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A NEURAL NETWORK MODEL FOR PREDICTING CHILDREN'S MATHEMATICAL GIFT

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SUMMARY

The aim of this paper was to model a neural network capable of detecting mathematically gifted fourth-grade elementary school pupils. The input space consisted of variables describing the five basic components of a child's mathematical gift identified in the body of previous research. The scientifically confirmed psychological evaluation of gift based on Raven's standard progressive matrices was used at the output. Three neural network models were tested on a Croatian dataset: multilayer perceptron, radial basis, and probabilistic network. The models' performances were measured according to the average hit rate obtained on the test sample. According to the results, the highest accuracy is produced by the radial basis neural network, which correctly recognizes all gifted children. Such high classification accuracy shows that neural networks have the potential to serve as an effective intelligent decision support tool able to assist teachers in detecting mathematically gifted children. This can be particularly useful in schools in which there is a shortage of psychologists.

INTRODUCTION

In the past few decades intelligent systems have been widely used in education for the purpose of creating tutoring tools. The detection of gifted children has mostly been conducted using the scientifically approved standard Raven progressive matrices in the process of the psychological evaluation of giftedness. However, in schools where psychologists are not available, an effective intelligent tool is needed to support teachers in their estimation of a child's gift.

Neural networks (NNs) are aimed at prediction, classification and association, and have the ability to deal with robust data. Hence, they are tested in order to obtain an accurate model for detecting a child's mathematical gift in the fourth grade of elementary school. A survey has been conducted at ten Croatian elementary schools where the psychologists' estimates were obtained for each child in the sample, and used as the output in NNs. Besides finding the most accurate NN model, some differences among the estimations of children's mathematical gift obtained by teachers as well as the neural network model are discussed. The results show that the estimations of the best NN model are more similar to the psychologists' findings than to the teachers' estimations. The NN model also finds more pupils to be mathematically gifted than teachers do. Therefore, the NN model can be used as a methodological tool to assist teachers in deciding on a child's gift.

PREVIOUS RESEARCH

Since the first appearance of *artificial intelligence* as a scientific discipline, a number of techniques have been developed with the aim of creating intelligent machines (Russell and Norvig, 2002). Some of those techniques are expert systems, problem solving, machine learning, natural language understanding, speech recognition, pattern recognition, robotics, neural networks, genetic algorithms, intelligent agents, and others. The research in the area of intelligent systems in education has mostly been focused on developing tutoring systems that could support learning and teaching a specific topic, with the ability of including multimedia and a personalized approach. For example, Stathacopoulou et al. (Stathacopoulou et al., 2005) have proposed to use the combination of NNs and fuzzy logic for advanced student diagnosis process in an intelligent learning system. Their model has enabled the system to "imitate" a teacher in diagnosing student characteristics, and selecting the appropriate learning style. The system has been tested in learning vector construction in physics and mathematics. Results obtained by the system have been compared to the recommendations of a group of experienced teachers, showing that the system was able to manage the diagnostic process, especially for marginal cases, where it was difficult even for the teacher to provide the accurate evaluation of the student. Canales et al. (Canales et al., 2007) developed an adaptive and intelligent web-based education system (WBES), which takes into account individual student learning requirements and enables the usage of different techniques, learning styles, learning strategies, and ways of interaction. The architecture of their system follows the standards proposed by the IEEE – LTSA (Learning Technology Systems Architecture), according to which the education systems should be structured into five layers: (1) the interaction of the learner with the environment, (2) learner-related design features, (3) system components, (4) implementation perspectives and priorities, and (5) operational components and interoperability (code, interfaces, protocols). Zeleznikov and Nolan (Zeleznikov and Nolan, 2001) created a decision support system based on fuzzy logic and predicated rules to assist teachers in grading essays. Saito et al. (Saito et al., 2007) investigated the influence of collaboration among schools and universities on the effectiveness of teaching and learning in schools and universities. Their results can be summarized in the following way: (1) joint lesson planning, observation, and reflection contribute to the improvement of teaching methodologies, (2) university faculty members and school teachers observe that students included in collaboration are more participative, (3) the interconnectedness between students and materials as well as among students themselves is necessary, (4) collaboration resulted in the development of collegiality within schools and among university faculty members and school teachers.

However, less research attention has been given to the area of intelligent systems used for detecting children's gift in particular areas such as mathematics. Gorr et al. (Gorr et al., 1994) tried to predict student grade point averages (GPAs) by using linear regression, stepwise polynomial regression, and NNs, and compared the predictions with an index used by an admissions committee for predicting student GPAs in a professional school. They have shown that none of the tested methods was significantly better than the practitioners' index. Hardgrave et al. (Hardgrave et al., 1994) have investigated NNs in predicting students' success in the graduate program. They have shown that non-parametric procedures such as NNs perform at least as well as traditional methods and are worthy of further investigation. Pavlekovic et al. (Pavlekovic et al., 2009) have also studied the use of the intelligent methods in talent detection. They have proposed an expert system which categorizes children into one of the four categories of talent on the basis of the five components of talent.

Johnson (Johnson, 2007) emphasized the importance and need for the accurate detection and further development of the mathematical gift as well as for the inclusion of criteria other than the mathematical competencies. Identification and development of giftedness was also investigated in Sterberg (Sterberg, 2001) and Tannenbaum (Tannenbaum, 1983). The shortage of psychologists in some countries makes the process of detecting gifted children even more difficult. For example, only 140 psychologists are employed in 931 elementary schools in Croatia (Vlahovic-Stetic, 2006). Previous research implies that there has been a great expansion of artificial intelligence methodology usage in tutoring tools in the past few years. However, the area of determining the giftedness in mathematics should be more investigated and it is necessary to design an intelligent system that would include competencies other than the mathematical one.

NEURAL NETWORK METHODOLOGY

Previous research has shown that NNs as a non-parametric method have the ability to perform at least as well as statistical methods in predicting student success (Hardgrave et al., 1994). They often outperform classical statistical methods due to their abilities to analyze incomplete, noisy data, to deal with problems that have no clear cut solution, and to learn from historical data. Most papers in this area deal with testing one NN network only – the multi-layer perceptron (MLP). In this paper we compare the performance of three NN models: the multilayer perceptron, the radial basis function, and the probabilistic network.

MLP is a general purpose feedforward network, and one of the most frequently used NN models. In order to optimize the error function, it uses the classical backpropagation algorithm based on the deterministic gradient descent algorithm originally developed by Paul Werbos in 1974, extended by Rumelhart, Hinton, Williams (in Masters, 1995), and a more advanced conjugate gradient algorithm. The concept of this algorithm is in minimizing the error in a direction that is conjugate to the previous directions, while the detailed computation is given in Masters (Masters, 1995). Conjugate gradient is combined with the classical backpropagation such that backpropagation is used in the first 100 epochs, while the conjugate gradient is used in the next 500 epochs. The standard delta rule was used for learning, while the learning rates and the momentum were dynamically optimized during the learning process (learning rate ranged from 0.08 to 0.01, while the momentum ranged from 0.8 to 0.1). Overtraining is avoided by a cross-validation process which alternatively trains and tests the network (using a separate test sample) until the performance of the network on the test sample does not improve for n number of attempts (n=10). The maximum number of epochs in our experiments was set to 1000. After the best network is selected, the model is tested on a hold-out validation sample to determine its generalization ability.

Radial basis function network (RBFN) is based on a clustering procedure for computing distances among each input vector and a center, represented by a radial unit. It has an advantage over the multi-layer perceptron algorithms as it does not suffer from the local minima problem, and is characterized by fast training and reasonably compact networks. Since it uses a radially symmetric and radially bounded transfer functions in its hidden layer, it is a general form of probabilistic and general regression network. The ability of RBFN with one hidden layer to approximate any nonlinear function has been proven by Park and Sandberg (in Karayianis et al., 1997). Michelli (in Karayianis et al., 1997) showed how this network can produce an interpolating surface which passes through all the pairs of the training set. RBFN algorithm uses Euclidean distance and Gaussian transfer function in the hidden layer which maps the output of the distance function according to:

$$f(x) = \varphi(\|x-c\|) = e^{\left(\frac{I_k^2}{\sigma_k^2}\right)} \quad (1)$$

where *x* is an input vector, *c* is the center determined by a clustering algorithm, and parameter σ is determined by the nearest neighbor technique. Learning through the architecture can be described in the following steps: (1) the clustering phase is present from the input to the hidden layer, where the incoming weights to the prototype layer learn to become the centers of clusters of input vectors using a dynamic k-means algorithm, (2) in the hidden layer the radii of the Gaussian functions at the cluster centers are computed using a 2-nearest neighbor technique, where the radius of a given Gaussian is set to the average distance to the two nearest cluster centers, and finally (3) the error is computed at the output layer. RBFN algorithm also suffers from some disadvantages, such as that the number of radial units must be decided in advance, and their centers and deviations must be set. In order to overcome those limitations, a pruning procedure for gradually decreasing the number of hidden units is used in our experiments. The initial number of hidden units is set to the size of the training sample. In order to use RBFN for the classification type of problems, a softmax activation function was added in its output layer to obtain probabilities in the output classes. Overtraining was avoided by a cross-validation process which alternatively trains and tests the network (using a separate test sample) until the performance of the network on the test sample does not improve for *n* number of iterations. After the best network is selected, it was tested on a new validation sample to determine its generalization ability.

Masters (Masters, 1995) suggests the probabilistic neural network as a good selection for classification problems when there are outliers in data, and when the learning speed is important, since this algorithm does not learn iteratively, but use only one pass through the dataset. For these reasons, the probabilistic network is also tested in this paper. It is a classifier that uses Parzen windows for clustering and produces a number of classes in the output. Euclidean summation function in the pattern unit is used with a competitive mode in the output layer.

The output layer in all NN architectures consisted of one processing unit (valued as 1 for gifted pupils and 0 for pupils that were not found gifted in mathematics). Equal prior probabilities for both gifted and non-gifted pupils were used in all networks (p=0.5). Sensitivity analysis is performed on the test sample in order to determine the significance of input variables to the model.

Psychologists used a classic set of Standard Progressive Raven's matrices (SPM) to find the category of mathematical gift for each child. It is a widely used, nonverbal test of analytic intelligence designed to assess a person's intellectual and reasoning ability and the ability to make sense of complex data (Carpenter et al., 1990). SPM test was used in this research due

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to its proven validity. Pind et al. (Pind et al., 2003) examined the criterion-related validity of the SPM with respect to scholastic achievement. Their results show that the highest correlation was obtained for mathematics, and lower correlations for the language subjects. Correlations ranged from 0.38 to 0.75. The test also showed high correlations with the national examinations. Their research included children in Iceland in the fourth, seventh and tenth grades who are required to take national exams in Icelandic and Mathematics; and additionally children from the tenth grade who also take national exams in two foreign languages, English and Danish. Laidra et al. (Laidra et al., 2007) used SPM on Estonian children in grades 2, 3, 4, 6, 8, 10, and 12 and found a high correlation between intelligence, as measured by the Raven's SPM, and students' grade point average (GPA) in all grades. The Raven's matrices are also suitable for all ability levels, they have extensive norms for different ages and cultures, they are easy to administer and score, and they overcome cultural and language bias.

DATA

The initial sample for the survey consisted of 247 pupils at the age of 10 (fourth grade of elementary school) from ten schools in Osijek, Croatia in December 2006. The survey consisted of 60 items describing each child and grouped into five groups of variables. Mathematical gift of those pupils was estimated by their teachers. According to legislative regulations, we were obliged to ask for parental permission in order to do the psychological evaluations of each child. The permission was obtained for 106 pupils, and further analysis is focused on that smaller sample. Table 1 shows the average grades of pupils from the smaller sample for each of the first three years of elementary school. A five-point discrete numeric evaluation is used for grading in Croatian schools (5=excellent or superior, 4=very good or above average, 3=good or average, 2=sufficient or below average, minimum passing grade, 1=insufficient or failing grade).

Table 1. Average grades of all the pupils from the sample

Description	Year 1	Year 2	Year 3
Average grade of all courses	4.7453	4.7170	4.7264
Average grade of mathematics course	4.5094	4.4340	4.3868

The neural network model used the total of 60 input variables describing the five components of a child's gift divided into the following groups of variables (as suggested in Pavlekovic et al. (Pavlekovic et al., 2007)): (1) assessment of mathematical competencies, (2) cognitive components of gift, (3) personal components that contribute the development of gift, (4) environmental factors, and (5) efficiency of active learning and exercising methods that enhance the development of mathematical competencies and possible realization of gift. All variables and their descriptive statistics are given in Appendix A. The psychological evaluation obtained by standard Raven's progressive matrices was used as the output variable in the model, since it is scientifically recognized as an efficient instrument in detecting a child's gift (Pind et al., 2003).

Both teachers and psychologists were requested to categorize each child into one of the four categories of gift (1=potentially gifted child in mathematics, 2=child with a special interest in mathematics, 3=child with average mathematical competencies, and 4=child with mathematical competencies below the average). The structure of teachers' assessments and psychologists' findings was the following:

Table 2. Frequencies of pupils assigned to categories of gift as obtained by the teachers and the psychologists

Category	Teachers' estimations		Psychologists' findings	
	No. of pupils	%	No. of pupils	%
1	7	6.67	20	18.87
2	40	37.73	37	34.91
3	36	34.29	21	19.81
4	23	21.91	28	26.41
Total	106	100.00	106	100.00

Since the primary objective of the paper is to extract children that are gifted in mathematics, the categories were further regrouped into two main groups: (1) the group of pupils found to be mathematically gifted, consisted of pupils assigned to categories 1 and 2 by psychologists, further referred to as *"gifted"* pupils, and (2) the group of pupils that were not found to be mathematically gifted, consisted of pupils assigned to categories 3 and 4, further referred as *"non-gifted"* pupils. Therefore, it was possible to create a binomial model of giftedness in mathematics used by NNs.

In order to train and test NNs, the total sample was divided into three subsamples such that 70% of the data was used for training the network, 10% of the data was used to find the optimal learning time and network structure in a cross-validation procedure, while the remaining 20% of data was used to finally test the network. The train and cross-validation samples contained an approximately equal distribution of gifted and non-gifted children, while the test sample consisted of 53.77% gifted pupils and 46.2% non-gifted pupils. The distribution of gifted and non-gifted pupils across all three samples is presented in Table 3.

Table 3. Sample distribution of the gifted and the non-gifted pupils

Sample	1 (gifted)		0 (non-gifted)		Total	
	No. of pupils	%	No. of pupils	%	No. of pupils	%
Train	37	50.00	37	50.00	74	100
CV	6	54.55	5	45.45	11	100
Test	14	66.67	7	33.33	21	100
Total	57	53.77	49	46.23	106	100

RESULTS

In order to find the best NN model, 25 NN architectures were created by each of the three NN algorithms (MLP, radial basis, and probabilistic). The following parameters were varied in each of the five architectures: number of layers, number of hidden units in the hidden layer, transfer functions, learning rules, and the number of iterations. NN architectures were evaluated on the basis of the average hit rates, while the hit rates for gifted and non-gifted pupils were also reported. The best model is selected on the basis of the highest average hit rate obtained on the test sample. The results of the best NN models produced by each NN algorithm are presented in Table 4.

Table 4. Results of the neural network models obtained on the test sample

NN algorithm	NN structure	Hit rate for gifted pupils (%)	Hit rate for non-gifted pupils (%)	Average hit rate (%)
MLP	60-32-1	78.57	85.71	82.14
RB	41-7-1	100.00	71.43	85.72
Probabilistic	60-107-1	57.14	71.43	64.29

It can be seen from Table 4 that the overall best NN model is obtained by the radial basis NN algorithm, which produced the average hit rate of 85.72%. The model accurately categorized all (100%) of the gifted children from the test sample, and 71.43% of the non-gifted children. The NN architecture of the best model contained 7 hidden processing units, and 41 input units as the result of the pruning procedure.

The sensitivity analysis performed on the test sample produced sensitivity ratios for 41 variables extracted as important for the model. The 10 variables with the highest sensitivity ratio are: (1) V57 – *Does the pupil have practical skills (bringing thoughts into action)?* (ratio=1.0076), (2) V33 - *Is the pupil open to new experience?* (ratio= 1.0069), (3) V29 - *Does the pupil have the ability of focused attention?* (ratio=1.0048), (4) V54 - *Does the pupil have the skill of selective comparison and connecting new information with the existing ones in the long term memory*? (ratio=1.0047), (5) V56 – *Does the pupil have the creative thinking skill (discovering, imagination, creating something new)*? (ratio=1.0046), (6) V4 – *Can the pupil follow the correct order of different arithmetic operations in calculation*? (ratio=1.0043), (7) V44 – *Does the pupil have the support of her/his family*? (ratio=1.0040), (8) V15 – *Does the pupil distinguish 2-dimensional shapes from 3-dimensional shapes*? (ratio=1.0039), (9) V37 – *Does the pupil get additional assignments from the teacher during regular courses in mathematics*? (ratio=1.0037), (10) V2 – *Does the pupil know how to graphically read large numbers?* (ratio=1.0033). Figure 1 graphically shows the sensitivity ratios of the 10 top ranked variables. Ratios of all the 41 extracted variables are presented in Appendix B. It can be seen that the largest number of variables (21) are selected from the group 1 (mathematical competencies), although at least two variables from each group of variables are extracted (2 variables from the cognitive components of gift, 3 variables from the components of gift that contribute gift realization, 7 variables from environment factors, and 8 variables from active learning and practicing).

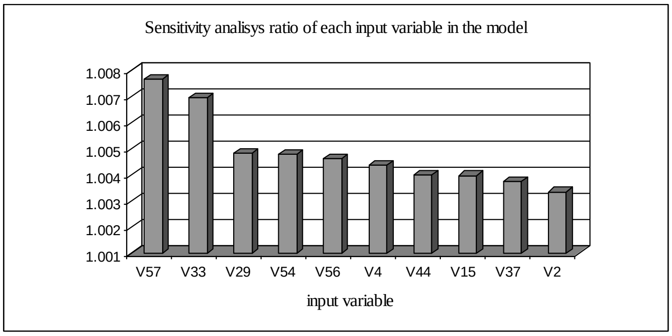

Figure 1. Sensitivity analysis ratio of 10 input variables with the highest rank in the model

The confusion matrix presented in Table 5 shows the absolute number of pupils from the test sample assigned to each class (gifted or non-gifted) by the radial basis NN model compared to the actual number of pupils assigned to each class by the psychologists.

Table 5. The confusion matrix of the estimations performed by the neural network and the psychologists

Gift – NN estimations	Gift – psychologists' findings		Total number of pupils
	0 (non-gifted)	1 (gifted)	
0 (non-gifted)	5	0	5
1 (gifted)	2	14	16
Total	7	14	21

When the estimations of teachers is compared to the estimations of psychologists, for the reason of possible comparison with the NN model, the hit rates of teachers' estimations are also computed. The average hit rate of the teachers’ estimations is 60.71%, where teachers accurately classify only 21.43% of the gifted pupils, and 100% of the non-gifted pupils. The confusion matrix of teachers and psychologists’ estimations is given in Table 6.

Table 6. The confusion matrix of the estimations provided by the teachers and the psychologists

Gift – teachers’ estimations	Gift – psychologists' findings		Total number of pupils
	0 (non-gifted)	1 (gifted)	
0 (non-gifted)	7	11	18
1 (gifted)	0	3	3
Total	7	14	21

The results show that the NN model recognizes gifted pupils more accurately, and yields higher average hit rate of estimating children’s gift. The average hit rate of teachers’ estimates is lower, although teachers are more successful in recognizing non-gifted pupils. The test of differences in average hit rates shows that the difference between the NN average hit rate and the teachers' average hit rate is statistically significant (p=0.014). Therefore the estimations of the best NN model are more close to psychological findings than teachers’ evaluations.

CONCLUSION

The paper deals with neural network modeling in the area of mathematical gift detection in elementary school pupils. The multi layer perceptron, the radial basis, and the probabilistic neural network algorithms were tested in order to find the best model. The best average hit rate was obtained by the radial basis function network which successfully detected all gifted pupils in the test sample. Besides finding the most accurate neural network model, the paper discusses some differences among the estimations of children's mathematical gift obtained by teachers and the neural network model. The results show that the neural network methodology is able to learn the connection among psychologists’ findings of gift and input variables. Since the estimation of teachers and psychologists’ findings differs, the model can be suggested as a methodological tool to assist teachers in making decision about a child’s mathematical gift especially in schools which have a shortage of psychologists. The sensitivity analysis showed that the neural network model extracted practical skills, openness to new experience, the ability of focused attention and others as the most important predictors, although the variables

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from all five components of gift are found to be important predictors of mathematical gift.

The abilities of neural network methodology in gift detection could be further investigated by using more datasets, in order to provide more valuable generalization assessment of the model performance. Testing other intelligent methods in gift detection, such as intelligent agents, support vector machines, and others, could also be valuable in future research. Also, a decision support system should be developed. It would help teachers in detecting gifted children at an early stage as well as assist in improving the development of that gift. Therefore, it would be beneficial for the children and the community.

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Appendix A. Statistical descriptive analysis of the input variables on the whole sample

Variable code	Variable description	Frequencies
Group 1: Mathematical competencies – Numbers and calculation		
V1	Can the pupil write large numbers graphically? (1=yes, 2=no)	1= 85.85% 2= 14.15%
V2	If the answer to V1 is „yes“: Can the pupil read large numbers graphically? (1=yes, 2=no)	0=13,21% 1= 77.36% 2= 9.43%
V3	Can the pupil successfully solve assignments that include one arithmetic operation? (1=yes, 2=no)	1= 87.74% 2= 12.26%
V4	If the answer to V3 is „yes“: Can the pupil follow the correct order of different arithmetic operations in calculation? (1=yes, fast, 2=yes, slow, 3=no)	0=12.26% 1= 47.17 % 2= 32.08% 3= 8.49%
V5	If the answer to V4 is „yes, fast“: Does the pupil successfully apply associative and distributive laws?" (1=yes, 2=no)	0= 53.77% 1= 40.57% 2= 5.66%
V6	If the answer to V4 is „yes, slow“: Does the pupil successfully apply associative and distributive laws? (1=yes, 2=no)	0= 71.70% 1= 13.21% 2= 15.09%
V7	How does the pupil estimate the result?" (1=The estimation improves by practicing, 2=Practice gives no improvement)	1= 88.68% 2= 11.32%
V8	Does the pupil recognize Roman digits and how does she/he use Roman numbers? (1=Yes, but only reads Roman numbers, 2=Yes, reads and writes Roman numbers, 3=Yes, reads, writes Roman numbers, and also solves assignments with matches used in calculating with Roman numbers, 4=Nothing of the above)	1= 16.04% 2= 43.40% 3= 33.02% 4= 7.55%
Measurements and measuring		
V9	Is the pupil familiar with the metric units of length and how does she/he apply them? (1=Yes, only familiar with the metric units of length, 2=Yes, familiar with the metric units of length and also estimates length accurately, 3=Yes, familiar with the metric units of length, estimates length accurately, and solves problem assignments that include length, 4=Nothing of the above)	1= 45.28% 2= 28.30% 3= 17.92% 4= 8.49%
V10	Is the pupil familiar with the metric units for liquid volume? (1=yes, 2=no)	1= 85.85% 2= 14.15%
V11	If the answer to V10 is „yes“: Can the pupil estimate and measure liquid? (1=yes, 2=no)	0= 9.43% 1= 38.68% 2= 51.89%

Variable code	Variable description	Frequencies
V12	If the answer to V11 is „yes“: Can the pupil “discover” the volume of an irregular 3-D shape by diving it into liquid in a gauge glass? (Perceiving the relationship between cubic decimetre and litre, as well as between cubic centimetre and millilitre.) (1=yes, 2=no)	0= 61.32% 1= 9.43% 2= 29.25%
V13	If the answer to V12 is „yes“: Can the pupil solve problem assignments that include liquid? (1=yes, 2=no)	0= 79.25% 1= 10.38% 2= 10.38%
V14	Is the pupil familiar with the metric units for mass and how does she/he apply them? (1=Yes, only familiar with the metric units for mass, 2= Yes, familiar with the metric units for mass and estimates mass properly, 3=Yes, familiar with the metric units for mass, estimates mass properly, and solves problem assignments which include mass, 4=Nothing of the above)	1= 50.94% 2= 31.13% 3= 11.32% 4= 6.60%
Plane and shape		
V15	Does the pupil distinguish 2-dimensional shapes from 3-dimensional shapes? (1=yes, 2=no)	1=84.91% 2= 15.09%
V16	Can the pupil distinguish different 2-D shapes (1=yes, 2=no)	1= 87.74% 2= 12.26%
V17	If the answer to V16 is „yes“: Does the pupil understand the terms: perimeter, flooring? (1=yes, 2=no)	0= 6.60% 1= 30.19% 2=63.21%
V18	If the answer to V17 is „yes“: Can the pupil split a shape into similar shapes? (1=yes, 2=no)	0= 68.87% 1=23.58% 2= 7.55%
V19	If the answer to V18 is „yes“: Does the pupil understand the relationship between perimeter and flooring (area)? (1=yes, 2=no)	0= 76.42% 1= 18.87% 2= 4.72%
V20	If the answer to V19 is „yes“: Does the pupil solve problem assignments including shapes? (1=yes, 2=no)	0= 81.13% 1= 13.21% 2= 5.66%
V21	Does the pupil recognize the parallel and perpendicular lines in the environment and in 2-D and 3-D shapes? (1=Yes, recognizes only, 2=Yes, recognizes and constructs given positions, but does not solve position assignments, 3=Yes, recognizes and solves position assignments, but is not able to construct given positions, 4=Yes, recognizes, constructs given positions, and is able to construct position assignments, 5=No, nothing of the above)	0= 0.94% 1= 28.30% 2= 33.02% 3= 25.47% 4= 6.60% 5= 5.66%
Data manipulation and problem solving		
V22	Does the pupil know the elements of statistics (counting)? (1=yes, 2=no)	1= 40.57% 2=59.43%

Variable code	Variable description	Frequencies
V23	If the answer to V22 is „yes“: Can the pupil read graphical data (for example charts)? (1=yes, 2=no)	0= 55.66% 1= 31.13% 2= 13.21%
V24	If the answer to V23 is „yes“: Can the pupil interpret text graphically? (1=yes, 2=no)	0= 68.87% 1= 20.75% 2= 10.38%
V25	If the answer to V24 is „yes“: Can the pupil solve problem assignments with graphical interpretation of text? (1=yes, 2=no)	0= 78.30% 1= 16.98% 2= 4.72%
V26	Can the pupil solve problem assignments using step by step method? (1=yes, 2=no)	0= 0.94% 1= 35.85% 2= 63.21%
V27	Can the pupil solve problem assignments using backward method? (1=yes, 2=no)	1= 30.19% 2= 69.81%
V28	Can the pupil solve problem assignments presented in the form of equation while understanding that addition is the inverse of subtraction, and multiplication is the inverse of division? (1=yes, 2=no)	1= 38.68% 2= 61.32%
Group 2: Cognitive components of gift		
V29	Does the pupil have the ability of focused attention? (1=yes, 2=no)	1= 51.89% 2= 48.11%
V30	Does the pupil have the ability of finding a path towards the solution (convergent thinking)? (1=yes, 2=no)	1= 39.62% 2= 60.38%
V31	Does the pupil have the ability of constructing complex problem situations after solving the simple ones (divergent thinking)? (1=yes, 2=no)	1= 32.08% 2= 67.92%
V32	Does the pupil have the ability of memorizing the facts and searching the long term memory? (1=yes, 2=no)	1= 30.19% 2= 69.81%
Group 3: Components of gift that contribute to gift realization		
V33	Is the pupil open to new experience? (1=Yes, she/he is open to constant changes, 2=Yes, accepts some changes, and then "freezes", 3=No)	1= 49.06% 2= 37.74% 3= 13.21%
čV34	How does the pupil see herself/himself in comparison to other people? (1=positive, 2=negative)	1= 82.08% 2= 17.92%
V35	How does the pupil perceive the evaluation of herself/himself provided by the people in her/his environment (evaluation coming from the people important to her/him)? (1=positive, 2=negative)	1= 88.68% 2= 11.32%
V36	How does the pupil react to stress (does she/he accept problems and failure as an opportunity for acquiring new experience)? (1=positive, 2=negative)	1= 72.64% 2= 27.36%
Group 4: Environment factors		
V37	Does the pupil get additional assignments from the teacher during regular math periods? (1=yes, 2=no)	1= 51.89% 2= 48.11%

Variable code	Variable description	Frequencies
V38	Does the pupil have parental support in terms of additional practicing of mathematics at home? (1=yes, 2=no)	1= 72.64% 2= 27.36%
V39	Does the pupil have her/his parents' financial support (such as financing additional individual classes in mathematics or similar)? (1=yes, 2=no)	1= 33.02% 2= 66.98%
V40	Does the pupil have her/his mentor's support towards achieving the necessary mathematical knowledge and skills? (1=yes, 2=no)	1= 47.17% 2= 52.83%
V41	Does the pupil have her/his mentor's support towards achieving the average mathematical competencies? (1=yes, 2=no)	0= 0.94% 1= 37.74% 2= 61.32%
V42	Does the pupil have her/his mentor's support in acknowledging her/his competencies (preparing for math competitions)?" (1=yes, 2=no)	1= 29.25% 2= 70.75%
V43	Does the pupil have a support of her/his school?" (1=yes, 2=no)	1= 53.77% 2= 46.23%
V44	Does the pupil have a support of her/his family? (1=yes, 2=no)	1= 80.19% 2= 19.81%
V45	Does the pupil have a support of the society? (1=yes, 2=no)	1= 78.30% 2= 21.70%
V46	Has the pupil received any prizes or acknowledgements in the area of mathematics? (1=yes, 2=no)	1= 0.00% 2= 100.00%
V47	Has the pupil received any prizes or acknowledgements in other areas? (1=yes, 2=no)	1= 33.96% 2= 66.04%
V48	Has the pupil participated at any meetings in the area of mathematics? (1=yes, 2=no)	1= 5.66% 2= 94.34%
V49	Is the pupil involved in any additional mathematical (or computer or technical) courses at school? (1=yes, 2=no)	1= 28.30% 2= 71.70%
V50	Is the pupil involved in any additional mathematical (or computer or technical) courses out of school? (1=yes, 2=no)	1= 16.04% 2= 83.96%
V51	Does the pupil use any other professional services in the area of mathematics (private lessons, computer or technical courses, mathematical magazines, or something similar)? (1=yes, 2=no)	1= 16.98% 2= 83.02%
Group 5: Active learning and exercising		
V52	Is the pupil skilled in distinguishing the important from the unimportant? (1=yes, 2=no)	1= 67.92% 2= 32.08%
V53	Is the pupil skilled in combining and organizing information into a meaningful structure? (1=yes, 2=no)	1= 47.17% 2= 52.83%
V54	Is the pupil skilled in performing selective comparison and connecting new pieces of information with the existing ones in the long term memory? (1=yes, 2=no)	1= 52.83% 2= 47.17%
V55	Does the pupil possess the analytical thinking skill (reasoning, comparing, estimating, evaluating)? (1=yes, 2=no)	1= 55.66% 2= 44.34%

Variable code	Variable description	Frequencies
V56	Does the pupil possess the creative thinking skill (discovering, imagination, creating new)? (1=yes, 2=no)	1= 57.55% 2= 42.45%
V57	Does the pupil possess practical skills (bringing thoughts into action)? (1=yes, 2=no)	1= 51.89% 2= 48.11%
V58	Does the pupil possess planning skills? (1=yes, 2=no)	1= 40.57% 2= 59.43%
V59	Is the pupil skilled in "keeping track" of self improvement? (1=yes, 2=no)	1= 48.11% 2= 51.89%
V60	Is the pupil skilled in regulating her/his behavior (is she/he prepared to make changes in the way of learning if it does not give certain results)? (1=yes, 2=no)	1= 48.11% 2= 51.89%
V61	Teacher estimation of pupil's mathematical gift: 1= potentially gifted child in mathematics, 2=child with a special interest in mathematics, 3=child with average mathematical competencies, 4=child with mathematical competencies below the average	1=6.61% 2=36.79% 3=33.96% 4=22.64%

Appendix B. Sensitivity ratios of input variables obtained by the sensitivity analysis

Variable code	Variable ratio	Variable rank	Variable group
V57	1.007649	1	5
V33	1.006947	2	3
V29	1.004836	3	2
V54	1.004785	4	5
V56	1.004632	5	5
V4	1.004388	6	1
V44	1.004007	7	4
V15	1.003977	8	1
V37	1.003741	9	4
V2	1.003361	10	1
V16	1.003296	11	1
V14	1.003263	12	1
V38	1.003009	13	4
V1	1.002873	14	1
V17	1.002862	15	1
V3	1.002852	16	1
V21	1.002475	17	1
V5	1.002466	18	1

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V10	1.002375	19	1
V9	1.002315	20	1
V7	1.001709	21	1
V8	1.001689	22	1
V53	1.00161	23	5
V58	1.001607	24	5
V26	1.001488	25	1
V28	1.001394	26	1
V18	1.001349	27	1
V55	1.000944	28	5
V49	1.000715	29	4
V34	1.0007	30	3
V59	1.000603	31	5
V60	1.000502	32	5
V30	1.000369	33	2
V46	1	34	4
V32	0.999494	35	1
V22	0.999169	36	1
V45	0.998574	37	4
V42	0.997625	38	4
V35	0.997502	39	3
V20	0.996506	40	1
V23	0.995907	41	1

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MODEL NEURONSKIH MREŽA ZA PREDVIĐANJE MATEMATIČKE DAROVITOSTI U DJECE

SAŽETAK

Cilj ovoga rada bio je modeliranje neuronske mreže kojom bi se mogla otkriti matematička darovitost u učenika četvrtih razreda osnovnih škola. Ulaz se sastojao od varijabli izvedenih za opis pet osnovnih komponenata matematičke darovitosti u djece, a koje su ustanovljene u prethodnim istraživanjima. Kao izlazni rezultat upotrijebljena je znanstveno potvrđena psihološka evaluacija darovitosti utemeljena u Ravenovim progresivnim matricama. Tri modela neuronskih mreža testirana su na hrvatskim podatcima: višeslojni perceptron, mreža s radijalno zasnovanom funkcijom i probabilistička (vjerojatnosna) mreža. Rad mreža mjeren je u odnosu na prosječnu stopu pogodaka prikupljenih na testnom uzorku. Analiza je pokazala da je najvišu točnost postigla neuronska mreža s radijalno zasnovanom funkcijom, kojom se mogu točno prepoznati sva darovita djeca. Tako visoka točnost u klasifikaciji pokazuje da neuronske mreže imaju potencijal služiti kao efektivan alat inteligentne odluke pomoću kojega bi učitelji mogli otkriti djecu s darovitošću za matematiku. To može biti osobito korisno u školama s manjkom psihologa.

UVOD

Tijekom proteklih nekoliko desetljeća inteligentni su sustavi u edukaciji imali široku uporabu u svrhu izrade alata za poučavanje. Otkrivanje darovite djece uglavnom se provodilo uz pomoć znanstveno odobrenih Ravenovih progresivnih matrica upotrijebljenih u procesu psihološke evaluacije darovitosti. Međutim, u školama u kojima nema psihologa, potreban je učinkovit inteligentan alat za podršku učiteljima u procjeni darovitosti u djece.

Cilj neuronskih mreža (NM) jest predviđanje, klasifikacija i asocijacija. One su sposobne raditi s robustnim podatcima. Stoga ih se testira kako bi se

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postigao točan model za otkrivanje matematičke darovitosti u djece četvrtih razreda osnovnih škola. U deset hrvatskih škola provedeno je istraživanje u kojemu su prikupljene procjene psihologa za svako dijete u uzorku, a koje su potom upotrijebljene kao izlazni rezultat u neuronskim mrežama. Uz pronalaženje najtočnijega NM modela, raspravlja se o nekim razlikama u procjenama matematičke darovitosti djece od strane učitelja te o modelu neuronskih mreža. Rezultati pokazuju da su procjene najboljega NM modela sličnije nalazima psihologa, nego procjenama učitelja. Za razliku od procjena učitelja, NM model svrstava veći broj djece u kategoriju darovitih. Zaključujemo da se NM model može koristiti kao metodološki alat za pomoć učiteljima u procjeni darovitosti učenika.

PRETHODNA ISTRAŽIVANJA

Od prvog pojavljivanja umjetne inteligencije kao znanstvene discipline razvijene su brojne tehnike kojima je cilj bio stvaranje inteligentnih strojeva (Russel i Norvig, 2002). Neke od tih tehnika su ekspertni sustavi, rješavanje problema, strojno učenje, razumijevanje prirodnoga jezika, prepoznavanje govora, prepoznavanje uzoraka, robotika, neuronske mreže, genetski algoritmi, inteligentni agenti i dr. Istraživanje inteligentnih sustava u edukaciji većinom je bilo usredotočeno na razvoj sustava za poučavanje kojima bi se mogla dati podrška učenju i poučavanju određene teme, uz mogućnost uključivanja multimedijskog i individualiziranoga pristupa. Primjerice, Stathacopoulou i dr. (Stathacopoulou i dr., 2005) predložili su uporabu kombinacije neuronskih mreža i fuzzy logike za napredne studentske dijagnostičke procese u inteligentnom sustavu učenja. Njihov model omogućio je sustavu „oponašanje“ učitelja u dijagnosticiranju karakteristika studenta te u odabiru primjerenoga stila učenja. Sustav je testiran u učenju vektorske konstrukcije u fizici i matematici. Dobiveni rezultati su uspoređeni s preporukama skupine iskusnih učitelja; pokazalo se da je sustav u stanju osmisliti dijagnostički proces, osobito u marginalnim slučajevima, gdje je čak i učitelju teško točno ocijeniti studenta. Canales i dr. (Canales i dr., 2007) su razvili inteligentni mrežno utemeljen edukacijski sustav (WBES) koji obuhvaća individualne potrebe svakoga studenta pri učenju te omogućuje uporabu različitih tehnika, stilova učenja, strategija učenja i načina interakcije. Arhitektura njihova sustava izrađena je u skladu s IEEE – LTSA (Learning Technology Systems Architecture) standardima, prema kojima bi edukacijski sustavi trebali biti strukturirani u pet slojeva: (1) interakcija učenika i okoline, (2) dizajnerske karakteristike koje utječu na učenika, (3) komponente sustava, (4) provedba perspektiva i prioriteta i (5) operacijske komponente i interoperabilnosti (kodiranje programa, sučelja, protokoli). Zeleznikov i Nolan (Zeleznikov i Nolan, 2001) su izradili sustav podrške u donošenju odluka utemeljen u fuzzy logici te su utvrdili pravila za pomoć učiteljima u ocjenjivanju pisanih sastavaka. Saito i dr. (Saito i dr., 2007) istražili su utjecaj suradnje među školama i fakultetima na učinkovitost učenja i poučavanja u ovim institucijama. Njihovi rezultati mogu se sažeti na sljedeći način: (1) zajedničko planiranje nastave, promatranje i promišljanje doprinose unaprjeđenju metodologija poučavanja, (2) profesori na fakultetima i učitelji u školama primjećuju da su studenti/učenici uključeni u suradnju aktivniji, (3) nužna je povezanost između učenika/studenata i materijala te između samih učenika/studenata (4) ishod suradnje jest razvoj kolegijalnosti među školama i između profesora na fakultetima i učitelja u školama.

Međutim, manje je istraživačke pozornosti posvećeno području inteligentnih sustava koji se koriste u otkrivanju darovitosti djece u određenim područjima kao što je matematika. Gorr i dr. (Gorr i dr., 1994) pokušali su predvidjeti prosjeke studentskih ocjena služeći se linearnom regresijom, stepwise polinominalnom regresijom i neuronskim mrežama te su usporedili predviđanja s indeksom kojim se koristio Odbor za upis u svrhu predviđanja prosjeka ocjena u profesionalnoj školi. Njihovi rezultati pokazali su da ni jedna od testiranih metoda nije bila značajno bolja od indeksa Odbora. Hardgrave i dr. (Hardgrave i dr., 1994) su istražili neuronske mreže u predviđanju studentskog uspjeha na diplomskom studiju. Pokazali su da su rezultati koje daju neparametrijske procedure poput neuronskih mreža barem jednako dobri kao rezultati dobiveni tradicionalnim metodama te stoga zavrjeđuju daljnje proučavanje. Pavleković i dr. (Pavleković i dr., 2009) također istražuju uporabu inteligentnih metoda u detekciji darovitosti, pri čemu predlažu eskpertni sustav koji na temelju pet komponenti darovitosti svrstava dijete u jednu od četiriju kategorija darovitosti.

Johnson (Johnson, 2007) je naglasila važnost i potrebu za točnim otkrivanjem i daljnjim razvojem matematičke darovitosti te uključivanjem ostalih kriterija uz matematičke kompetencije. Utvrđivanje i razvoj darovitosti istraženi su i u Sterberg (Sterberg, 2001) i Tannenbaum (Tannenbaum, 1983). U nekim zemljama manjak psihologa još više otežava proces otkrivanja darovite djece. Primjerice, samo 140 psihologa radi na 931 hrvatskoj osnovnoj školi (Vlahović-Štetić, 2006). Istraživanja pokazuju da je u proteklih nekoliko godina došlo do velikog širenja uporabe metodologije umjetne inteligencije u alatima za poučavanje. Međutim područje utvrđivanja matematičke darovitosti bi trebalo detaljnije istražiti te je nužno izraditi inteligentan sustav koji bi uz matematičke uključio i druge kompetencije.

METODOLOGIJA NEURONSKIH MREŽA

Prethodna istraživanja su pokazala da su neuronske mreže kao neparametrijska metoda barem jednako učinkovite u predviđanju studentskog uspjeha kao i statističke metode (Hardgrave i dr., 1994). One su često bolje od klasičnih statističkih metoda s obzirom na njihovu sposobnost za analizu nepotpunih i nejasnih podataka, rad na problemima koji nemaju jasno rješenje te sposobnost za učenje iz povijesnih podataka. Većina radova u ovome području bavi se testiranjem samo jedne neuronske mreže – višeslojnoga perceptrona (VSP). U ovome radu uspoređujemo rad tri NM modela: višeslojnoga perceptrona, mreže s radijalno zasnovanom funkcijom i vjerojatnosnu mrežu.

VSP je neuronska mreža s unaprijednom propagacijom funkcijskog signala. Namijenjen je u opće svrhe te je jedan od najčešće korištenih NN modela. U cilju optimizacije funkcije grješke, VSP koristi algoritam širenja unatrag koji se temelji na determinističkom algoritmu gradijentnog opadanja koji je godine 1974. razvio Paul Werbos, a proširio Rumelhart, Hinton, Williams (u Masters, 1995) te napredniji konjugirani gradijentni algoritam. Ovaj algoritam minimizira grešku u smjeru koji je konjugiran s prethodnim smjerovima, dok se detaljna komputacija daje u Masters (Masters, 1995.) Konjugirani gradijent se kombinira s klasičnim širenjem unatrag, tako da se širenje unatrag koristi u prvih 100 epoha, dok se konjugirani gradijent koristi u sljedećih 500 epoha. Standardno delta pravilo upotrijebljeno je pri učenju, dok su stope učenja i momentum bili dinamički optimizirani tijekom procesa učenja (stopa učenja se kretala od 0,08 do 0,01, a momentum od 0,8 do 0,1). Prekomjerno uvježbavanje se izbjeglo uključivanjem procesa unakrsne provjere koji naizmjence uvježbava i testira mrežu (uporabom posebnog testnog uzorka) sve dok se izvedba mreže na testnom uzorku ne poboljša za n broj pokušaja (n=10). Maksimalni broj epoha u našem eksperimentu bio je namješten na

1000. Nakon što je odabrana najbolja mreža, model je testiran hold-out metodom na uzorku za provjeru kako bi se utvrdila njegova sposobnost uopćavanja.

Model neuronskih mreža s radijalno zasnovanom funkcijom (RBF) utemeljen je u procedurama klasteriranja za izračunavanje razdaljina između svakog vektora ulaza i središta, koje je predstavljano radijalnom jedinicom. Njegova prednost u odnosu na algoritme višeslojnog perceptrona jest u tome što ne podliježe problemu lokalnih minimuma, a karakteriziraju ga razmjerno kompaktne mreže koje se brzo uvježbavaju. S obzirom na to da koristi radijalno simetrične i radijalno vezane funkcije transfera u njegovu skrivenom sloju, on predstavlja opći oblik vjerojatnosne i opće regresijske mreže. Park i Sandberg su dokazali sposobnost RBF-a s jednim skrivenim slojem da približi bilo koju nelinearnu funkciju (u Karayianis i dr., 1997). Michelli (u Karayianis i dr., 1997) je pokazao kako ta mreža može proizvesti interpolirajuću površinu koja prolazi kroz sve parove skupa podataka za treniranje. RBF algoritam koristi Euklidovu udaljenost i Gaussovu prijenosnu funkciju u skrivenom sloju koji mapira izlazni rezultat funkcije udaljenosti prema:

$$f(x) = \varphi(\|x - c\|) = e^{\left(\frac{I_k^2}{\sigma_k^2}\right)} \quad (1)$$

gdje je *x* vector ulaza, *c* je središte određeno cluster algoritmom, a parameter σ je određen tehnikom najbližega susjeda. Učenje kroz arhitekturu može se opisati u sljedeća tri koraka: (1) faza je prisutna od ulaza u skriveni sloj, gdje ulazne težine u sloj prototipa uče postati središtem klastera vektora ulaza koristeći dinamični k-means algoritam, (2) u skrivenom sloju se izračunavaju polumjeri Gaussove funkcije u središtima klastera uporabom tehnike 2-najbliža susjeda, gdje je polumjer danog Gaussa postavljen na prosječnu udaljenost o dva najbliža središta klastera, konačno (3) greška se izračunava na izlaznom sloju. I RBF algoritam ima neke nedostatke pa se broj radijalnih jedinica mora unaprijed odrediti, a njihova središta i devijacije moraju biti postavljeni. U cilju prevladavanja tih ograničenja, u našim je eksperimentima upotrijebljena procedura odbacivanja pojedinih čvorova za postupno smanjivanje broja skrivenih jedinica. Početni broj skrivenih jedinica podešen je prema veličini uzorka za uvježbavanje. Kako bi se RBF upotrijebio u svrhu klasifikacijskih problema, softmax aktivacijska funkcija je dodana izlaznom sloju da bi se dobile vjerojatnosti u izlaznim razredima. Prekomjerno uvježbavanje se izbjeglo uključivanjem procesa unakrsne provjere koji naizmjence uvježbava i testira mrežu (uporabom posebnog testnog uzorka) sve dok se izvedba mreže na testnom uzorku ne poboljša za *n* broj ponavljanja. Nakon što je odabrana najbolja mreža, model je testiran na novom uzorku za provjeru u svrhu određivanja njegove mogućnosti uopćavanja.

Masters (Masters, 1995) predlaže vjerojatnosnu neuronsku mrežu kao dobar izbor u klasifikacijskim problemima gdje u podacima postoje netipične vrijednosti i gdje je brzina učenja važna, s obzirom na to da taj algoritam ne uči ponavljanjem, već koristi samo jedan prolaz kroz skup podataka. Iz tih razloga u ovom se radu testira i vjerojatnosna mreža. To je klasifikator koji za izradu klastera koristi Parzenove prozore te proizvodi brojne razrede kao izlazni rezultat. Euklidova funkcija sumacije u jedinici uzorka koristi se kompetitivnim modalitetom u izlaznom sloju.

Izlazni sloj u svim NM arhitekturama sastojao se od jedne jedinice procesiranja (označene s 1 za darovite učenike i 0 za učenike koji nemaju darovitosti za matematiku). Jednake prethodne vjerojatnosti za darovite i nedarovite učenike su upotrijebljene u svim mrežama (p=0,5). Analiza osjetljivosti je izrađena za testni uzorak kako bi se odredila značajnost ulaznih varijabli za model.

Kako bi pronašli kategoriju matematičke darovitosti za svako dijete, psiholozi su upotrijebili uzorak Ravenovih progresivnih matrica (RPM). To je neverbalni test analitičke inteligencije široke uporabe. Osmišljen je za procjenu intelektualnih sposobnosti i sposobnosti zaključivanja te razumijevanja kompleksnih podataka (Carpenter i dr., 1990). U ovome istraživanju RPM test je upotrijebljen zbog njegove potvrđene valjanosti. Pind i dr. (Pind i dr., 2003) su istražili kriterijsku valjanost RPM testa u odnosu na akademski uspjeh. Pokazali su da je najviša korelacija dobivena za matematiku, a niže korelacije za jezične predmete. Korelacije su se kretale između 0,38 i 0,75. Test je pokazao i visoke korelacije s nacionalnim ispitima. Njihovo istraživanje provedeno je na Islandu, u četvrtim, sedmim i desetim razredima u kojima djeca moraju polagati nacionalne ispite iz islandskog i matematike. Osim toga, djeca u desetom razredu moraju polagati i nacionalne ispite iz dva strana jezika: engleskog i danskog. Laidra i dr. (Laidra i dr., 2007) su RPM testom testirali estonsku djecu u 2., 3., 4., 6., 8., 10. i 12. Razredu. Ustanovili su visoku korelaciju između inteligencije, mjerene RPM testom, i učeničke prosječne ocjene u svim razredima. Ravenove matrice su prikladne za testiranje svih stupnjeva sposobnosti, imaju ekstenzivne norme za različite dobi i kulture, lako ih je administrirati i ocijeniti, a nisu kulturološki ni jezično pristrani.

PODACI

Početni uzorak u istraživanju sačinjavalo je 247 učenika u dobi od deset godina (četvrti razred osnovne škole) iz deset osječkih škola. Istraživanje je provedeno u prosincu 2006. godine. U istraživanju je upotrijebljeno 60 elemenata grupiranih u pet skupina varijabli i upotrijebljenih za opis svakoga djeteta. Matematičku darovitost tih učenika procijenili su njihovi učitelji. Prema legislativnim regulativama, morali smo zatražiti roditeljski pristanak kako bismo proveli psihološku evaluaciju svakoga djeteta. Dobili smo roditeljski pristanak za 106 učenika te je daljnja analiza usredotočena na taj manji uzorak. U Tablici 1 prikazan je prosjek ocjena učenika iz manjega uzorka za svaku od prve tri godine osnovne škole. U hrvatskim školama se koristi sustav ocjenjivanja od pet brojčanih ocjena (5=izvrstan ili superioran, 4=vrlo dobar ili iznad prosjeka, 3=dobar ili prosječan, 2=dovoljan ili ispod prosjeka, najmanja prolazna ocjena, 1=nedovoljan ili pad).

Tablica 1.

Model neuronskih mreža upotrijebio je svih 60 varijabli iz ulaza kojima su opisane komponente dječje darovitosti, a koje su podijeljene u sljedeće skupine varijabli (kao što je predloženo u Pavleković i dr. (Pavleković i dr., 2007)): (1) procjena matematičkih kompetencija, (2) kognitivne komponente darovitosti, (3) osobne komponente koje doprinose razvoju darovitosti, (4) okolni čimbenici te (5) učinkovitost aktivnog učenja i provođenja metoda kojima se pospješuje razvoj matematičkih kompetencija te moguća realizacija darovitosti. Sve varijable i deskriptivna statistika nalaze se u Prilogu A. Psihološka evaluacija dobivena Ravenovim progresivnim matricama upotrijebljena je kao izlazna varijabla u modelu, s obzirom na to da je znanstveno dokazana kao učinkovit instrument u otkrivanju darovitosti u djece (Pind i dr., 2003).

Zatražili smo učitelje i psihologe da svako dijete smjeste u jednu od četiri kategorije darovitosti (1=potencijalno matematički nadareno dijete, 2=dijete osobito zainteresirano za matematiku, 3=dijete s prosječnim matematičkim kompetencijama i 4=dijete s matematičkim kompetencijama nižim od prosjeka). Struktura procjena učitelja i zaključaka psihologa bila je sljedeća:

Tablica 2.

S obzirom na to da je primarni cilj ovoga rada bio izdvojiti djecu koja su matematički darovita, kategorije su dalje podijeljene u dvije glavne skupine: (1) skupina matematički darovitih učenika sastojala se od učenika koje su psiholozi smjestili u kategorije 1 i 2 (u daljnjem tekstu se nazivaju „darovitim“ učenicima) te (2) skupina učenika koji nisu matematički daroviti sastojala se od učenika koji su smješteni u kategorije 3 i 4 (u daljnjem tekstu se nazivaju „nedarovitim“ učenicima). Dakle, bilo je moguće izraditi binominalni model matematičke darovitosti koji su upotrijebile neuronske mreže.

Kako bi se uvježbale i testirale neuronske mreže, ukupan je uzorak podijeljen u tri poduzorka tako da je 70% podataka upotrijebljeno za uvježbavanje mreže, 10% podataka je upotrijebljeno za pronalaženje optimalnog vremena potrebnog za učenje te mrežne strukture u proceduri unakrsne provjere, dok je preostalih 20% podataka upotrijebljeno za završno testiranje mreže. Uzorci za uvježbavanje i unakrsnu provjeru sadržavali su otprilike jednaku distribuciju darovite i nedarovite djece, dok se testni uzorak sastojao od 53,77% darovitih i 46,2% nedarovitih učenika. Distribucija darovitih i nedarovitih učenika u sva tri uzorka predstavljena je u Tablici 3.

Tablica 3.

REZULTATI

S ciljem pronalaženja najboljeg NM modela, svaki od tri NM algoritma (VSP, mreža s radijalno zasnovanom funkcijom i vjerojatnosna mreža) izradio je 25 NM arhitektura. U svakoj od pet arhitektura varirani su sljedeći parametri: broj slojeva, broj skrivenih jedinica u skrivenom sloju, funkcije transfera, pravila za učenje i broj ponavljanja. NM arhitekture su ocijenjene na temelju prosječnih stopa pogodaka, a prikazane su i stope pogodaka za darovite i nedarovite učenike. Najbolji model izabran je na temelju najviše prosječne stope pogodaka dobivene na testnom uzorku. Rezultati najboljih NM modela koje je proizveo svaki NM algoritam predstavljene su u Tablici 4.

Tablica 4.

Kao što je prikazano u Tablici 4, najbolji ukupni NM model postigao je algoritam mreže s radijalno zasnovanom funkcijom, koji je proizveo prosječnu stopu pogotka od 85,72%. Model je točno kategorizirao svu (100%) darovitu djecu iz testnog uzorka te 71,43% nedarovite djece. NM arhitektura najboljega modela sadržavala je 7 skrivenih procesnih jedinica i 41 ulaznu jedinicu koje su rezultat procedure odbacivanja pojedinih čvorova.

Analizom osjetljivosti, provedenoj na testnom uzorku, dobiveni su omjeri osjetljivosti za 41 varijablu koje su izdvojene kao važne za model. Deset varijabli s najvišim omjerom osjetljivost su: (1) V57 – *Posjeduje li učenik praktične vještine (provođenje ideje u praksu)?* (omjer=1,0076), (2) V33 – *Je li učenik otvoren za nova iskustva? (omjer=1,0069)*, (3) V29 – *Posjeduje li učenik sposobnost usredotočenosti? (omjer=1,0048)*, (4) V54 – *Posjeduje li učenik vještinu selektivne usporedbe i povezivanja novih informacija s već postojećima u dugoročnom pamćenju? (omjer=1,0047)*, (5) V56 – *Posjeduje li učenik vještinu kritičkog promišljanja (otkrivanja, mašte, stvaranja novoga)? (omjer=1,0046)*, (6) V4 – *Može li učenik pratiti točan redoslijed različitih aritmetičkih operacija tijekom izračuna? (omjer=1,0043)*, (7) V44 – *Ima li učenik podršku svoje obitelji? (omjer=1,0040)*, (8) V15 – *Razlikuje li učenik dvodimenzionalne od trodimenzionalnih oblika? (omjer=1,0039)*, (9) V37 – *Daje li učitelj učeniku dodatne zadatke tijekom redovne nastave matematike? (omjer=1,0037)*, (10) V2 – *Može li učenik grafički čitati velike brojeve? (omjer=1,0033)*. Slika 1 predstavlja grafički prikaz osjetljivosti omjera prvih 10 varijabli. Omjeri svih 41 izvedenih varijabli predstavljeni su u Prilogu B. Zamjetno je da je najveći broj varijabli (21) izveden iz skupine 1 (matematičke kompetencije), iako su iz svake skupine izvedene barem po dvije varijable (2 varijable iz kognitivnih komponenata darovitosti, 3 varijable iz komponenti koje doprinose realizaciji darovitosti, 7 varijabli iz čimbenika okoline i 8 varijabli iz aktivnog učenja i uvježbavanja).

Slika 1.

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Iz matrice grešaka (Tablica 5) vidi se apsolutan broj učenika iz testnog uzorka koje je NM s radijalno zasnovanom funkcijom dodijelila svakoj skupini (daroviti ili nedaroviti) u usporedbi sa stvarnim brojem učenika koje su psiholozi podijelili u ove skupine.

Tablica 5.

Kad se procjene učitelja usporede s procjenama psihologa, izračunavaju se i stope pogodaka procjena učitelja u svrhu moguće usporedbe s NM modelom. Prosječna stopa pogodaka procjena učitelja je 60,71%, gdje učitelji točno klasificiraju samo 21,43% darovitih učenika i 100% nedarovitih učenika. Matrica grješaka procjena učitelja i psihologa predstavljena je u Tablici 6.
Tablica 6.
Rezultati pokazuju da je NM model točniji u prepoznavanju darovitih učenika te da daje višu prosječnu stopu pogodaka u procjeni darovitosti u djece. Prosječna stopa pogodaka procjena učitelja je niža, iako učitelji uspješnije prepoznaju nedarovite učenike. Test razlika u prosječnim stopama pogodaka pokazuje da je razlika između prosječne stope pogodaka NM modela i učitelja statistički značajna (p=0,014). Iz navedenoga proizlazi da su procjene najboljeg NM modela bliže zaključcima psihologa nego procjenama učitelja.

ZAKLJUČAK

Ovaj rad bavi se modeliranjem neuronskih mreža u području otkrivanja matematičke darovitosti u osnovnoškolskih učenika. S ciljem pronalaženja najboljeg modela testirani su algoritmi višeslojnog perceptrona, mreže s radijalno zasnovanom funkcijom i vjerojatnosne neuronske mreže. Najbolju prosječnu stopu pogotka postigla je mreža s radijalno zasnovanom funkcijom, koja je uspješno otkrila sve darovite učenike u testnom uzorku. Osim pronalaženja najtočnijeg modela neuronskih mreža, rad se bavi i nekim razlikama između učiteljskih procjena matematičke darovitosti u djece i modela neuronskih mreža. Rezultati pokazuju da je NM metodologija sposobna naučiti povezanost između zaključaka psihologa o darovitosti i varijabli ulaza. S obzirom na to da se procjene učitelja i zaključci psihologa razlikuju, model se može predložiti kao metodološki alat za pomoć učiteljima u odlučivanju o matematičkoj darovitosti učenika, a osobito u školama s manjkom psihologa.

Analiza osjetljivosti pokazala je da je model neuronskih mreža izlučio praktične vještine, otvorenost za novo iskustvo, sposobnost usredotočenosti i ostale kao najvažnije prediktore, iako je zaključeno da su varijable svih pet komponenti darovitosti važni prediktori matematičke darovitosti.

Sposobnosti koje pokazuje metodologija neuronskih mreža u otkrivanju darovitosti mogu se dalje istražiti uporabom više skupova podataka, s ciljem pružanja vrjednije procjene uopćavanja rada modela. Testiranje drugih inteligentnih metoda u otkrivanju darovitosti, kao što su inteligentni agenti, strojevi s potpornim vektorima i dr. mogla bi biti vrijedna tema budućih istraživanja. Trebalo bi razviti i sustav podrške u donošenju odluka koji bi pomogao učiteljima u ranom otkrivanju darovite djece te im pružio podršku u razvoju te darovitosti. Takav sustav bio bi koristan za djecu i zajednicu.