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ANALYTICAL EVALUATION OF HEAT TRANSFER CONDUCTIVITY WITH VARIABLE PROPERTIES

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Original scientific paper

The homotopy analysis method (HAM) as a new technique which is powerful and easy-to-use, is applied to solve heat transfer problems. In this paper, we use HAM for heat transfer conductivity equation with variable properties which may contain highly nonlinear terms.The obtained results are also compared with the Adomian decomposition method (ADM). The homotopy analysis method contains the auxiliary parameter ħ, which provides us with a simple way to adjust and control the convergence region of solution series.

Keywords: *homotopy analysis method, multi-layer composite, transfer conductivity equation, unsteady*

Analitička evaluacija vodljivosti prijenosa topline s različitim svojstvima

Izvorni znanstveni članak

Metoda homotopne analize (HAM) kao nova tehnika, moćna i jednostavna za upotrebu, primjenjuje se kod rješavanja problema prijenosa topline. U ovom radu koristimo HAM za jednadžbu vodljivosti prijenosa topline s promjenljivim svojstvima koja može sadržavati krajnje nelinearne članove. Dobiveni su rezultati također uspoređeni s Adomian metodom dekompozicije (ADM). Metoda homotopne analize sadržava pomoćni parameter ħ koji na jednostavan način omogućuje podešavanje i praćenje područja konvergencije serije rješenja.

Ključne riječi: *jednadžba prijenosa vodljivosti, metoda homotopne analize, nestalan, višeslojni kompozit*

1 Introduction
Uvod

Most scientists believe that the combination of numerical and semi-exact analytical methods can also end with useful results. Recently, several approximate methods have been developed to analyze the nonlinear problems [1-12]. Homotopy analysis method (HAM) which was recently developed by [13] is one of the most successful and efficient methods in solving non-linear equations.

In comparison with previous analytical techniques, the HAM has the following advantages. Firstly, unlike all previous analytical techniques, the HAM provides us with great freedom to express solutions of a given non-linear problem by means of different base functions. Secondly, the HAM always provides us with a family of solution expressions in the auxiliary parameter *h*, even if a non-linear problem has a unique solution. Thirdly, unlike perturbation techniques, the HAM is independent of any small or large quantities and eventually, through previous works [14-18], it is also shown that the HAM method logically contains some previous techniques such as Adomian's decomposition method, Lyapunov's artificial small parameter method, and the d-expansion method.

Many authors [19-24] have successfully applied the HAM in solving different types of non-linear problems i.e. coupled, decoupled homogeneous and non-homogeneous equations arising in different physical problems such as heat transfer, fluid flow, oscillatory systems, etc.

Ghasemi et al. [22] analyzed the steady two-dimensional laminar forced MHD Hiemenz flow against a flat plate with variable wall temperature in a porous medium. They utilized homotopy analysis method for solving the transformed nonlinear boundary layer equations. The velocity and temperature profiles for various values of Prandtl number *Pr*, the Hartmann number *Ha*, exponent of wall temperature λ, the permeability parameter Ω and suction and injection parameter f_w were presented within their research. An approximate analytical solution was later established for the well known Richards' equation for unsaturated flow in soils [23]. Indeed, it was found that using homotopy analysis method (HPM) and differential transform method (DTM) yields a non converging series to the problem under consideration as time elapses. However, convergence was adequately achieved within the HAM [23].

Moreover, there are no rigorous theories to direct us to choose the initial approximations, auxiliary linear operators, auxiliary functions, and auxiliary parameter *h*. From the practical viewpoints there are some fundamental rules such as the rule of solution expression, coefficient ergodicity, and the rule of solution existence, which play important roles within the HAM. Unfortunately, the rule of solution expression implies such an assumption that we should have, more or less, some prior knowledge about a given non-linear problem. So, theoretically, this assumption impairs the HAM, although we can always attempt some base functions even if a given non-linear problem is completely new for us.

In this research the basic idea of the HAM is introduced and then its application to the diffusion equation with variable physical properties such as density and conductivity is studied and compared with Adomian decomposition method in [25].

2 Mathematical modeling of M-layer unsteady heat conduction
Matematički prikaz nestalne toplinske vodljivosti M-sloja

Consider a composite solid consisting of *M* parallel layers in perfect thermal contact, as shown in Fig. 1. Let k_i and α_i be the thermal conductivity and the thermal diffusivity of the *i*-th layer, respectively (i=1, 2,…, M). Initially (t=0), the body, which is confined to the domain $x_1 \le x \le x_{M+1}$, is at a specified temperature $f(x)$. Suddenly, at t=0, both boundary surfaces of the composite solid are subjected to convection heat flux. In particular, a fluid at a temperature T_∞ with a heat transfer coefficient h_1 flows over the outer surface x=x_1, and another fluid at the same temperature T_∞ but with a different heat transfer coefficient h_{M+1} flows over the other outer surface $x = x_{M+1}$.

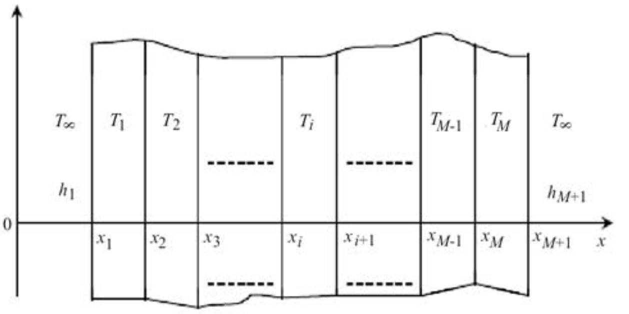

***Figure 1** Schematic representation of a multi-layer composite medium* [26]
***Slika 1.** Schematski prikaz medija od višeslojnog kompozita* [26]

The assumptions made in deriving the mathematical modeling of the proposed unsteady heat conduction process are:

(a) There is no heat generation within the body.
(b) The thermal properties, i.e., conductivity and diffusivity, are independent of temperature and are uniform within each of the M_{layers}.
(c) The temperature T_∞ of the fluid surrounding the medium is spatially uniform and maintained constant for times $t > 0$.
(d) The multi-layer solid is sufficiently large in the y and z directions in comparison to its thickness in the x direction.
(e) The heat transfer coefficients h_1 and h_{M+1} are uniform and constant.

Therefore, the heat conduction problem at issue can be considered linear, one-dimensional, and also homogeneous setting $\theta_i(x,t) = T_\infty - T_i(x,t)$ $(i = 1,2,...,M)$ [27]. Its final mathematical formulation in a generic coordinate system, namely rectangular, cylindrical or spherical, may be given as $(t \geq 0)$.

- Heat conduction differential equations:

$$\frac{1}{x^q}\frac{\partial}{\partial x}\left(x^q \frac{\partial \theta_i}{\partial x}\right) = \frac{1}{\alpha_i}\frac{\partial \theta_i}{\partial t}, \quad x \in \left[x_i, x_{i+1}\right] \quad (i = 1,2,...,M). \tag{1}$$

Where q=0, 1, 2 for plate, cylinder and sphere, respectively.

- Outer boundary condition (x=x_1):

$$-k_1\left(\frac{\partial \theta_1}{\partial x}\right)_{x_1} + h_1\theta_1(x_1,t) = 0. \tag{2}$$

- Inner boundary conditions (x=x_i):

$$\theta_{i-1}(x_i,t) = \theta_i(x_i,t) \qquad (i = 2,3,...,M), \tag{3}$$

$$k_{i-1}\left(\frac{\partial \theta_{i-1}}{\partial x}\right)_{x_i} = k_i\left(\frac{\partial \theta_i}{\partial x}\right)_{x_i} \qquad (i = 2,3,...,M). \tag{4}$$

- Outer boundary condition:

$$-k_M\left(\frac{\partial \theta_M}{\partial x}\right)_{x_{M+1}} + h_{M+1}\theta_M(x_{M+1},t) = 0. \tag{5}$$

- Initial conditions:

$$\theta_i(x,t=0) = F(x), \quad x \in \left[x_i, x_{i+1}\right] \ (i = 1,2,...,M). \tag{6}$$

3 Basic idea of HAM
Osnovna ideja HAM-a

In this paper, we apply the homotopy analysis method [14-18] to the discussed problem. To show the basic idea, let us consider the following differential equation

$$N[u(\tau)] = 0. \tag{7}$$

Where N is a nonlinear operator, τ denotes independent variable, $u(\tau)$ is an unknown function, respectively. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. Generalizing the traditional homotopy method [14], constructs the so-called zero-order deformation equation.

$$(1-p)L[\phi(\tau;p) - u_0(\tau)] = p\hbar H(\tau)N[\phi(\tau;p)], \tag{8}$$

where $p \in [0,1]$ is the embedding parameter, $\hbar \neq 0$ is a nonzero auxiliary parameter, $H(\tau) \neq 0$ is an auxiliary function, L is an auxiliary linear operator, $u_0(\tau)$ is an initial guess of $u(\tau)$, $\phi(\tau;p)$ is a unknown function, respectively. It is important that one has great freedom to choose auxiliary things in HAM. Obviously, when p=0 and p=1, it holds respectively.

$$\phi(\tau;0) = u_0(\tau), \ \phi(\tau;1) = u(\tau),$$

Thus, as p increases from 0 to 1, the solution $\phi(\tau;p)$ varies from the initial guess $u_0(\tau)$ to the solution $u(\tau)$. Expanding $\phi(\tau;p)$ in Taylor series with respect to p, one has

$$\phi(\tau;p) = u_0(\tau) + \sum_{m=1}^{+\infty} u_m(\tau)p^m \tag{9}$$

Where

$$u_m(\tau) = \frac{1}{m!}\left.\frac{\partial^m \phi(\tau;p)}{\partial p}\right|_{p=0} \tag{10}$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter $\hbar$, and the auxiliary function are so properly chosen, the series (9) converges at p=1, one has

$$u(\tau) = u_0(\tau) + \sum_{m=1}^{+\infty} u_m(\tau) \tag{11}$$

which must be one of solutions of original nonlinear equation, as proved by Liao [14]. According to the definition (4), the governing equation can be deduced from the zero-order deformation equation (2). Define the vector

$$\vec{u}_n = \{u_0(\tau), u_1(\tau), ..., u_n(\tau)\}.$$

Differentiating Eq. (8) m times with respect to the embedding parameter p and then setting p=0and finally dividing them by $m!$, we have the so-called m^{th}-order deformation equation

$$L[u_m(\tau) - \chi_m u_{m-1}(\tau)] = \hbar H(\tau) R_m(\vec{u}_{m-1}) \tag{12}$$

Where

$$R_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\phi(\tau, p)]}{\partial p^{m-1}}\bigg|_{p=0}$$

And

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases}$$

It should be emphasized that $u(\tau)$ for $m \geq 1$ is governed by the linear Eq. (12) with the linear boundary conditions that come from original problem, which can be easily solved by symbolic computation software such as Maple.

3.1 Application
Primjena

In the following, we apply HAM to solve one of the nonlinear heat transfer equations.

3.1.1 Problem description
Opis problema

Consider the heat transfer equation with variable properties:

$$c{\cdot}\rho(x)\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\frac{\partial u}{\partial x}\right), \quad 1 < x < \mathrm{e}^{\pi},\ t > 0 \tag{14}$$

Where the thermal coefficients ρ, k are function of x.

In order to assess the advantages and accuracy of the HAM for solving non-linear partial differential equations, we will consider the following examples.

3.1.2 Polynomial functions
Polinomske funkcije

Homotopy analysis method can be expressed by many different base functions [14]. According to the governing equation it is straightforward to use the set of base functions

$$\{t^m | m = 1,2,3,...\}$$

In the form

$$u(x,t) = \sum_{m=1}^{+\infty} b_n t^m. \tag{15}$$

Where b_n is function of x.

3.2 Case study
Analiza slučaja

In special case assume that $\rho(x) = \frac{1}{x}$, $k(x) = x$. So we have the following equation:

$$\frac{c}{x}\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(x\frac{\partial u}{\partial x}\right). \tag{16}$$

Consider the following boundary and initial condition:

$$\begin{aligned} &u(1,t) = 0, && t > 0 \\ &u(\mathrm{e}^{\pi}, t) = k t, && t > 0 \\ &u(x,0) = 0, && 1 < x < \mathrm{e}^{\pi}. \end{aligned} \tag{17}$$

3.2.1 HAM implementation
Implementacija HAM-a

With the aid of the initial condition and boundary condition Eq. (17), we choose the initial approximation

$$u_0(x,t) = \frac{k t}{\pi}\ln x + t\sin(\ln x). \tag{18}$$

By expanding $u_0(x,t)$ in Taylor series and under the rule of solution expression (15), we have:

$$\begin{aligned} u_0(x,t) = {} & \frac{t}{\pi}(4k x - \frac{25}{12}k - 3k x^2 + \frac{4}{3}k x^3 - \frac{1}{4}k x^4) + \\ & + t(\frac{10}{4}x - \frac{5}{3} - x^2 + \frac{1}{6}x^3) \end{aligned} \tag{19}$$

and the auxiliary linear operator

$$L[\phi(x,t;p)] = \frac{\partial u(x,t;p)}{\partial t}. \tag{20}$$

Furthermore, Eq. (1) suggests defining the nonlinear operator

$$\begin{aligned} N[\phi(x,t;p)] = {} & \frac{\partial \phi(x,t;p)}{\partial t} - \left(\frac{x}{c}\right)\left(\frac{\partial \phi(x,t;p)}{\partial x} + \right. \\ & \left. + \ x\frac{\partial^2 \phi(x,t;p)}{\partial x^2}\right). \end{aligned} \tag{21}$$

From Eq. (7) and (4) we have:

$$\begin{aligned} R_m(\vec{u}_{m-1}) = {} & \frac{\partial u_{m-1}(x,t)}{\partial t} - \left(\frac{x}{c}\right)\left(\frac{\partial u_{m-1}(x,t)}{\partial x} + \right. \\ & \left. + \ x\frac{\partial^2 u_{m-1}(x,t)}{\partial x^2}\right). \end{aligned} \tag{22}$$

Now, the solution of the mth-order deformation Eq. (12), for $m \geq 1$ becomes:

$$u_m(x,t) = X_m u_{m-1}(x,t) + \hbar \int_0^t H(\tau) R_m(\vec{u}_{m-1}) d\tau + F(x) \quad (23)$$

In order to obey the rule of solution expression and the rule of the coefficient ergodicity, which was expressed by Liao [14] the corresponding auxiliary function can be determined uniquely $H(\tau)=1$.

We now successively obtain

$$\begin{aligned} u_1(x,t) &= \frac{\hbar x t^2}{\pi c}\left(2kx^3 - 6kx^2 + 6kx - 2k + 2\right) + \\ &+ \frac{\hbar x t^2}{c}\left(-\frac{3}{4}x^2 + 2x - \frac{5}{4}\right) + t\hbar\left(\frac{10}{4}x - \frac{5}{3} - x^2 + \frac{1}{6}x^3\right) + \\ &+ \frac{t\hbar}{\pi}\left(4kx - \frac{25}{12}k - 3kx^2 + \frac{4}{3}kx^3 - \frac{1}{4}kx^4\right) \\ u_2(x,t) &= -\frac{\hbar^2 t^3 k x}{c^2 \pi}\left(\frac{32x^3}{3} - 18x^2 + 8x\right) - \\ &- \frac{\hbar^2 t^3 x}{c^2}\left(-\frac{9}{4}x^2 + \frac{8}{3}x - \frac{5}{12}\right) + \frac{\hbar t k}{\pi}\left(4x - \frac{25}{6}\right) + \\ &+ \frac{\hbar^2 t^2 x}{c}\left(-\frac{3}{2}x^2 + 4x - \frac{5}{2}\right) - \frac{\hbar^2 t k x}{\pi}\left(-x^2 + 3x - 4\right) - \\ &- \hbar^2 t x\left(-\frac{1}{6}x^2 + x - \frac{5}{2}\right) + \hbar t\left(\frac{5}{2}x - \frac{10}{3}\right) - \\ &- \frac{\hbar t^2 x}{c}\left(\frac{3}{4}x^2 - 2x + \frac{5}{4}\right) + \frac{\hbar t k x}{\pi}\left(-\frac{1}{4}x^2 - 3x + \frac{4}{3}\right) + \\ &+ \hbar t k x\left(-1 + \frac{1}{6}x\right) + \frac{\hbar^2 t^2 k x}{c\pi}\left(4x^3 - \frac{9}{2}x^2 + \frac{9}{2}x - 4\right) - \\ &- \frac{\hbar t^2 k x}{c\pi}\left(-2x^3 + 6x^2 - 6x + 2\right) \end{aligned} \quad (24)$$

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Consequently, the obtained solution by 8th-order approximation of solution is

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \ldots + u_8(x,t). \quad (25)$$

As pointed by Liao [14], the auxiliary parameter $\hbar$ can be employed to adjust the convergence region of the series (15) in homotopy analysis solution. In general, by means of the so called $\hbar$-curve, it is straightforward to choose an appropriate range for $\hbar$ which ensures the convergence of the solution series (Fig. 2).

The behavior of solution of Eq. (25) obtained by HAM for $\hbar = -1$ and the ADM solution [25] are shown in Fig. 3 – Fig. 6 .

As shown in Figs. 7 and 8, difference between HAM and ADM becomes zero as the order of approximation tends from 3[th] to 5[th] and 8[th].

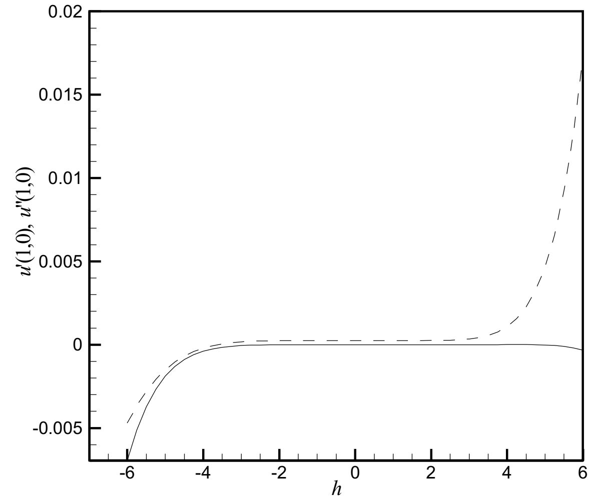

Figure 2 *The $\hbar$-curve for c=1 and k=2, solid line : 8th-order approximation of u'(1,0); dashed line: 8 th-order approximation of u''(1,0)*

Slika 2. *$\hbar$- krivulja za c=1 i k=2, puna linija: aproksimacija 8. reda za u'(1,0); isprekidana linija: approksimacija 8. reda za u''(1,0)*

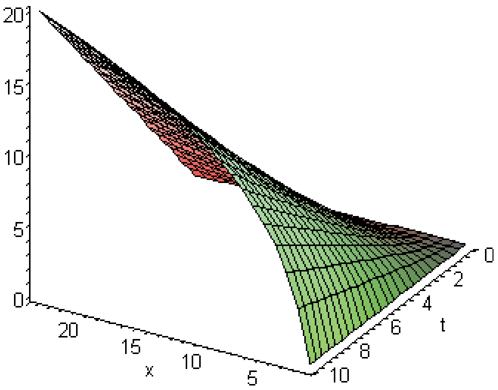

Figure 3 *The obtained solution by HAM for various x and t by 8th-order approximation of solution, $\hbar = -1$, c=1 i k=2*

Slika 3. *Rješenje dobiveno primjenom HAM za različite x i t pomoću aproksimacije rješenja 8. reda, $\hbar = -1$, c=1 i k=2*

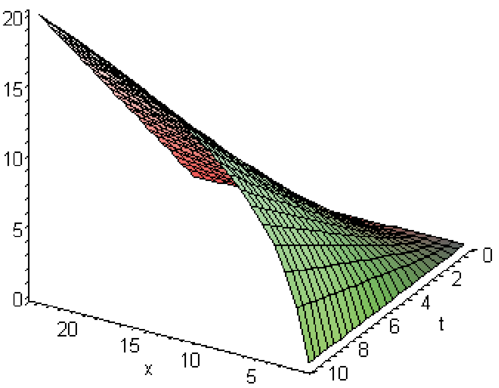

Figure 4 *The obtained solution by ADM for various x and t, c=1 and k=2* [25]

Slika 4. *Rješenje dobiveno primjenom ADM za različite x i t, c=1 i k=2* [25]

4 Conclusions
Zaključci

In this paper, we have successfully developed the HAM for solving heat equation with variable properties. It is apparently seen that the HAM is a very powerful and

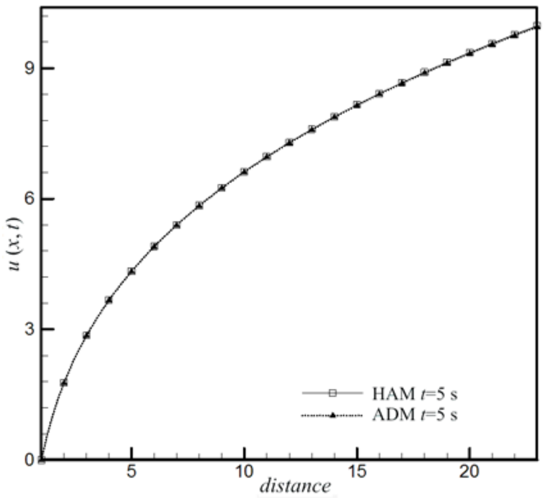

***Figure 5** The obtained solution by HAM for various x by 8th-order approximation of solution, t=5 s, ħ = −1, c=1 and k=2 compared with ADM* [25]
***Slika 5.** Rješenje dobiveno primjenom HAM za različiti x pomoću aproksimacije rješenja 8. reda, t=5 s, ħ = −1, c=1 i k=2 u usporedbi s ADM* [25]

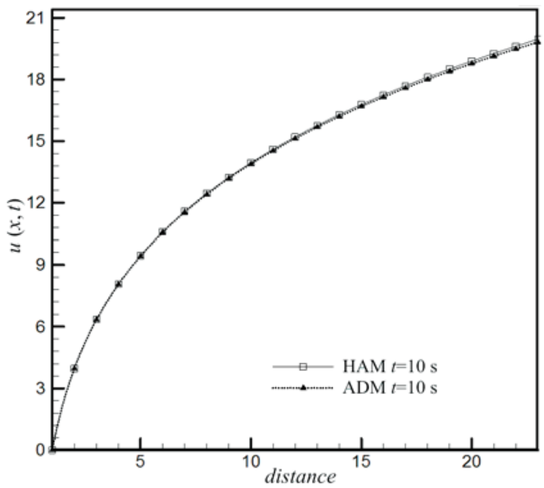

***Figure 6** The obtained solution by HAM for various x by 8th-order approximation of solution, t=10 s, ħ = −1, c=1 and k=2 compared with ADM* [25]
***Slika 6.** Rješenje dobiveno primjenom HAM za različiti x pomoću aproksimacije rješenja 8. reda, t=10 s, ħ = −1, c=1 i k=2 u usporedbi s ADM* [25]

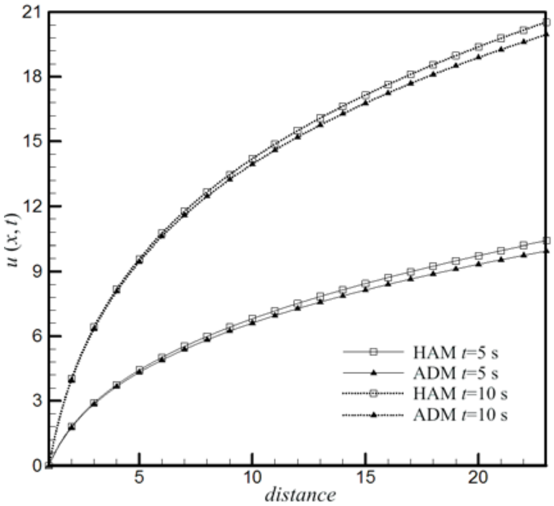

***Figure 7** The obtained solution by HAM for various x in t=5 s and t=10 s for 3th-order approximation compared with ADM* [25]
***Slika 7.** Rješenje dobiveno pomoću HAM za različiti x u t=5 s i t=10 s za aproksimaciju 3. reda u usporedbi s ADM* [25]

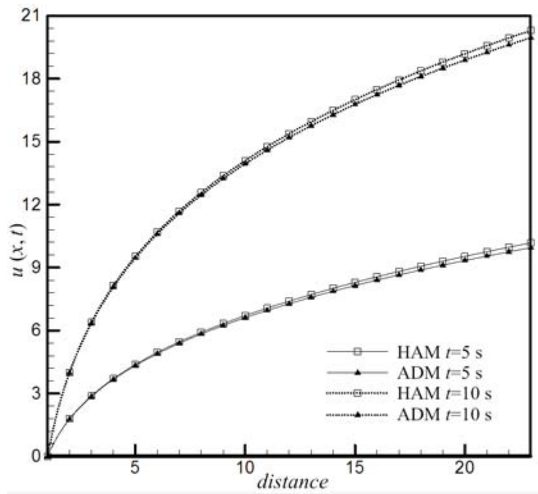

***Figure 8** The obtained solution by HAM for various x in t=5 s and t=10 s for 5th-order approximation compared with ADM* [25]
***Slika 8.** Rješenje dobiveno primjenom HAM za različiti x u t=5 s i t=10 s za aproksimaciju 5. reda u usporedbi s ADM* [25]

efficient technique in finding analytical solutions for wide classes of nonlinear problems. It is worth pointing out that this method presents a rapid convergence for the solutions. In conclusion, the HAM provides accurate numerical solution for nonlinear problems in comparison with other methods.

The results show that the HAM is a powerful mathematical tool for solving nonlinear partial differential equations and systems of nonlinear partial differential equations having wide applications in engineering.

Also the results show that:

1. HAM can give much better approximations for nonlinear differential equations than the previous solutions.
2. The comparison of the methods reveals that the approximations obtained by the HAM converge to the exact solution quite fast.
3. In HAM, the auxiliary parameter *ħ* provides us with a convenient way to adjust and control the convergence and its rate for the solutions series.
4. Solutions of HAM can be expressed with different functions and therefore they can be originated from the nature of the problems.
5. When small parameter of ε is increased, the error of HAM is less than HPM in comparison with exact solution.

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