NEW DOCTORAL DEGREES

IN THE DEPARTMENT OF MATHEMATICS UNIVERSITY OF OSIJEK

Dr. Mirela Jukić Bokun received her PhD in Mathematics from the Department of Mathematics of the University of Zagreb on 5 July 2011 with the dissertation entitled "ELLIPTIC CURVES OF LARGE RANK OVER QUADRATIC FIELDS" (Mentor: Prof. A. Dujella).

Abstract

In this thesis, we study the construction of elliptic curves of positive and relatively large rank with a fixed torsion group over quadratic fields. If K is a quadratic field and $E|K$ an elliptic curve over the field K, the possible torsion groups are:

- $\mathbb{Z}/n\mathbb{Z}$, where $1 \leq n \leq 16$, $n = 18$,
- $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2mZ$, where $1 \leq m \leq 6$,
- $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/3k\mathbb{Z}$, where $k = 1, 2$, $K = \mathbb{Q}(\sqrt{-3})$,
- $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$, for $K = \mathbb{Q}(i)$.

First we construct curves of relatively large rank with torsion groups $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$, $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/6\mathbb{Z}$ and $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$. For the first two groups, curves of rank at least 3 were already known, and for the third group, the already known result was that there exists a curve with rank ≥ 2. In the thesis we construct an elliptic curve over $\mathbb{Q}(i)$ with torsion group $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$ and rank equal to 7 and a family of elliptic curves with the same torsion and rank ≥ 2. In the case of elliptic curves over the quadratic field $\mathbb{Q}(\sqrt{-3})$, we construct an elliptic curve with torsion group $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$ and rank equal to 7 and an elliptic curve with torsion group $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/6\mathbb{Z}$ and rank equal to 6. Mestre's conditional upper bound for rank plays a significant role in the methods we use, so it is studied in more detail.

For torsion groups $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/10\mathbb{Z}$, $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/12\mathbb{Z}$, $\mathbb{Z}/15\mathbb{Z}$, $\mathbb{Z}/11\mathbb{Z}$ and $\mathbb{Z}/14\mathbb{Z}$ we study elliptic curves of positive rank over the quadratic field $\mathbb{Q}(\sqrt{d})$, where the absolute value of the discriminant of the quadratic field is minimal. Specifically, we construct curves of positive rank with torsion groups $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/10\mathbb{Z}$, $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/12\mathbb{Z}$ and $\mathbb{Z}/15\mathbb{Z}$ over the fields $\mathbb{Q}(\sqrt{-2})$, $\mathbb{Q}(\sqrt{13})$ and $\mathbb{Q}(\sqrt{-7})$, respectively, and we determine curves of conditionally positive rank over the fields $\mathbb{Q}(\sqrt{-7})$ and $\mathbb{Q}(\sqrt{3})$ for torsion groups $\mathbb{Z}/11\mathbb{Z}$ and $\mathbb{Z}/14\mathbb{Z}$, respectively. We also determine new families of elliptic curves with torsion groups $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/10\mathbb{Z}$ and $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/12\mathbb{Z}$.

Additionally, we construct elliptic curves with a fixed torsion group and maximal rank over quadratic fields $\mathbb{Q}(\sqrt{d})$, $|d| \leq 10^{100}$, where $|d|$ is minimal, and we give an overview of current results.

Published papers

[1] **M. Jukić Bokun**, *On the rank of elliptic curves over* $\mathbf{Q}(\sqrt{-3})$ *with torsion groups* $\mathbf{Z}/3\mathbf{Z} \times \mathbf{Z}/3\mathbf{Z}$ *and* $\mathbf{Z}/3\mathbf{Z} \times \mathbf{Z}/6\mathbf{Z}$, Proc. Japan Acad. Ser. A Math. Sci. 87 (2011), 61-64.

[2] A. Dujella, M. Jukić Bokun, *On the rank of elliptic curves over* $\mathbf{Q}(i)$ *with torsion group* $\mathbf{Z}/4\mathbf{Z} \times \mathbf{Z}/4\mathbf{Z}$, Proc. Japan Acad. Ser. A Math. Sci. 86 (2010), 93-96.