

Treatment of relative permeabilities for application in hydrocarbon reservoir simulation model

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REVIEW

Measurements of relative permeabilities and their analysis and modification by means of appropriate relative permeability models represent one of the bases for development and verification of reservoir simulation models. It requires careful selection of these curves providing that a statistically sufficiently relevant groups of similar curves are available for the selected reservoir or for individual rock types within the same reservoir. In order to be able to correctly apply these curves in reservoir simulations, previous knowledge of all reservoir rock parameters is required, i.e. porosity, absolute and effective permeability, saturation, etc. Objective of the paper is to present procedures of relative permeability curve measurements and analysis methods of data experimentally obtained from different models which describe such data, to be able to apply them with high degree of quality in numerical reservoir simulations. In that regard, the final part of the paper through a concrete example shows the procedure of relative permeability data analysis and their processing for application in hydrocarbon reservoir model construction.

Key words: hydrocarbon reservoir, reservoir model, relative permeability, numerical simulation, rock type

1. INTRODUCTION

In several chapters the paper briefly presents data sources for obtaining of relative permeability curves and techniques for measurement and analysis of these data. Interpretation and modification of such curves by means of relative permeability models with the objective to continue using them within the scope of construction of integrated hydrocarbon reservoir model is also described. In the process care was taken that the ultimate shape of individual sets of relative permeability curves is in agreement with relevant parameters that characterize particular reservoir rock types within uniform and heterogeneous hydrocarbon reservoirs. Processing of relative permeability data within the scope of measurements on six reservoir rock samples from different parts of the same hydrocarbon reservoir is presented in tabular form and as diagram with the objective to confirm it.

2. SOURCES OF RELATIVE PERMEABILITY DATA

There are different techniques and methods for measurement of relative permeabilities and their ratios. Out of the following methods briefly presented in further text, the first is based on measurement of field production data and on direct calculation of relative permeability ratios from them and others are performed in the laboratory through experiments on reservoir rock samples, i.e. cores.

2.1. Production measurements

Here is presented a short derivation which refers to obtaining of relative permeabilities from production data.⁶ On the basis of production data it is possible to calculate

the ratios of effective and relative permeabilities applying the Darcy's law on radial flow. Providing PVT ratios of fluids are known, it is possible to define the ratio of free and dissolved gas at different pressures in total gas production according to the following equation:

Produced gas = free gas + petroleum gas

i.e.:

$$R_p = R + R_s \quad (1)$$

In accordance with equation (1) and applying the ratios from equations for gas and oil radial flow:

$$R_p = \frac{k_{rg}}{k_{ro}} \frac{\mu_o}{\mu_g} \frac{B_o}{B_g} + R \quad (2)$$

equation (2) can be adjusted for calculation of gas and oil relative permeability ratios:

$$\frac{k_{rg}}{k_{ro}} = (R_p - R_s) \frac{B_o}{B_o} \frac{\mu_g}{\mu_o} \quad (3)$$

oil saturation which corresponds to this ratio of relative permeabilities is determined according to material balance:

$$S_o = \left(1 - \frac{N_p}{N}\right) \frac{B_o}{B_{oi}} (1 - S_{wi}) \quad (4)$$

which corresponds to total saturation with fluids according to equation (5):

$$S_L = S_w + (1 - S_{wi}) \left(\frac{N - N_p}{N_p}\right) \left(\frac{B_o}{B_{oi}}\right) \quad (5)$$

Following this calculation, dependence between k_{rg}/k_{ro} , from equation (3), and S_L , from equation (5), is plotted on the log-lin diagram with k_{rg}/k_{ro} on logarithmic scale. Values k_{rg}/k_{ro} can be used to verify the accuracy of relative permeability obtained on the basis of empirical, i.e. laboratory procedures.⁵

2.2. Laboratory measurements on cores

Laboratory measurements of relative permeabilities can be categorized into three major groups: unsteady-state and steady-state measurements performed directly on cores, and indirect through measurement of capillary pressures on cores. It is important to note that the mentioned measurement techniques are performed on different laboratory apparatuses.⁶ The main differences in approach and in measurement techniques between these methods will be mentioned in further text.

2.2.1. Unsteady-state methods

Unsteady-state relative permeability measurements can be performed faster than steady-state measurements, but mathematical description of such measurements is much more complex.⁶ A combination of Darcy's law and equation that defines capillary pressure in differential form, which takes into consideration the gravity effect, is used for computation of fractional flow of water in the fluid at the outlet end of the core, often called the fractional water flow equation with gravity effects.^{5,6} A variant of this equation will be presented here for the horizontal flow case at negligible capillary pressure, and such conditions are considered satisfied when this measurement method is used:

$$S_{w,av} - S_w = f_{o2} \cdot Q_w \quad (6)$$

Q_w and $S_{w,av}$ can be measured during the experiment, and f_{o2} can be determined from the slope of the curve for dependence between Q_w and $S_{w,av}$. According to the definition:

$$f_{o2} = \frac{q_o}{q_o + q_w} \quad (7)$$

From the combination of this equation and Darcy's law it follows that:

$$f_{o2} = \frac{1}{1 + \frac{\mu_o / k_{ro}}{\mu_w / k_{rw}}} \quad (8)$$

Since viscosities μ_o and μ_w are known, the ratio of relative permeabilities k_{ro}/k_{rw} can be determined from equation (8). A similar expression can be derived in case of oil displacement by gas.

When performing experiments for determination of relative permeability according to unsteady-state measurement method, attention should be paid to the following:

1. Pressure gradient through the core must be sufficiently high to minimize capillary effects;
2. Pressure difference through the core must be sufficiently low in comparison with total working pressure to make the impact of compressibility negligible;

3. Core must be homogeneous; and
4. During the experiment, pressure and fluid properties must be maintained at constant level.

The method is most frequently used to determine effective (relative) permeabilities k_w/k_o , k_g/k_o and k_g/k_w .

2.2.2. Steady-state methods

Several different methods and authors, who conducted tests on cores applying the steady-state method are mentioned.⁶ The main assumption of the steady-state method is that a fixed ratio of fluid (e.g. water and oil) is injected through the core until the equilibrium between saturation and pressure is achieved. After the equilibrium is achieved, flow velocities of the two phases are recorded and saturation is measured. The procedure is repeated until the equilibrium at higher flow velocity of the second phase is reached, and the data at new saturation value are obtained. The procedure is repeated at higher and higher saturation values until a sufficient number of points are obtained for generation of relative permeability curves. Besides the measurement of saturation by weighing, there are faster and simpler alternatives, such as measurement of core's electrical resistance, in addition to other methods.

When using steady-state relative permeability measurement methods, primary concern is elimination or at least reduction of saturation gradient, caused by impact of capillary pressure at the outlet end of the core, in English original quoted as *outflow boundary effect*, or abbreviated as *end-effect*. It is achieved in several ways, which constitute one of the main differences between the steady-state measurement methods.⁶ Abbreviated term end-effect is common in the Croatian practice and will be used in further text.

2.2.3. Indirect capillary pressure curve measurement methods

Procedures used for calculation of relative permeabilities from capillary pressure measurement data are based on drainage processes where the non-wettable phase (gas) displaces the wettable phase (oil or water).⁶ Because of that these test are limited to reservoirs where production regime is based on the process which proceeds in the drainage direction of flow. Although such methods are generally not preferred for generation of relative permeability curves, they are still used in cases when core sample is too small for flow test, but sufficient for injection of mercury. They are also used for cores with too low permeability for flow tests, or in cases when the capillary pressure is measured but the core is no longer available for flow tests.

Process of injection of mercury as a non-wettable phase into the previously drained core is usual for indirect obtainment of relative permeability curves through capillary pressure measurement. Mercury is injected in measured quantities and at increasing pressures. Usually approximately 20 points of measurement of value pairs (P_c , S_{Hg}) are needed to obtain the complete capillary pressure curve needed for calculation of relative permeability curves. Several authors developed similar equations for calculation of relative permeability curves.⁶ Equations generated by combining Burdin's and

Purcell's equation for wettable and non-wettable phase are shown here:

$$k_{rwt} = \left(\frac{S_L - S_{wi}}{1 - S_{wi}} \right)^2 \frac{\int_{S_{wi}}^{S_L} dS_w / P_c^2}{\int_{S_{wi}}^1 dS_w / P_c^2} \quad (9)$$

$$k_{mwt} = \left(\frac{S_L - S_L}{1 - S_{wi}} \right)^2 \frac{\int_{S_L}^1 dS_w / P_c^2}{\int_{S_{wi}}^1 dS_w / P_c^2} \quad (10)$$

Derivation of these equations is explained and shown in the book by Amyx et al.²

2.2.4. Centrifuge methods

After uniform saturation of reservoir rock samples with one or two fluids (for example oil and water), they are loaded into core holders in an automatic centrifuge.⁶ During centrifuging fluids are expelled from the core and collected into see-trough containers connected to core holders. During centrifugation produced quantities of individual fluids are measured. Works which, together with derivations, specify equations needed for calculation of relative permeability curves from measurement results obtained by centrifuge methods are listed in references of Honarpour et al.⁶

Although centrifuge methods are not often used, they have certain advantages over steady-state methods, i.e. they are much faster. Besides, centrifuge measurements are not accompanied by viscous fingering, which can occur during unsteady-state measurement methods. On the other hand, centrifuge methods are affected by the capillary end-effect problem, and there is no possibility to define the relative permeability of displacing phase (e.g. water).⁶

3. RELATIVE PERMEABILITY MODELS

Over many years experimental relative permeability curve measurements made on cores in the laboratories have created a need to describe these physical processes by equations.⁶ Based on knowledge of physical processes, measured parameters which affect the course of experiments, and applying mathematical and statistical methods with the aid of computers, numerous authors developed different equations. These equations which describe physical processes of flow through the core, and thus the actual flow in the hydrocarbon reservoir, are known as relative permeability models. Basic classifications of these models are based on the following:

1. Do they refer to measurement of two-phase or three-phase flow processes, i.e. do the corresponding equations describe the two-phase or three-phase relative permeability curves;
2. Which combination of two-phase fluids is the subject of testing (oil-water, gas-oil or gas-water);
3. Which experimental methods are used and under which p, T conditions (laboratory or formation conditions) to obtain the relative permeability curves, later on used as basis for description of the process by means of relative permeability models;
4. What is the value of interfacial tension and wettability, i.e. wetting angle, of the system where the flow process occurs;
5. In which direction is the flow measured: in drainage or imbibition direction;
6. On which lithological type of core is the test performed: carbonate rock, consolidated or unconsolidated sandstones, dolomites, conglomerates, fractured rocks, etc.; and
7. On which theoretical foundations and calculation methods is the obtained relation based: on physical laws, statistical-empirical relation or a combination of the first two.

The last seventh point leads to the following classification, which includes the largest number of relative permeability models:

1. Capillary models, which are based on the assumption that porous medium consists of a bundle of capillary tubes of different diameters, whose length is bigger than the length of the core sample (with presence of tortuosity effect);
2. Statistical models, described similarly to capillary models as a bundle of capillary tubes with random distribution of different diameters. These models are described by a large number of thin sections perpendicular to the direction of capillary tubes. Sequence and distribution of these sections is considered accidental;
3. Empirical models, based on empirically obtained relations which describe experimentally determined relative permeabilities; and
4. Network models, most often based on modelling of fluid flow through the porous medium by means of a network of electrical resistors used as an analog model. Mathematical models, used to simulate the flow through the core on computers, can be classified into this group.

The complexity of multiphase flow through the porous medium and different shortcomings of measurement methods (e.g. disregarding of end-effect or ignoring of hysteresis effect) result in only approximate equations of relative permeability curves. Three different relative permeability models, well known and often used in world practice are described in further text. Among them the first two are also used as models for three-phase relative permeability curves.

3.1. Corey correlation

The work of Corey et al describing the measurement and interpretation of three-phase relative permeabilities is among the first important studies in this sphere.⁴ The work is often quoted in literature and particularly appreciated since he took into consideration both end-effect and hysteresis effect in his experiments. In his work relative permeability curves are shown on ternary diagrams, which were a great novelty at the time of publication. Three different ternary diagrams show the curves of equal three-phase relative permeabilities, i.e. isoperms for oil, water and gas. Isoperms for water and gas are

shown as straight lines, analogous to equal saturation lines. Isoperms for oil were calculated according to the following equation:

$$k_{ro} = \frac{(S_L - S_w)^3}{(1 - S_{Lr})^4} (S_w + S_L - 2S_{Lr}) \quad (11)$$

Equation derivation (11) is presented and explained in original Corey's manuscript.⁴ According to ternary diagram for oil, Fig. 4 in that manuscript, it is evident that isoperms, calculated according to equation (11), are in good agreement with points of experimentally measured values of relative permeabilities to oil, k_{ro} . Corey's work is a practical method for calculation of permeability to oil and water in a three-phase system based on measured relative permeabilities to gas.

3.2. Stone correlation

Stone defined a method for estimating three-phase relative permeability based on statistical probability model.¹¹ The data required for calculation of relative permeability to oil, k_{ro} , are two sets of two-phase relative permeabilities in water-oil systems with values of k_{rw} and k_{row} , and in gas-oil systems with values of k_{rg} and k_{rog} . Hysteresis effects were taken into consideration as much as possible using appropriate two-phase data. The main assumption of Stone's method is that relative permeability for wettable phase (water) and relative permeability for non-wettable phase (gas) are functions only of own values for water and gas saturations. At the same time, relative permeability for average or transitional wettability phase (to oil) has much more complex behaviour, and statistical probability model is used with sets of two-phase data for prediction. Equations (12) and (13) show normalized saturations with fluids defined by treating irreducible water saturation, S_{wir} , and residual oil saturation, S_{or} , as saturations that represent immobile fluids:

$$S_o^* = \frac{S_o - S_{or}}{1 - S_{wir} - S_{or}} \quad \text{for } S_o \geq S_{or} \quad (12)$$

and:

$$S_w^* = \frac{S_w - S_{wir}}{1 - S_{wir} - S_{or}} \quad \text{for } S_w \geq S_{wir} \quad (13)$$

Attention should be paid to the fact that:

$$S_g^* = \frac{S_g}{1 - S_{wir} - S_{or}} \quad (14)$$

and:

$$S_g^* + S_w^* + S_o^* = 1 \quad (15)$$

At normalized oil saturation $S_o^* = 100\%$, k_{ro} is also 100%, but decrease of S_o^* (by increasing water and/or gas saturation) results in decrease of k_{ro} which exceeds the decrease of S_o^* . β_w is defined as a factor for multiplication with S_o^* to eliminate the disproportion in decrease between k_{ro} and S_o^* due to the presence of mobile water, and introduces the assumption that β_w is only a function of water saturation. Factor $\beta_g = \beta_g(S_g^*)$ is defined in a similar way. It is assumed that combinations of oil-water and

gas-water flow are mutually independent events, and equation (16) can be defined as a relation between relative permeability to oil and normalized saturation with oil as follows:

$$k_{ro} = S_o^* \cdot \beta_w \cdot \beta_g \quad (16)$$

Values of factor β_w , which is a function of water saturation, are obtained from experimental measurements of k_{row} values by inserting into equation (16) that $\beta_g = 1$ and $S_g^* = 0$. When β_w is derived from thus modified equation (16) we obtain:

$$\beta_w = \frac{k_{row}}{1 - S_w^*} \quad (\text{from two-phase data}) \quad (17)$$

It is assumed that in equation (17) k_{row} is only a function of water saturation, as it is defined in two-phase experiments. Equation for factor β_g can be derived in a similar manner as a function of S_g^* from experimentally determined k_{rog} values:

$$\beta_g = \frac{k_{rog}}{1 - S_g^*} \quad (\text{from two-phase data}) \quad (18)$$

Equations from (12) to (18) define relative permeability to oil in a three-phase system. The assumption on identical microscopic distributions of fluid in the environment of two-phase interfaces is the basis for the assumption that factor β_w is only a function of water saturation, and β_g is only a function of gas saturation. Likewise, the assumption that water and gas are spatially separated leads us to the assumption that dependence of oil flow on water or gas are two mutually independent phenomena. Due to that, probability theory is linked to the assumption that probability of oil flow blockage by water, gas or both fluids is equal to the product of these factors, $\beta_w \cdot \beta_g$, and k_{ro} was so defined in equation (16). This probability model was verified on experimental data from previously published works of renowned authors in that sphere (Corey, Dalton and Saraf) confirming good agreement with measured values.¹¹

Besides the previously described article, later on Stone published another work in which he revised the results presented in the first paper.¹² In that work he gave a new equation, known as three-phase relative permeability estimation equation, based on channel flow theory:

$$k_{ro} = (k_{row} + k_{rw}) \cdot (k_{rog} + k_{rg}) - (k_{rw} + k_{rg}) \quad (19)$$

A derivation of that equation is given in the same work, as well as the theoretical basis for explanations. The following explanation is needed to clarify obtaining of data required for insertion into this equation and their use in ternary diagram plotting. If calculated values of S_w and S_g in a three-phase system are given, then S_w is used to obtain k_{rw} and k_{row} values from the two-phase water-oil system; S_g is used to obtain k_{rg} and k_{rog} values from the two-phase gas-oil system. Relative permeabilities to gas and oil, k_{rg} and k_{rw} , are directly inserted into a three-phase system, while three-phase relative permeability to oil, k_{ro} , is calculated according to equation (19). In the later work Stone explained that he improved

his probability model given in the previous work. Relative permeabilities predicted by this model are in better agreement with experimental data. To confirm it, he applied again the data of the same authors (Corey, Dalton and Saraf).¹² In this work he also used residual oil saturation data, S_{or} , and compared it against the data given by Holmgren and Morse to additionally verify validity of equation (19).¹²

3.3. Chierici correlation

In his work Chierici used 4- and 5-parameter equations to obtain relative permeability curves of gas-oil system in drainage direction, and water-oil system in imbibition direction.³ As the author notes, these curves are in good agreement with experimental data, particularly in initial and end points, better than Corey's model and polynomial approximation. The main characteristics of Chierici's model that some of the parameters used in that model have a physical meaning, the others can be determined by non-linear regression on the experimental data points and adjusted to represent pseudorelative permeability curves. In the following equations are symbols used by Chierici in his work.³

For gas-oil drainage he proposed the following equations:

$$k_{ro} = e^{-A \cdot R_g^L} \quad (20)$$

and:

$$k_{rg} = e^{-B \cdot R_g^M} \quad (21)$$

where A , B , L and M are positive numbers, and R_g is normalized saturation according to equation:

$$R_g = \frac{S_g - S_{gc}}{1 - S_{iw} - S_{gc}} \quad (22)$$

where:

$$S_g - S_{gc} = 0 \text{ for } S_g \leq S_{gc}$$

Equations (20) and (21) are 4-parameter equations in which empirical coefficients A , L , S_{gc} and S_{iw} are used to determine functional dependence $k_{ro}(S_o)$, and empirical coefficients B , M , S_{gc} and S_{iw} are used to determine functional dependence $k_{rg}(S_g)$. Only S_{gc} and S_{iw} have a physical meaning. The values of empirical coefficients A , L , B and M are determined by nonlinear regression on the sets of experimental data points. It was confirmed in practice that it is better not to apply the regression process to obtain coefficients A , L , B and M directly to equations (20) and (21), but to their logarithmic forms:

$$-\ln k_{ro} = A \cdot R_g^L \quad (23)$$

$$-\ln k_{rg} = B \cdot R_g^M \quad (24)$$

The following equations for water-oil imbibition have been found to reproduce very well the experimentally determined relative permeability curves, including their behaviour at the initial and end points:

$$k_{ro}^* = e^{-A \cdot R_w^L} \quad (25)$$

$$k_{rw}^* = e^{-B \cdot R_w^M} \quad (26)$$

where A , B , L and M are positive numbers, while:

$$R_w = \frac{S_w - S_{iw}}{1 - S_{or} - S_w} \quad (27)$$

$$k_{rw}^* = \frac{k_{rw}}{k_{rw}(S_{or})} \quad (28)$$

and

$$k_{ro}^* = \frac{k_{ro}}{k_{ro}(S_{iw})} \quad (29)$$

Equations (25) and (26) are 5-parameter equations, where empirical coefficients A , L , S_{iw} , S_{or} and $k_{ro}(S_{iw})$ are used to calculate the functional dependence $k_{ro}(S_o)$, and empirical coefficients B , M , S_{iw} , S_{or} and $k_{rw}(S_{or})$ to calculate the functional dependence $k_{rw}(S_w)$. Out of these coefficients only S_{iw} , S_{or} , $k_{ro}(S_{iw})$ and $k_{rw}(S_{or})$ have a physical meaning, and for statistically homogeneous reservoir zones their values can be evaluated by using correlations. The values of empirical coefficients A , L , B and M are determined by nonlinear regression on the sets of experimental data points. In this case, as in the previous one, equations (25) and (26) are used in logarithmic form to achieve better matching of curves and measured points. This is particularly important for achievement of good matching near the initial and end points of curves, where previously used methods did not succeed to sufficiently approach the measured points.³

4. DATA PROCESSING PRIOR TO INPUT INTO RESERVOIR MODEL

Prior to input of relative permeability data into the hydrocarbon reservoir simulation model, particular attention should be paid to analysis and selection of curves obtained on the basis of laboratory measurements. If a sufficient number of correctly measured and analysed curves are available for the relevant heterogeneous reservoir, we can continue with their statistical analysis. Following is an example which shows the processing of relative permeability data.

4.1. Summary of core measurements

Laboratory measurements of oil-water system relative permeabilities were performed on six core samples. The cored rock was consolidated sandstone and all samples belonged to the same reservoir, i.e. had the same or similar lithology, as explained in further text. Measurements were carried out applying the porous plate technique, from the previously described steady-state core measurement methods. The samples were restored to their original condition under vacuum with purified mineral oil of 20 mPa·s viscosity. Each sample was placed into a hydraulic core holder and saturated with oil until all trapped gas was eliminated (these measurements were preceded by gas-oil system relative permeability measurements). Effective permeability to oil was again measured afterwards. Measurements were performed by displacing oil with synthesised brine, whose physical properties are equal or very similar to actual formation water. Incremental volumes of produced oil and water were recorded as a function of time and experiments

Table 1. Relative and normalized relative permeability data for sample No. 1 depending on water saturation and normalized water saturation⁷

S_w [parts of unit]	S_w^N [parts of unit]	$k_{row}=k_{row}^N$ [parts of unit]	$(k_{row}^N)_{av}$ [parts of unit]	k_{rw} [parts of unit]	k_{rw}^N [parts of unit]	$(k_{rw}^N)_{av}$ [parts of unit]
0.153	0.000	1.000 0	1.000 0	0.000 0	0.000 0	0.000 0
0.233	0.168	0.425 0	0.419 8	0.088 0	0.162 7	0.140 9
0.291	0.290	0.231 0	0.203 3	0.130 0	0.240 3	0.256 5
0.364	0.443	0.095 0	0.071 9	0.227 0	0.419 6	0.409 0
0.386	0.489	0.065 0	0.051 4	0.270 0	0.499 1	0.456 1
0.406	0.532	0.049 0	0.037 6	0.305 0	0.563 8	0.499 3
0.434	0.590	0.032 0	0.024 4	0.351 0	0.648 8	0.560 3
0.466	0.658	0.022 0	0.015 2	0.410 0	0.757 9	0.630 8
0.527	0.786	0.008 2	0.006 8	0.491 0	0.907 6	0.767 2
0.552	0.838	0.005 0	0.004 8	0.518 0	0.957 5	0.823 7
0.578	0.893	0.003 1	0.003 1	0.540 0	0.998 2	0.882 9
0.629	1.000	0.000 0	0.000 0	0.541 0	1.000 0	1.000 0

Table 2. Relative and normalized relative permeability data for sample No. 2 depending on water saturation and normalized water saturation⁷

S_w [parts of unit]	S_w^N [parts of unit]	$k_{row}=k_{row}^N$ [parts of unit]	$(k_{row}^N)_{av}$ [parts of unit]	k_{rw} [parts of unit]	k_{rw}^N [parts of unit]	$(k_{rw}^N)_{av}$ [parts of unit]
0.178	0.000	1.000 0	1.000 0	0.000 0	0.000 0	0.000 0
0.305	0.381	0.085 0	0.111 2	0.116 0	0.666 7	0.346 7
0.392	0.643	0.025 0	0.016 8	0.132 0	0.758 6	0.615 1
0.433	0.766	0.011 0	0.007 6	0.144 0	0.827 6	0.745 8
0.468	0.871	0.003 4	0.003 8	0.157 0	0.902 3	0.859 0
0.490	0.937	0.000 88	0.001 8	0.171 0	0.982 8	0.930 9
0.511	1.000	0.000 0	0.000 0	0.174 0	1.000 0	1.000 0

Table 3. Relative and normalized relative permeability data for sample No. 3 depending on water saturation and normalized water saturation⁷

S_w [parts of unit]	S_w^N [parts of unit]	$k_{row}=k_{row}^N$ [parts of unit]	$(k_{row}^N)_{av}$ [parts of unit]	k_{rw} [parts of unit]	k_{rw}^N [parts of unit]	$(k_{rw}^N)_{av}$ [parts of unit]
0.069	0.000	1.000 0	1.000 0	0.000 0	0.000 0	0.000 0
0.236	0.300	0.184 0	0.190 9	0.082 0	0.148 3	0.266 1
0.296	0.408	0.101 0	0.092 7	0.129 0	0.233 3	0.372 9
0.338	0.483	0.063 0	0.053 9	0.187 0	0.338 2	0.449 4
0.364	0.530	0.044 0	0.038 2	0.231 0	0.417 7	0.497 3
0.390	0.576	0.035 0	0.027 0	0.283 0	0.511 8	0.545 7
0.431	0.650	0.024 0	0.016 0	0.346 0	0.625 7	0.622 8
0.544	0.853	0.006 0	0.004 4	0.463 0	0.837 3	0.839 4
0.579	0.916	0.003 0	0.002 5	0.507 0	0.916 8	0.907 7
0.626	1.000	0.000 0	0.000 0	0.553 0	1.000 0	1.000 0

were discontinued after the k_{rw}/k_{ro} ratio exceeded 100. It was followed by measurement of effective permeability to water. Tables 1 to 6 show experimentally obtained core data.

4.2. Average curves from laboratory data

The practical use of non-linear regression method applied to the measured data in order to obtain average rel-

ative permeability curves will be briefly presented below. The well known previously presented Sigmund and McCaffery¹⁰ relative permeability data processing method, often used in addition to other three described methods, as another high-quality relative permeability model confirmed in practice, was used to obtain the average functional dependence of our data. In continuation all shown nomenclature is in agreement with symbols

Table 4. Relative and normalized relative permeability data for sample No. 4 depending on water saturation and normalized water saturation⁷

S_w [parts of unit]	S_w^N [parts of unit]	$k_{row}=k_{row}^N$ [parts of unit]	$(k_{row}^N)_{av}$ [parts of unit]	k_{rw} [parts of unit]	k_{rw}^N [parts of unit]	$(k_{rw}^N)_{av}$ [parts of unit]
0.122	0.000	1.000 0	1.000 0	0.000 0	0.000 0	0.000 0
0.184	0.160	0.211 0	0.438 6	0.244 0	0.366 9	0.133 7
0.190	0.176	0.189 0	0.402 1	0.251 0	0.377 4	0.147 9
0.252	0.336	0.102 0	0.151 0	0.305 0	0.458 6	0.301 5
0.366	0.630	0.026 0	0.018 3	0.419 0	0.630 1	0.602 4
0.405	0.731	0.012 0	0.009 4	0.476 0	0.715 8	0.709 0
0.437	0.814	0.005 7	0.005 7	0.522 0	0.785 0	0.797 5
0.461	0.876	0.002 9	0.003 6	0.569 0	0.855 6	0.864 6
0.470	0.899	0.002 3	0.002 9	0.583 0	0.876 7	0.889 8
0.509	1.000	0.000 0	0.000 0	0.665 0	1.000 0	1.000 0

Table 5. Relative and normalized relative permeability data for sample No. 5 depending on water saturation and normalized water saturation⁷

S_w [parts of unit]	S_w^N [parts of unit]	$k_{row}=k_{row}^N$ [parts of unit]	$(k_{row}^N)_{av}$ [parts of unit]	k_{rw} [parts of unit]	k_{rw}^N [parts of unit]	$(k_{rw}^N)_{av}$ [parts of unit]
0.112	0.000	1.000 0	1.000 0	0.000 0	0.000 0	0.000 0
0.209	0.299	0.221 0	0.191 5	0.157 0	0.219 3	0.005 7
0.222	0.340	0.168 0	0.147 5	0.201 0	0.280 7	0.008 4
0.246	0.414	0.110 0	0.088 9	0.240 0	0.335 2	0.017 4
0.283	0.528	0.062 0	0.038 7	0.322 0	0.449 7	0.049 0
0.315	0.627	0.032 0	0.018 8	0.398 0	0.555 9	0.107 3
0.333	0.682	0.019 0	0.012 9	0.450 0	0.628 5	0.159 8
0.353	0.744	0.011 0	0.008 7	0.502 0	0.701 1	0.241 0
0.377	0.818	0.005 6	0.005 5	0.576 0	0.804 5	0.379 3
0.402	0.895	0.002 5	0.003 1	0.648 0	0.905 0	0.585 3
0.417	0.941	0.001 2	0.001 7	0.683 0	0.953 9	0.746 5
0.436	1.000	0.000 0	0.000 0	0.716 0	1.000 0	1.000 0

Table 6. Relative and normalized relative permeability data for sample No. 6 depending on water saturation and normalized water saturation⁷

S_w [parts of unit]	S_w^N [parts of unit]	$k_{row}=k_{row}^N$ [parts of unit]	$(k_{row}^N)_{av}$ [parts of unit]	k_{rw} [parts of unit]	k_{rw}^N [parts of unit]	$(k_{rw}^N)_{av}$ [parts of unit]
0.103	0.000	1.000 0	1.000 0	0.000 0	0.000 0	0.000 0
0.178	0.182	0.290 0	0.388 0	0.092 0	0.146 0	0.153 8
0.198	0.231	0.212 0	0.292 6	0.140 0	0.222 2	0.199 4
0.235	0.320	0.125 0	0.167 2	0.171 0	0.271 4	0.286 3
0.293	0.461	0.045 0	0.063 2	0.233 0	0.369 8	0.427 2
0.367	0.641	0.019 0	0.017 0	0.333 0	0.528 6	0.613 2
0.430	0.794	0.007 3	0.006 4	0.430 0	0.682 5	0.775 7
0.472	0.896	0.002 9	0.003 1	0.501 0	0.795 2	0.885 9
0.495	0.951	0.001 2	0.001 4	0.539 0	0.855 6	0.946 8
0.515	1.000	0.000 0	0.000 0	0.630 0	1.000 0	1.000 0

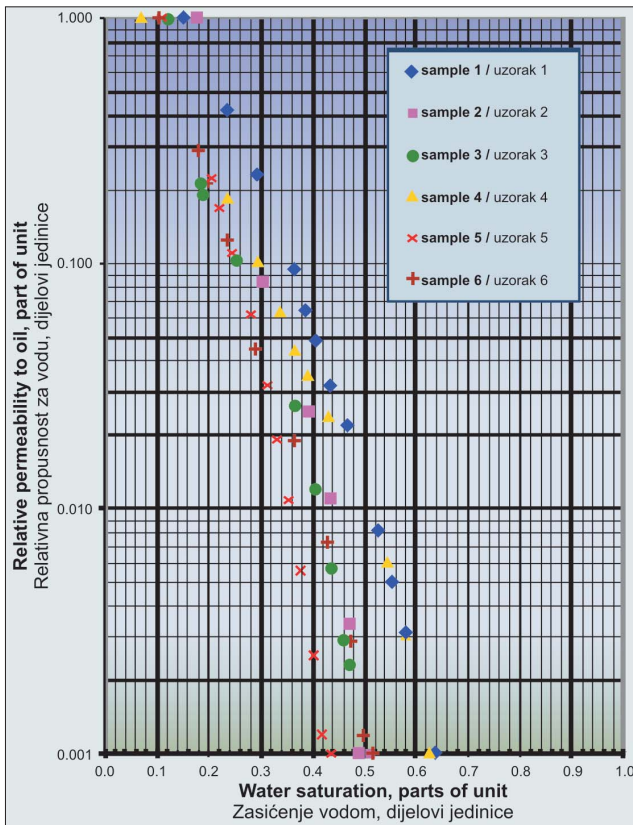


Fig. 1. Dependence of relative permeability to oil on water saturation⁷
 Sl. 1. Ovisnost relativne propusnosti za naftu o zasićenju vodom⁷

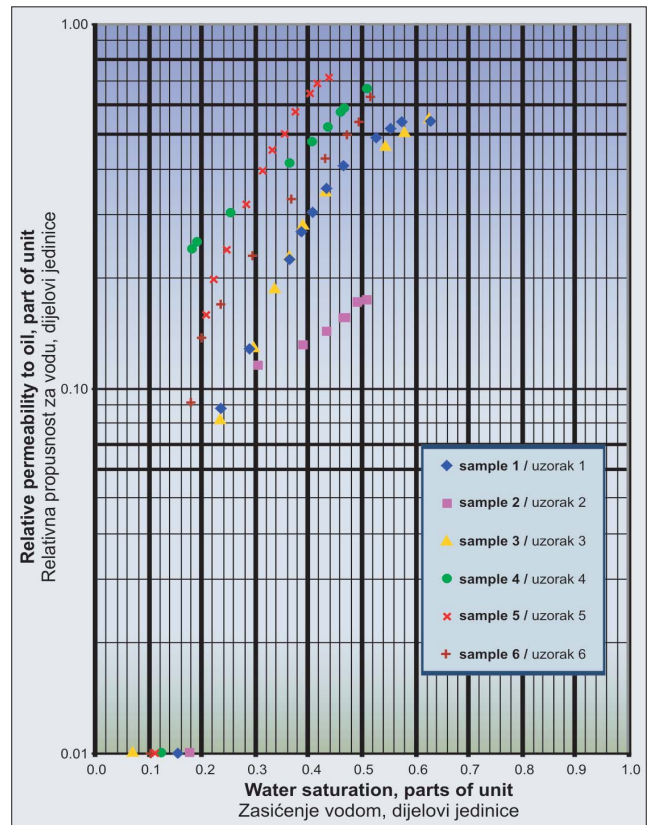


Fig. 2. Dependence of relative permeability to water on water saturation⁷
 Sl. 2. Ovisnost relativne propusnosti za vodu o zasićenju vodom⁷

used in the described work.¹⁰ The method is based on determination of average normalized curves k_{row}^N and k_{rw}^N in the function of normalized water saturation, S_w^N . Figs.1 and 2, for six previously described samples, show points of measured values k_{row} or k_{rw} in the function of saturation. These samples have different initial and end saturation points, and due to that significant scattering between value pairs (k_{row}, S_w) and (k_{rw}, S_w) is noticeable. Because of that, saturation and relative permeability values are normalized according to equation (31), which is similar to the previously given equation (13):

$$S_w^N = S_e = \frac{S_w - (S_w)_{min}}{(S_w)_{max} - (S_w)_{min}} \quad (31)$$

and previously shown equations (28) and (29). Regression analysis is applied to obtained value pairs (S_w^N, k_{row}^N) and (S_w^N, k_{rw}^N) according to the following relative permeability models for wettable:¹⁰

$$k_{rw}^* = k_{rw}^N = \frac{(S_e)^{\epsilon_w} + A \cdot S_e}{1 + A} \quad (32)$$

and non-wettable phase:

$$k_{ro}^* = k_{ro}^N = \frac{(1 - S_e)^{\epsilon_{nw}} + B \cdot (1 - S_e)}{1 + B} \quad (33)$$

Pairs of normalized values (k_{row}^N, S_w^N) and (k_{rw}^N, S_w^N) are shown in Figs. 3 and 4 where good correlation between the data is evident, in confirmation of the same lithological assignment, being at the same time a criterion for selection of samples for statistical processing. It was the basis for application of regression analysis to all 6 selected samples mentioned in the previous sub-section. Averaged normalized curves $k_{row}^N(S_w^N)$ and $k_{rw}^N(S_w^N)$ shown on lin-log diagram in the same figures are obtained as a result of regression analysis.

4.3. Construction of relative permeability curves based on known initial and end saturation points

Relative permeability curves $k_{row}(S_w)$ and $k_{rw}(S_w)$ are determined on the basis of known dependence trend of initial (S_{wir}) and end (S_{orw}) saturation points in the function of: 1. absolute permeability - $k(k_{air})$, 2. parameter $(k/\phi)^{1/2}$ or 3. flow zone indicator - F_{zi} .¹⁰ Flow zone indicator equation is:

$$F_{zi} = \frac{R_{qi}}{\phi_z} = \frac{0.0316 \cdot \sqrt{\frac{f}{\phi}}}{\left(\frac{\phi}{1 - \phi}\right)} \quad (34)$$

Factor 0,0316 is present knowing that in equation (34) permeabilities are entered in $10^{-3} \mu m^2$.

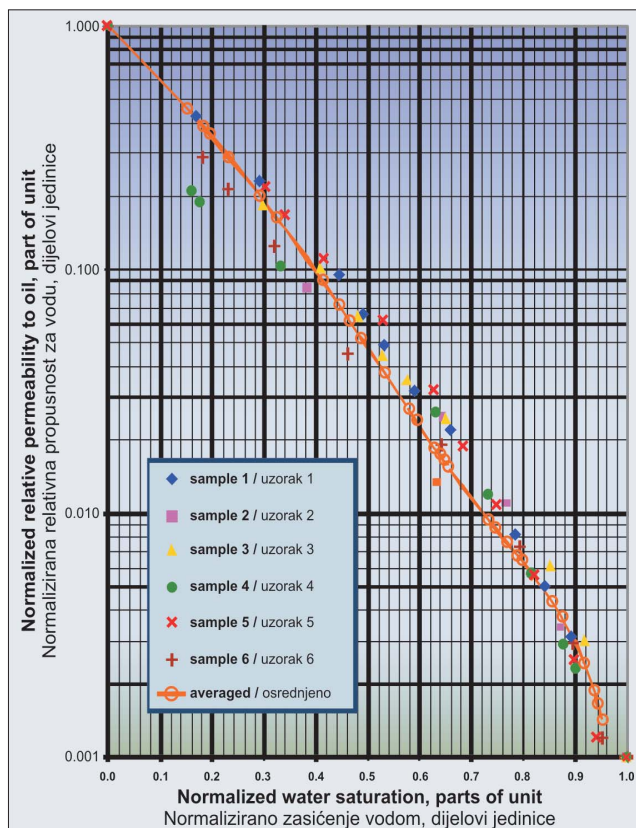


Fig. 3. Dependence of normalized relative permeability to oil on normalized water saturation^{7,10}

Sl. 3. Ovisnost normalizirane relativne propusnosti za ulje na normaliziranom zasićenju vodom^{7,10}

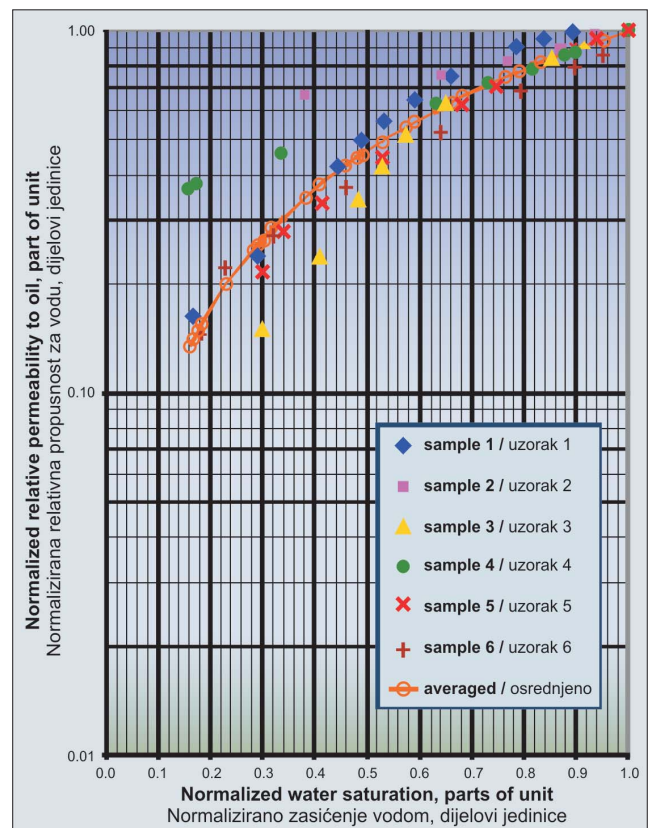


Fig. 4. Dependence of normalized relative permeability to water on normalized water saturation^{7,10}

Sl. 4. Ovisnost normalizirane relativne propusnosti za vodu o normaliziranom zasićenju vodom^{7,10}

Table 7. Data for dependence diagram of irreducible water and residual oil saturation on absolute permeability - k_{air} parameter (k_{air}/ϕ)^{1/2} and zone of flow indicator - F_{zi} ^{1,7,10}

Sample No.	S_{wir} [parts of unit]	S_{orw} [parts of unit]	k_{air} [$10^{-3} \mu m^2$]	$k_o(S_{wir})$ [$10^{-3} \mu m^2$]	ϕ [parts of unit]	$(k_{air}/\phi)^{1/2}$ [μm]	F_{zi}
1	0.153	0.371	194	133	0.212	30.25	3.531
2	0.178	0.489	3.9	0.23	0.119	5.72	1.331
3	0.069	0.374	787	519	0.171	67.84	10.327
4	0.122	0.491	356	236	0.145	49.55	9.174
5	0.112	0.564	117	74	0.159	27.13	4.505
6	0.103	0.485	13	8.1	0.143	9.53	1.794

Table 7 shows the data required to determine average trends of measured end saturation points in the function of three previously explained parameters. Figure 5 in lin-log diagram shows trends determined through exponential regression on 6 dependence combinations explained in the legend of the figure. It is followed by selection of combination k (k_{air}), $(k/\phi)^{1/2}$ or F_{zi} which indicates the best correlation on the basis of regression analysis, R^2 , and classification into statistical classes, i.e. hydraulic types of reservoir rocks (HT) within the same lithological unit is made on the basis of the selected combination, but with different k and ϕ . According to selected

statistical classes of HT , value pairs (S_{wir} , S_{orw}) required for denormalization according to S_w^N of $k_{row}^N(S_w^N)$ and $k_{rw}^N(S_w^N)$ curves described in previous section, are read from diagram in Fig. 5. Following the same principle, not described in detail in this paper, the curves are subsequently denormalized also according to relative permeabilities, k_{row}^N and k_{rw}^N . Families of relative permeability curves with different initial and end points are obtained on that basis, which describe different hydraulic types (HT), i.e. rock types (RT), and serve for input into the reservoir simulation model.

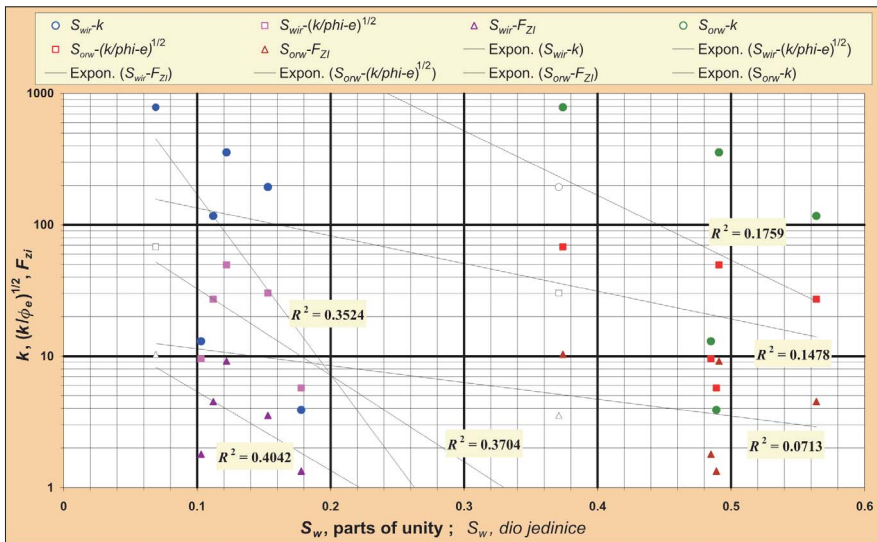


Fig. 5. Presentation of end saturation point trends on samples 1 to 6 in the function of k (k_{air}), $(k/\phi)^{1/2}$ and F_{zi} 1,7,10
 Sl. 5. Prikaz trenda završnih točaka zasićenja uzoraka od 1 do 6 u funkciji k (k_{air}), $(k/\phi)^{1/2}$ and F_{zi} 1,7,10

5. ANALYSIS OF RELATIVE PERMEABILITY TREATMENTS IN SIMULATION PROGRAMS

Individual hydraulic type, *HT*, or rock type, *RT*, is presented in reservoir model with one set of relative permeability and capillarity curves. The model can include one or more *HT* (*RT*), depending on more or less pronounced reservoir heterogeneity, i.e. reservoir rock structure complexity. In Black Oil simulator up to 40 different types of rock can be determined.⁸ Within each *RT* curve set, three types of curves can be simulated. i.e. memorized in the simulator:

- Correlation curves, which are automatically generated using different regression models according to industry standards,
- Laboratory curves, which represent direct measurements on cores, or are obtained from production data measurements, and
- Simulation curves, used in simulation.

After input of data into the model, any of these three types of curves can be manipulated in different ways. They can be changed graphically or applying numerical data from tables. Likewise, it is also possible to show different types of curves at the same time for comparison purposes. Besides, copying from one curve type to another is also possible. The treatment of hysteresis effect on relative permeability and capillary pressure curves is also possible.

To create the three-phase flow system, generated automatically according to the selected Stone's model, such as Stone II, actually used in reservoir simulation model, requires two different sets of relative permeability curves: water-oil and gas-fluid.⁹ After entering the two-phase sets of relative permeability curves, three-phase relative permeabilities can be generated by

means of a simulator. Description of use of different versions of Stone's three-phase flow model is shown in more detail in SimBest II manual,⁹ and theoretical basis is briefly explained in the central part of this paper.^{11,12}

CONCLUSION

The application of knowledge briefly presented in this paper can usefully contribute to better understanding of the importance that should be dedicated to preparation and analysis of data entered into hydrocarbon reservoir models. Relative permeability curves, whose shape considerably affects the simulation of hydrocarbon reservoir production behaviour, are among the most essential and delicate parts of the initializing input data sets, that constitute the reservoir model. The comprehensive insight into this segment of science will be useful in the

simulation phase known as history matching, where the quality of input data will be even more significant. In case of well defined relative permeability curves, history matching should not be a time-consuming process and only slight corrections of entered curves will lead to reliable verification of reservoir model.

NOMENCLATURE

- A* = empirical coefficient in relative permeability equation
- B* = empirical coefficient in relative permeability equation
- B_g* = gas volume factor
- B_o* = oil volume factor
- B_{oi}* = initial oil volume factor
- f_{o2}* = fractional oil flow in the liquid phase at the out let end, part of unit
- F_{zi}* = flow zone indicator
- HT* = hydraulic type of rock
- k* = absolute permeability, 10⁻³ μm²
- k_g* = effective permeability to gas, 10⁻³ μm²
- k_o* = effective permeability to oil, 10⁻³ μm²
- k_{rg}* = relative permeability to gas, part of unit
- k_{ro}* = relative permeability to oil in the three-phase fluid system, part of unit
- k_{ro}(S_{wi})* = relative permeability to oil at initial water saturation, part of unit
- k_{rog}* = relative permeability to oil in the two-phase oil-gas system, part of unit
- k_{row}* = relative permeability to oil in the two-phase oil-water system, part of unit
- k_{rw}* = relative permeability to water, part of unit
- k_{rw}(S_{or})* = relative permeability to oil at residual oil saturation, part of unit
- k_{ro}** = normalized relative permeability to oil, part of unit
- k_{rw}** = normalized relative permeability to water, part of unit
- k_{rw}^N* = normalized relative permeability for wetting phase, part of unit

k_{mw}^N	=	normalized relative permeability for non-wetting phase, part of unit
L	=	empirical coefficient in relative permeability equation
M	=	empirical coefficient in relative permeability equation
N	=	initial oil reserves, m ³
N_p	=	produced oil quantity, m ³
P_c	=	capillary pressure, Pa
q_o	=	oil quantity in the flow, cm ³ /s
q_w	=	water quantity in the flow, cm ³ /s
Q_w	=	cumulative injected water quantity, 1/(pore volume)
R	=	free gas and oil ratio
R_g	=	normalized gas saturation, part of unit
R_p	=	total produced gas and oil ratio
R_{qi}	=	reservoir quality index, μm
R_s	=	dissolved (petroleum) gas and oil ratio
R_T	=	rock type
R_w	=	normalized water saturation, part of unit
S_e	=	normalized wetting phase saturation, part of unit
S_g	=	gas saturation, part of unit
S_{gc}	=	critical gas saturation, part of unit
S_g^*	=	normalized gas saturation, part of unit
S_{iw}	=	initial water saturation, part of unit
S_L	=	liquid phase saturation, part of unit
S_{Lr}	=	residual liquid phase saturation, part of unit
S_o	=	oil saturation, part of unit
S_{or}	=	residual oil saturation, part of unit
S_{orw}	=	residual oil saturation in two-phase oil-water system, part of unit
S_o^*	=	normalized oil saturation, part of unit
S_w	=	water or wetting phase saturation, part of unit
$S_{w,av}$	=	average water saturation, part of unit
S_{w2}	=	water saturation at the out let end, part of unit
S_{wi}	=	initial water saturation, part of unit
S_{wir}	=	irreducible water saturation, part of unit
S_w^*	=	normalized water saturation, part of unit
S_w^N	=	normalized wetting phase saturation, part of unit
$(S_w)_{max}$	=	maximal wetting phase saturation, part of unit
$(S_w)_{min}$	=	minimal wetting phase saturation, part of unit
β_g	=	factor which takes into consideration oil blockage by gas
β_w	=	factor which takes into consideration oil blockage by water
ε_w	=	parameter in equation for relative permeability of wettable phase
ε_{nw}	=	parameter in equation for relative permeability of non-wettable phase
μ_g	=	gas viscosity, mPa·s
μ_o	=	oil viscosity, mPa·s
μ_w	=	water viscosity, mPa·s
ϕ	=	porosity, part of unit
ϕ_z	=	pore volume and grain volume ratio

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