

# MODELLING AND SIMULATIONS OF ELEVATOR DYNAMIC BEHAVIOUR

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Preliminary notes

Electric drive elevators represent a special group of transport machines for vertical lifting of a load with stressed dynamic features. Special attention should be paid to the behaviour of these devices in exploitation, as early as in the projecting phase. The paper shows the process of forming adequate mechanical models for elevators grouped according to their features, and with this an analysis can be performed of the influence of certain parameters on their behaviour in operation. Based on the formed elevator models, simulations have been performed, and the obtained results are graphically shown through diagrams, with varying "dynamic" parameters (drive characteristics, velocity and acceleration, the height of lifting, rated load and the weight of the cabin, mechanical characteristics of the rope...) with the conclusions on their influence on elevator behaviour.

**Keywords:** *elevators, dynamic models, simulations of elevator operating*

## Modeliranje i simulacije dinamičkog ponašanja dizala

Prethodno priopćenje

Dizala na električni pogon predstavljaju posebnu skupinu transportnih strojeva za vertikalno dizanje tereta s naglašenim dinamičkim značajkama. Posebnu pozornost treba obratiti na ponašanje ovih uređaja u eksploataciji još u ranoj fazi projektiranja. U članku je prikazan proces formiranja odgovarajućih mehaničkih modela za dizala grupirana prema njihovim značajkama, a s time se može obaviti analiza utjecaja pojedinih parametara na ponašanje u radu. Na temelju formiranih modela dizala, izvedene su simulacije, a dobiveni rezultati prikazani su grafički dijagramima, s različitim "dinamičkim" parametrima (pogonske značajke, brzina i ubrzanje, visina dizanja, ocijenjeno opterećenje i težina kabine, mehaničkih svojstava užeta...) sa zaključcima o njihovom utjecaju na ponašanje dizala.

**Ključne riječi:** *dinamički modeli, dizala, simulacija rada dizala*

## 1 Introduction

### Uvod

The problem of force transfer and driving by friction (driving pulley – supporting rope) and dynamic behaviour of certain elements of elevator device in exploitation is the subject of research in many studies and is still not satisfactorily solved.

For determining dynamic load in the case of vertical lifting by a driving pulley in [1], the reduced mass of the driving part, mass of the cabin and load and mass of counterweight are used. Driving torque (force) in the transient regime of operation is defined approximately as a time function, depending on the characteristic of a driving motor (soft, medium, and rigid regime). Final solutions for lifting by a driving pulley are obtained based on the equilibrium of torques of both sides. The model is applicable at low heights and low lifting velocities.

At low velocities and great heights we observe longitudinal oscillations of the dynamic model with an infinite number of degrees of freedom (oscillations of the steel rope as an elastic stick of constant length) [2].

At great lifting heights and great lifting velocities, a dynamic model is applied which considers the influence of changing the free rope length in relation to its dynamic behaviour [3] and [4] where different solutions are applied for solving the partial differential equations of movement for the cabin side and the counterweight side. Besides, in [3] there is an analysis of a connection problem between longitudinal and transversal oscillations of the steel rope in exploitation facilities in mining ("Köppe" system).

Ref. [5] shows the problem of longitudinal oscillations and stability of movement of an elastic string with concentrated masses.

For solving partial differential equations of a hyperbolic type in the function of one space coordinate and

time, a module has been developed ("hpde" function) which is applicable for a MATLAB software package [6].

As a special problem, from the point of view of technical system theory [7], elevators have to fulfil certain conditions. Since the market constantly sets new demands on riding security, capacity and comfort, in a fierce market competition the time from placing the demand to the coming out of a new product to the market is getting shorter and shorter [8, 9]. This problem is solved through development of new computer tools and computer applications in all construction phases [10, 11].

The aim of this paper is to show the forming of adequate analytical-computer procedures which make it possible, even in the phase of designing the elevators, to predict their behaviour during exploitation.

With the purpose of solving the stated problems, this paper shows different dynamic and suitable mathematical models of a drive module and supporting elements which are suitable for elevator analysis depending on their drive characteristics (height, lifting velocities, mechanical characteristics of the rope...). It also shows the simulations of elevator dynamic behaviour for mechanical models of a driving mechanism whose movement is defined through the law of changing the number of revolutions of the electrical motor (application of managing devices) and the change of acceleration on a driving pulley (application of classical motors with one and two velocities).

## 2 Dynamic models in the lifting system by a driving pulley

Dinamički modeli u sustavu podizanja pogonskom remenicom

Elevators represent a specific group of transport machines for vertical lifting. Needs of contemporary

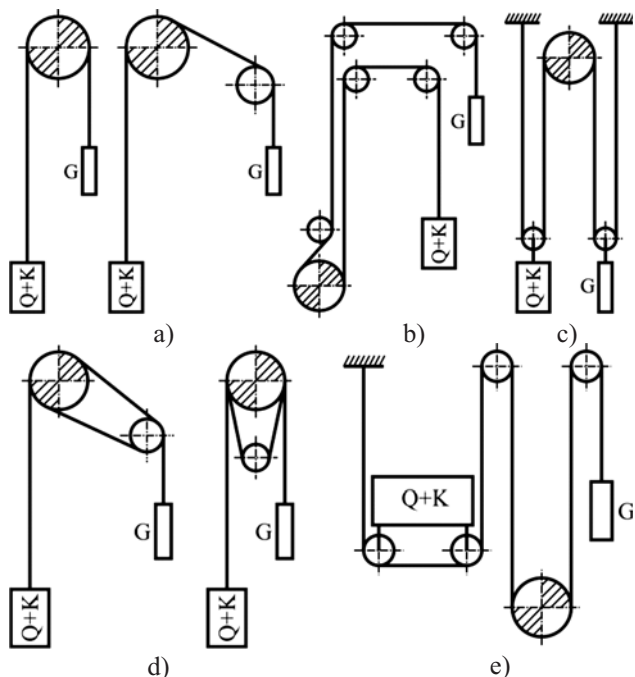


Figure 1 Kinematic solutions for hanging the cabin and counterweight  
Slika 1. Kinematička rješenja za vješanje kabine i protutega

society have influenced developments of different elevator constructions. Fig. 1 shows some kinematic solutions. Dynamic analysis is a specific problem and it is related to every single case. In real life, most often applied elevators are those whose kinematic scheme is shown in Fig. 1a.

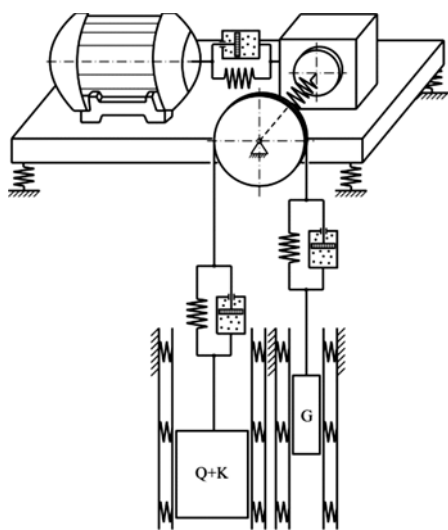


Figure 2 A general elevator model  
Slika 2. Opći model dizala

Since the parameters of a dynamic model depend on masses, mechanical features of the elements' materials, forces which affect the system, rope sliding conditions on a driving pulley, etc., the dynamic analysis of elevators is a very complicated problem. However, the problem can be greatly simplified [12], based on detailed research of drive lifting, especially the influence of element stiffness on its dynamic behaviour. Fig. 2 shows general scheme of an electric drive elevator. The stiffness of some elements depends on constructive performance of a certain device, and yet it is possible to omit the elasticity of driving mechanism elements with great accuracy, compared to the elasticity of a steel rope. Apart from the elevator elements'

masses, which are relatively easily defined, a dominant influence on the dynamic behaviour belongs to the rope stiffness and driving mechanism characteristics.

Rope modelling is most often done by combining a Hook's model (an ideal elastic body) and a Newton's one (an ideal viscose body), as shown in Fig. 3. For the drive systems with vertical lift, it is justified to observe the rope as a Calvin's or standard model (Fig. 3b and 3c), where the rope is considered as a spring of great stiffness (c) combined with damping (b). Those parameters depend on the construction and load of the rope and they are defined by an experimental method [13].

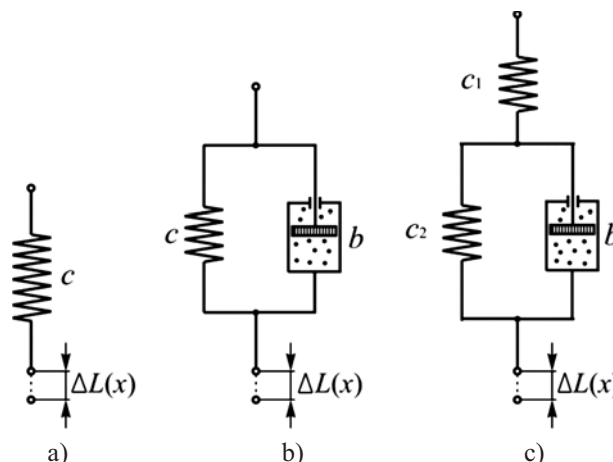


Figure 3 Rheological models of material (a – Hook's ideal elastic body, b – Calvin's model, c – Standard model)  
Slika 3. Reološki modeli materijala (a – Hookovo idealno elastično tijelo, b – Calvinov model, c – standardni model)

The elevator cabin and counterweight are modelled in most cases as concentrated masses (rigid elements). However, because of elasticity of the cabin frame, there is a possibility of forming different models, Fig. 4. One can also consider the elasticity of the upper and lower supporters in the cabin frame ( $m_4$ ) with the stiffness of  $c_3$ . That model can be expanded, so one can also consider the elasticity of side supporters ( $m_5$ ) with  $c_4$  stiffness, as well as elastic leaning on the cabin's guide rails ( $c_5$ ).

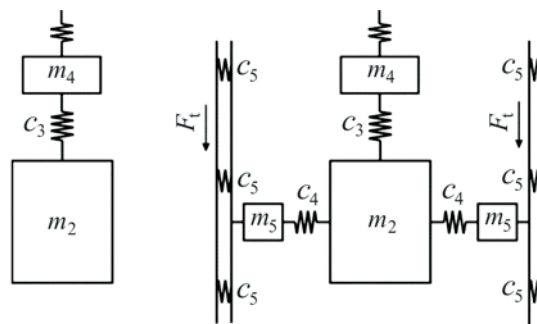


Figure 4 Cabin dynamic models  
Slika 4. Dinamički modeli kabine

Besides modelling of the elevator supporting elements and elevator cabin, modelling drive characteristics is also very important. As mentioned before, because of the significant difference between the rope stiffness and the stiffness of driving mechanism elements, when modelling elevators they are observed as absolutely rigid, with the reduction of masses and inertia moments on the shaft of a driving pulley.

Modelling of a driving moment, i.e. driving force, is complicated because it depends on the features of the driving motor, i.e. electromagnetic flux, balance masses, (especially of the first shaft, and the way of control (direct motor supply, motors with one or two velocities, control via frequency regulator, etc.). These influences are defined in electronics through a system of six differential equations which describe the interdependence of mechanical and electric parameters.

It is most common to approximately define the magnitude of driving force in the transferring operating regime [2], depending on the way of starting the electric motor, based on these relations:

$$F(t) = F_0 = \text{const.}, \text{ Fig. 5a (p. 1)}$$

$$F(t) = F_0 \left( 1 - \frac{t}{t_1} \right) - \text{soft drive regime, Fig. 5a (p. 2)}$$

$$F(t) = F_0 \left( 1 - \frac{t^2}{t_1^2} \right) - \text{medium drive regime, Fig. 5a (p. 3)}$$

$$F(t) = F_0 \left( 1 - \frac{t^4}{t_1^4} \right) - \text{rigid drive regime, Fig. 5a (p. 4)}$$

with:

$t_1$  – acceleration time, s

$F_0$  – pulled force, N

More precisely, but analytically more difficult, the driving moment can be defined by a so-called "static characteristic of an electric motor" in the velocity function, or number of revolutions –  $F=f(v)$ . The moment characteristic of an asynchronous machine in Fig. 5b, has this form according to [14]:

$$M = \frac{q}{2 \cdot \pi \cdot f} \cdot U^2 \cdot p \cdot \frac{\frac{R'_r}{s}}{\left( R_s + \frac{R'_r}{s} \right)^2 + (X_{ys} + X'_{yrk})^2} \quad (1)$$

with:

$q$  – number of phases, –

$f=50$  – network frequency, Hz

$U$  – voltage, V

$p$  – number of pole pairs

$R_s$  – stator resistance,  $\Omega$

$R'_r$  – narrowed down rotor resistance,  $\Omega$

$X_{ys}$  – reactance of overflowing of stator,  $\Omega$

$X'_{yrk}$  – narrowed down reactance of rotor overflowing,  $\Omega$ .

$s$  – slip, –

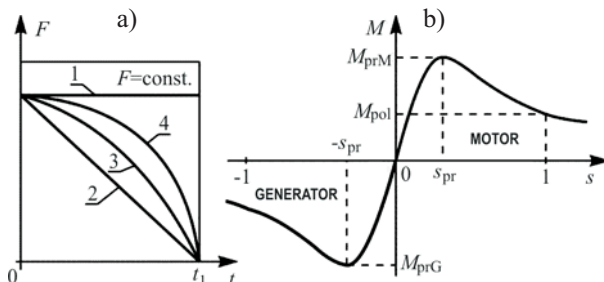


Figure 5 Different drive regimes (a) and static moment characteristic of an asynchronous machine (b), [14]

Slika 5. Različiti režimi pogona (a) i karakteristika statičkog momenta ansinhronog stroja (b), [14]

Apart from the before mentioned, this paper considers only the dynamic behaviour of the upgoing rope to a pulley for the analysis, which can be expanded to the other cases by analogue procedures, Fig. 7 and 8. Elevator models depend in the first line on the lifting height and velocity. As real combinations of the two parameters there occur:

- Passenger and freight elevators with small heights and low velocities of lifting, the so-called elevators with small lifting velocities.
- Passenger and freight elevators with great heights and medium velocities of lifting, the so-called elevators with high lifting velocities.
- Passenger elevators and exploitation facilities in mining with great heights and lifting velocities, the so-called express elevators.

The fourth combination (small height and great velocity) is not applied in practice.

### 2.1 Elevator models with small heights and lifting velocities

Modeli dizala s malim visinama i brzinama dizanja

Elevators with small lifting velocities are elevators with the riding velocity up to 0,85 m/s which are generally used at maximum heights of about 15-20 m. This group of elevators comprises most of the freight elevators and the passenger elevators in buildings which have less than 10 floors.

Elevator models which are suitable for small heights and velocities are shown in Fig. 6, where we have  $m_1$  – reduced mass of the driving part,  $m_2$  – load and cabin mass, and  $F$  – driving force. It is interesting to stress that the rope model, in the shape of an inelastic flexible element, defines the so-called rigid-kinetic elevator model, Fig. 6a, which is used for defining the optimal relation between the cabin weight and counterweight, and it is also very useful for defining the average values of dynamic parameters. Fig. 6b, 6c, 6d show dynamic models with two degrees of freedom, which are often used when analysing elevators. Those cases are analysed in great details in general literature and they are not going to be especially dealt with.

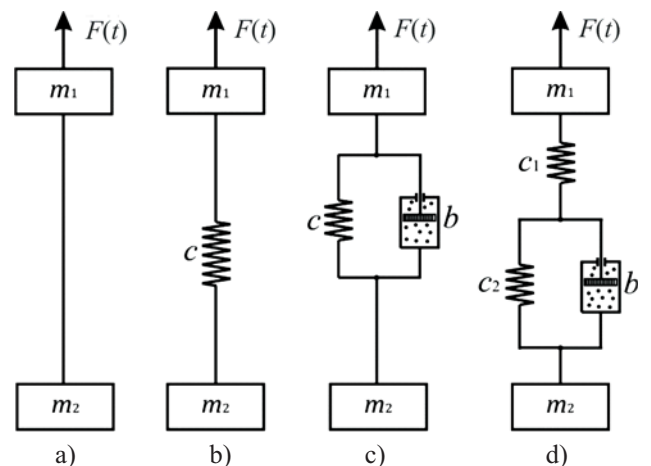


Figure 6 Dynamic model of a supporting element, inelastic element (a), Hook's model (b), Calvin's model (c) and a standard model (d)

Slika 6. Dinamički model potpornog elementa, elastičnog elementa (a), Hookov model (b), Calvinov model (c) i standardni model (d)

## 2.2 Elevator model with great heights and medium lifting velocities

Model dizala s velikim visinama i srednjim brzinama dizanja

This group embodies elevators with the lifting velocity up to 2 m/s and it embodies the greatest number of passenger elevators in skyscrapers. The problem can be modelled as a system with an infinite number of degrees of freedom (longitudinal stick oscillation) with suitable boundary conditions.

Forming differential equations of rope oscillations (rope with constant length) is to be done through considering one side (the side of the cabin), so basically one examines longitudinal oscillations of a stick with the length equal  $L$  [11], Fig. 7. By representing the rope as a Calvin's model, it can be written that:

$$S(x,t) = E \cdot A \cdot \frac{\partial}{\partial x} \left[ u(x,t) + b \cdot \frac{\partial u(x,t)}{\partial t} \right], \quad (2)$$

with:

- $E$  – elasticity modulus, Pa
- $A$  – rope cross-section,  $m^2$
- $u$  – rope elastic deformations, m
- $b$  – damping parameter, s [4].

Based on de Lambert's principle, it can be written by observing the equilibrium of the elementary rope part:

$$\frac{q}{g} \cdot \frac{\partial^2 u(x,t)}{\partial t^2} = E \cdot A \cdot \frac{\partial^2}{\partial x^2} \left( u(x,t) + b \cdot \frac{\partial u(x,t)}{\partial t} \right) + q \quad (3)$$

with:

- $q$  – rope weight pro meter, N/m.

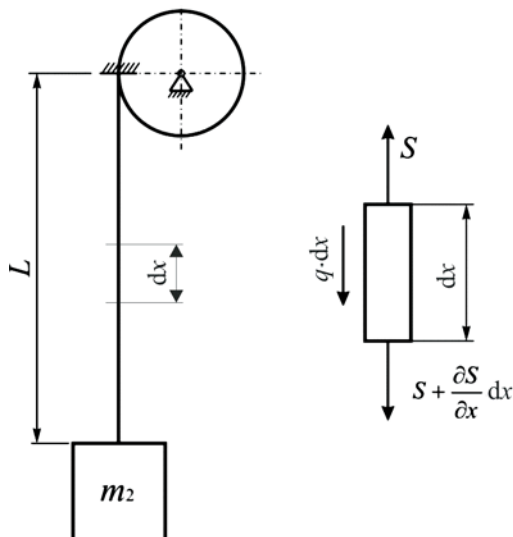


Figure 7 Oscillation of a rope with constant length  
Slika 7. Oscilacije užeta sa stalnom dužinom

If one observed oscillation around the equilibrium position (position of static equilibrium), where  $v(x,t)$  marks distance from the equilibrium position, and  $u(x,0)=u_{st}$  elongation of the rope caused by its own weight, the differential equation for oscillations of the rope with

constant length would look like this [12]:

$$\frac{\partial^2 v}{\partial t^2} = \frac{g \cdot E \cdot A}{q} \cdot \frac{\partial^2}{\partial x^2} \left( v + b \cdot \frac{\partial v}{\partial t} \right). \quad (4)$$

Solution of the equation (4) will be searched in a form of two functions which are functions of only one variable:

$$v(x,t) = X(x) \cdot T(t). \quad (5)$$

After differentiation of the expression and incorporation into the equation (4), two ordinary differential equations are obtained:

$$\begin{aligned} \ddot{T} + b \cdot k^2 \cdot C^2 \cdot \dot{T} + k^2 \cdot C^2 \cdot T &= 0, \\ \ddot{X} + k^2 \cdot X &= 0, \end{aligned} \quad (6)$$

with:

$$C^2 = \frac{g \cdot E \cdot A}{q} \text{ – propagation velocity of an elastic wave.}$$

These are well known equations for oscillations of a homogeneous beam with constant cross section, which is restrained on one side, and connected with mass on the other side [15].

Solution to the second equation has the following form:

$$X(x) = A_1 \cdot \cos(k \cdot x) + A_2 \cdot \sin(k \cdot x). \quad (7)$$

The constants  $A_1$  and  $A_2$  and the frequency equation are defined through boundary conditions:

- a) For  $x=0$ , based on Fig. 7, elongation equals zero,  $u(0,t)=0$ , so the first boundary condition is  $v(0,t)=0$ .
- b) For  $x=L$ , the second boundary condition is:

$$\frac{Q}{q} \cdot \left( \frac{\partial^2 v}{\partial t^2} \right)_{x=L} = -E \cdot A \cdot \frac{\partial}{\partial x} \left( v + b \cdot \frac{\partial v}{\partial t} \right)_{x=L}. \quad (8)$$

A complete solution of the system of regular differential equations (6), and therefore the equation (4) as well, is given in [3]. A frequency equation of rope oscillations with the load on its back end, in the case when a driving pulley is standing still, is obtained by using the second boundary condition. After some algebra, we get a frequency equation like this:

$$\beta \cdot \tan \beta = \alpha, \quad (9)$$

with:

$$\begin{aligned} \beta &= k \cdot (L-l), \\ \alpha &= \frac{q \cdot (L-l)}{Q}. \end{aligned}$$

It is possible to find solutions for the mentioned equation, when there are different weights of the rope and load. The equation has an infinite number of roots, so the number of eigenfrequencies is infinite, too. In practice, the most important items are the first eigenfrequency eigenmode and the first eigenfrequency. Based on an analysis for different weight connections between a free

jib of the rope, and cabin weight, where at elevators we have ( $\alpha \ll 1$ ), with satisfying accuracy one can say that the basic oscillation form is a straight line, i.e. the movement of rope points is the linear function of their distance from the point of rope arriving at the pulley, which is applied in the shown work situations.

### 2.3 Elevator model with great heights and velocities of lifting (express elevators)

Model dizala s velikim visinama i brzinama dizanja (ekspresna dizala)

It is a group of contemporary passenger models (Bujr Khalifa, a building in Dubai with the height of 828 metres, with over 160 floors. It has 57 elevators which do not go throughout the whole building. They are divided into three groups - till the 43<sup>rd</sup>, 76<sup>th</sup> and 123<sup>rd</sup> floor. The fastest elevator has a velocity of 18 m/s and the exploitation facilities in mining. The biggest velocity is 20 m/s and the pitch depth is 1 km at the most. For these elevators, the previously mentioned models are not suitable because of the fact that during lifting, by diminishing the free movement of a free rope jib, the basic parameter of a dynamic model is drastically changed – stiffness ( $E \cdot A/l$ ). Based on that, a suitable dynamic model for describing dynamic behaviour of devices with a driving pulley in application at elevator drive and for forming differential equations of movement according to [4] is shown in Fig. 8. Such a model should be applied for elevators with high lifting velocities without the machine room (a small free rope length in the upper station), while for the express elevator it is necessary because of the parametric oscillations (a change in a free rope length) which can cause unstable movement and unpermitted load of elevator supporting elements with critical consequences (tearing the rope, human victims and material damage).

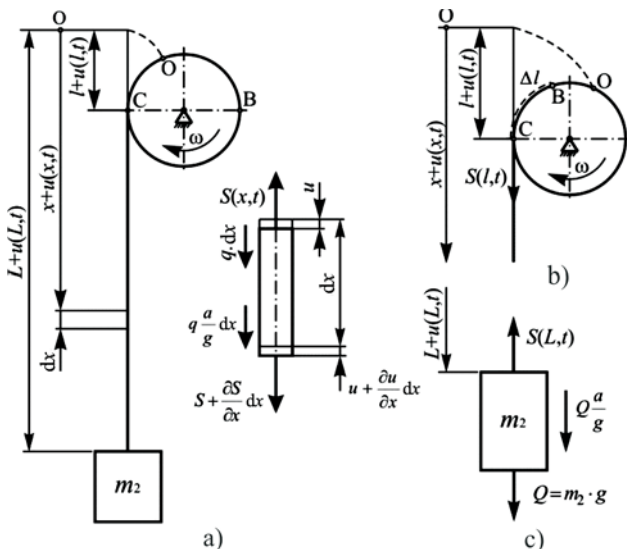


Figure 8 Dynamic elevator model with boundary conditions: a) on a pulley without slipping, b) on a pulley with slipping, c) on a cabin [4]  
 Slika 8. Dinamički model dizala s rubnim uvjetima: a) na užnici bez klizanja, b) na užnici s klizanjem, c) na kabini [4]

Just as in the previous example, the equilibrium equation of the elementary rope part is:

$$\frac{q}{g} \cdot \frac{\partial^2 u(x,t)}{\partial t^2} = E \cdot A \cdot \frac{\partial^2}{\partial x^2} \left[ u(x,t) + b \cdot \frac{\partial u(x,t)}{\partial t} \right] + q \cdot \left( 1 \pm \frac{a}{g} \right) \quad (10)$$

with:

$a$  – driving mechanism acceleration,  $m/s^2$ .

As a boundary condition at the point of contact between the rope and a driving pulley out of the equilibrium of elements, Fig. 8a, we get:

$$M_m = \frac{R}{i \cdot \eta} \cdot E \cdot A \cdot \frac{\partial}{\partial x} \left[ u(l,t) + b \cdot \frac{\partial u(l,t)}{\partial t} \right] + J_r \cdot \frac{a \cdot i}{R} \quad (11)$$

with:

$M_m$  – driving motor torque,  $N \cdot m$

$i$  – gear ratio, –

$\eta$  – driving mechanism efficiency, –

$J_r$  – moment of inertia of rotating masses, reduced onto the motor shaft,  $kg \cdot m^2$

$R$  – driving pulley radius,  $m$ .

In equation (11), the terms in the brackets are not constant, but they depend on the length, that is, the velocity of rolling the rope onto a pulley:

$$u(l,t) \text{ and } \frac{\partial u(l,t)}{\partial t}$$

The size of deformation on the rolled part of the rope can be defined with integration over the observed time period according to:

$$u(l,t) = \int_0^t \frac{\partial u(l,t)}{\partial x} \left( \frac{dl}{dt} \right) \cdot dt \quad (12)$$

with:

$l$  – the rope part rolled on a pulley,  $m$

$\frac{dl}{dt}$  – the velocity of rolling the rope (lifting),  $m/s$ .

Expression (12) shows the case when the rope sliding on a driving pulley is neglected. The problem is greatly complicated because of elastic sliding of the rope on a driving pulley  $\Delta l$ , Fig. 8b, where the above expression can be written as:

$$u(l,t) = \int_0^t \frac{\partial u(l,t)}{\partial x} \left[ \frac{dl}{dt} - \frac{d(\Delta l)}{dt} \right] dt + \Delta l \cdot \frac{\partial u(l,t)}{\partial x} \quad (13)$$

with:

$\frac{d}{dt}(\Delta l)$  – velocity of elastic sliding,  $m/s$ .

Boundary condition on the connection point of rope and cabin, Fig. 8c, is:

$$Q = E \cdot A \cdot \frac{\partial}{\partial x} \left( u(L,t) + b \cdot \frac{\partial u(L,t)}{\partial t} \right) + \left( \frac{Q}{g} \cdot \frac{\partial^2 u(L,t)}{\partial t^2} - a \right) \quad (14)$$

An analytical solution of a system of differential equations with uncholosom boundary conditions is extremely complicated, and it demands application of different procedures shown in [16] and [4].

It is noticeable that the analysis scheme (modelling and solving mathematical models) is complex because it is essential to form adequate dynamic models with acceptable

simplifications for various kinds of elevators according to their specific characteristics. Analytical solutions of a closed form are possible only for the simplest cases, so numerical procedures have been applied lately, i.e. suitable software packages for dynamic analysis, such as MATLAB-Simulink, ADAMS, Mathcad, Mathematica etc.

### 3 Numeral process and elevator operating simulations

#### Numerički postupci i simulacije rada dizala

Based on the previous analysis it can be deduced that it is possible to apply with satisfying accuracy the elevator models with reduced cabin masses, counterweight and elements of driving mechanism with the rope models shown in Fig. 6. At the express elevators, a rope can be seen as a visco-elastic body with varying stiffness. The rope stiffness changes in the function of the free rope length, and the basic oscillation form is an approximately straight line. However, sliding between the ropes and a driving pulley in the point of contact is neglected.

Modelling of movement can be done in various ways through models of driving mechanisms. These are the ways:

- Defining the function for changing the number of revolutions of electric motor (through the velocity of a driving pulley),
- Defining acceleration on a driving pulley,
- Assigning driving force depending on the way of releasing a driving motor, Fig. 5a,
- Modelling of a driving moment on the rotor of an electrical motor through a "static characteristic", Fig. 5b, etc.

This paper represents the case of assigning movement through a change in the number of revolutions of electric motor and defining acceleration on a driving pulley. In such cases a simulation of dynamic behaviour of elevator elements is performed for different operating regimes (acceleration, stationary movement and breaking).

#### 3.1 Defining the change in number of revolutions and the change of acceleration on a driving pulley

##### Definiranje promjene broja okretaja i promjene ubrzanja na pogonskoj remenici

Basic kinematic values which occur in elevators are velocity and acceleration. Due to the comfort in passenger elevators, there are specific acceleration values and velocity shifts, the so-called hitch. By various examinations, boundary values of acceleration and velocity of its shift (hitch) have been determined, and they are:  $a_{max} = 1,4 \text{ m/s}^2$  and  $\dot{a}_{max} = 1,0 \text{ m/s}^3$ . Kinematic diagrams characteristic for special elevators are given in Fig. 9 and Fig. 10. Such movement diagrams are available when there is a regulated moment of revolution of a driving motor, which is applied on elevators with high lifting velocities and express elevators.

To solve the problem of setting the movement at elevators is not easy and simply. Most software does not contain tools for representing the supporting rope – driving pulley system. In such cases one has to use a combination of existing tools in order to get satisfying results. After various attempts of movement, several suitable solutions emerged. Setting a function for changing the number of revolutions

of an electric motor can be seen as setting a function for changing the position of a certain marker on the rope in the direction of lifting the cabin. That enables setting a translatory movement on a translatory or cylindric joint, which can simply model the connection of a rope to a driving pulley.

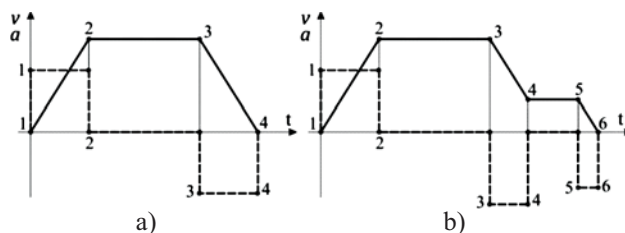


Figure 9 Ideal diagrams of elevator movement for one (a) and two (b) velocities of motor  
Slika 9. Idealni dijagrami gibanja dizala za jednu (a) i dvije (b) brzine motora

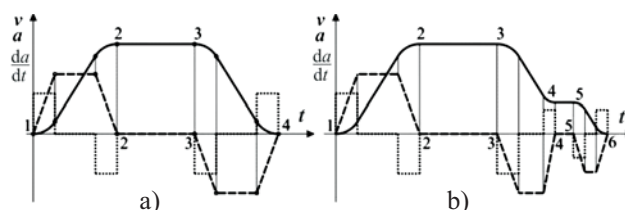


Figure 10 Ideal diagrams of elevator movement for one (a) and two (b) velocities of motor  
Slika 10. Idealni dijagrami gibanja dizala za jednu (a) i dvije (b) brzine motora

Description of a movement in time can be assigned in many ways, although a combination of commands *If* and *Step* is the most often applied. The *If* command has the following inscription:

*IF* (expression1 : expression2, expression3, expression4).

*Expression1* is actually a variable in a movement function, which is time in this case. If *expression1* is less than a zero, the function equals *expression2*. If it equals zero, the function is equal to *expression3* and when it is bigger than zero, it equals *expression4*.

*Step* command defines the *Step* function, which is approximately like Heaviside function with a cube polynome. Such function is given in Fig. 11, which is approximately suitable for the velocity change in Fig. 10.

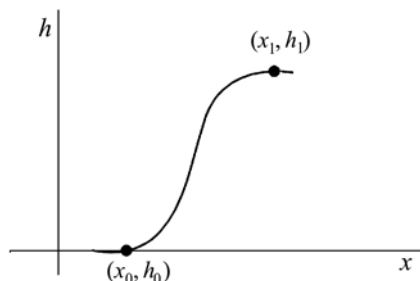


Figure 11 Step function  
Slika 11. Koracična funkcija

This equation defines *Step*:

$$Step = \begin{cases} h_0 & : x \leq x_0 \\ h_0 + (h_1 - h_0) \cdot \left[ \frac{x - x_0}{x_1 - x_0} \right]^2 \cdot \left\{ 3 - 2 \cdot \left[ \frac{x - x_0}{x_1 - x_0} \right] \right\} & : x_0 < x < x_1 \\ h_1 & : x \geq x_1 \end{cases}$$

Step command format goes like this:

$$STEP(x, x_0, h_0, x_1, h_1)$$

with:

- $x$  – independent variable,
- $x_0$  – real variable which defines  $h$  value where the *step* function begins,
- $x_1$  – real variable that defines  $h$  value where the *step* function ends,
- $h_0$  – the *step* function value at the beginning,
- $h_1$  – the final value of the *step* function.

It should be noted that the movements can be given by displacement, by velocity or acceleration, depending on what is easier to define at a specific moment. In this paper, movements are defined by a *Step* function through a big velocity of the driving pulley and the *If* command through acceleration on the driving pulley.

### 3.2 Movement simulation results given through a change in the number of revolutions in an electrical motor

Rezultati simulacije gibanja dani preko promjene broja okretaja elektromotora

The first case (lifting an elevator cabin with the loading capacity of 1000 kg, velocity 10 m/s, lifting height 100 m and damping 1 N·s/mm).

The function of changing the velocity is set like this:

$$IF(time-9: step(time,0,0,5,10000), step(time,9,10000,14,0), step(time,9,10000,14,0))$$

The given function means that when the time is less than 5 s, the velocity changes according to a *step* function (Fig. 11) and it changes so that at the starting point, the velocity equals zero, and within 5 s the velocity is 10 m/s. After that, the cabin moves at a velocity of 10 m/s in the next 4 s. Then breaking begins. That part in the mentioned expression is represented by another *step* function, according to which the velocity changes from 10 m/s at 9 s to 0 m/s at 14 s. The time of acceleration is determined on the basis of the recommended values of acceleration according to elevator standards.

As far as the rope stiffness is concerned, some cases were observed when it is constant and when it changes as a function of a free rope length (the remaining lifting height).

When discussing the lifting of a cabin mass oscillating on the cabin side  $m_2 = 1000$  kg, the following results were obtained by a dynamic simulation. Fig. 12 presents a character of acceleration changes in the case of the driving rope is observed as an ideal elastic body. It can be seen in the diagram that the cabin oscillation amplitudes during its breaking are bigger than the amplitudes at its moving period. The influence of the moment of changing the acceleration, i.e. way of managing the oscillation amplitudes while riding at a constant velocity is provided in Fig. 13. The discussion is provided in three cases and it can be seen that with keeping all of the oscillation parameters constant, the amplitude changes depending on the moment of "shifting" from an accelerated to stationary movement.

Fig. 12 also presents changes of the cabin positions during the simulation, as well as an ample velocity of the pulley and pulley acceleration.

Fig. 14 also shows change of the cabin acceleration during the simulation when the rope is observed as an ideal elastic body, but with changeable stiffness. At the

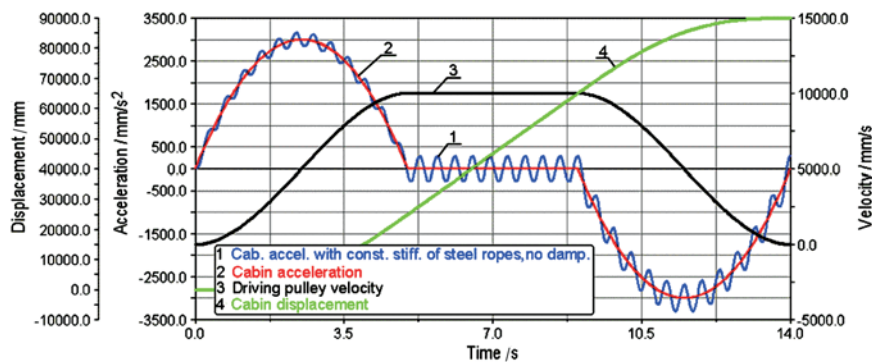


Figure 12 The change of acceleration and position of the cabin for the example of set movement through a change in the number of spinning of a pulley

Slika 12. Promjene ubrzanja i položaja kabine kod gibanja određenog promjenom broja okretaja užnice

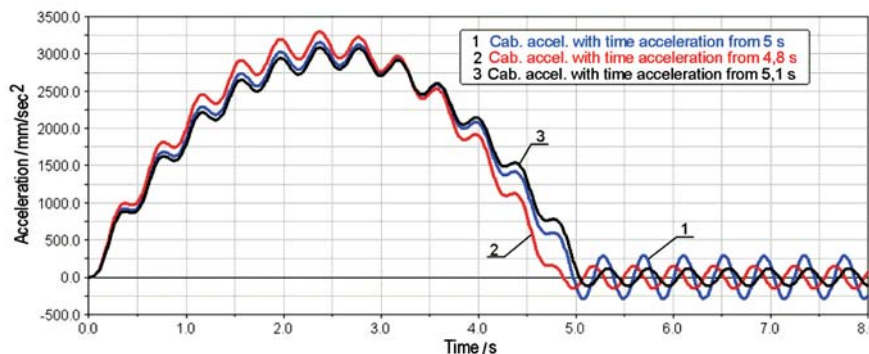


Figure 13 The change of the cabin acceleration during different managements of the driving pulley

Slika 13. Promjena ubrzanja kabine pri promjenama upravljanja pogonskom remenicom

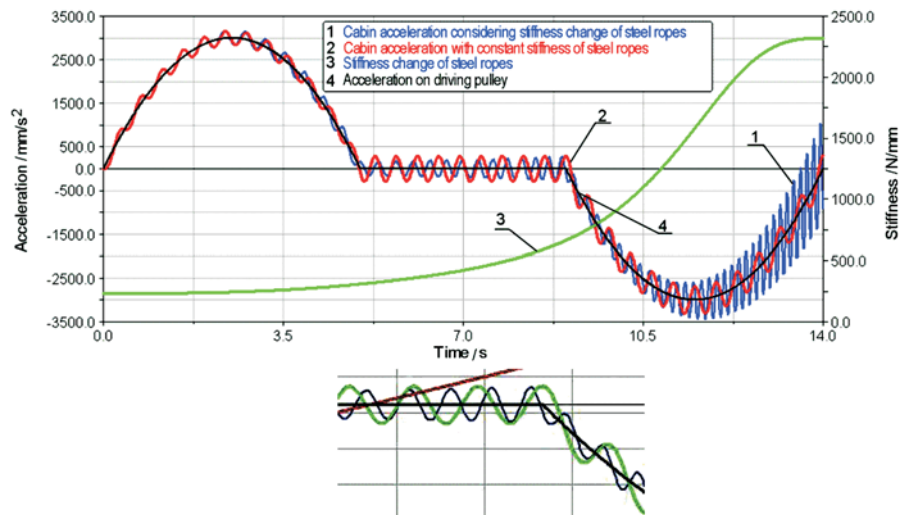


Figure 14 The change of cabin acceleration in two different cases and change in stiffness of the supporting ropes  
Slika 14. Promjena ubrzanja kabine u dva različita slučaja i kod promjene krutosti potporne užadi

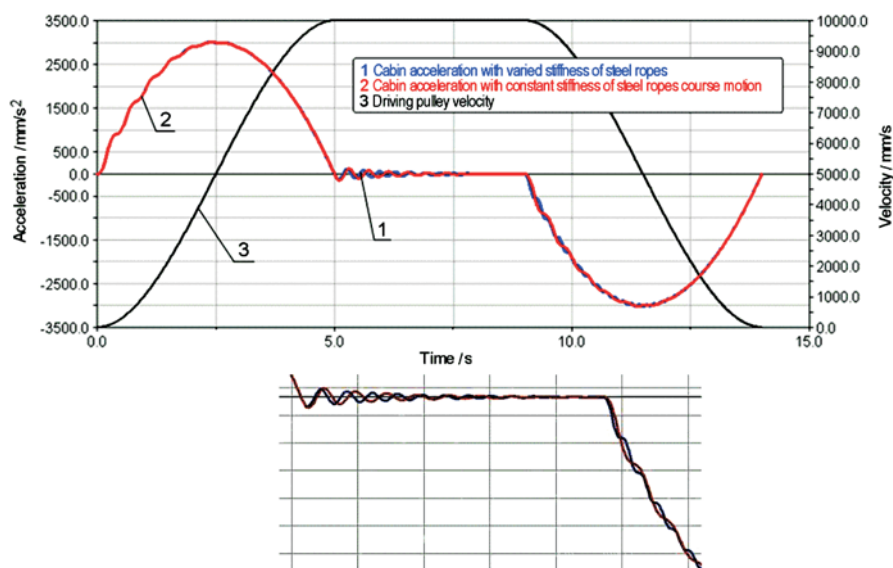


Figure 15 The change of acceleration for cases with the constant and variable stiffness of ropes  
Slika 15. Promjena ubrzanja kod konstantne i promjenljive krutosti užadi

beginning of the simulation, it can be seen that the character of acceleration change in the case of constant stiffness matches with the change character in the case when one considers the change of rope stiffness because of reduction of their free length. As the free rope length gets smaller, the greater difference emerges between these two cases. It is noticeable that when the stiffness changes, the frequency increases as well as the amplitude of cabin oscillations, Fig. 14 (unstable movement).

In case that a driving rope is observed as the Calvin's model (with damping), Fig. 15, a similar comment on the character of change in the amplitude and the cabin frequency oscillations for driving ropes with a variable and constant stiffness could be made as in the previous case, while the damping reduces the effects of variable stiffness on oscillation amplitudes and contributes to the movement stability.

Fig. 16 represents a compared diagram of the cabin acceleration change when being lifted, in cases when the rope stiffness changes (increases) in a way shown in the same figure, with and without damping.

The second case (lifting the cabin with the carrying capacity of 1000 kg, at the velocity of 20 m/s, for the lifting

height of 500 m, and the damping of  $1 \text{ N} \cdot \text{s}/\text{mm}$ ).

Function of change of cabin lifting velocity is represented like this:

$$IF(\text{time}-24: \text{step}(\text{time}, 0, 0, 8, 20000), \text{step}(\text{time}, 24, 20000, 32, 0), \text{step}(\text{time}, 24, 20000, 32, 0))$$

When the time is less than 8 s, then the velocity changes according to the step function (Fig. 11), in a way that at the beginning the velocity equals zero, and within 8 s the velocity is 20 m/s. After that the cabin moves at the velocity of 20 m/s for the following 16 s. Then the braking starts. This part in the previous expression is represented by another *step* function according to which the velocity of 20 m/s at 24 s, changes to 0 mm/s at 32 s.

The mass oscillating at the cabin side is  $m_2 = 1000 \text{ kg}$ . The simulation result is shown in the following figures. Fig. 17 represents a pulley velocity through which the movement was given, as well as the change in cabin acceleration in cases when rope stiffness is a constant and a variable with reducing the free length at the side of the cabin. Fig. 19 shows how the amplitude of cabin oscillations significantly increases in the part where the change of stiffness is great.



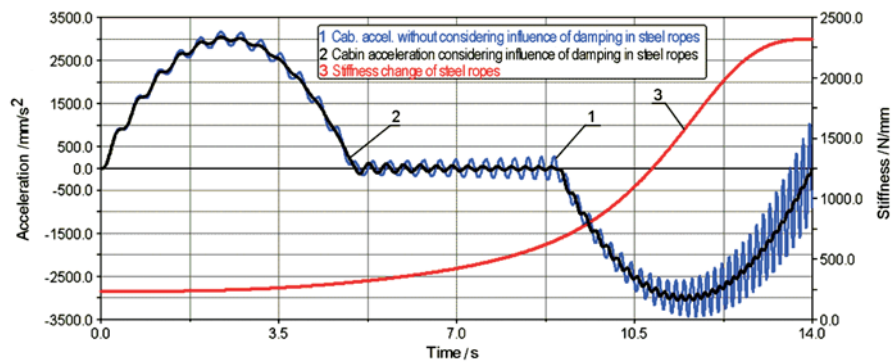


Figure 16 Diagram of rope acceleration and stiffness in cases with and without damping  
Slika 16. Dijagram ubrzanja i krutosti užadi u slučajevima sa i bez prigušenja

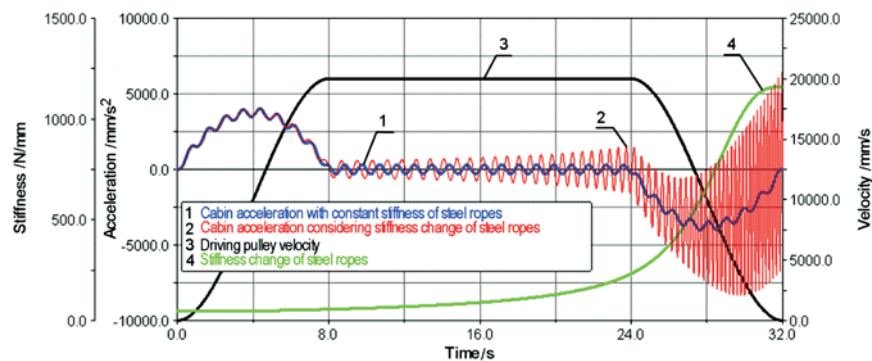


Figure 17 The change of ample pulley velocity and acceleration with a constant and varying rope stiffness  
Slika 17. Promjena brzine i ubrzanja užnice uz stalnu i promjenjivu ukrućenost užeta

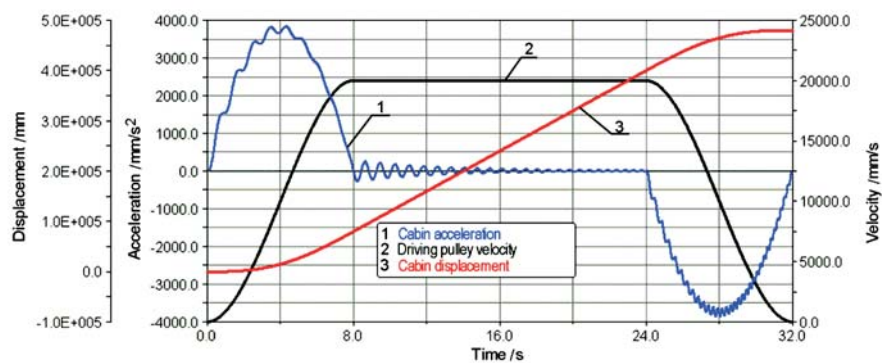


Figure 18 The change of cabin position and acceleration considering inner friction within the ropes  
Slika 18. Promjena položaja i ubrzanja kabine uzimajući u obzir unutarnje trenje među užadi

Fig. 18 represents the change of the cabin position during simulation, as well as the change in acceleration when the damping in ropes is considered. The figure shows that in the part where the cabin stops, an increase in oscillation frequency occurs, which is directly related to a significant increase in stiffness in that part.

Besides the change in stiffness of driving ropes in Fig. 19, there is also represented an elongation of ropes which is changed with the change in free length at the cabin side.

### 3.3 Movement simulation results defined through acceleration of a driving pulley

Rezultati simulacije gibanja za kretanje dani preko ubrzanja pogonske remenice

The following figures contain diagrams of certain parametres, where instead of the velocity at the driving pulley, acceleration was defined, and it was for cases with

and without damping, with the constant and variable rope stiffness.

When defining the movement through rope acceleration on a driving pulley, we discussed the lifting situation where the lifting capacity was 1000 kg, lifting height 100 m, acceleration 2 m/s<sup>2</sup>, and damping 4 N·s/mm.

The function of the change of acceleration is set like this (Fig. 20):

$$IF(time-5:2000,0,IF(time-9:0,-2000,IF(time-14:-2000,0,0)))$$

The function shows that within the time span 0 to 5 s, the acceleration is constant and equals 2 m/s<sup>2</sup>. After that, the cabin moves at a constant velocity (acceleration equals zero) for the next 4 s.

Then the breaking begins. In the mentioned expression it is represented with the last *IF* function, according to which the slowing down is constant, amounts to -2 m/s<sup>2</sup>, and lasts for the next 5s.

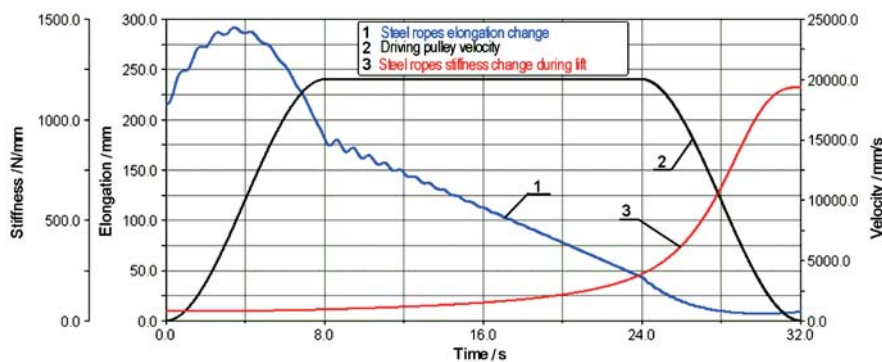


Figure 19 The change of rope stiffness and elongation during a simulation when the lifting height is 500 m  
Slika 19. Promjena krutosti i izduženja užadi tijekom simulacije kod visine podizanja od 500 m

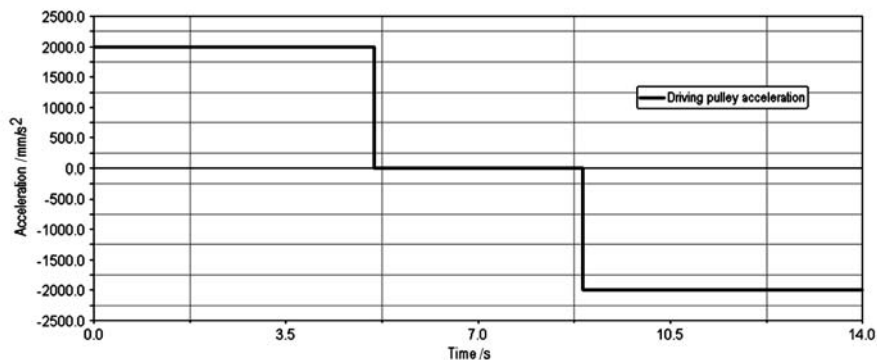


Figure 20 The change of rope acceleration at the point of meeting the driving pulley  
Slika 20. Promjena ubrzanja užeta u točki spajanja s pogonskom remenicom

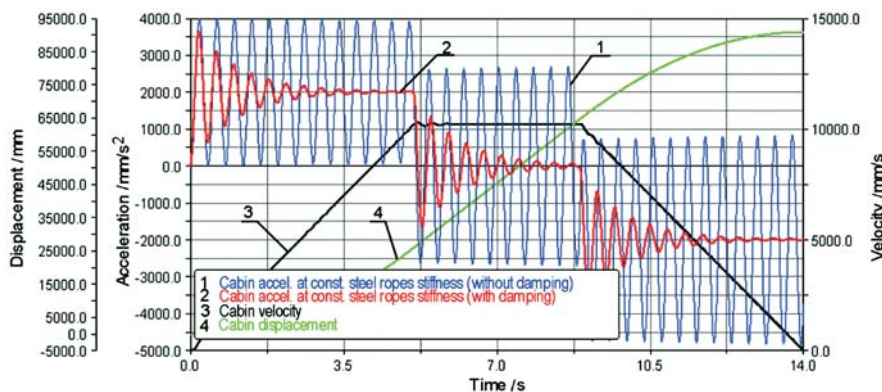


Figure 21 The change of position, velocity and acceleration of the cabin with constant rope stiffness, with and without damping  
Slika 21. Promjena položaja, brzine i ubrzanja kabine uz konstantnu ukrućenost užadi, sa ili bez prigušenja

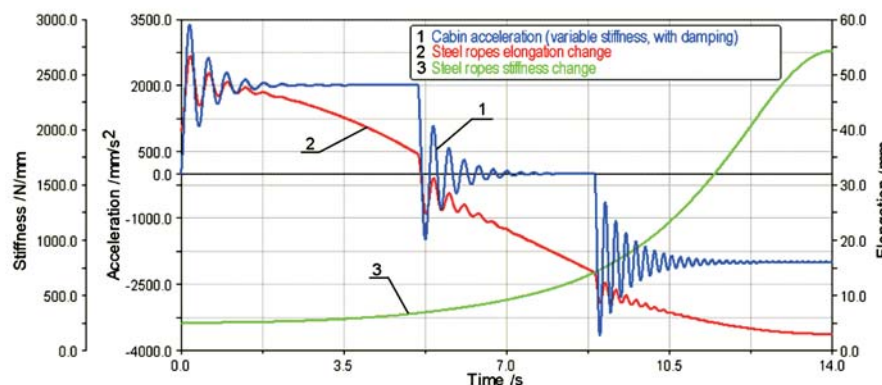


Figure 22 The change of cabin acceleration with variable rope stiffness, and rope elongation and stiffness  
Slika 22. Promjena ubrzanja kabine s promjenljivom krutosti užadi, te izduženje i ukrućenost užadi

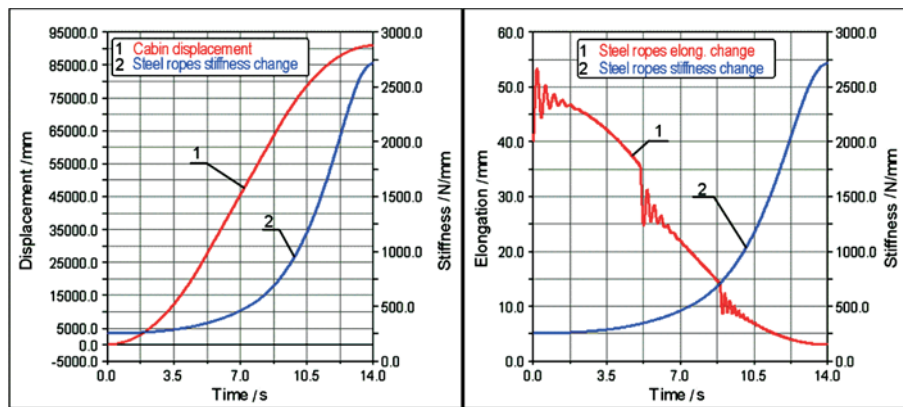


Figure 23 Diagram of the change in cabin position, elongation and stiffness of driving pulleys  
Slika 23. Dijagram promjene položaja kabine, izduženja i ukrućenosti pogonskih remenica

Fig. 21 shows the change of the cabin position and velocity, and the change of the cabin acceleration at the constant rope stiffness during the simulation, with and without inner friction within them. As in the previous example, there is a difference whether the movement was defined through the change in the number of revolutions (velocity at the pulley). Nevertheless, the cause of the difference is a *step* function which makes a "slight" transfer from accelerated to constant velocity movement, which is closer to the real drive operating.

Fig. 22 shows the change of cabin acceleration during the simulation considering the change of stiffness and the quotient of inner friction of supporting ropes. Also, the figure shows time dependence of rope elongation, as well as the change of stiffness. Fig. 23 shows, in the same diagrams, the changes of cabin positions and rope stiffness, as well as the changes of stiffness and rope elongations.

#### 4

### Conclusion Zaključak

The paper represents the procedures and methods of elevator modelling, depending on their features, with the application of numerical methods and contemporary software packages. The represented dynamic elevator models enable to grasp the most influential parameters which occur during their exploitation.

The dynamic models provided in the paper enable simulations of elevator behaviour with a defined pulley movement (control through the number of revolutions of an electric motor). Based on the previous analysis, we came to the conclusion that:

- It is possible to analyse the influence of weight of the cabin, cabin load, lifting height, rheological rope features ( $E, A, b, c, \dots$ ) influences of various movement diagrams (the defined movement through a control system).
- It is possible to authentically simulate the movement of a driving pulley, in accordance with the control system of a real elevator,
- The influence of the change in stiffness can be neglected at small lifting heights, especially in the cases when there is bigger inner friction within supporting ropes,
- At greater lifting heights, one must take into consideration the influence of the change in stiffness, because it strongly influences the size of dynamic load, especially at high movement velocities and low inner friction, i.e. damping,

- Simulation results can be used for determining control parameters (determining an optimal moment of change from accelerated to stationary movement and breaking), aiming at decreasing the dynamic load,
- The given simulations make it possible to determine the critical lifting velocity (through varying it) depending on the relations between the change of stiffness and the size of inner friction within ropes.

With the aim of verification and comparing the results obtained from the simulations of dynamic models for different parameters, represented in this paper, the paper predicts experimental research on a lab elevator model, the exploitation facility of a mine at the city of Bor and passenger elevators of FTS in the city of Novi Sad.

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#### 5

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