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EXPERIMENTAL INVESTIGATION OF THE ADDED MASS OF THE CANTILEVER BEAM PARTIALLY SUBMERGED IN WATER

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Preliminary notes

The phenomena of added (virtual) mass of submerged structures are analyzed numerically and experimentally in this paper. The classical concept of modal added mass is analyzed in parallel with local added mass that is evaluated using pseudo-residual force vector. A finite element structure model is used to determine the local added mass. The model update and the model reduction techniques are necessary, and their respective roles are described and applied in determining the local added mass. The procedure is implemented to the cantilever beam partially submerged into water. Experimental modal analysis is performed on a cantilever beam in the air and on the same beam partially submerged in the water. This results in two sets of modal parameters, and the cause of their differences is presented and explained by the added mass. The results obtained in this way can be used as benchmark for further study and comparison with other simulations.

Keywords: *added mass, modal mass, model reduction, partially submerged cantilever beam*

Eksperimentalno istraživanje dodane mase na gredi djelomično uronjenoj u vodu

Prethodno priopćenje

U radu se numerički i eksperimentalno analizira fenomen dodane (virtualne) mase konstrukcije uronjene u tekućinu. Standardni model dodane modalne mase se analizira usporedo s prividno dodanom lokalnom masom koja se određuje korištenjem vektora pseudo-ostatka sile. U određivanju lokalno dodane virtualne mase koristi se konstrukcija modelirana metodom konačnih elemenata. Procedura zahtijeva popravljeni model konačnih elemenata na temelju eksperimenta kao i redukciju modela. Način primjene popravke i redukcije modela u analizi lokalno dodane virtualne mase su prikazani u radu te je diskutirana njihova uloga. Postupak je primijenjen na konzolu djelomično uronjenu u vodu. Eksperimentalna modalna analiza izvršena je na konzoli u zraku te na istoj konzoli djelomično uronjenoj u vodu. Ova analiza daje dva seta modalnih parametara, a uzrok njihove razlike je dodana masa koja je prezentirana i objašnjena u ovom radu. Rezultati dobiveni na ovaj način mogu se koristiti u daljim istraživanjima kao polazna točka u simulacijama.

Ključne riječi: *djelomično uronjena greda, dodana masa, modalna masa, redukcija modela*

1 Introduction

Elastic submerged structures have wide engineering application especially in ocean engineering. Understanding the influence of the fluid is important to predict the dynamic behavior of the submerged structures. It is well known that the natural frequencies of the submerged structures are different from those in vacuum. The effect of fluid forces on the submerged structures is represented as added mass [1], which decreases the natural frequencies of the structures from those which would be measured in the vacuum. This decrease in the natural frequencies of the submerged structures is caused by the increase of the kinetic energy of the fluid-structure system without a corresponding increase in strain energy.

Lindholm et al. [2] investigated the vibrations of cantilever plates in air and water. They compared the results with theoretical predictions using simple beam theory and thin plate theory. A comparison between numerical simulations using the finite element method and experimental measurements of partially and fully submerged cantilever plates is shown in [3]. Different numerical and experimental boundary conditions were the source of a 15 % discrepancy. The influence of the free surface, submerged length and boundary conditions was investigated by many researchers [4-6]. Numerical simulation of the fluid-structure interaction effect, calculated in terms of the generalized added mass values independent of frequency using the boundary integral equation method [7] also shows at least 5 % error compared with experimental measurements. Liang et al. [8] adopted an empirical added-mass formulation to determine the frequencies and mode shapes of submerged cantilever plates. A particular experimental investigation using a reduced scale model of a Francis turbine [9] has shown how to detect the modal added mass. Continued research on the Francis turbine [10] includes numerical simulations, where the added mass effect of the surrounding water has been evaluated by comparing the respective frequencies in air and water. The added mass effect is physically explained based on the energy theory.

In this paper, experimental investigation- modal analysis is performed on the cantilever beam in air and partially submerged in water. Also, the beam is submerged in two different tanks to investigate the influence of tank size. The modal added mass for the first four modes is calculated. Further research is heading to numerical analysis and detection of the local added mass which requires knowledge of system properties like mass and stiffness matrices and modal properties i.e. natural frequencies. In order to compare experimental and numerical data, model reduction is necessary. The reduced model validation has been done by comparing the experimental and numerical natural frequencies. The required model update is carried out using optimization of the objective function by genetic algorithms.

2 Theoretical background

In this paper, the beam partially submerged in water will be called the modified beam, and the beam that is not submerged the genuine beam. The equation of motion for the genuine beam can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{F}, \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices respectively, $\boldsymbol{x}$ is the displacement vector and $\boldsymbol{F}$ the excitation force vector. For small vibration amplitudes of the beam, the fluid-structure interaction affects only the structural terms of the motion equation. In this case the mass, damping and stiffness matrices are modified. The equation of motion for the modified beam is obtained using equation (1) and appending additional terms $\boldsymbol{M}_{\mathrm{a}}$, $\boldsymbol{C}_{\mathrm{a}}$ and $\boldsymbol{K}_{\mathrm{a}}$ to the genuine mass, damping and stiffness matrices respectively. These added mass, damping and stiffness matrices describe the effect of water

$$\begin{aligned} &\boldsymbol{M}_{\mathrm{w}}\ddot{\boldsymbol{x}} + \boldsymbol{C}_{\mathrm{w}}\dot{\boldsymbol{x}} + \boldsymbol{K}_{\mathrm{w}}\boldsymbol{x} = \boldsymbol{F} \\ &(\boldsymbol{M} + \boldsymbol{M}_{\mathrm{a}})\ddot{\boldsymbol{x}} + (\boldsymbol{C} + \boldsymbol{C}_{\mathrm{a}})\dot{\boldsymbol{x}} + (\boldsymbol{K} + \boldsymbol{K}_{\mathrm{a}})\boldsymbol{x} = \boldsymbol{F}. \end{aligned} \tag{2}$$

The structure defined using equation (1) possesses modal parameters e.g. damped natural frequencies $\omega_{\mathrm{d}r}$, natural frequencies ω_r, damping ratios ζ_r and mode shapes Ψ_r. The index r indicates the particular vibration mode. The modal parameters are usually determined by experimental modal analysis and they make the modal model of the structure. The modal model also consists of the modal mass m_r, modal damping c_r and modal stiffness k_r

$$\begin{aligned} m_r &= \boldsymbol{\Psi}_r^{\mathrm{T}} \boldsymbol{M} \boldsymbol{\Psi}_r \\ c_r &= \boldsymbol{\Psi}_r^{\mathrm{T}} \boldsymbol{C} \boldsymbol{\Psi}_r \\ k_r &= \boldsymbol{\Psi}_r^{\mathrm{T}} \boldsymbol{K} \boldsymbol{\Psi}_r . \end{aligned} \tag{3}$$

There are no unique values for the modal mass, modal damping and modal stiffness as they are directly related to the scaling method which has been used to define the mode shape. However the ratio between these values is unique and the modal parameters can be expressed as

$$\omega_{\mathrm{d}r} = \omega_r \sqrt{1-\zeta_r^2} \tag{4}$$

$$\omega_r^2 = \frac{k_r}{m_r} \tag{5}$$

$$\zeta_r = \frac{c_r}{2\sqrt{k_r m_r}}. \tag{6}$$

The equations (3) - (6) are also valid for the modified beam. The modal mass, modal damping and modal stiffness of the modified beam can be expressed using the respective modal mass, modal damping and modal stiffness of the genuine beam and adding the additional terms in a similar way as in equation (2)

$$\begin{aligned} m_{\mathrm{w}r} &= m_r + m_{\mathrm{a}r} \\ c_{\mathrm{w}r} &= c_r + c_{\mathrm{a}r} \\ k_{\mathrm{w}r} &= k_r + k_{\mathrm{a}r} . \end{aligned} \tag{7}$$

If the respective equation (5) for the modified and genuine beam is divided

$$\frac{\omega_{\mathrm{w}r}^2}{\omega_r^2} = \frac{k_{\mathrm{w}r}}{k_r} \cdot \frac{m_r}{m_{\mathrm{w}r}}, \tag{8}$$

and equation (7) substituted into equation (8), the equation (9) is obtained

$$\frac{\omega_{\mathrm{w}r}^2}{\omega_r^2} = \left(1 + \frac{k_{\mathrm{a}r}}{k_r}\right)\left(1 + \frac{m_{\mathrm{a}r}}{m_r}\right)^{-1}. \tag{9}$$

If the stiffness effect of the water is neglected ($k_{\mathrm{a}r} = 0$), which is a reasonable assumption for most structures, it is possible to obtain the ratio $m_{\mathrm{a}r}/m_r$. This ratio describes the percentage of modal added mass and can be calculated for known undamped natural frequencies of the genuine and modified beam. However, experimental modal analysis provides the damped natural frequencies and damping ratio, and substituting equation (4) into equation (9) it leads to the

$$\frac{m_{\mathrm{a}r}}{m_r} = \left(\frac{1-\zeta_{\mathrm{w}r}^2}{1-\zeta_r^2}\right)\frac{\omega_{\mathrm{d}r}^2}{\omega_{\mathrm{dw}r}^2} - 1. \tag{10}$$

The above equation can be used to find the percentage of modal added mass for all measurement modes. The implementation of this equation does not require the use of mode shapes. Practically this means that only one frequency response function is sufficient to calculate the percentage of modal added mass. Of course, measuring more frequencies response functions will lead to more reliable results.

3 Experimental analysis

Experimental modal analysis is carried out using a cantilever beam in the air and again the same beam partially submerged in water. Measurements on the partially submerged beam were carried out in two tanks of different sizes. In this way the influence of the amount of water around the beam is investigated, i.e. the influence of the distance between the beam and the wall of the tank is studied. The length of the beam is 0,65 m and cross-section of the beam is 0,05 × 0,01 m. The beam is made of steel, Fig. 1.

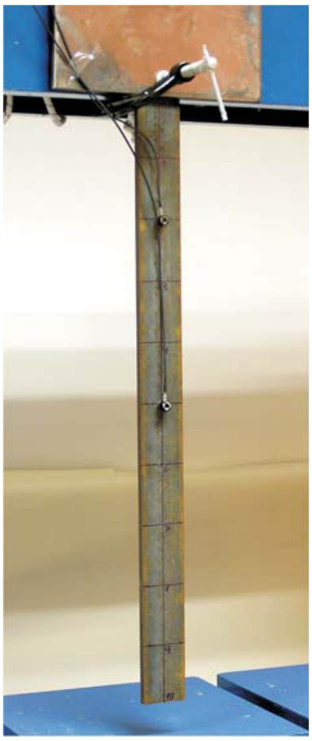

Figure 1 Genuine beam

Table 1 Measured natural frequencies and damping ratio

	Genuine beam		Modified beam - larger tank		Modified beam - smaller tank	
Mode	ω_{dr} / Hz	ζ_r / %	ω_{dwr} / Hz	ζ_{wr} / %	ω_{dwr} / Hz	ζ_{wr} / %
1	18,14	0,49894	16,93	0,87303	17,00	0,65432
2	112,07	0,67156	106,99	2,37840	108,34	1,14640
3	327,57	0,29140	320,71	0,54096	321,66	0,34052
4	629,59	0,38025	618,94	0,87303	621,00	0,88930

The beam is divided into ten elements and eleven nodes. The experimental modal analysis is performed by the impact test. For this purpose, two light accelerometers are used with a mass of 3,2 grams each to avoid their influence on the measurement data. The accelerometers were placed in the third and sixth node.

When measuring in the tank, the last element of the beam is immersed in the water. The distance between the end of the beam and the bottom of the tank is 3 cm, so the level of the water in the tank is 9,5 cm. The length and width of the larger and smaller tank are 50×25 cm and 25×14 cm respectively. The beam is placed in the middle of the larger tank, Fig. 2, and 2 cm from the edge in the smaller tank, Fig. 3.

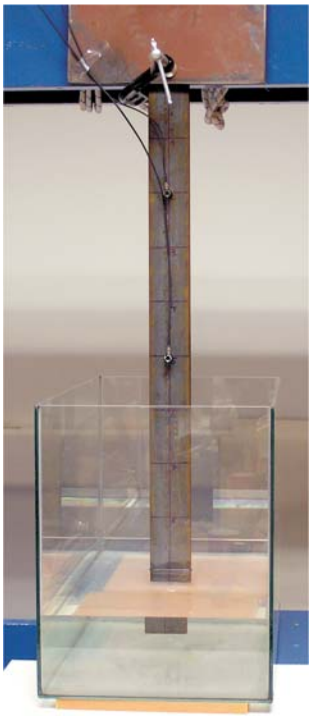

Figure 2 Modified beam, larger tank

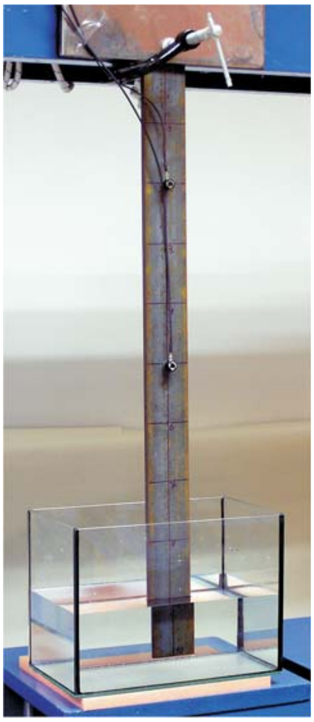

Figure 3 Modified beam, smaller tank

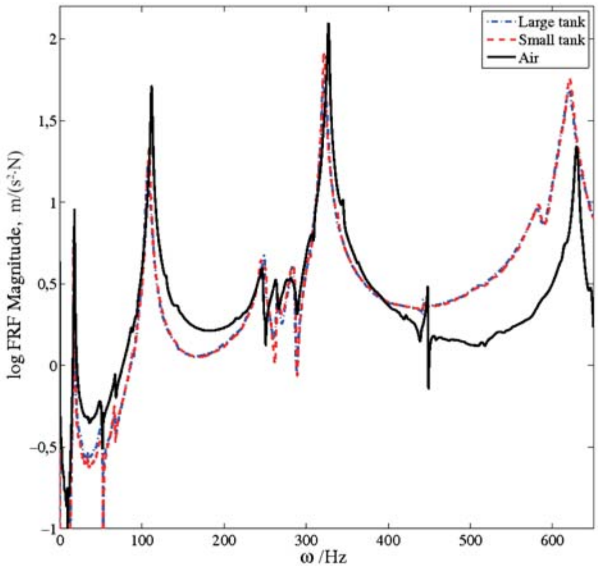

Figure 4 Example of measured frequency response functions

Modal analysis is performed using the multi-reference polynomial method. The duration of one measurement is 8,19 seconds, during which 16 384 points are collected. Linear averaging is used four times. The results of the measurement for the first four modes are shown in Tab. 1. In Fig. 4, an example of the measured frequency response function is given. Fig. 5 shows the magnification of the second mode from Fig. 4, in which the difference between the genuine and the modified beam can be observed.

The results of experiments are summarized in Tab. 1 where the modal damping ratio ξ, according to the commercial software for experimental modal analysis is given in percentages to be more readable, so for example ξ_r = 0,49894 % has meaning of ξ_r = 0,0049894.

The results in Tab. 1 were expected, the damped natural frequencies of the modified beam are lower and the corresponding damping ratio is higher. Also, comparing the results between the larger and the smaller tank, it is obvious that the larger tank damps the beam more and reduces the damped natural frequencies. Now, the modal added mass can be calculated using the equation (5). The percentages of modal added mass for both cases, larger and smaller tank, are shown in Tab. 2.

Tab. 2 shows that the highest modal added mass is in

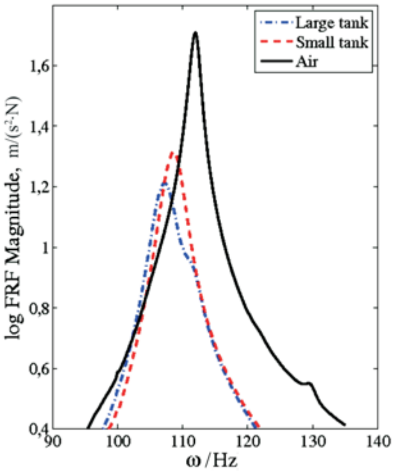

Figure 5 Zoom of one mode of measured frequency response functions

Table 2 Percentage of modal added mass

Mode	Larger tank / %	Smaller tank / %
1	0,1479	0,1388
2	0,0971	0,0700
3	0,0432	0,0371
4	0,0347	0,0279

the first mode and it gradually decreases for both cases.

4 Numerical analysis

The finite element model of the beam is required to determine the virtual local added mass on the immersed part of the beam. The background of numerical analysis is based on a pseudo-residual force vector [11]. The equation of motion for the modified beam is given by (2) and the corresponding characteristic equation (neglecting the damping) can be derived and written as

$$\left[(\boldsymbol{K}+\boldsymbol{K}_{\mathrm{a}})-(\omega_r^2-\Delta\omega_r^2)(\boldsymbol{M}+\boldsymbol{M}_{\mathrm{a}})\right](\boldsymbol{\Psi}_r+\Delta\boldsymbol{\Psi}_r)=\boldsymbol{0}, \tag{11}$$

where $\Delta\omega_r$ is the difference in natural frequencies and $\Delta\Psi_r$ the difference in mode shapes between the genuine and the modified beam. Upon expansion, equation (11) gives

$$\begin{aligned}&(\boldsymbol{K}-\omega_r^2\boldsymbol{M})\boldsymbol{\Psi}_r+(\boldsymbol{K}-\omega_r^2\boldsymbol{M})\Delta\boldsymbol{\Psi}_r+(\boldsymbol{K}_{\mathrm{a}}-\omega_r^2\boldsymbol{M}_{\mathrm{a}})\boldsymbol{\Psi}_r-\\&-\Delta\omega_r^2\boldsymbol{M}\boldsymbol{\Psi}_r+(\boldsymbol{K}_{\mathrm{a}}-\omega_r^2\boldsymbol{M}_{\mathrm{a}})\Delta\boldsymbol{\Psi}_r-\\&-\Delta\omega_r^2(\boldsymbol{M}\Delta\boldsymbol{\Psi}_r+\boldsymbol{M}_{\mathrm{a}}\boldsymbol{\Psi}_r+\Delta\boldsymbol{M}\Delta\boldsymbol{\Psi}_r)=\boldsymbol{0}.\end{aligned} \tag{12}$$

The first term is the characteristic equation for the genuine beam and thus it is equal to zero. Neglecting high order terms and pre-multiplying all terms by $\boldsymbol{\Psi}_r^{\mathrm{T}}$ yields

$$\begin{aligned}&\boldsymbol{\Psi}_r^{\mathrm{T}}(\boldsymbol{K}-\omega_r^2\boldsymbol{M})\Delta\boldsymbol{\Psi}_r+\boldsymbol{\Psi}_r^{\mathrm{T}}(\boldsymbol{K}_{\mathrm{a}}-\omega_r^2\boldsymbol{M}_{\mathrm{a}})\boldsymbol{\Psi}_r-\\&-\boldsymbol{\Psi}_r^{\mathrm{T}}\Delta\omega_r^2\boldsymbol{M}\boldsymbol{\Psi}_r=\boldsymbol{0}.\end{aligned} \tag{13}$$

Again, the first term is the characteristic equation for the genuine beam, hence it is equal to zero. If $\boldsymbol{\Psi}_r$ is mass normalized, the equation (13) can be simplified to

$$\boldsymbol{\Psi}_r^{\mathrm{T}}\boldsymbol{K}_{\mathrm{a}}\boldsymbol{\Psi}_r-\omega_r^2\boldsymbol{\Psi}_r^{\mathrm{T}}\boldsymbol{M}_{\mathrm{a}}\boldsymbol{\Psi}_r=\Delta\omega_r^2. \tag{14}$$

It is evident from equation (14) that the difference in frequencies is caused by the change in stiffness and mass of the structure. Remembering that $\boldsymbol{M}_{\mathrm{a}}$ and $\boldsymbol{K}_{\mathrm{a}}$ are matrices with values equal to zero except for those that belong to modified elements, equation (14) can be reduced to modified elements, and $\boldsymbol{M}_{\mathrm{a}}$ and $\boldsymbol{K}_{\mathrm{a}}$ can be expressed as functions of mass and stiffness matrices of finite elements

$$\begin{aligned}\boldsymbol{M}_{\mathrm{a}}&=\sum_{j=1}^{n}\boldsymbol{M}_j^e\delta m_j\\\boldsymbol{K}_{\mathrm{a}}&=\sum_{j=1}^{n}\boldsymbol{K}_j^e\delta k_j\end{aligned} \tag{15}$$

where δm_j and δk_j are the proportional mass and stiffness modification factors for element j respectively, and n is the number of modified elements. In this way, the problem is reduced to two unknowns per modified element (δm_j and δk_j). Since equation (14) can be a system of over-determined equations, (for 2×number of measured modes > number of modified elements), it is necessary to write equation (14) in some adequate way. Using

$$\begin{aligned}\boldsymbol{A}_{rj}&=\boldsymbol{\Psi}_r^{\mathrm{T}}\boldsymbol{K}_j^e\boldsymbol{\Psi}_r\\\boldsymbol{B}_{rj}&=-\omega_r^2\boldsymbol{\Psi}_r^{\mathrm{T}}\boldsymbol{M}_j^e\boldsymbol{\Psi}_r\end{aligned} \tag{16}$$

$$\begin{aligned}\boldsymbol{\delta k}&=\left[\delta k_1,\delta k_2,\dots,\delta k_n\right]^{\mathrm{T}}\\\boldsymbol{\delta m}&=\left[\delta m_1,\delta m_2,\dots,\delta m_n\right]^{\mathrm{T}}\end{aligned} \tag{17}$$

$$\Delta\boldsymbol{\lambda}=\left[\Delta\omega_1^2,\Delta\omega_2^2,\dots,\Delta\omega_p^2\right] \tag{18}$$

where p is the number of measured modes, the equation (14) can be expressed in the form

$$\left[\boldsymbol{A},\boldsymbol{B}\right]\cdot\left[\boldsymbol{\delta k},\boldsymbol{\delta m}\right]^{\mathrm{T}}=\Delta\boldsymbol{\lambda}. \tag{19}$$

Applying the least square method in solving equation (19), the unknowns can be found from the following expression

$$\left[\boldsymbol{\delta k},\boldsymbol{\delta m}\right]^{\mathrm{T}}=\left(\left[\boldsymbol{A},\boldsymbol{B}\right]^{\mathrm{T}}\cdot\left[\boldsymbol{A},\boldsymbol{B}\right]\right)^{-1}\cdot\left[\boldsymbol{A},\boldsymbol{B}\right]^{\mathrm{T}}\Delta\boldsymbol{\lambda}. \tag{20}$$

In order to find the proportional modification factors according to equation (20), it is necessary to have the finite element model of the genuine structure (mass and stiffness matrices). Also, the mode shapes and natural frequencies of the genuine structure are required. The only data required for the modified structure are the respective natural frequencies, and that is the advantage of this procedure. Another advantage of the procedure is the detection of stiffness modification. If there is no significant modification in stiffness, as with submerged structures, this information can serve as validation because in this case it is expected to be zero. The drawback of this procedure is the requirement that two natural frequencies need to be known in order to obtain the proportional modification factors for only one element. This means that it is necessary to have a structure with localized modification and finite element model with a small number of elements. The partially submerged beam (structures) is a very suitable pattern because the modification is localized to only the submerged part. A small number of finite elements causes lower resolution, and introduces an error that may be crucial in detecting modification, apropos virtual local added mass. To solve this problem, in this paper, model reduction will be used. In this way the accuracy will be retained in spite of the smaller size of the mass and stiffness matrices due to the reduced number of degrees of freedoms (DOF). In the model reduction the number of elements to be submerged in water must be kept less or equal to the half of the number of modes that are measured.

4.1 Model reduction

The system equivalent reduction-expansion process (SEREP) [12] will be applied. In order to implement the reduction of the model it is necessary to partition the terms in the equation (1) to the master DOFs '*m*' that are measured and slave DOFs '*s*' that are not measured.

$$\begin{bmatrix} \boldsymbol{M}_{mm} & \boldsymbol{M}_{ms} \\ \boldsymbol{M}_{sm} & \boldsymbol{M}_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{x}}_m \\ \ddot{\boldsymbol{x}}_s \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{mm} & \boldsymbol{K}_{ms} \\ \boldsymbol{K}_{sm} & \boldsymbol{K}_{ss} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_m \\ \boldsymbol{x}_s \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f}_m \\ \boldsymbol{f}_s \end{Bmatrix}. \tag{21}$$

In the same way, the corresponding mode shapes should be partitioned.

$$\boldsymbol{\varPsi} = \begin{Bmatrix} \boldsymbol{\varPsi}_m \\ \boldsymbol{\varPsi}_s \end{Bmatrix}. \tag{22}$$

The relationship between the full set of finite element DOFs and the reduced set of master DOFs can be written as (23):

$$\boldsymbol{x} = \boldsymbol{T}\boldsymbol{x}_m, \tag{23}$$

where $\boldsymbol{T}$ is the transformation matrix and is given by (24):

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{\varPsi}_m \\ \boldsymbol{\varPsi}_s \end{bmatrix} \left(\boldsymbol{\varPsi}_m^{\mathrm{T}} \boldsymbol{\varPsi}_m \right)^{-1} \boldsymbol{\varPsi}_m^{\mathrm{T}}. \tag{24}$$

The mass and stiffness matrices of the reduced model are obtained by using the transformation matrix $\boldsymbol{T}$

$$\begin{aligned} \boldsymbol{M}_{\mathrm{R}} &= \boldsymbol{T}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{T} \\ \boldsymbol{K}_{\mathrm{R}} &= \boldsymbol{T}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{T}. \end{aligned} \tag{25}$$

Thus the reduced mass and stiffness matrices can be used in equation (20). The modal parameters of the reduced system are equal to the modal parameters of the full system.

4.2 Updated finite element model of the genuine beam

As stated in the previous section, in order to calculate the local added mass, a finite element model of the genuine beam is required. Beam elements with two DOFs per node are used, one translational and one rotational. The beam is divided into hundred elements and after that, the beam is reduced to ten elements and eleven nodes, whereby in each node only the translational DOF is left. The dimensions of the beam are the same as in the section 2. The beam is made of steel, and its material properties are taken as: elastic modulus is 210 GPa and density 7 800 kg/m^3. The obtained natural frequencies are shown in the Tab. 3.

Table 3 Numerical natural frequencies, model of genuine beam

Mode	Natural frequencies / Hz
1	19,84
2	124,33
3	348,12
4	682,18

Comparing experimental and numerical natural frequencies for the genuine beam, Tab. 1 and Tab. 3, a difference can be observed. The average difference is about 10 %. Such a large difference can cause problems in the detection of the local added mass. The problem can be especially well observed in Tab. 1, i.e. the difference between the genuine and modified beam in natural frequencies is less than 5 %. In this way we try to detect changes of 5 % using the data which initially deviate 10 %. It is obvious that the finite element model of the genuine beam must be updated so that its natural frequencies correspond to those measured.

4.2.1 Finite element model update

It is very important to set the degree of the problem properly and select the optimization parameters and their values needed to successfully update the model. When these parameters are chosen, it should be taken into account that they provide most of the information about the experimental model. As optimization parameters, the material and the geometric characteristics of the model are usually selected. In this case, five parameters will be updated: width, height and length of the beam, elastic modulus and density. The height, width and length of the beam are selected as update parameters because the surface of the beam is not ideally flat. However, it is unlikely to expect their significant deviation from the measured values given in section 2 and their values are bounded to ±5 % of their respective initial values. On the other hand, the material properties are less certain and it is expected that their values may vary more. Therefore, the elastic modulus and density are bounded to ±8 % of their respective initial values.

The difference between the measured and numerical natural frequencies for the genuine beam is selected as the objective function [13]

$$f = \sum_{r=1}^{p} \frac{\left| \omega_{\mathrm{A}r} - \omega_{\mathrm{X}r} \right|}{\omega_{\mathrm{X}r}}, \tag{26}$$

where the index A denotes the numerical natural frequencies and index X the experimentally obtained natural frequencies. The objective function is minimized using genetic algorithms. Because of their stochastic nature, optimization by genetic algorithms is repeated five times. Those design parameters values that yield the minimum objective function are chosen as final, Tab. 4.

The natural frequencies of the finite element model of the genuine beam obtained using the optimized design parameters are given in Tab. 5.

If we now compare experimental and numerical natural frequencies of the genuine beam, Tab. 1 and Tab. 5, a much smaller difference than before can be noted. The average difference is just 1 %. It is expected that applying this more

Table 4 Update values of design parameters

Design parameters	Initial	Update
Width / m	0,01	0,009497
Height / m	0,05	0,049740
Length / m	0,65	0,655755
Elastic modulus / GPa	210	210,02799489
Density / kg/m^3	7800	8106,7788

Table 5 Numerical natural frequencies of the genuine beam obtained using updated design parameters

Mode	Natural frequencies / Hz
1	18,14
2	113,69
3	318,39
4	624,36

accurate numerical model in equation (20) will give good results in detecting the local added mass.

4.3 Local added mass

As aforesaid, the finite element model is reduced to ten elements that are equal to the experimental model. The lowest element at the free end of the modified beam is immersed in the water. This means that we should find the proportional modification factors $\boldsymbol{\delta k}$ and $\boldsymbol{\delta m}$, according to equation (20) of one (immersed) element. To do this, it is enough to know the natural frequencies of two modes, but for more accurate solutions the first four modes were used. The mass and stiffness matrices of the immersed element are extracted from the reduced finite element model. The obtained virtual local added mass for the modified beam in the larger and smaller tank is shown in Tab. 6.

The obtained proportional modification factors confirm the expectation that there is no structural change in stiffness, i.e. the proportional modification factor is equal to zero. This means that the water does not represent a significant resistance proportional to the beam displacements. The mass proportional modification factor is higher for the beam submerged in the larger tank than in the smaller tank. This was expected and agrees with results obtained for the modal added mass in Tab. 2.

Table 6 Local added mass, proportional modification factors

Mode	Larger tank	Smaller tank
δk / %	0,0000	0,0002
δm / %	0,3122	0,2547

5 Conclusion

The manifestation of added mass for the beam submerged in water is investigated experimentally and numerically. The obtained results of the modal added mass and local added mass show the same tendency, the added mass effect is more dominant in the larger tank than in the smaller tank. The proposed procedure for determining the local added mass is very useful because beyond the added mass it also gives the added stiffnesses that can be used as control parameters. It is shown that the main problem may arise from the mismatch between the finite element model and the experimental model, which is here solved by applying the model update. The obtained proportional modification factors for stiffness can be used as process control.

The results for added mass obtained in this experiment will be used as a benchmark for further numerical investigation.

6 References

[1] Brennen, C. E. A Review of Added Mass and Fluid Inertial Forces, Department of the Navy, Port Hueneme, (1982) USA. http://resolver.caltech.edu/CaltechAUTHORS:BREncel82 (21. 01. 2011.)
[2] Lindholm, U. S.; Kana, D. D.; Chu W. H.; Abramson, H. N. Elastic vibration characteristics of cantilever plates in water. // Journal of Ship Research 9, 1(1965), pp. 11-22.
[3] Volcy, G.C.; Morel. P.; Bureau, M.; Tanida, K. Some studies and researches related to the hydro-elasticity of steel work. // Proceedings of the 122nd Euromech Colloquium on numerical analysis of the dynamic of ship structures, Paris, 1979, pp. 403-406.
[4] Muthuveerappan, G.; Ganesan, N.; Valuswami, M. Note on vibration of a cantilever plate immersed in water. // Journal of Sound and Vibration, 63, (1979), pp. 385-391.
[5] Fu, Y.; Price, W. G. Interaction between a partially or totally immersed vibrating cantilever plate and the surrounding fluid. // Journal of Sound and Vibration, 118, 3(1987), pp. 495-513.
[6] Kwak, M. K. Hydroelastic vibration of circular plates. // Journal of Sound and Vibration, 201, 3(1997), pp. 293-303.
[7] Ergin, A.; Ugurlu, B. Linear vibration analysis of cantilever plates partially submerged in fluid. // Journal of Fluids and Structures, 17, 7 (2003), pp. 927-939.
[8] Liang, C. C.; Liao, C. C.; Tai, Y. S.; Lai, W. H. The free vibration analysis of submerged cantilever plates. // Ocean Engineering, 28, 9, (2001), pp. 1225-1245.
[9] Rodriguez, C. G.; Egusquiza, E.; Escaler, X.; Liang, Q. W.; Avellan, F. Experimental investigation of added mass effect on a Francies turbine runner. // Journal of Fluids and Structures, 22, 5(2006), pp. 699-712.
[10] Liang, Q. W.; Rodriguez, C. G.; Egusquiza, E.; Escaler, X.; Farhat, M.; Avellan, F. Numerical simulation of fluid added mass effect on a francis turbine runner. // Computers and Fluids, 36, (2007), pp. 1106-1118.
[11] Ge, M; Lui, E. M. Structural damage identification using system dynamic properties. // Computers and Structures, 83, (2005), pp. 2185-2196.
[12] O'Callahan, J.; Avitable, P.; Riemer, R. System equivalent reduction expansion process. // Proceedings of VII International Modal Analysis Conference, Las Vegas, 1989, pp. 29-37.
[13] Schwarz, B.; Richardson, M. FEA Model Updating Using SDM. // Proceedings of XXV International Modal Analysis Conference, Orlando, 2007., pp. 18-22.

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