CODEN STJSAO ISSN 0562-1887
ZX470/1491 UDK 621.785.36:539.374

# Unified Theory of Transformation Plasticity and the Effect on Quenching Simulation

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Original scienfic paper

A phenomenological mechanism of transformation plasticity (TP) is discussed, in the first part of the paper, why it takes place under a stress level even lower than the characteristic yield stress of mother phase: This is principally based on the difference in thermal expansion coefficient in both phases. Bearing in mind that it is also a kind of plastic strain, a unified plastic flow theory is derived by introducing the effect of progressing new phase into the yield function of stress, temperature and plasticity related parameters. Experimental results of the TP coefficient will be shown in the following section for some steels under tensile and compressive stress. As examples of the application of the theory and the data, metallo-thermo-mechanical simulation for a quenched blank gear wheel is executed to demonstrate the effect of TP.

**Jedinstvena teorija transformacijske plastičnosti i utjecaj na simulaciju gašenja**

Izvornoznanstveni članak

Fenomenološki mehanizam transformacijske plastičnosti (TP) istraživan je u prvom dijelu rada, kada se dešava kod napreznaja nižeg od granice razvlačenja osnovne faze: To se temelji na razlici koeficijenta toplinskog širenja dviju faza. Uzimajući u obzir da je to vrsta plastične deformacije jedinstvena plastična teorija tečenja izvedena je uvođenjem efekta napredovanja nove faze u funkciju granice tečenja, temperature I parametara povezanih s plastičnosti.

Eksperimentalni rezultati koeficijenta transformacijske plastičnosti prikazat će se u slijedećem dijelu za neke neke čelike izložene vlačnom i tlačnom naprezanju. Kao primjer primjene teorije, izvršena je metalo-termo-mehaničke simulacija za zakaljeni neobrađeni zupčanik kako bi se pokazao efekt TP-a.

**Keywords**
*Metallo-thermo-mechanics*
*Quenching*
*Transformation plasticity*
*Unified transformation-thermoplasticity theory*

**Ključne riječi**
*Jedinstvena transformacijsko-termoplastična teorija*
*Kaljenje*
*Metalo-termo mehanika*
*Transformacijska plastičnost*

**Received (primljeno)**: 2009-09-10
**Accepted (prihvaćeno)**: 2010-12-30

## 1. Introduction

Most constitutive laws for transformation plasticity, or TP [1, 3-4, 13], which is known to contribute a drastic effect on the simulation of some practical engineering courses of thermo-mechanical processes, have been treated to be independent of ordinal thermo-plasticity. Considering that the mechanisms for both plastic strains are essentially with no difference from metallurgical viewpoint, the constitutive equation for transformation plastic strain rate is expected to be described in relation with plasticity theory [7-9].

This paper motivates to propose a mechanism for TP and to formulate unified constitutive equation of transformation and thermoplasticity by introducing the effect of increasing phase in mother phase. Application of the theory is made to numerical simulation of quenching process of a blank gear wheel to demonstrate the effect of TP, by use of the TP coefficients identified by experiments.

## 2. Phenomenological Mechanism of Transformation Plasticity

As mentioned above, yielding of *mother phase* (or austenite in this case) is considered to occur due to progressive *new phase* (such as pearlite or martensite. See Figure 1) even under small tensile stress below yield stress of the mother phase.

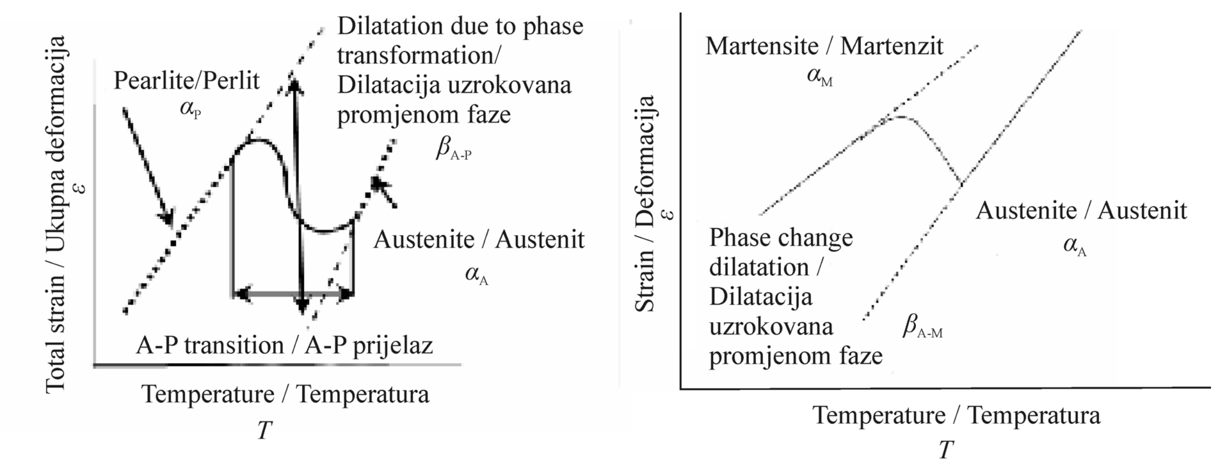
Denote the volume fractions of new and mother phases $\xi_n$ and $\xi_m$. (In more general cases to be discussed in the next section, the volume fraction of new phase will be denoted by $\xi_n$). When both elements are supposed to be connected parallely as indicated in Figure 2 being subjected to applied stress $\sigma$, then the following relations hold [8] when considering the phase change dilatation $\beta\xi_m$;

**Symbols/Oznake**

| | |
|---|---|
| TP | - transformation plasticity<br>- transformacijska plastičnost |
| $E$ | - Young's modulus<br>- Youngov model elastičnosti |
| $H$ | - strain hardening parameter<br>- parametar deformacijskog kaljenja |
| $\tau$, $T$ | - temperature<br>- temperatura |
| $\tau_s$, $T_s$ | - transformation start temperature<br>- temperatura na početku promjene |
| $k$ | - transofrmation constant<br>- konstanta transformacije |
| $n$ | - transformation exponent<br>- eksponent transformacije |
| $\hat{G}$ | - hardening modulus<br>- modul kaljenja |
| $F_I$ | - yield function<br>- funkcija popuštanja |
| $K$ | - coefficient of transformation plasticity<br>- koeficijent transformacijske plastičnosti |
| $r$ | - radius<br>- polumjer |
| $z$ | - distance<br>- udaljenost |
| $\xi$ | - phase volume fraction<br>- volumni udio faze |
| $\sigma$ | - stress<br>- naprezanje |
| $\beta$ | - expansion coefficient due to phase change<br>- koeficijent rastezanja uzorkovanog promjenom faze |
| $\alpha$ | - linear expansion coefficient<br>- koeficijent linearnog rastezanja |
| $\varepsilon$ | - strain<br>- deformacija |
| $\delta$ | - yield condition<br>- stanje popuštanja |
| $\sigma_{ps}$ | - yield stress of pearlite<br>- naprezanje popuštanja za perlit |
| $\sigma_{ms}$ | - yield stress of martensite<br>- naprezanje popuštanja martenzita |
| $\Phi$ | - martensite reaction constant<br>- konstanta reakcije martenzita |
| $\chi$ | - phase fraction<br>- udio faze |
| $\kappa$ | - plastic hardening parameter<br>- parametar plastičnog otvrdnjivanja |
| $\dot{\varepsilon}_I^{tp}$ | - transformation plastic strain rate<br>- brzina plastične deformacije transformacije |

**Indices/Indeksi**

| | |
|---|---|
| n | - new phase<br>- nova faza |
| m | - mother phase<br>- ishodišna, početna faza |
| P | - pearlite<br>- perlit |
| M | - martensite<br>- martenzit |




(a) Austenite-pearlite (b) Austenite-martensite

**Figure 1.** Schematic illustration of phase transformation.

**Slika 1**. Shematski prikaz pretvorbe

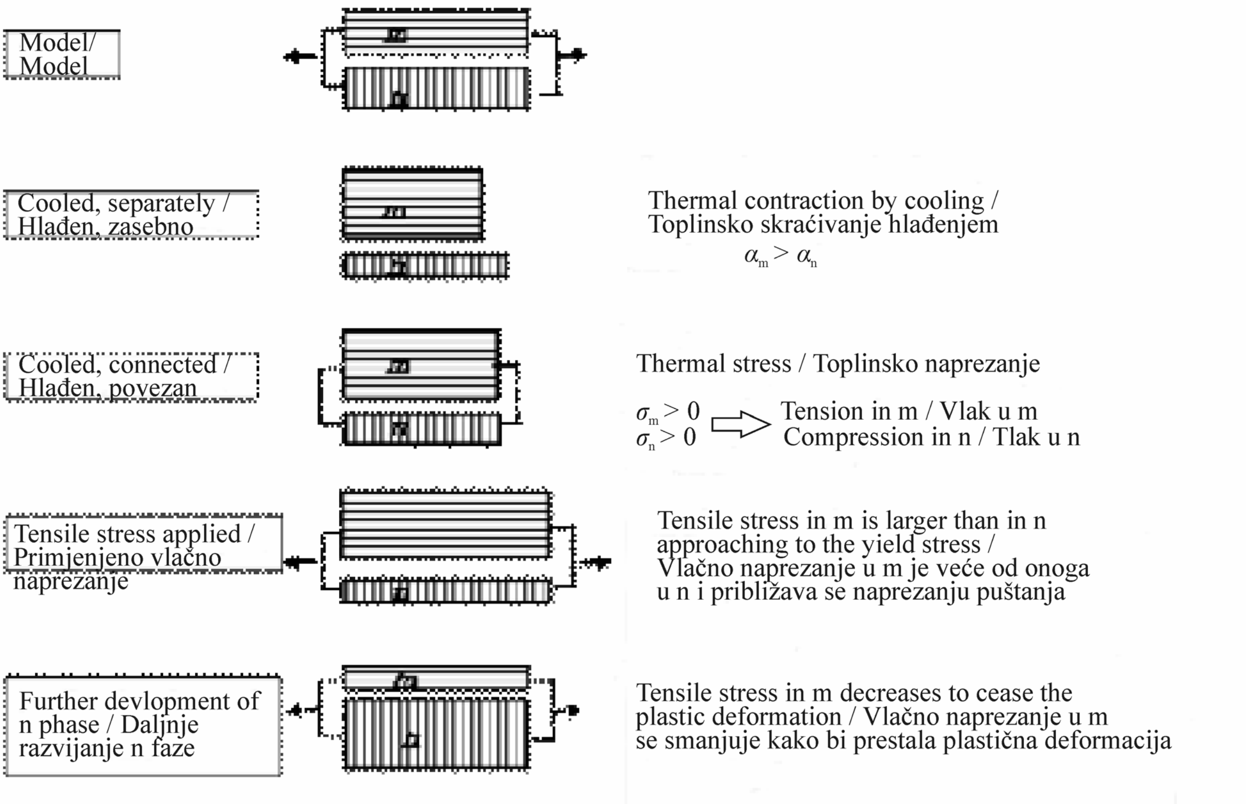



**Figure 2.** A model for transformation plasticity
**Slika 2.** Model transformacijske plastičnosti

Elastic-plastic constitutive equation;

$$d\varepsilon_m = \left(\frac{1}{E_m} + \frac{\delta_m}{H_m}\right) d\sigma_m + \alpha_m dT,$$

in mother phase

$$d\varepsilon_n = \left(\frac{1}{E_n} + \frac{\delta_n}{H_n}\right) d\sigma_n + \alpha_n dT + \beta d\xi_n, \quad (1)$$

in new phase,
with $\delta_m$ and $\delta_n$ representing yield condition

$$\delta_m = \begin{cases} 1, if\, \sigma_m \geq \sigma_{ms} \\ 0, if\, \sigma_m \leq \sigma_{ms} \end{cases}, \delta_n = \begin{cases} 1, if\, \sigma_n \geq \sigma_{ns} \\ 0, if\, \sigma_n \leq \sigma_{ns} \end{cases}, \quad (2)$$

where $\sigma_{ms}$ and $\sigma_{ns}$ stand for yield stress of mother and new phase.

Stress equilibrium condition;

$$\sigma = \sigma_m \xi_m + \sigma_n \xi_n, \text{ with}$$

$$\xi_m + \xi_n = 1, \text{ or } \xi_n \equiv \xi \text{ and } \xi_m \equiv 1 - \xi. \quad (3)$$

Compatibility condition;

$$\varepsilon_m = \varepsilon_n. \quad (4)$$

Here, $\sigma$, $\varepsilon$, E and $\alpha$ with suffices $m$ and $n$ denote the stress, strain, Young' modulus and linear expansion coefficient for mother and new phase, $\beta$ is the expansion coefficient due to phase change, and $T$ and $T_s$ respectively stand for varying temperature and transformation start temperature.

Figures 3(a) and (b) depict the results of temperature dependence of both stresses in mother and new phase with volume fraction of new phase respectively for pearlite and martensite transformation from austenite. The figures show that the stress in the mother phase increases with progressing volume fraction to reach the estimated yield stress of pearlite $\sigma_{ps}$= 80 MPa and for martensite $\sigma_{ms}$=100MPa, larger than initially applied stress $\sigma$ = 40 MPa with $E$=150 GPa, $\alpha_m$=2.15x10$^{-5}$ K$^{-1}$ $\alpha_n$=1.61x10$^{-5}$ K$^{-1}$ and $T_s$=700 °C for pearlite transformation and $\alpha_n$=2.0x10$^{-5}$ K$^{-1}$ and $T_s$=420 °C for martensite reaction.

The kinetic relation of the transformation employed in the analysis are [11]

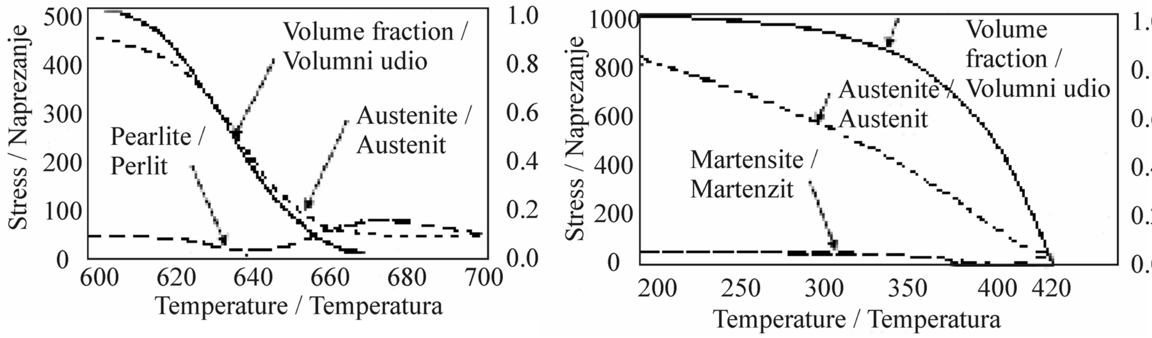
$$\xi_P(\tau) = 1 - \exp\left[-k(\tau - \tau_s)^n\right],$$

$$k = 2 \times 10^{-6},\ n = 4.2,\ \tau_s = 352 \quad (5)$$

and for martensite reaction [Magee, 1968]

$$\xi_M(T) = 1 - \exp[\phi(T_s - T)], \quad (6)$$

where $\phi = 0.0219$.




(a) Austeite-pearlite reaction. (b) Austenite-martensite reaction.

**Figure 3.** Variation of stress in mother and new phase with the progressive fraction of the phases when applied tensile stress.

**Slika 3.** Razlika naprezanja u osnovnoj i novoj fazi s porastom udjela faza nakon primjene vlačnog naprezanja.

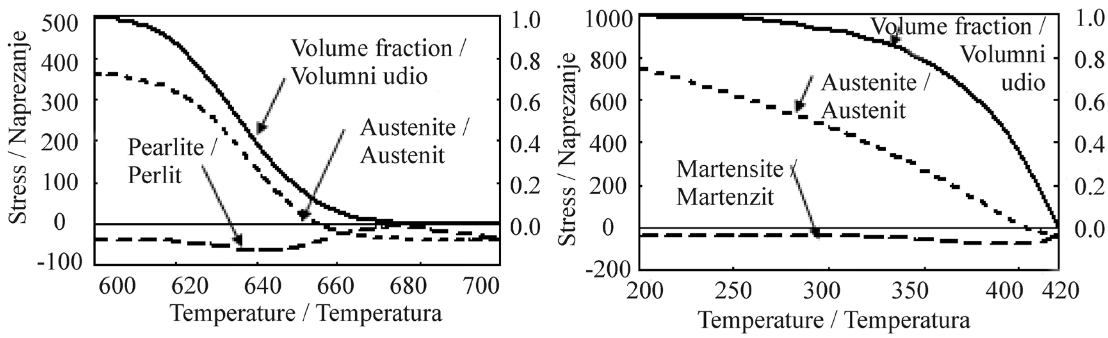
in terms of the units; stress in MPa, time in sec, and temperature in °C with dimensionless strain and volume fraction. It is seen from the figures that the stress in the mother phase increasing with progressing volume ftraction of new phase reaches the estimated yield stress of pearlite $\sigma_{ps}$=80 MPa and martensite $\sigma_{ms}$= 100 MPa. On the contrary to the previous case of tensile applied stress, Figure 4(a) and (b) show the results of progressive stress in mother and new phases when compressive stress is applied. In the cooling process, phase transformation strain is generally increased since $\alpha_m > \alpha_n$, then the stresses in both phases tend to be positive in the final stage of phase change. Nevertheless, compressive stress in new phase is rather increased in the first stage, which would be possible to reach the compressive yield stress.

($I$=1,2,3,...,N) as is shown in Figure 5(a) and that the mechanical and thermophysical property $\chi$ is represented by the *mixture law* [2] such that

$$\chi = \sum_{I=1}^{N} \xi_I \chi_I, \quad \text{with} \quad \sum_{I=1}^{N} \xi_I = 1. \tag{7}$$

Some phases associating plastic deformation.

Stress state related to the yielding of the $I$-th phase (say, mother phase) is assumed to be affected by other phases (new phase) with the volume fraction $\xi_J$ ($J$=1,2,3,...,M) as indicated in Fig.5(b). Then, the plasticity of the $I$-th phase is controlled by the yield function in the form,




(a) Austenite-pearlite reaction. (b) Austenite-martensite reaction

**Figure 4.** Variation of stress in mother and new phase with the progressive fraction of the phases when applied compressive stress.

**Slika 4.** Razlika naprezanja u osnovnoj i novoj fazi s porastom udjela faza nakon primjene tlačnog naprezanja .

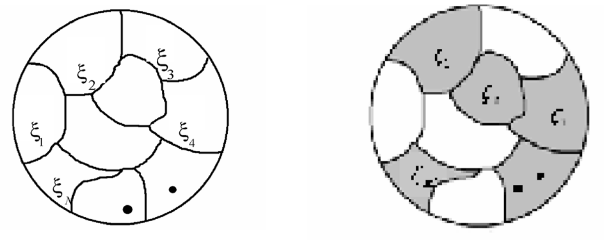
## 3. Unified Transformation-Thermo-Mechanical Plasticity

In order to formulate a constitutive equation of a body under phase transformation, we assume that the material point focused is composed of $N$ kinds of phases, which include all phases with the volume fraction $\xi_I$

$$F_I = F_I\,(\sigma_{ij},\ T,\ \varepsilon_{Iij}^{p},\ \kappa_I,\ \zeta_J), \tag{8}$$

($I$=1,2,3,...,N; $J$=1,2,3,...,M)

Here, $\sigma_{ij}$, $T$ and $\kappa_I$ are respectively stand for uniform stress and temperature and plastic hardening parameter.




(a) All phases / Sve faze  b) Some phases associating plastic deformation / Neke faze povezane s plastičnom deformacijom

**Figure 5.** Volume fraction of phases consisting a material point.

**Slika 5.** Volumni udio faza

Applying the consistency relation and the normality law, we have the form of plastic strain rate of the $I$-th phase as

$$\dot{\varepsilon}^{p}_{Iij} = \Lambda_I \frac{\partial F_I}{\partial \sigma_{ij}} = \hat{G}_I [\left( \frac{\partial F_I}{\partial \sigma_{kl}} \dot{\sigma}_{kl} + \frac{\partial F_I}{\partial T} \dot{T} \right) + \sum_{J=1}^{M} \frac{\partial F_I}{\partial \zeta_J} \dot{\zeta}_J ] \frac{\partial F_I}{\partial \sigma_{ij}}, \tag{9}$$

with the hardening modulus $\hat{G}_I$.

The first term in Eq. (9) is the ordinal thermo-mechanical plastic strain rate in the $I$-th phase, which is summation of mechanical and thermal parts. It is noted that the second term in Eq.(9) occurring in the $I$-th phase is originated from the progressing the new $J$-th phase $\xi_J$, which is found to be so-called *transformation plastic strain rate*

$$\dot{\varepsilon}^{tp}_{Iij} = \hat{G}_I \left( \sum_{J=1}^{M} \frac{\partial F_I}{\partial \zeta_J} \dot{\zeta}_J \right) \frac{\partial F_I}{\partial \sigma_{ij}}. \tag{10}$$

Adopting the mixture law, we finally have the global strain rate in the form.

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{th}_{ij} + \dot{\varepsilon}^{m}_{ij} + \dot{\varepsilon}^{p}_{ij}. \tag{11}$$

Attention is focused on the unified plastic strain rate

$$\dot{\varepsilon}^{p}_{ij} = \sum_{I=!}^{N} \zeta_I \dot{\varepsilon}^{p}_{Ij} = \sum_{I=1}^{N} \hat{G}_I \left[ \left( \frac{\partial F_I}{\partial \sigma_{kl}} \zeta_I \dot{\sigma}_{kl} + \frac{\partial F_I}{\partial T} \xi_I \dot{T} \right) + \right] \left[ \xi_I \left( \sum_{J=1}^{M} \frac{\partial F_I}{\partial \zeta_J} \dot{\zeta}_J \right) \right] \frac{\partial F_I}{\partial \sigma_{ij}}, \tag{12}$$

As the sum of thermo-mechanical and transformation plastic parts. The result indicates that both plastic strain rates related to thermo-mechanical and phase transformation effect is automatically derived from yield function in the form of Eq.(9) and that the Eq.(12) is the *unified plastic strain rate*.

Now, consider a special case when uniaxial stress $\sigma$ is applied. Then we have the transformation plastic strain rate from Eq.(10)

$$\dot{\varepsilon}^{tp} = \sum_{I=1}^{N} \frac{1}{H'_I} \left[ \sum_{J=1}^{M} \frac{H^{\zeta}_{IJ} \dot{\zeta}_J}{\bar{\sigma}_I} \xi_I \right] \sigma. \tag{13}$$

Here, hardening parameters are defined with the flow stress of the $I$-th phase $\bar{\sigma}_I$,

$$H'_I = \frac{\partial \bar{\sigma}_I}{\partial \bar{\varepsilon}^{p}_I}. \tag{14}$$

Strain hardening parameter of the $I$-th phase

$$H^{\zeta}_{IJ} = \frac{\partial \bar{\sigma}_I}{\partial \zeta_J}. \tag{15}$$

Dependence of flow stress in the $I$-th phase on $J$-th phase and the constitutive equation for TP strain in case between the mother and new phase reads

$$\dot{\varepsilon}^{tp} = \frac{1}{H'_m} \left[ \left( \frac{H^{\zeta}_{mm} \dot{\zeta}_m}{\bar{\sigma}_m} + \frac{H^{\zeta}_{mn} \dot{\zeta}_n}{\bar{\sigma}_m} \right) \xi_m \right] \sigma + \frac{1}{H'_n} \left[ \left( \frac{H^{\zeta}_{nm} \dot{\zeta}_m}{\bar{\sigma}_n} + \frac{H^{\zeta}_{nn} \dot{\zeta}_n}{\bar{\sigma}_n} \right) \xi_n \right] \sigma, \tag{16}$$

with the flow stress $\bar{\sigma}_m$, $\bar{\sigma}_n$ of mother and new phase. Since the effect of structure on flow stress is possible;

$$H^{\zeta}_{mn} \neq 0, \text{ and } H^{\zeta}_{mm} = H^{\zeta}_{nm} = H^{\zeta}_{nn} = 0. \tag{17}$$

And putting

$$\xi_m = 1 - \xi_n \equiv 1 - \xi, \text{ and } \zeta_n \equiv \xi, \tag{18}$$

then we have:

$$\dot{\varepsilon}^{tp} = \frac{1}{H'_m} \frac{H^{\zeta}_{mn} \dot{\xi}}{\bar{\sigma}_m} (1 - \xi) \sigma \equiv 3K (1 - \xi) \sigma. \tag{19}$$

This is the well known formula of Greenwood-Johnson for TP [4].

## 4. Application to Strain Response for Stress-Temperature Variation

The theory developed is now applied to some cases under varying stress and temperature. Total strain in such cases of varying temperature is expressed as

$$\varepsilon(T)=\varepsilon^{e}+\varepsilon^{th}+\varepsilon^{m}+\varepsilon^{tp}=\frac{\sigma}{E}+ \qquad (20)$$
$$+\int_{T_i}^{T}\left\{\left[\alpha_M(1-\zeta)+\alpha_N\zeta\right]+\beta\frac{\partial\zeta}{\partial T}\right\}dT+3\int_{0}^{\zeta(T)}K(1-\zeta)\sigma d\zeta.$$

The first case is to draw so-called temperature-elongation diagram. Figure 6 depicts the experimental diagram during pearlite transformation of a Cr-Mo steel (SCM420) [16], which show the strong stress dependence. Result of numerical calculation of Eq.(20) is plotted as temperature-elongation diagram is possible to be represented in Figure 7 for paerlite reaction. Here, Eqs. (5) and (6) are employed for diffusional and martensite transformation with the data of MATEQ [15], a material database accumulated by our group of Materials Database, JSMS. The coefficient of transformation plasticity ( $K=\sigma/\varepsilon^{tp}$ =9x10$^{-5}$1/MPa) for the steel is adopted by our experiment [15].

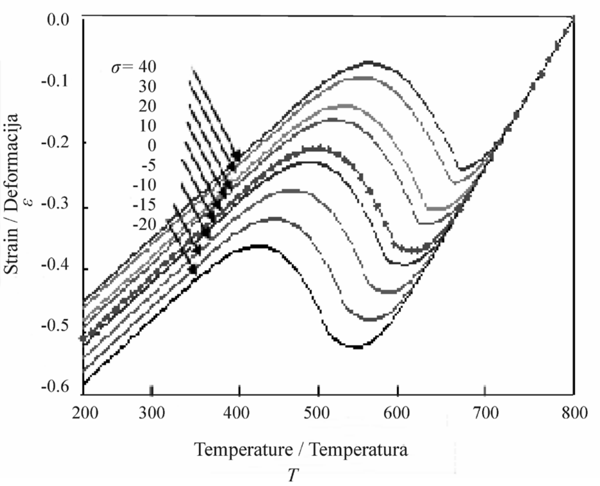



**Figure 7.** Simulated temperature-elongation diagram.

**Slika 7.** Simulacijski dijagram temperaturnog produljenja

## 5. Effect of Transformation Plasticity on Metallo-thermo-mechanical Simulation of Quenching Process

By employing the experimentally determined data for the transformation plasticity coefficient in the previous section, metallo-thermo-mechanical simulation is carried out for a quenching process of a blank gear wheel. Use is made of a code COSMAP developed by Ju and the present author [12].

The wheel is made of Chromium steel (SCr420) with outer and inner diameter of 34 and 24 mm, and the height of 30 mm [16]. The work is quenched by an oil, and the data for heat transfer coefficient depending on the metal surface temperature is adopted from Japanese cooperation group [10]. Some results of the simulation are represented in Figures 8-11. Here, attention is focused on the effect on transformation plasticity. Figure 8 plots the distribution of martensite fraction after quenching process along the line A in the small figure. Similar situation is also seen in the fraction in entire region.

On the contrary to the structure distribution, there are quite large difference in stress distribution. Figure 9 (a) duplicates the tangential residual stress pattern as well as the distortion in the cross section of the work when considering the TP effect, while Figure (b) is the case without the effect. Tangential and axial stress distribution along the line A is shown in Figure 10, and the difference in tangential stress along the line A and B is presented in Figure 11.

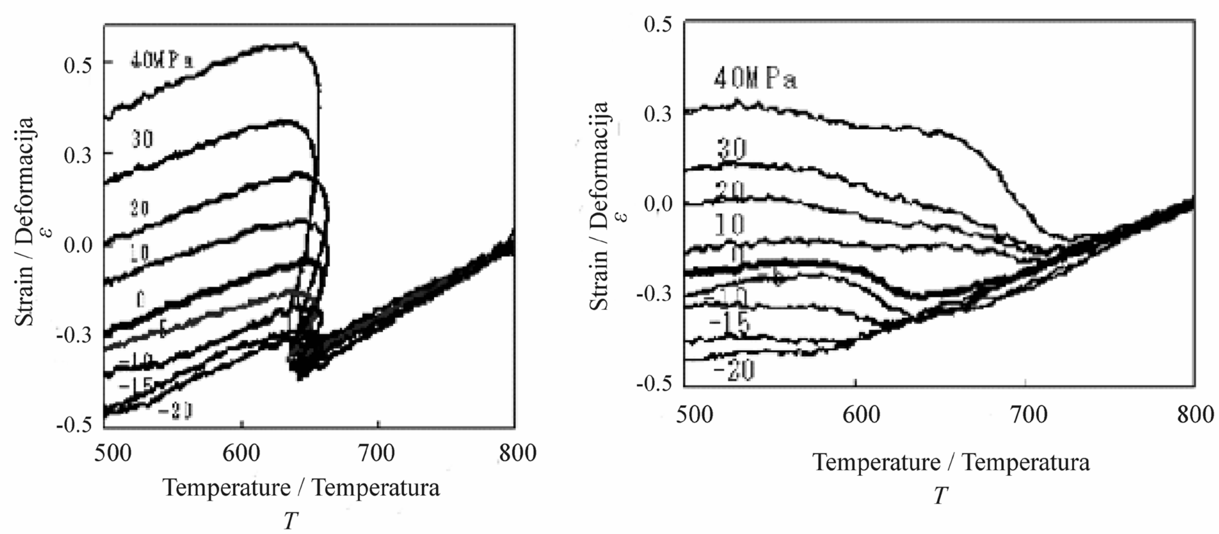



**Figure 6.** Experimental temperature-elongation diagrams during pearlite transformation

**Slika 6.** Eksperimentalni dijagram temperaturnog produljenja tijekom perlitne pretvorbe

From the results of the simulation, the TP is known to affect drastically the induced stress, while the progressing structure is less influenced.

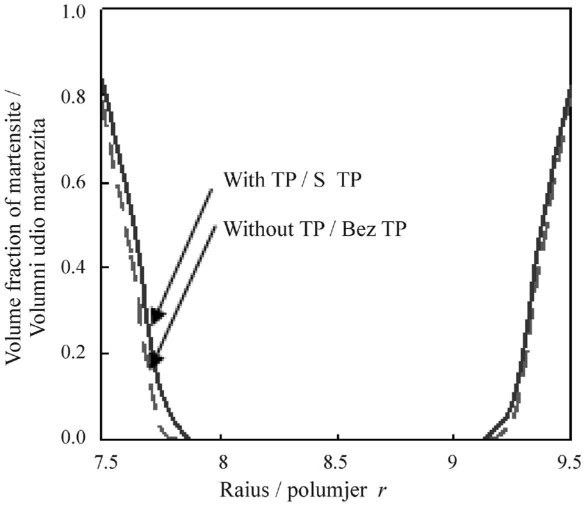
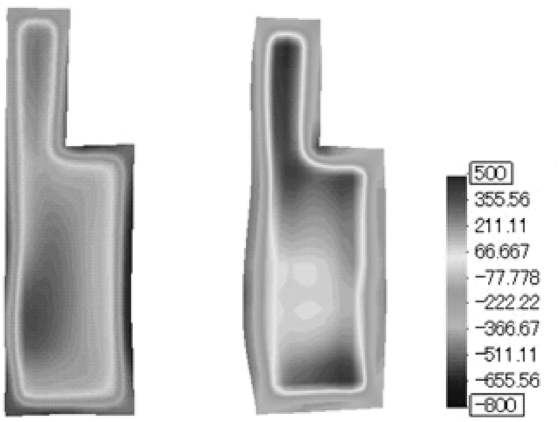



**Figure 8.** Martensite fraction along line B and distortion.

**Slika 8.** Udio martenzita duž linije B i distorzija.




(a) With TP effect (b) Without TP

**Figure 9.** Difference in tangential stress

**Slika 9.** Razlika u tangencijalnom naprezanju

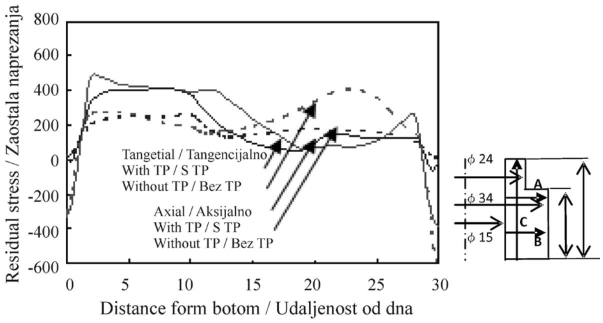



**Figure 10.** Tangential and axial residual stress effect along line C

**Slika 10.** Efekt tangencijalnog i aksijalnog zaostalog naprezanja duž linije C

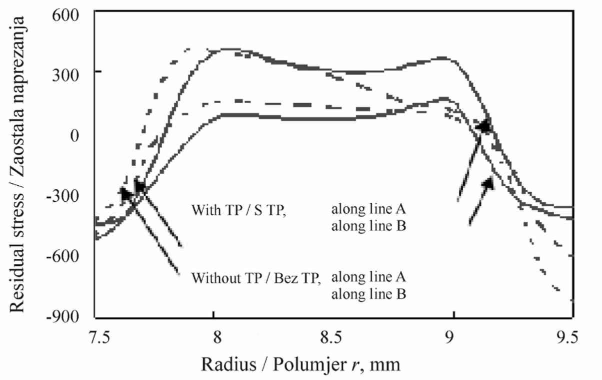



**Figure 11.** Difference of tangential residual along lines A and B.

**Slika 11** Razlika tangencijalnog zaostalog naprezanja duž linija A i B

## 6. Concluding Remarks

A discussion on the mechanism how the transformation plasticity is introduced under applied stress is carried out from thermo-mechanical viewpoint in the first part of the paper based on the assumption that the TP strain is also a kind of plastic one, and the constitutive law is derived from unified thermo-mechanical-transformation plasticity theory. Application of the theory is made to simulate a quenched gear wheel to demonstrate the effect of TP.

## Acknowledgement

The author is indebted to express his gratitude to IMS-Japan, NEDO and METI for their financial supports to this project.

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