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J-INTEGRAL AS POSSIBLE CRITERION IN MATERIAL FRACTURE TOUGHNESS ASSESSMENT

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Abstract: *Numerical fracture toughness assessment of two types of steel, AISI 420 and AISI 431, has been conducted in this paper. The J-integral is chosen as a criterion for fracture behaviour comparison of compact type (CT) specimens made of investigated materials. The values of J-integral are determined through a newly developed algorithm using finite element (FE) stress analysis results of numerically simulated single specimen test method, which is usually employed for experimental J-integral assessment. J-integral values are presented in dependence on specimen crack growth size (Δa) for three initial measures of specimen's crack size, a/W = 0.25, 0.5, 0.75.*

Keywords:
– *J*-integral
– CT specimen
– Fracture
– Crack growth
– FE analysis

1. INTRODUCTION

Crack appearance and growth can seriously endanger reliability of structures and components in operation. Therefore, it is important to assess their influence on the structural integrity. Several parameters can be used for such a task, one of the most common being the stress intensity factor, *K*. A limiting fact is that the stress intensity factor is relevant for elastic behaviour of material in the zone ahead of the crack tip. In ductile fractures, when material exhibits elastic-plastic behaviour ahead of the crack tip, different fracture mechanics parameters should be used.

One of them is the *J*-integral, a parameter that successfully characterizes stress and strain fields around the crack tip with material exhibiting significant plasticity. The *J*-integral can also be applied when dealing with elastic behaviour of material, being thus a widely acceptable criterion for fracture behaviour comparison of different materials.

When considering material fracture resistance, the appearance of growing cracks should be accounted. With cracks growing from initial length a, *J*-integral values should be obtained for a number of crack extension values (Δa). The applicability of *J*-integral to measurement of crack driving force during crack extension can be presented in J-Δa sets of values.

Usually, J values for growing cracks are obtained in laboratory environment following some of the standardized experimental procedures, such as ASTM E1820 [1]. Prescribed single specimen test method is a common technique for determining J values on standardized specimens such as single-edge bend (SENB) or compact type specimen (CT). In this technique, crack length extension is estimated from the compliance at certain deflections of the specimen. The compliance is determined from the elastic slopes of the load – deformation curve, which occur during partial unloading. This deformation can be described by the crack opening displacement [2]. Collected values can be presented in terms of crack resistance curves.

Single specimen test method can also be performed by using finite element (FE) method. FE analysis results, verified by some kind of experimental results, can either be an addition or substitute method for costly experiments. Some of the previous works concerning simulation of the single specimen test method and obtained predicted crack resistance curves are listed here. Thus, a wide applicability of *J*-integral is shown when *J*-integral method, along with FE analysis has been successfully used in computing stress intensity

factor [3]. Numerical simulations of experimental techniques for J determination have been performed [4] by employing cohesive elements. Local ductile fracture criterion has been developed in modelling crack growth and J-R curves simulation [5] Besides, FE models of single-edge notched tension specimens have been developed for crack size evaluation by using unloading compliance [6].
Fracture behaviour of two different types of steel, based on the criterion of J-integral, is also compared in this research. The J-integral is numerically obtained by using a newly developed numerical algorithm based on FE analysis results. Resulting J values are presented as a measure of crack driving force versus crack growth (Δa).

2. *J*-INTEGRAL

The J-integral was first introduced by Rice [7] as a path-independent integral which can be drawn around the crack tip and viewed both as an energy release rate parameter and a stress intensity parameter. In a two-dimensional form and with reference to Figure 1, it can be written as:

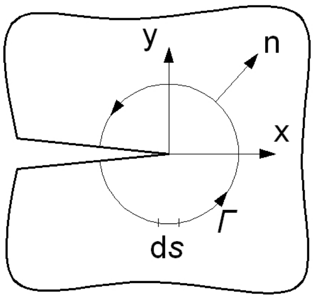

Figure 1. J-integral arbitrary contour path enclosing the crack tip

$$J = \int_{\Gamma} \left(w dy - T_i \frac{\partial u_i}{\partial x} ds \right) \quad (1)$$

where W is strain energy density, $T_i = \sigma_{ij} n_j$ are components of the traction vector, u_i are the displacement vector components and ds is an incremental length along the contour Γ.

When applied to FE models, the J-integral can be written as [8]:

$$J = \int_{\Gamma} \left\{ \frac{1}{2} \left[\sigma_{xx} \frac{\partial u_x}{\partial x} + \sigma_{xy} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \frac{\partial u_x}{\partial x} + \sigma_{yy} \frac{\partial u_y}{\partial y} \right] \frac{\partial y}{\partial \eta} - \left[\left(\sigma_{xx} n_1 + \sigma_{xy} n_2 \right) \frac{\partial u_x}{\partial x} + \left(\sigma_{xy} n_1 + \sigma_{yy} n_2 \right) \frac{\partial u_y}{\partial x} \right] \sqrt{\left(\frac{\partial x}{\partial \eta} \right)^2 + \left(\frac{\partial y}{\partial \eta} \right)^2} \right\} \mathrm{d}\eta . \quad (2)$$

With Gauss integration rule along the contour Γ, the J-integral is:

$$J = \sum_{g=1}^{ng} W_g I_g \left(\xi_g , \eta_g \right) \quad (3)$$

where W_g is the Gauss weighting factor, ng is the number of integration points and I_g is the integrand evaluated at each Gauss point g:

$$I_g = \left\{ \frac{1}{2} \left[\sigma_{xx} \frac{\partial u_x}{\partial x} + \sigma_{xy} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \frac{\partial u_x}{\partial x} + \sigma_{yy} \frac{\partial u_y}{\partial y} \right] \frac{\partial y}{\partial \eta} - \left[\left(\sigma_{xx} n_1 + \sigma_{xy} n_2 \right) \frac{\partial u_x}{\partial x} + \left(\sigma_{xy} n_1 + \sigma_{yy} n_2 \right) \frac{\partial u_y}{\partial x} \right] \sqrt{\left(\frac{\partial x}{\partial \eta} \right)^2 + \left(\frac{\partial y}{\partial \eta} \right)^2} \right\}_g . \quad (4)$$

The numerical algorithm has been written in MATLAB by employing these relations. It uses FE stress analysis results from integration points of finite elements surrounding the crack tip. The total value of J, Figure 2, is calculated by evaluating J-integral values in these points and by summing them along a path enclosing the crack tip. To account for results dissipation in the vicinity and away from the crack tip, three different paths around crack tip have been defined, and their average value taken as the final one.

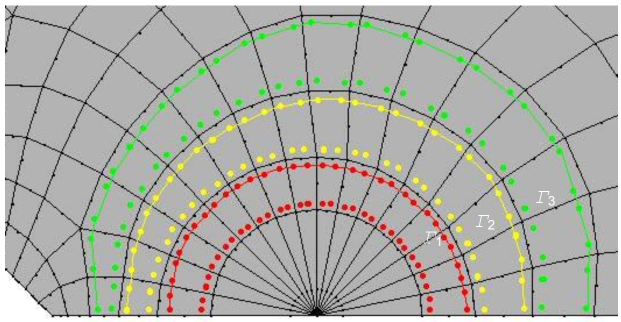

Figure 2. J-integral contour paths (Γ_1, Γ_2, Γ_3) surrounding the crack tip through the integration points of finite elements

3. CONSIDERED MATERIALS

Two types of steel have been investigated: AISI 420 and AISI 431. The former is martensitic stainless steel with typical application in pump components, turbine blades, piston rods, shafting fittings, steel balls, bolts, nuts, valve parts, glass and plastics processing tools, cutlery, dental and surgical instruments and various hand tools. Typical applications of the latter include: nuts and bolts, pump shafts, propeller shafting, machine building industry, paper industry, aircraft fittings fasteners, chemical process equipment, gears, etc. It has a good resistance to oxidation.

Mechanical properties of the mentioned materials are given in Table 1 [9, 10] and their composition (σ_{YS} - yield strength, σ_{TS} - tensile strength) in Table 2. Stress-strain curves, important for modelling elastic-plastic behaviour of materials, are shown in Figure 3.

Table 1. Mechanical properties of considered materials

Material	σ_{YS} [MPa]	σ_{TS} [MPa]
AISI 420	526	828
AISI 431	758.9	943.4

Table 2. Chemical composition of considered materials (w_t %)

Material	C	Mn	P	Si	S	Ni	Cr	Rest
AISI 420	0.2	1.50	0.04	1.00	0.03	-	13.00	84.23
AISI 431	0.2	1.00	0.04	1.00	0.03	1.50	16.00	80.23

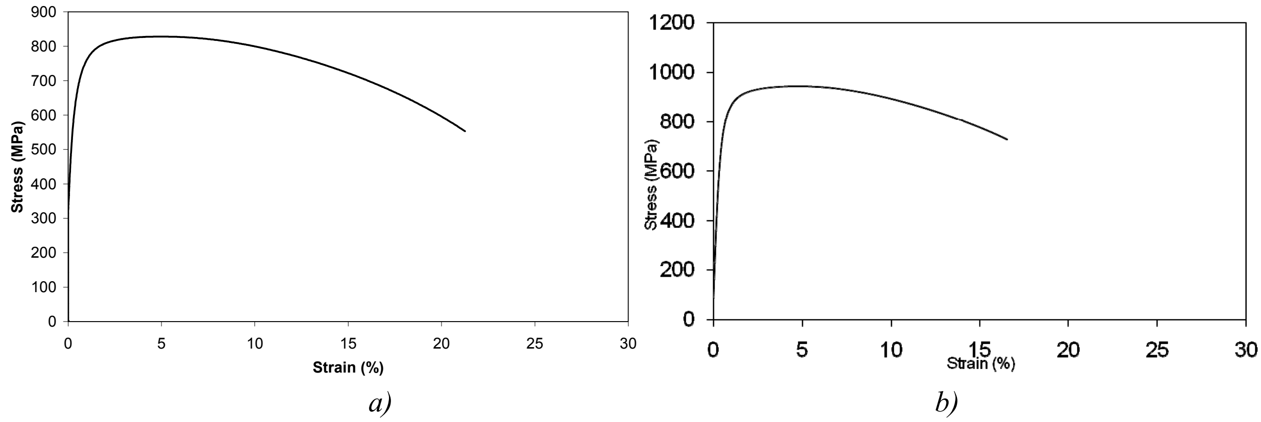

Figure 3. Stress-strain curve for: a) AISI 420. b) AISI 431.

4. FE MODEL

To determine values of the *J*-integral, experimental single specimen test method is simulated by using FE method, test method is defined with ASTM E1820 [1] and to estimate growing crack size, measured crack mouth opening displacement of CT specimen is employed. Geometry and dimensions of CT specimen are shown in Figure 4, with W = 50 mm. Measured J values can be presented versus crack extension.

Two-dimensional FE models of compact type (CT) specimens were modelled in Ansys using 8-node isoparamateric quadrilateral elements, Figure 5. Particular care was taken in discretization of the crack tip. Material behaviour was considered to be multilinear isotropic hardening type. Quasi-statical load was imposed on specimens to simulate compliance procedure of single specimen test method. Several initial crack size were modelled, a/W = 0.25, 0.5, 0.75. Gradual release of node constraints was used to simulate crack propagation.

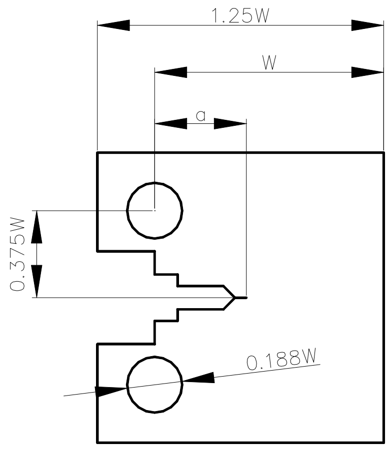

Figure 4. Geometry and dimension of CT specimen

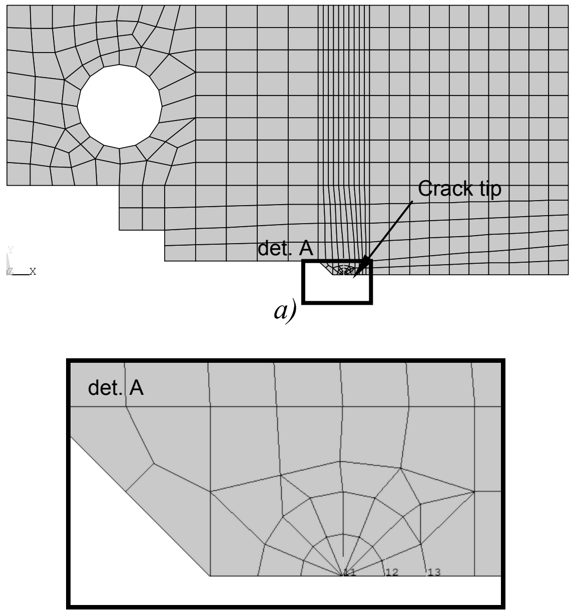

Figure 5. a) FE model of CT specimen. B) Detail of FE mesh around crack tip

After having run FE stress analysis, results were recorded in integration points of elements around the crack tip. This was done for every crack extension size and the results were used as input for developed numerical algorithm that calculates *J* values along three paths that enclose the crack tip. Their average value represents final value of *J* for specific crack size.

5. RESULTS

Numerically obtained *J* values used as a measure of crack driving force can be presented versus crack growth size (Δa). When first using an algorithm that simulates experimental procedure, it is important to verify it, usually by comparing values calculated through it with available experimental results from authors. Since there were no available experimental results for AISI 420 and AISI 431, the algorithm for *J*-integral calculation was verified on the AISI 304LN steel for which experimental results existed [11], Figure 6.

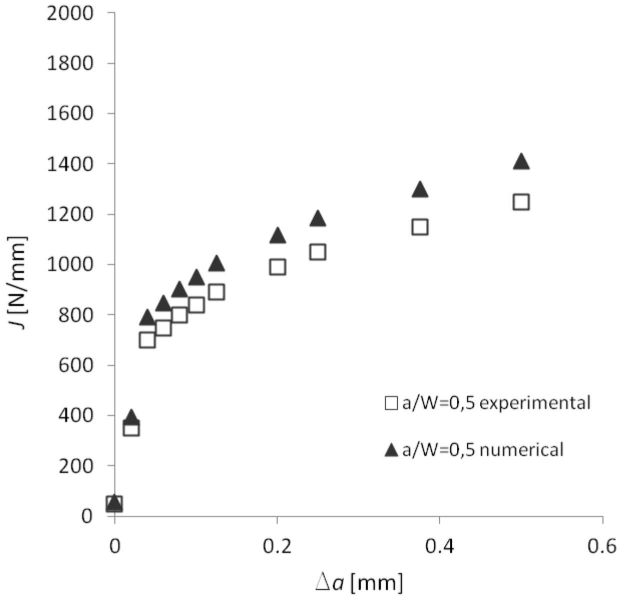

Figure 6. Verification of numerically obtained J-integral values on AISI 304LN

Figure 5 shows good compatibility of experimental and numerical results with experimental ones, which gives a bit more emphasis on conservative values. This correspondence of values encouraged the use of numerical procedure for AISI 420 and AISI 431, for which there were no available fracture toughness test results. Figures 7 and 8 show final *J* values for AISI 420 and AISI 431, as a measure of crack driving force versus crack growth size (Δa).

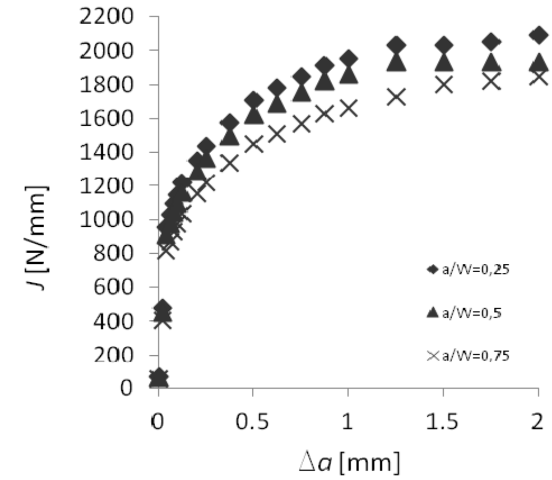

Figure 7. AISI 420: predicted J values for crack extension Δa

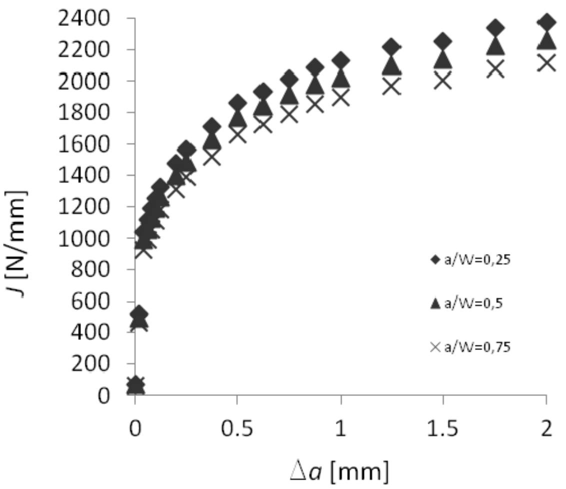

Figure 8. AISI 431: predicted J values for crack extension Δa.

Observing Figures 7 and 8, it can be noted that AISI 431 steel has higher resulting values than AISI 420 steel, a fact that might be contributed to the difference in mechanical properties and chemical composition of considered materials. When observing individual figures, it is evident that higher *a/W* ratios correspond to lower *J*-integral values of materials and vice versa, a behaviour being observed in the earlier researches [12].

6. CONCLUSION

Numerical algorithm, based on finite element stress analysis, was developed for *J*-integral calculation. Good correspondence with available experimental results gives confidence in using developed algorithm for evaluating *J* values of other materials. Further research can concentrate on applying algorithm on various materials and types of specimens proposed in [1], as well as on applying numerical models of real cracked structures and components.

Obtained *J* values provide an insight into values of crack driving force for modeled CT specimens made of various materials containing a range of crack sizes. Extensive experimental procedures can be reduced when having numerical results as a starting point in the investigation into *J* values for new materials. Obtained results give valuable insight into fracture behaviour of CT specimens made of considered materials. Such results can be of great help in the process of material selection during the design of structures.

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