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INFLUENCE OF LEARNING PROCESS ON THE RELATION BETWEEN CHOSEN ANTHROPOMETRIC DIMENSIONS VIA LINEAR, PARABOLIC AND CUBIC RELATION MODEL[3]

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SUMMARY

The aim of this paper was to compare, analyze and explain the change in linear, parabolic and cubic models of the relation between two chosen anthropometric soft tissue variables through a learning process implemented as a kinesiological treatment. A homogenous sample which homogenised further as the learning process advanced was used for the purpose of this research. In accordance with this, the linear model ($y = \beta_1 x$ *), the parabolic model (* $y = ax^2 + bx + c$ *) and the cubic model which has never been used before (* $y = ax^3 + b\sqrt{x} + c$ *) depicting the relation between the thigh skinfold criterion variable and the upper arm circumference predictor variable were observed at 4 points in time. The convergence of statistically insignificant nonlinear models to statistically significant nonlinear model was observed and explained, providing a new methodological tool for quantitative analysis of the success of the learning process.*

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The research has shown that the cubic model is the most appropriate one at the highest point of sample homogenization. It also suggests the introduction of a mathematical expression for an interval of numbers which encompasses those predictor variable values for which it is reasonable to calculate the criterion variable value – the domain of the (non) linear regression model. Furthermore, the process of calculating the extreme points of the general nonlinear model was mathematically described and applied.

Key words: *methodology, anthropometry, nonlinear regression, cubic regression, education*

INTRODUCTION

It is well known that the theoretical model of human morphology is conceptualised through 4 latent dimensions – longitudinal dimensionality, transversal dimensionality, voluminosity and subcutaneous fat. However, research on general samples of the male population has not always completely confirmed this model (Stojanović et al. 1975, Szirovica et al. 1980). In accordance with this instability of structure, some authors have warned about the problems of the classical approach and methods which are based on the linearity assumption relation of different anthropometric dimensions (Blahuš, 1988). It is also important to point out that relations between the variables that describe anthropological dimensions and their behaviour influenced by educational and kinesiological motor learning process have often been the subject of recent body of research (Sertić, Segedi and Prskalo, 2010; Babin, Bavčević, Prskalo, 2010). Historically speaking, it can be said that detailed research on the relation between anthropological dimensions by the use of nonlinear methodological tools began with Ambrožić. On a representative sample consisting of 686 males aged 19-27, by analysing the linear and nonlinear relations between 23 anthropometric measures, Ambrožić (1996) concluded that 91 of 253 observed pairs of variables fit the nonlinear parabolic model. Furthermore, the inclusion of quadratic terms in the polynomial regression provided a higher percentage of the explained variance, so the nonlinear model seems to be appropriate for describing relations between the stated variables. Nowadays, nonlinear models are becoming an inevitable methodological tool in advanced scientific kinesiological and educational research (Sekulić, Viskić-Štalec, Rausavljević, 2003, Trninić, Jelaska, Papić, 2010). Furthermore, the quantitative approach used in the research on educational process has undoubtedly made a significant contribution to scientific research and improvement of education.

A well known fact that should be noted is that on a given interval of real numbers, every function can be approximated with arbitrary precision by the polynomial function of some power. This theorem also gives us a hint for the inclusion of quadratic and cubic power into regression models. Moreover, Jelaska, Trninić and Papić (2010) emphasize the necessity of using nonlinear cybernetic models and differential equations within the scope of humanities and social sciences and offer guidelines for their future use.

It is important to emphasize that until now there has not been a simultaneous analysis of the temporal change in the linear and various nonlinear relation models of chosen anthropometric variables under the influence of educational and training processes, which is the primary aim of this research. That is, the aim is to simultaneously analyze and explain the temporal evolution of the linear, parabolic ($y=ax^2+bx+c$) and cubic ($y=ax^3+b\sqrt{x}+c$) relation models of two anthropometric variables under the influence of the educational kinesiological treatment. In the process, the researchers used a homogenous sample of subjects which, influenced by the learning process – an extremely intensive and extensive several-months-long treatment, underwent further homogenisation. The secondary aim is to establish general methodological guidelines for the future analyses and applications of nonlinear, especially parabolic models, in education and kinesiology.

It is important to emphasize that in practical applications the explication of calculated model domains is the key piece of information, but authors rarely mention it. When we talk about the domain of a one-dimensional model (linear or nonlinear) we refer to the set of all those values (number interval) which a predictor variable may assume and for which it is reasonable to calculate the criterion variable values (Jelaska, 2005). Because of this, a general expression for the calculation of the regression model domain has been proposed in this paper. Apart from this, the process of calculating extreme values of nonlinear models by the use of differential calculus has been described, and the described process has been applied to the nonlinear model used in this paper.

METHODS

In this research, the entity sample consisted of 25 Croatian Army commandos who were 22 years of age on average and who underwent measurements at 4 different points in time. In the research process, the subjects went through a special educational and training process the efficiency of which was manifested in the soft tissue morphological subsegment as well. Precisely because of this, for the purposes of this research the subjects underwent measurements of subcutaneous adipose tissue – thigh skinfold (Y) and voluminosity – the upper arm circumference (X). The chosen variables are excellent representatives of the dimensions most sensitive to impulses produced by transformational processes. More precisely, the subject sample underwent a specially programmed four-and-a-half-month kinesiological treatment which, apart from contributing to the subjects' acquisition of special motor knowledge and skills, has also had an effect on the development of aerobic endurance and repetitive strength as the bases for the enhancement of specific urgent situation responding abilities. (Table 1)

Table 1: Plan and program of the kinesiological and educational treatment

PLAN	1st PHASE	2nd PHASE	3rd PHASE
Duration (weeks)	6	6	6
Total No. of workdays	30	30	30
No of work hours per day	4	6	7
Total No. of work hours	120	180	210
Total number of training sessions	84	78	72
Number of training sessions per week	14	13	12

Type of training	Times per week	Training duration	Times per week	Training duration	Times per week	Training duration
Basic training						
Morning march	5	30	4	30	4	30
Aerobic endurance training (long range running)	4	60	3	60	2	90
Aerobic endurance Training (forced march with weight)	1	180	1	240	1	240
Aerobic endurance Training (long range swimming)	2	300	2	30	1	60
Aerobic endurance Training (long range rowing)	1	60	2	45	1	90
Repetitive strength training	4	60	3	60	3	60
Specific training						
Forcing obstacles	2	45	2	60	2	60
Combat training	2	60	2	60	2	60
Breath diving training	2	30	2	25	2	20
Depth plunging			1	40	1	50
High diving			1	180	2	210
Situation training						
Tactical tasks			1	420	1	540

At each point in time, the descriptive statistics parameters (M ± *σ*) and the minimum and maximum results were calculated, whereas the distribution normality was tested using a KS test. The linear regression model coefficients were calculated and the lowest square method acquired via the Levenberg-Marquardt method for solving the obtained system was used to calculate the *a*, *b* and *c* parameters of the parabolic and cubic regression models, as well as the corresponding significance levels, the standard error and the proportion of interpreted variance. Furthermore, the chart displays both nonlinear regression models at each point in time. To conclude, by using the differential calculus for the functions of one variable, a general process for calculating nonlinear model extreme values has been proposed.

RESULTS

As it has been previously noted, the variables observed in this research were the thigh skinfold criterion variable (Y) and the upper arm circumference predictor variable (X). Throughout the research process the descriptive statistics parameters of the educational and training process were calculated at each point in time, whereas the KS test was used to test the distribution normality (Table 2). In accordance with the subsequently provided and explained formula, the linear and non linear model domain $y_i=f(x_i)$ was calculated.

Table 2**:** Descriptive statistics parameters in the initial point, the two transitive points and the final point. M- mean, σ – standard deviation, KS – empirical significance level by using Kolmogorov-Smirnov test, min – minimal result, max – maximal result, domain – regression model domain.

	M±σ	KS	Min	Max	Domain
Y_1	14.3±6.0	>0.20	6.0	26.4	-
Y_2	11.6±14.4	>0.20	4.8	19.6	-
Y_3	9.4±3.2	>0.20	4.2	16.4	-
Y_4	8.6±2.4	>0.20	4.2	13.2	-
X_1	30.3±1.7	>0.20	26.0	33.5	<25.2,34.4>
X_2	30.1±1.6	>0.20	26.0	33.0	<25.2,33.8>
X_3	30.4±1.5	>0.20	27.0	33.0	<28.2,33.8>
X_4	29.9±1.3	>0.20	27.0	32.0	<26.3,32.7>

LEGEND: *Y_i- thigh skinfold (criterion), X_i - upper arm circumference (predictor) in four measurements, i=1 initial state, i=2 first transitive point, i=3 second transitive point and i=4 final state.*

For each point in time, the *a*, *b* and *c* parameters of the linear model, the nonlinear parabolic model $y = ax^2 + bx + c$ and the cubic model $y = ax^3 + b\sqrt{x} + c$ were calculated, together with the corresponding

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significance levels (*p*), standard errors (*Se*) and the proportion of interpreted variance (R). (Table 3)

Table 3: Evolution of parameters within the linear regression model $y = \beta_1 x$, the parabolic regression model $y = ax^2 + bx + c$ and the cubic regression model $y = ax^3 + b\sqrt{x} + c$ in the initial, the two transitive points and the final point. Se –standard error, p – significance level, R – proportion of variance accounted for in the dependent variable by the model

Linear $y = \beta_1 x$ Model												
	Initial			1. Point			2. Point			Final		
	R=0.40			R=0.39			R=0.36			R=0.38		
	Value	P	Se	Value	P	Se	Value	p	Se	Value	p	Se
Beta	0.40	**0.04**	0.19	0.39	0.05	0.19	0.36	0.08	0.19	0.38	0.06	0.37
Parabolic $y = ax^2 + bx + c$ model												
	R=0.42			R=0.48			R=0.56			R=0.53		
	Value	P	Se	Value	p	Se	Value	p	Se	Value	P	Se
a	-0.18	0.53	0.29	-0.36	0.14	0.24	-0.49	**0.03**	0.20	-0.45	**0.04**	0.21
b	12.37	0.48	17.29	22.70	0.13	14.24	30.34	**0.02**	12,31	27.23	**0.04**	12.57
c	-191.39	0.47	257.53	-341.95	0.12	211.82	-458.64	**0.02**	185,50	-406.93	**0.04**	187.18

Cubic $y = ax^3 + b\sqrt{x} + c$ model												
	R=0.42			R=0.48			R=0.48			R=0.48		
	Value	P	Se	Value	p	Se	Value	p	Se	Value	P	Se
a	-0.002	0.55	0.002	-0.003	0.14	0.002	-0.004	**0.02**	0.001	-0.004	**0.04**	0.001
b	60.73	0.43	75.84	104.98	0.10	62.59	137.19	**0.01**	54.17	122.89	**0.04**	55.06
c	-276.29	0.43	345.00	-476.05	0.10	284.54	-625.29	**0.01**	247.81	-555.86	**0.03**	250.36

Figures 1 and 2 present a chart of parabolic and cubic regression functions at all 4 measured points respectively. Model in the *1*th measurement point is described as $Y_i=f(X_i)$.

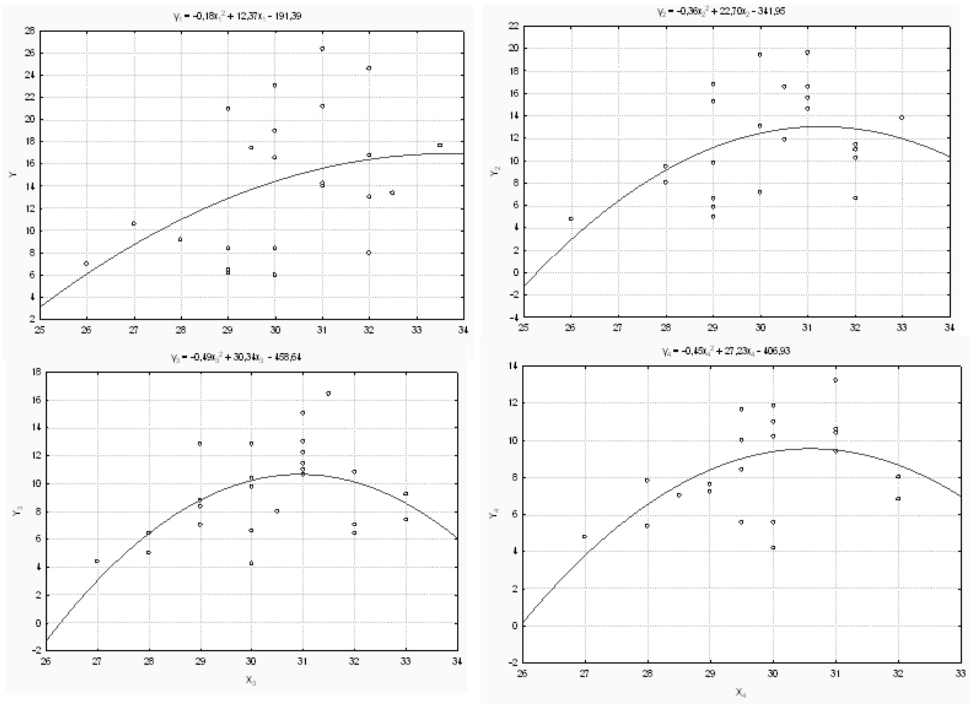

Figure 1: Parabolic regression model for the thigh skinfold (ThSF-Y) variables and the upper arm circumference (UAC-X) variables at all 4 points of the observation process.

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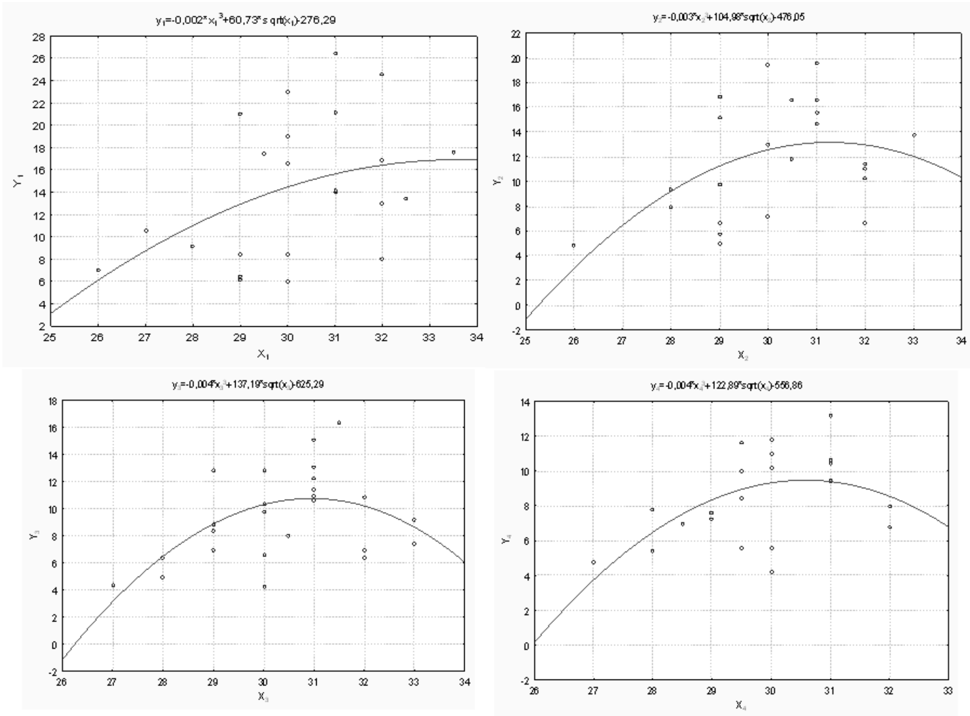

Figure 2: Cubic regression model for the thigh skinfold (ThSF-Y) variables and the upper arm circumference variables at all 4 points of the observation process.

DISCUSSION

It can be seen from Table 3 that *p* levels for the regression coefficients *a*, *b* and *c* of both nonlinear models considered at the 4 points in time have a rapidly decreasing trend. During the process of education and sample homogenisation, the second transitive point and the final observation have generated a reliable parabolic and cubic model ($p<0,05$). It is also obvious that the parabolic coefficient absolute value (a) increases with time. Furthermore, the standard error trend of coefficient a (by quadratic term - x^2) is decreasing as well and it can be seen at the final point of the treatment that there is a small standard error increase. The stated facts clearly show the existence of a real nonlinear relation between the selected variables (Figure 1). It can be seen from figure 2 that the cubic model is geometrically similar to the parabolic one but it has more significance.

Domain analysis of nonlinear models

The interval $I =< \mathbf{min}, \mathbf{max} >$ clearly contains all the predictor variable values for individual models, and the arithmetic mean of the predictor variable equals $AS \in I$. Moreover, it is known that approximately 68% of subjects fall within the interval of the length of one standard deviation around the arithmetic mean $\overline{x} - \sigma, \overline{x} + \sigma >$. Thus the extended interval $I_{ext} =< \min - \frac{\sigma}{2}, \max + \frac{\sigma}{2} >$ contains all the predictor variable values and is one standard deviation wider than the above mentioned interval *I*, the length of which is $l = max - min = R_x$. Therefore, in future nonlinear model applications the above mentioned interval I_{ext} might be considered as a set of those predictor variable values for which it is possible to calculate the criterion variable – the observed model domain. This is due to the fact that the model usually has to be applied somewhat outside the interval of the length of a variation range R_x, i.e. the interval *I*. The length of a domain model interval defined in this way is clearly $l = max - min + \sigma = R_x + \sigma$. Table 1 contains the domains of the calculated nonlinear models at all observed points in time.

Extreme nonlinear model analysis

In anthropological and kinesiological scientific applications, it is often important to know the values of the predictor variable for which the minimum/maximum criterion variable value is achieved, especially because this is the point at which a change occurs in the behaviour of the model in terms of increase and decrease. It is known that the extreme value of the general nonlinear model (a model with n predictor variables) can be found by solving the equation $\nabla f(x_1,\ldots, x_n) = \nabla f(\vec{x}) = \vec{0}$ where ∇ is *Hamilton's* differential operator. Specifically, if we observe the function of one variable and the parabolic model, we calculate the extreme model point using differential calculus in the following way:

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$$y = ax^2 + bx + c \quad \Big| \ {}^{d}\!\big/_{dx} \Rightarrow y' = 2ax + b = 0 \Rightarrow x = \frac{-b}{2a}$$

We calculate the extreme point of the cubic model in analogy to this:

$$y = ax^3 + b\sqrt{x} + c \quad \Big| \ {}^{d}\!\big/_{dx} \Rightarrow y' = 3ax^2 + \frac{b}{2\sqrt{x}} = 0 \Rightarrow x = \sqrt[5]{\left(-\frac{b}{6a}\right)^2}$$

By including the obtained predictor variable values, the model assumes the extreme value of the model criterion variable:

$y(-\frac{b}{2a}) = \frac{4ac - b^2}{4a}$. This provides us with a general form of the extreme value of the parabolic criterion variable $T\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$. If the nonlinear coefficient is positive, the minimum value is obtained and if the nonlinear coefficient is negative, the maximum value is obtained. It is also important to emphasize that the value has to be $-\frac{b}{2a} \in I_{ext}$ in order for us to be able to conclude that the generated model has the extreme point. That is, the extreme point x-axis has to be within the model domain. Otherwise, the model only has a parabolic increase/decrease in a given domain. Similarly, it has to be noted that the extreme value for the cubic model is $y(\sqrt[5]{\left(\frac{b}{6a}\right)^2}) = \sqrt[5]{\left(\frac{b}{6a}\right)^6} + b\sqrt[5]{-\frac{b}{6a}} + c$.

For parabolic models in general, if the coefficient value at the quadratic term is positive, the increase of the predictor variable value to the extreme value (generated by the criterion variable minimum) leads to the decrease of the criterion variable value, whereas the increase of the predictor variable value beyond the extreme value leads to the increase of the criterion variable value. Inversely, if the coefficient value at the quadratic term is negative, the increase of the predictor variable value to the extreme value (generated by the criterion variable maximum) leads to the increase of the criterion variable value, whereas the increase of the predictor variable value beyond the extreme value leads to the decrease of the criterion variable value (Figure 3).

On this particular sample it is important to emphasize that *f'(x)>0* means that the model increases until it reaches the point where the predictor variable equals *0*. When the predictor variable is beyond 0, then the predictor variable increase causes the decrease of the criterion variable i.e. *f'(x)<0*. The maximum point of the simple model provided here is, obviously, *T (0, 3)*.

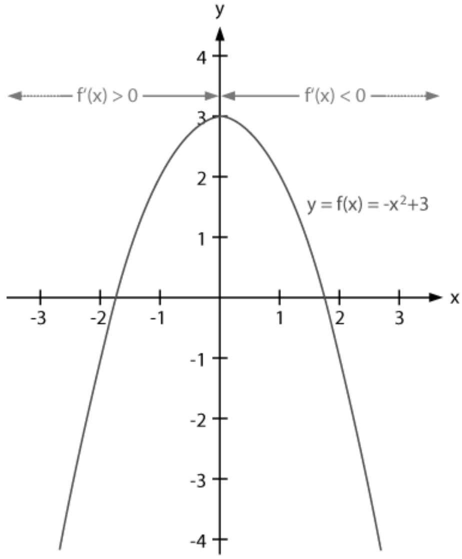

Figure 3: Example of a simple parabolic curve – y= f(x)= - x^2+3

For instance, the first and fourth parabolic models are expressed by the formulas:

$$y_1 = -0.18x_1^2 + 12.37x_1 - 191.39$$

$$y_4 = -0.45x_4^2 + 27.23x_4 - 406.93$$

By using the above mentioned derived formulas we get

$$x_1 = -\frac{12.37}{2\cdot(-0.18)} = 34.4 \quad and \quad x_4 = -\frac{27.23}{2\cdot(-0.45)} = 30.3$$

predictor variable values for which the extreme criterion value is achieved beyond the above mentioned values, whereas the predictor increase causes the decrease of the criterion variable (Figure 1). Furthermore, it can be seen in Table 1 that the first model domain is described with the interval I_1=<25.2, 34.4> and the fourth model domain with I_4=<26.3, 32.7>. Also, the initially obtained extreme point of the model is at the very edge of the model domain I_1 ($x_1 = 34.4 \notin I_1$), whereas the extreme point of the model obtained at the end

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falls within the model domain ($x_4 = 30.3 \in I_4$). We can, therefore, conclude that at its initial point the nonlinear model does not have the value at which the transition from an increase to a decrease occurs, whereas the model observed at the final point has achieved this value. A similar analysis can be done for a cubic model. The observed facts provide us with additional information on the temporal instability of the relation observed, that is, information on the true nonlinear relation between the observed variables.

CONCLUSION

This research analysed the temporal evolution of the relation between anthropometric characteristics through a process of intensive and extensive treatment. The sample, although initially homogenous, underwent further homogenization under the influence of the educational kinesiological treatment, which is why the presented results truly display the convergence of the relation models according to their true nonlinear relation. It is important to emphasize that all members of staff with the corresponding specialization can contribute substantially to the development of particular professions and enhance further education of an individual in different areas in the future. In this process, new methodological tools can make a fundamental contribution to the evaluation of the educational process. A mathematical expression has been proposed for the regression model domain in the form of an interval within which we can obtain the predictor variable values through a model for criterion variable assessment using the following formula:

$$I = < \min - \frac{\sigma}{2}, \max + \frac{\sigma}{2} >$$

In the future uses, this expression should enable a simpler scientific application of general regression models in educational, anthropologic and kinesiological sciences.

Also, by using the differential calculus, a process for nonlinear models has been derived and an explicit formula has been proposed for calculating the predictor variable value for which the minimum or maximum criterion variable value is achieved. This is a crucial piece of information for regression analysis because the maximum point constitutes the predictor variable value after which there is a change in the behaviour of the regression curve – in terms of increase and decrease.

Nonlinear regression procedures, if used appropriately, can enable the researcher to interpret relations among the data more precisely. It is also important to note that it is not methodologically appropriate to use exclusively linear models in educational, kinesiological and anthropological scientific research.

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UTJECAJ PROCESA UČENJA NA POVEZANOST ODABRANIH ANTROPOLOŠKIH VARIJABLI LINEARNIM, PARABOLIČNIM I KUBIČNIM MODELOM[4]

SAŽETAK

Cilj ovog rada bio je usporediti, analizirati i objasniti promjenu linearnog, paraboličnog i kubičnog modela povezanosti dvije odabrane antropometrijske varijable mekih tkiva, kroz proces učenja implementiranog kao kineziološki tretman. Pritom je korišten homogen uzorak koji se procesom učenja dodatno homogenizira. U skladu s tim, promatrani su linearni ($y = \beta_1 x$ *), parabolični (* $y = ax^2 + bx + c$ *) i do sada nekorišteni kubični model (* $y = ax^3 + b\sqrt{x} + c$ *) povezanosti kriterijske varijable kožnog nabora natkoljenice i prediktorske varijable opsega nadlaktice u 4 vremenske točke. Uočena i objašnjena je konvergencija statistički neznačajnih nelinearnih modela prema statistički značajnom nelinearnom modelu, što nam daje novi metodološki alat za kvantitativnu analizu uspješnosti procesa učenja. Pritom je pokazano da je kubični model najprikladniji kod najvišeg stupnja homogenizacije uzorka. Također je predložen matematički izraz za interval brojeva koji obuhvaća one vrijednosti prediktorske varijable za koje je smisleno izračunavati vrijednost kriterijske varijable - domenu (ne)linearnog regresijskog modela. Nadalje, matematički je opisan i primijenjen postupak izračuna ekstremnih točaka općeg nelinearnog modela.*

***Ključne riječi**: metodologija, antropometrija, nelinearna regresija, kubična regresija, edukacija*

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UVOD

Poznata je činjenica da je teorijski model ljudske morfologije konceptualiziran s 4 latentne dimenzije: longitudinalnom i transverzalnom dimenzionalnošću, voluminoznošću i potkožnim masnim tkivom. Istraživanja na općim uzorcima nisu uvijek u potpunosti potkrjepljivala taj model (Stojanović i sur. 1975, Szirovica i sur. 1980). U skladu sa nestabilnošću navedene latentne strukture neki su autori upozorili na probleme klasičnog pristupa i metoda temeljenih na pretpostavci linearne povezanosti različitih antropometrijskih dimenzija (Blahuš, 1988). Također, važno je naglasiti da je problematika istraživanja povezanosti varijabli koje opisuju antropološki status te njihovo ponašanje pod utjecajem kineziološkog procesa motoričkog učenja često istraživan problem u recentnim znanstvenim istraživanjima (Sertić, Segedi i Prskalo, 2010, Babin, Bavčević, Prskalo, 2010). Povijesno gledano, sustavna istraživanja povezanosti antropoloških dimenzija korištenjem nelinearnih metodoloških alata su počela s F. Ambrožić. Ambrožić (1996) korištenjem uzorka od 686 muškaraca, starih 19-27 godina, analiziranjem linearnih i nelinearnih relacija između 23 promatrane antropometrijske varijable by zaključuje da kod 91 od 253 promatrana para varijabli postoji nelinearna povezanost. Nadalje, uključivanje kvadratnih potencija u regresijski model daje viši postotak protumačene varijance pa zaključuje da je nelinearni model prikladan za opisivanje relacija između korištenih varijabli.

U recentnim znanstvenim istraživanjima, nelinearni modeli postaju nezaobilazan metodološki alat u naprednim kineziološkim i edukacijskim istraživanjima (Sekulić, Viskić-Štalec, Rausavljević, 2003, Trninić, Jelaska, Papić, 2010). Nadalje, kvantitativni pristup u istraživanju obrazovnog procesa je zasigurno dao značajan doprinos znanstvenom istraživanju i unapređivanju obrazovanja i odgoja.

Također potrebno je naglasiti dobro poznatu činjenicu da funkcija definirana danom na intervalu realnih brojeva može biti aproksimirana proizvoljno precizno s polinomom dovoljno visokog stupnja. Ovaj teorem nam daje poticaj za uključivanje kvadratičnih i kubičnih... potencija u regresijski model. U skladu s tim, Jelaska, Trninić i Papić (2010) naglašavaju nužnost upotrebe modernih nelinearnih kibernetičkih modela i diferencijalnih jednadžbi u društvenim i humanističkim znanostima te daje smjernice za njihovo buduće korištenje.

Važno je naglasiti da se do sada nije usporedno analizirala vremenska promjena linearnih i raznih nelinearnih modela povezanosti odabranih antropometrijskih varijabli pod djelovanjem edukacijskog i trenažnog procesa, što je i primarni cilj ovog istraživanja. Odnosno, cilj je usporedno analizirati i objasniti vremensku evoluciju linearnog, paraboličnog ($y = ax^2 + bx + c$) i kubičnog modela ($y = ax^3 + b\sqrt{x} + c$) povezanosti dvije antropometrijske varijable i to pod djelovanjem edukacijskog kineziološkog tretmana. Pritom je korišten homogen uzorak ispitanika, koji se pod djelovanjem procesa učenja - višemjesečnog iznimno intenzivnog i ekstenzivnog tretmana dodatno homogenizira. Sekundarni cilj je utemeljiti opće metodološke smjernice pri budućim analizama i primjenama u edukaciji i kineziologiji nelinearnih, posebno paraboličnih modela.

Važno je istaknuti da je u praktičnim primjenama ekspliciranje domene izračunatih modela ključni podatak, a autori je generalno ne navode. Pritom pod pojmom domene jednodimenzionalnog modela (linearnog ili nelinearnog) podrazumijevamo skup svih onih vrijednosti (interval brojeva) koje može poprimiti prediktorska varijabla a da za nju ima smisla izračunavati vrijednost kriterijske varijable (Jelaska, 2005). Stoga je u radu predložen opći izraz kojim se može izračunati domena regresijskog modela. Također je korištenjem diferencijalnog računa opisan postupak računanja ekstremnih vrijednosti nelinearnih modela te je opisani postupak primijenjen na u ovom radu korištene nelinearne modele.

METODE

U ovom istraživanju uzorak entiteta sastojao se od 25 diverzanata Hrvatske vojske prosječne dobi od 22 godine koji su mjereni u 4 vremenske točke. Pritom su ispitanici između pojedinih mjerenja bili podvrgnuti specijalnom edukacijskom i trenažnom procesu čija se učinkovitost manifestirala i preko morfološkog subsegmenta mekih tkiva. Upravo zbog toga su na ispitanicima za potrebe ovog istraživanja izvršena mjerenja potkožnog masnog tkiva - kožni nabor natkoljenice (Y) i voluminoznosti - opsega nadlaktice (X) - odabrane varijable su izvrsni reprezentanti dimenzija koje su najosjetljivije na podražaje izazvane transformacijskim postupcima. Preciznije, uzorak ispitanika je bio podvrgnut 4,5-mjesečnom posebno programiranom kineziološkom tretmanu u kojem se uz svladavanje specijalnih motoričkih znanja i vještina utjecalo i na razvoj aerobne izdržljivosti i repetitivne snage kao osnove za nadgradnju specifičnih sposobnosti za djelovanje u urgentnim situacijama.

U svakoj vremenskoj točki izračunati su parametri deskriptivne statistike (AM ± σ), minimalni i maksimalni rezultat te je KS testom ispitan normalitet distribucija. Izračunati su koeficijenti linearnog regresijskog modela te su metodom najmanjih kvadrata, korištenjem Levenberg-Marquardt metode za rješavanje dobivenog sustava, izračunati koeficijenti *a*, *b* i *c* paraboličnog i kubičnog regresijskog modela te pripadni nivoi signifikantnosti, standardna grješka te količina protumačene varijance. Također su u svakoj pojedinoj točki grafom prikazana oba nelinearna regresijska modela. Zaključno, korištenjem diferencijalnog računa za funkcije jedne varijable predložen je opći postupak izračunavanja ekstremnih vrijednost nelinearnih modela.

REZULTATI

Kao što je i prethodno rečeno, promatrane su kriterijska varijabla kožni nabor natkoljenice (Y) i prediktorska varijabla opseg nadlaktice (X). Pritom se u svim vremenskim točkama edukacijskog i trenažnog procesa izračunalo parametre deskriptivne statistike te je KS testom ispitan normalitet distribucija. U skladu s naknadno danom i objašnjenom formulom, izračunata je domena linearnih i nelinearnih modela $y_i=f(x_i)$.

Nadalje, za svaku vremensku točku izračunati su parametri *a*, *b* i *c* linearnog, nelinearnog paraboličnog modela $y = ax^2 + bx + c$ i kubičnog modela $y = ax^3 + b\sqrt{x} + c$ skupa sa pripadnim nivoima signifikantnosti (*p*), standardnim greškama (*Se*) i količinom protumačene varijance (R).

Također, grafički su prikazani parabolični i kubični regresijski modeli redom u 4 promatrane točke.

RASPRAVA

Iz rezultata vidljivo je da *p* nivoi za regresijske koeficijente a, b i c, promatrani u 4 vremenske točke, imaju brzo opadajući trend. Tijekom procesa edukacije i homogenizacije uzorka druga tranzitivna točka i finalna točka su dale pouzdane parabolične i kubične modele ($p<0.05$). Također, očito je da se apsolutne vrijednosti koeficijenta *a* u paraboličnom modelu povećavaju tijekom vremena. Nadalje, trend standardne greške navedenog koeficijenta je opadajući i, kao što može biti vidljivo u finalnoj točki, postoji malo povećanje standardne greške.

Navedene činjenice jasno ukazuju na realnu egzistenciju nelinearne povezanosti između odabranih varijabli.

Analiza domene (ne)linearnih modela

Očito je da se u intervalu $I=<\mathbf{min},\mathbf{max}>$ nalaze sve vrijednosti prediktorske varijable za pojedini model te da za aritmetičku sredinu prediktorske varijable vrijedi $AS \in I$. Nadalje, poznato je da se u intervalu duljine jedne standardne devijacije u okolini aritmetičke sredine $\bar{x}-\sigma, \bar{x}+\sigma >$ nalazi približno 68% ispitanika. Stoga prošireni interval $I_{ext}=< \min-\frac{\sigma}{2}, \max+\frac{\sigma}{2}>$ sadrži sve vrijednosti prediktorske varijable te je za jednu standardnu devijaciju širi od navedenog intervala *I* čija je duljina $l = max - min = R_x$. Stoga bi se u daljnjim primjenama nelinearnih modela navedeni interval I_{ext} moglo uzeti kao skup onih vrijednosti prediktorske varijable za koje je moguće izračunavati kriterijsku varijablu - domenu promatranog modela. To stoga što je model najčešće potrebno primijeniti malo i izvan intervala duljine raspona varijacije R_x (*range*), odnosno intervala *I*. Duljina ovako definiranog intervala domene modela je očito $l = max - min + \sigma = R_x + \sigma$.

Analiza ekstremnih vrijednosti nelinearnih modela

Nadalje, u antropološkim i kineziološkim znanstvenim aplikacijama često je važno znati vrijednost prediktorske varijable za koju se postiže minimalna/maksimalna vrijednost kriterijske varijable. Naročito zbog toga što je to točka u kojoj dolazi do promjene ponašanja modela u smislu rasta i pada. Poznato je da se ekstremna vrijednost općeg nelinearnog modela (modela s *n* prediktorskih varijabli) može pronaći rješavanjem jednadžbe $\nabla f(x_1,..., x_n) = \nabla f(\vec{x}) = \vec{0}$ pri čemu je ∇ *Hamiltonov* diferencijalni

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operator. Specijalno, ako promatramo funkciju jedne varijable i parabolični model, ekstremnu točku modela računamo korištenjem diferencijalnog računa na sljedeći način:

$$y = ax^2 + bx + c \quad \Big| \; {}^{d}\!/_{dx} \Rightarrow y' = 2ax + b = 0 \Rightarrow x = \frac{-b}{2a}$$

Analogno izračunavamo ekstremnu točku kubičnog modela:

$$y = ax^3 + b\sqrt{x} + c \quad \Big| \; {}^{d}\!/_{dx} \Rightarrow y' = 3ax^2 + \frac{b}{2\sqrt{x}} = 0 \Rightarrow x = \sqrt[5]{\left(-\frac{b}{6a}\right)^2}$$

Uvrštavanjem dobivene vrijednosti prediktorske varijable u model dobivamo ekstremnu vrijednost kriterijske varijable modela:

$y(-\frac{b}{2a}) = \frac{4ac - b^2}{4a}$. Ovaj izraz nam daje općenitu formu za izračun ekstremne vrijednosti paraboličnog modela $T\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$. Ako je koeficijent uz kvadratnu potenciju pozitivan, tada ekstremna vrijednost predstavlja minimum, a ako je negativan ekstremna vrijednost predstavlja maksimum. Također je važno podcrtati da mora vrijediti $-\frac{b}{2a} \in I_{ext}$ da bismo mogli zaključiti da generirani model ima točku ekstrema. Odnosno, apscisa ekstremne točka mora biti u domeni modela. U protivnom, model samo ima parabolični rast/pad na danoj domeni. Analognim računom za kubični model se dobiva da je ekstremna vrijednost modela

$$y(\sqrt[5]{\left(\frac{b}{6a}\right)^2}) = \sqrt[5]{\left(\frac{b}{6a}\right)^6} + b\sqrt[5]{-\frac{b}{6a}} + c\,.$$

Općenito za parabolične modele, ako je vrijednost koeficijenta uz kvadratni član pozitivna, tada se povećanjem vrijednosti prediktorske varijable do ekstremne vrijednosti (koja generira minimum kriterijske varijable) smanjuje vrijednost kriterijske varijable, a povećanjem vrijednosti prediktorske varijable nakon ekstremne vrijednosti se povećava vrijednost kriterijske varijable. Obratno, ako je vrijednost koeficijenta uz kvadratni član negativna, tada se povećanjem vrijednosti prediktorske varijable do ekstremne vrijednosti (koja generira maksimum kriterijske varijable) povećava vrijednost kriterijske varijable, a povećanjem vrijednosti prediktorske varijable nakon ekstremne vrijednosti se smanjuje vrijednost kriterijske varijable.

Tako su primjerice prvi i četvrti parabolični modeli dani formulama

$$y_1 = -0{,}18\,x_1^2 + 12{,}37\,x_1 - 191{,}39$$

$$y_4 = -0{,}45\,x_4^2 + 27{,}23\,x_4 - 406{,}93$$

Iz čega korištenjem gore izvedene formule dobivamo

$$x_1 = -\frac{12{,}37}{2\cdot(-0{,}18)} = 34{,}4 \quad i \quad x_4 = -\frac{27{,}23}{2\cdot(-0{,}45)} = 30{,}3$$

vrijednosti prediktorske varijable za koju se postiže ekstremna vrijednost kriterija odnosno nakon navedenih vrijednosti, povećanjem prediktora dolazi do padanja vrijednosti kriterijske varijable. Očito je domena prvog modela opisana intervalom I_1=<25.2,34.4> a četvrtog I_4=<26.3,32.7> te da je inicijalno dobivena točka ekstrema modela na samom rubu domene modela I_1 ($x_1 = 34{,}4 \notin I_1$) dok je finalno dobivena točka ekstrema modela unutar domene modela ($x_4 = 30{,}3 \in I_4$). Stoga možemo zaključiti da nelinearni model u inicijalnoj točki nema vrijednost u kojoj se događa prijelaz iz rasta u pad, dok je promatran u finalnoj točki ima. Slična analiza se može realizirati za kubični model. Uočena činjenica nam dodatno govori vremenskoj nestabilnosti promatrane povezanosti, odnosno o istinskoj nelinearnoj povezanosti promatranih varijabli.

ZAKLJUČAK

U ovom istraživanju je istražena vremenska evolucija povezanosti antropometrijskih karakteristika kroz proces intenzivnog i ekstenzivnog tretmana. Uzorak, iako inicijalno homogen, dodatno se homogenizirao pod utjecajem edukacijskog kineziološkog tretmana pa prezentirani rezultati uistinu daju prikaz konvergencije modela povezanosti prema njihovoj istinskoj nelinearnoj povezanosti. Važno je istaknuti da kadrovi s odgovarajućom specijalizacijom mogu značajno pridonijeti razvoju pojedine struke te pomoći budućoj daljnjoj edukaciji pojedinaca u raznim područjima. Pritom novi metodološki alati mogu biti od fundamentalnog doprinosa za evaluaciju edukacijskog procesa. Predložen je matematički izraz za domenu regresijskih modela kao interval unutar kojeg možemo uzeti vrijednosti prediktorskih varijabli u modelu za procjenu kriterijske varijable formulom

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$$I = < \min - \frac{\sigma}{2}, \max + \frac{\sigma}{2} >$$

Korištenje navedene formule trebalo bi pojednostavniti buduću znanstvenu primjenu općih regresijskih modela u edukacijskoj, antropološkoj i kineziološkoj znanosti.

Također je, korištenjem diferencijalnog računa, za nelinearne modele izveden postupak te je dana eksplicitna formula za izračun vrijednosti prediktorske varijable za koju se postiže minimalna odnosno maksimalna vrijednost kriterijske varijable. To je u nelinearnoj regresijskoj analizi ključan podatak jer je točka maksimuma ona vrijednost prediktorske varijable nakon koje dolazi do promjene ponašanja regresijske krivulje – u smislu rasta i pada.

Nelinearni regresijski modeli, prikladno korišteni, mogu dati mogućnost istraživaču da interpretira rezultate među podacima više precizno. Također, važno je naglasiti da metodološki nije prikladno koristiti isključivo linearne modele u znanstvenim istraživanjima u polju edukacije, kineziologije i antropologije.