaka, određena je veličina i smjer pomaka, ukazano je na njihova skupna svojstva i empirijske zakonitosti ponašanja. Pokazuje se da su pomaci repera signifikantnih iznosa u odnosu na točnost izmjere visinskih razlika u nivelmanskim mrežama te da se korelirano s položajnom distribucijom repera može uočiti dugovalna i kratkovalna komponenta njihove varijacije. Obavljeno je modeliranje pomaka pomoću tzv. grid modela, kreiranih uz kombiniranje regresijskog modeliranja i modeliranja plohom minimalne zakrivljenosti te su na temelju modeliranih vrijednosti pomaka određene brzine jednolikog visinskog gibanja zemljine kore. Usprkos činjenici da je primijenjena vrlo jednostavna metodologija određivanja pomaka i gibanja, koja se temelji na računanju razlika apsolutnih visina istih repera koji su visinski pozicionirani u različitim vremenskim trenucima – mrežama, dobivena saznanja su primjereno polazište za odabir i primjenu sofisticiranijih metodologija određivanja i modeliranja pomaka u predstojećim istraživanjima, odnosno polazište koje omogućuje interdisciplinarno tumačenje geodinamičkih procesa koji dovode do promjene oblika i geometrije Zemljine kore na razmatranom području.

Ključne riječi: recentna gibanja kore, visinski pomaci, reperi, nivelmanske mreže, grid model