GREAT CIRCLE SAILING - CALCULATION OF INTERMEDIATE POSITIONS

Ortodromska plovidba – izračun međutočaka

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Summary

This paper deals with the realization of the great circle navigation. In practice, the main problem occurs because the great circle is a curve on the Mercator navigation chart, which has to be broken down into a number of smaller rhumb line parts. Besides computer programs, the simplest way of realization of the great circle navigation is by using the Gnomonic chart as the great circle is a straight line on this chart. Conventional numerical and tabular methods based on spherical trigonometry are quite complicated and time consuming. In order to simplify the way of computing intermediate positions along the great circle, this paper suggests the use of the Latitude Equation of the Mid-longitude and appropriate tables based on this method. Also, using spherical trigonometry, the paper presents a way of obtaining the Latitude Equation of the Mid-longitude. Originally, the Latitude Equation of the Mid-longitude is derived without the use of spherical trigonometry.

Keywords: great circle, calculation of intermediate positions, latitude equation of the mid-longitude

Sažetak

Ovaj se rad bavi problemom realizacije ortodromske plovidbe. Glavni problem u praksi pojavljuje se zbog toga što je ortodroma krivulja na Mercatorovoj navigacijskoj karti i što se neizostavno mora razbiti u više manjih loksodromskih dijelova. Ako se izuzmu računalni programi, najjednostavniji način realizacije ortodromske plovidbe je upotrebom gnomonske karte, jer je na njoj ortodroma pravac. Ostali numerički i tablični načini dosta su komplicirani i zahtjevaju vremena za rješavanje. Kako bi se pojednostavnio način računanja međutočaka ortodrome, predlaže se uporaba računa zemljopisne širine za srednju zemljopisnu dužinu dviju točaka ortodrome te prema ovoj metodi izrada odgovarajućih tablica. Jednadžba širine za srednju zemljopisnu dužinu izvorno je izvedena bez korištenja sfernom trigonometrijom, a u ovom radu ona se izvodi upravo s pomoću sferne trigonometrije.

Ključne riječi: ortodroma, izračun međutočaka ortodrome, jednadžba širine za srednju zemljopisnu dužinu.

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INTRODUCTION / *Uvod*

A great circle track is the shortest distance between two points on the Earth's surface, assuming the Earth as a perfect spherical model[1]. One of the fundamental features of the great circle is that it intersects the meridians at different angles. On the other hand, determining the direction and the orientation at sea (or in the air) are based on the use of the compass, i.e. defining the direction that intersects the meridians at the same angle. For this purpose the Mercator navigation chart is also in use. On this chart a straight line (rhumb line) intersects the meridians at the same angle. This means that the great circle on the Mercator chart is a curve. It is very difficult to draw the great circle on the Mercator chart, but even if it is drawn, it is not practical to sail along an exact great circle route using the classic compass as, to follow a great circle track, the navigator needs to adjust the ship's course continuously. Also, every time a ship sails in one course, even for a little while, she is navigating the rhumb line. Therefore, in practice, the great circle is divided into a number of smaller parts, i.e. the intermediate positions, between which a ship sails along the rhumb line, are determined. The greater the number of intermediate positions, the closer the sailing will be to the ideal great circle.

Intermediate positions can be determined in many ways, but the simplest one is by using the Gnomonic chart. On this chart the great circle track is a straight line and the coordinates of the intermediate positions can be easily read and transferred to the Mercator navigation chart. On the other hand, numerical methods, even the simplest ones, require a lot of time. For example, commonly used models of spherical trigonometry, in which the Earth is an ideal sphere, necessarily imply the calculation of a number of additional elements without which it is not possible to determine intermediate positions. This involves determining the great circle (orthodromic) distance, initial orthodromic course, final orthodromic course, and latitude and longitude of the vertex. These data are useful, but they are not sufficient in accomplishing the great circle navigation which requires intermediate positions. In order to shorten the process of determining the elements of the great circle navigation, this paper presents the usefulness of Latitude Equation of the Mid-longitude. This method is directly aimed at determining intermediate positions, where the geographical coordinates of points of departure and arrival are the only necessary inputs.

[1] A great circle is defined as a circle on the Earth's surface whose plane passes through the centre of the Earth.

USE OF SPHERICAL TRIGONOMETRY / *Korištenje sfernom trigonometrijom*

Figure 1 shows orthodromic spherical triangle. From this triangle, by use of few basic laws of spherical trigonometry [4, 101-105] it is possible to obtain orthodromic distance, initial and final course, vertex and intermediate positions.

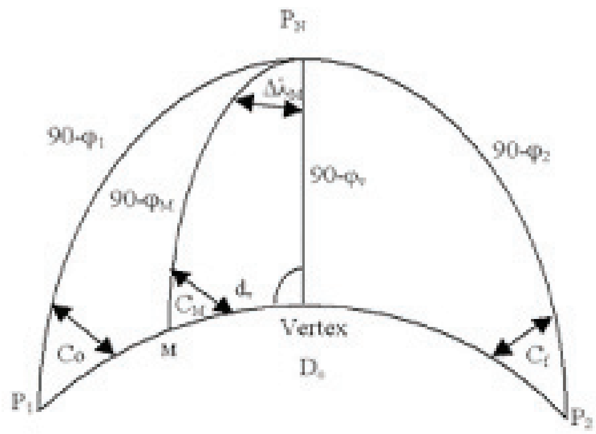

Figure 1. Orthodromic spherical triangle
Slika 1. Ortodromski sferni trokut

P_1 - starting position (standpoint)
P_2 - ending position (forepoint)
D_o - orthodromic distance (P_1P_2)
C_o - initial course
C_f - final course
φ_V - latitude of Vertex
M - intermediate position
φ_M - latitude of point M
λ_M - longitude of point M
$\Delta\lambda_M$ (λ_V-λ_M)
C_M - orthodromic course at point M
d_v - distance from Vertex to point M

Orthodromic Distance / *Ortodromska udaljenost*

According to the law of cosines:

$$\cos D_o = \sin\varphi_1 \cdot \sin\varphi_2 + \cos\varphi_1 \cdot \cos\varphi_2 \cdot \cos\Delta\lambda \, . \qquad (1)$$

Initial and Final Course / *Početni i završni kurs*

a) According to the law of cosines:

$$\cos C_o = \frac{\sin\varphi_2 - \sin\varphi_1 \cdot \cos D_o}{\cos\varphi_1 \cdot \sin D_o} \, , \qquad (2)$$

$$\cos C_f = \frac{\sin\varphi_1 - \sin\varphi_2 \cdot \cos D_o}{\cos\varphi_2 \cdot \sin D_o} \, . \qquad (3)$$

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b) According to the law of sines:

$$\sin C_o = \frac{\cos\varphi_2 \cdot \sin\Delta\lambda}{\sin D_o}, \tag{4}$$

$$\sin C_f = \frac{\cos\varphi_1 \cdot \sin\Delta\lambda}{\sin D_o}. \tag{5}$$

c) According to the law of cotangent:

$$\cos C_o = \frac{\cos\varphi_1 \cdot \tan\varphi_2}{\sin\Delta\lambda} - \sin\varphi_1 \cdot \cot\Delta\lambda \;. \tag{6}$$

Vertex / *Vrh ortodrome*

According to Napier's Rule:

$$\cos\varphi_v = \cos\varphi_1 \cdot \sin K_o, \tag{7}$$

$$\cot\Delta\lambda_v = \sin\varphi_1 \cdot \tan K_o, \tag{8}$$

$$\lambda_v = \lambda_1 \pm \Delta\lambda_v. \tag{9}$$

Intermediate Positions / *Međutočke ortodrome*

a) Determining the φ_M of the selected λ_M (according to Napier's Rule):

$$\tan\varphi_M = \tan\varphi_V \cdot \cos(\lambda_v - \lambda_M). \tag{10}$$

b) Determining the position of a waypoint by distance d_v from Vertex (according to Napier's Rule):

$$\sin\varphi_M = \cos d_v \cdot \sin\varphi_V, \tag{11}$$

$$\cot\Delta\lambda_M = \cos\varphi_v \cdot \cot d_v, \tag{12}$$

$$\lambda_M = \lambda_V \pm \Delta\lambda_M. \tag{13}$$

c) Determining the positions of the waypoints for course change (C_M) of 1° (according to Napier's Rule):

$$\cos\varphi_M = \frac{\cos\varphi_V}{\sin C_M} \qquad C_M = C_o \pm 1,\ C_o \pm 2,\ \ldots, \tag{14}$$

$$\sin\Delta\lambda_M = \frac{\cos C_M}{\sin\varphi_V} \qquad \lambda_M = \lambda_V \pm \text{Ä}\lambda_M \;. \tag{15}$$

In addition to the above formulas there are other models, also based on spherical trigonometry[2].

GRAPHICAL SOLUTION / *Grafičko rješenje*

Figure 2 shows how to use Gnomonic chart to obtain waypoints of a great circle. Procedure:

- Join the two places on the Gnomonic chart by a straight line.
- Choose intermediate positions (waypoints) – those, it is recommend, where the great circle intersects the drawn meridians (for the same $\Delta\lambda$).
- Transfer the waypoints (latitude and longitude) on the Mercator navigation chart.
- Join the waypoints on the Mercator chart by straight lines.

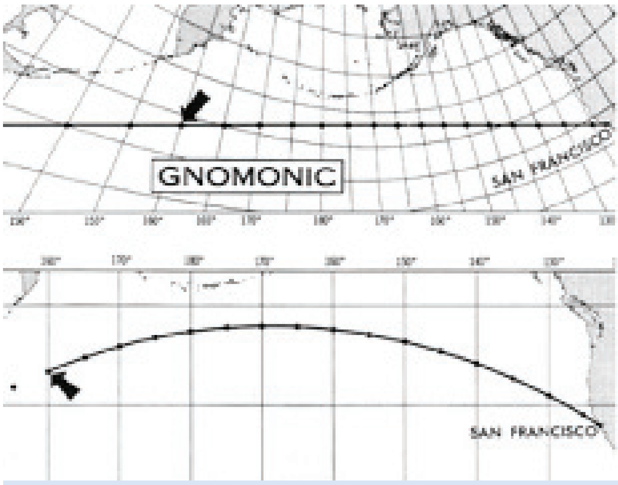

Figure 2. Using the Gnomonic chart to construct a great circle track on a Mercator projection
Slika 2. Upotreba gnomonske karte za konstrukciju ortodrome na Mercatorovoj karti [2, 372)]

LATITUDE EQUATION OF THE MID LONGITUDE / *Jednadžba širine za srednju zemljopisnu dužinu*

Waypoints of the great circle can be determined directly, i.e. without calculating the initial course, orthodromic distance, vertex, etc. One way is to calculate the latitude at the longitude halfway between the start longitude and the end longitude (latitude equation of the mid longitude)[6]. Having the coordinates of that middle-point, it is possible to split each half further, and so on, using the same method, until point-to-point legs are short enough to be treated as rhumb-lines.

The latitude equation of the mid longitude can

[2] See different types of tables for calculation of great circle elements (haversines [3, 83], ABC tables [1, 587], PRω [4, 41], etc.).

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be derived in several ways[3]. However, the same result (formula) can be obtained by using spherical trigonometry.

Cotanges theorem (for spherical triangle with lines a, b, c and angles α, β, γ):

$$\frac{\cot\alpha}{\sin b} = \frac{\cot a}{\sin\gamma} - \cot\gamma \cdot \cot b \,. \tag{16}$$

If we apply Expression (16) to the Orthodromic spherical triangle (Figure 1) to find the latitude of point M which has longitude difference Δλ/2 from P_1 (and P_2), then:

$$\frac{\cot C_o}{\sin(90\text{-}\varphi_1)} = \frac{\cot(90\text{-}\varphi_M)}{\sin\frac{\Delta\lambda}{2}} - \cot\frac{\Delta\lambda}{2}\cdot\cot(90\text{-}\varphi_1)\,,$$

$$\frac{\tan\varphi_M}{\sin\frac{\Delta\lambda}{2}} = \frac{\cot C_o}{\cos\varphi_1} + \frac{\tan\varphi_1}{\tan\frac{\Delta\lambda}{2}}\,,$$

$$\tan\varphi_M = \sin\frac{\Delta\lambda}{2}\cdot\left(\frac{\cot C_o}{\cos\varphi_1} + \frac{\tan\varphi_1}{\tan\frac{\Delta\lambda}{2}}\right). \tag{17}$$

If we apply Expression (16) to the Orthodromic spherical triangle (P_1, P_2, P_N) it is possible to obtain a formula for the initial course C_o (Expression 6):

$$\cot C_o = \frac{\cos\varphi_1\cdot\tan\varphi_2}{\sin\Delta\lambda} - \sin\varphi_1\cdot\cot\Delta\lambda\,.$$

If the above Expression for C_o (6) replaces the initial course C_o in Expression 17:

$$\tan\varphi_M = \sin\frac{\Delta\lambda}{2}\cdot\left(\frac{\cos\varphi_1\cdot\tan\varphi_2 - \sin\varphi_1\cdot\cos\Delta\ddot{e}}{\sin\Delta\lambda\cdot\cos\varphi_1} + \frac{\tan\varphi_1}{\tan\frac{\Delta\lambda}{2}}\right). \tag{18}$$

Double-angle formula for sinα:

$$\sin\alpha = 2\cdot\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}\,, \tag{19}$$

$$\tan\varphi_M = \sin\frac{\Delta\lambda}{2}\cdot\left(\frac{\cos\varphi_1\cdot\tan\varphi_2 - \sin\varphi_1\cdot\cos\Delta\lambda}{2\cdot\sin\frac{\Delta\lambda}{2}\cdot\cos\frac{\Delta\lambda}{2}\cdot\cos\varphi_1} + \frac{\tan\varphi_1}{\tan\frac{\Delta\lambda}{2}}\right) =$$

$$\frac{\tan\varphi_2}{2\cdot\cos\frac{\Delta\lambda}{2}} - \frac{\tan\varphi_1\cdot\cos\Delta\lambda}{2\cdot\cos\frac{\Delta\lambda}{2}} + \tan\varphi_1\cdot\cos\frac{\Delta\lambda}{2} =$$

$$\frac{\tan\varphi_2 + \tan\varphi_1\cdot\left(2\cdot\cos^2\frac{\Delta\lambda}{2} - \cos\Delta\lambda\right)}{2\cdot\cos\frac{\Delta\lambda}{2}}\,,$$

$$\tan\varphi_M = \frac{\tan\varphi_1 + \tan\varphi_2}{2\cdot\cos\frac{\Delta\lambda}{2}}\,. \tag{20}$$

For $\varphi_1 = \varphi_2$:

$$\tan\varphi_M = \frac{\tan\varphi_1}{\cos\frac{\Delta\lambda}{2}}\,. \tag{21}$$

According to the final formula (20), the tangent of latitude at mid-longitude is equal to the sum of the tangents of two latitudes divided by the double cosine of mid-longitude. If using this formula, it is easy to split up the great circle into smaller parts, without any approximation. Also, this formula enables making a table containing the latitudes of the mid longitude for various starting and ending positions (Table 1). These results can be used for further rough estimation of waypoints, i.e. for an approximate calculation of the waypoints.

[3] Equation of mid-longitude derived by the equation of the plane determined by the two points and the centre of the sphere or equation derived by the equation of straight line on the Polar Gnomonic [6].

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Latitude 2		Latitude 1																
	ΔLong	-80	-70	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80
30	10	-68,638	-47,445	-30,096	-17,133	-7,484	0	6,113	11,380	16,160	20,721	25,289	30,096	35,410	41,603	49,215	59,068	72,316
	15	-68,731	-47,681	-30,214	-17,218	-7,520	0	6,142	11,433	16,234	20,811	25,395	30,214	35,540	41,739	49,350	59,189	72,364
	20	-68,861	-47,773	-30,381	-17,325	-7,579	0	6,165	11,506	16,337	20,939	25,544	30,381	35,722	41,930	49,540	59,356	72,505
	25	-69,027	-48,020	-30,599	-17,467	-7,635	0	6,237	11,606	16,472	21,106	25,738	30,599	35,958	42,177	49,786	59,575	72,647
	30	-69,231	-48,324	-30,867	-17,643	-7,718	0	6,303	11,727	16,639	21,312	25,978	30,867	36,249	42,482	50,087	59,842	72,820
	35	-69,471	-48,686	-31,189	-17,854	-7,814	0	6,383	11,873	16,840	21,560	26,266	31,189	36,597	42,845	50,445	60,157	73,025
	40	-69,749	-49,107	-31,567	-18,103	-7,929	0	6,477	12,045	17,077	21,852	26,605	31,567	37,005	43,269	50,861	60,522	73,260
	45	-70,062	-49,587	-32,002	-18,393	-8,063	0	6,587	12,245	17,352	22,190	26,995	32,002	37,473	43,754	51,337	60,937	73,527
	50	-70,412	-50,129	-32,499	-18,725	-8,217	0	6,714	12,476	17,668	22,577	27,444	32,499	38,005	44,304	51,872	61,402	73,824
	55	-70,799	-50,735	-33,060	-19,103	-8,393	0	6,859	12,738	18,027	23,016	27,951	33,060	38,605	44,920	52,469	61,917	74,150
	60	-71,221	-51,405	-33,690	-19,531	-8,594	0	7,023	13,036	18,435	23,510	28,523	33,690	39,276	45,606	53,130	62,483	74,507
	65	-71,678	-52,143	-34,394	-20,014	-8,821	0	7,210	13,373	18,895	24,076	29,164	34,394	40,021	46,365	53,856	63,100	74,893
	70	-72,171	-52,950	-35,177	-20,557	-9,077	0	7,421	13,754	19,413	24,704	29,880	35,177	40,846	47,198	54,648	63,768	75,309
	75	-72,699	-53,827	-36,045	-21,167	-9,367	0	7,659	14,184	19,995	25,407	30,679	36,045	41,755	48,111	55,509	64,489	75,752
	80	-73,260	-54,776	-37,005	-21,852	-9,695	0	7,929	14,668	20,648	26,194	31,567	37,005	42,754	49,107	56,439	65,260	76,224
	85	-73,856	-55,805	-38,064	-22,620	-10,066	0	8,234	15,214	21,382	27,073	32,553	38,064	43,849	50,189	57,442	66,083	76,722
	90	-74,484	-56,909	-39,232	-23,482	-10,486	0	8,580	15,832	22,208	28,055	33,648	39,232	45,045	51,361	58,518	66,959	77,247
	95	-75,144	-58,092	-40,517	-24,453	-10,963	0	8,974	16,531	23,137	29,152	34,864	40,517	46,351	52,629	59,669	67,884	77,799
	100	-75,836	-59,356	-41,930	-25,544	-11,508	0	9,424	17,325	24,185	30,381	36,212	41,930	47,773	53,995	60,897	68,861	78,374
	105	-76,558	-60,706	-43,483	-26,777	-12,133	0	9,941	18,231	25,370	31,759	37,709	43,483	49,319	55,464	62,202	69,888	78,974
	110	-77,309	-62,139	-45,188	-28,173	-12,853	0	10,537	19,269	26,716	33,305	39,371	45,188	50,997	57,039	63,585	70,964	79,597
	115	-78,088	-63,656	-47,058	-29,759	-13,689	0	11,231	20,465	28,248	35,046	41,218	47,058	52,814	58,724	65,047	72,089	80,242
	120	-78,893	-65,260	-49,107	-31,567	-14,668	0	12,045	21,852	30,000	37,005	43,269	49,107	54,776	60,522	66,587	73,260	80,908
40	10	-67,593	-43,786	-24,141	-10,037	0	7,484	13,413	18,400	22,859	27,006	31,125	35,410	40,108	45,549	52,228	60,947	72,964
	15	-67,659	-43,903	-24,243	-10,085	0	7,520	13,475	18,482	22,937	27,117	31,246	35,540	40,243	45,685	52,360	61,063	73,061
	20	-67,824	-44,096	-24,389	-10,151	0	7,570	13,562	18,598	23,076	27,273	31,417	35,722	40,432	45,877	52,546	61,226	73,169
	25	-67,997	-44,344	-24,576	-10,238	0	7,636	13,676	18,749	23,255	27,476	31,639	35,958	40,676	46,126	52,786	61,435	73,306
	30	-68,209	-44,650	-24,806	-10,345	0	7,716	13,817	18,936	23,478	27,727	31,913	36,249	40,981	46,431	53,080	61,692	73,473
	35	-68,459	-45,014	-25,086	-10,475	0	7,814	13,987	19,161	23,745	28,029	32,241	36,597	41,342	46,795	53,430	61,995	73,670
	40	-68,747	-45,439	-25,414	-10,628	0	7,929	14,188	19,425	24,060	28,382	32,625	37,005	41,763	47,218	53,835	62,345	73,898
	45	-69,074	-45,925	-25,793	-10,805	0	8,063	14,421	19,732	24,424	28,791	33,068	37,473	42,247	47,703	54,297	62,743	74,156
	50	-69,438	-46,474	-26,220	-11,010	0	8,217	14,688	20,085	24,840	29,259	33,573	38,005	42,795	48,250	54,817	63,189	74,442
	55	-69,840	-47,090	-26,718	-11,245	0	8,393	14,995	20,486	25,314	29,790	34,144	38,605	43,410	48,862	55,395	63,682	74,757
	60	-70,280	-47,773	-27,273	-11,509	0	8,594	15,340	20,939	25,848	30,381	34,784	39,276	44,096	49,540	56,034	64,223	75,102
	65	-70,757	-48,527	-27,896	-11,809	0	8,821	15,731	21,451	26,448	31,040	35,498	40,021	44,854	50,288	56,733	64,812	75,475
	70	-71,271	-49,355	-28,592	-12,148	0	9,077	16,173	22,026	27,120	31,791	36,291	40,846	45,690	51,107	57,495	65,450	75,876
	75	-71,822	-50,256	-29,370	-12,531	0	9,367	16,670	22,670	27,871	32,616	37,170	41,755	46,605	51,999	58,320	66,135	76,303
	80	-72,408	-51,242	-30,235	-12,962	0	9,695	17,230	23,393	28,709	33,535	38,141	42,754	47,606	52,969	59,210	66,869	76,756
	85	-73,028	-52,306	-31,198	-13,450	0	10,066	17,860	24,203	29,642	34,553	39,211	43,849	48,696	54,017	60,166	67,651	77,236
	90	-73,687	-53,460	-32,269	-14,002	0	10,486	18,571	25,110	30,682	35,679	40,388	45,045	49,879	55,148	61,188	68,480	77,744
	95	-74,378	-54,701	-33,459	-14,628	0	10,963	19,374	26,129	31,841	36,925	41,681	46,351	51,161	56,363	62,277	69,357	78,276
	100	-75,102	-56,034	-34,784	-15,340	0	11,508	20,284	27,273	33,133	38,304	43,101	47,773	52,546	57,665	63,435	70,280	78,830
	105	-75,858	-57,462	-36,257	-16,154	0	12,133	21,319	28,563	34,574	39,829	44,658	49,319	54,039	59,057	64,661	71,249	79,407
	110	-76,645	-58,989	-37,897	-17,088	0	12,853	22,498	30,017	36,184	41,514	46,363	50,997	55,646	60,540	65,956	72,263	80,007
	115	-77,462	-60,616	-39,725	-18,168	0	13,689	23,852	31,665	37,984	43,378	48,228	52,814	57,367	62,115	67,318	73,321	80,627
	120	-78,308	-62,346	-41,763	-19,425	0	14,668	25,414	33,535	40,000	45,439	50,266	54,776	59,210	63,784	68,747	74,421	81,268

Table 1. Latitudes of the mid-longitudes (example)

Tablica 1. Zemljopisne širine za srednje zemljopisne dužine (primjer)

Source [Author]

In the Table 1 the input parameters (Lat_1 and $Long_2$) are given with a 10° alteration, while ΔLong is given with a 5° alteration. Results are in degrees. By selecting this density of input parameters with relatively small number of offered final results (latitudes), a large part of the Earth's surface can be covered in a satisfactory way. With these final results it is possible to approximately determine any great circle waypoints, by using linear interpolation. An error occurs solely because of using linear interpolation (when real coordinates of starting and ending positions do not match the ones for which the final results have been offered). The linear interpolation error is reduced, or eliminated, by using tables featuring a higher density of input parameters or by selecting starting and ending coordinates according to the ones contained in the tables.

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EXAMPLE 1 / *Primjer 1.*

From Lat. 30°00,0’ N., Long. 060°00,0’ W., to Lat. 40°00,0’ N., Long. 020°00,0’ W., find the total distance on the great circle and 3 waypoints by using Latitude Equation of the Mid-longitude (i.e., find the waypoints on Long. 050°00,0’ W., Long. 040°00,0’ W. and Long. 030°00,0’ W.).

a) Use of Equation (20):

$$\tan\varphi_M = \frac{\tan 30^\circ + \tan 40^\circ}{2\cdot\cos 20^\circ} \quad \rightarrow \quad \phi_M = 37^\circ 00{,}3'\ \mathrm{N}\ \ (\text{for } \lambda = 040^\circ 00{,}0'\ \mathrm{W}),$$

$$\tan\varphi_M = \frac{\tan 30^\circ + \tan 37^\circ 00{,}3'}{2\cdot\cos 10^\circ} \quad \rightarrow \quad \phi_M = 34^\circ 03{,}0'\ \mathrm{N}\ \ (\text{for } \lambda = 050^\circ 00{,}0'\ \mathrm{W}),$$

$$\tan\varphi_M = \frac{\tan 37^\circ 00{,}3' + \tan 40^\circ}{2\cdot\cos 10^\circ} \quad \rightarrow \quad \phi_M = 38^\circ 57{,}7'\ \mathrm{N}\ \ (\text{for } \lambda = 030^\circ 00{,}0'\ \mathrm{W}).$$

b) Use of Table 1:

With Lat.1 = +30°00,0’, Lat.2 = +40°00,0’ and ΔLong. = 040° → Table 1→ ϕ_M = 37,005°= 37°00,3’ N (for Long. = 040°00,0’ W).

With Lat.1 = +30°00,0’, Lat.2 = +37,005° and ΔLong. = 020° → Table 1→ ϕ_M = 34,122°= 34°07,3’ N (for Long. = 050°00,0’ W; ϕ_M obtained by linear interpolation between results for Lat.2 = 30° N and Lat.2 = 40° N).

With Lat.1 = +37,005°, Lat.2 = +40°00,0’ and ΔLong. = 020° →Table 1→ ϕ_M = 39,021°= 39°01,3’ N (for Long. = 030°00,0’ W; ϕ_M obtained by linear interpolation between results for Lat.1 = 30° N and Lat.1 = 40° N).

Table 2. Calculation of waypoints - Example 1

Tablica 2. Izračun međutočaka - primjer 1

	Waypoints – from Table 1				**Waypoints – from Equation (20)**			
No	Lat	Long	Course RL	Dist (n.m.)	Lat	Long	Course RL	Dist (n.m.)
0	**30-00,0 N**	**060-00,0 W**	064,2	567,5	**30-00,0 N**	**060-00,0 W**	064,6	565,8
1*	**34-07,3 N** φ1=30° φ2=37,005°	**050-00,0 W** ΔLong=20°	070,6	519,8	**34-03,0 N** φ1=30° φ2=37,005°	**050-00,0 W** ΔLong=20°	070,1	521,4
2	**37-00,3 N** φ1=30° φ2=40°	**040-00,0 W** ΔLong=40°	075,7	489,8	**37-00,3 N** φ1=30° φ2=40°	**040-00,0 W** ΔLong=40°	076,1	489,1
3*	**39-01,3 N** φ1=37,005° φ2=40°	**030-00,0 W** ΔLong=20°	082,8	468,4	**38-57,7 N** φ1=37,005° φ2=40°	**030-00,0 W** ΔLong=20°	082,4	469,1
4	**40-00,0 N**	**020-00,0 W**			**40-00,0 N**	**020-00,0 W**		
		Total distance	**2045,5**			**Total distance**		**2045,4**
GC distance 2036,6 n.m. // RL distance 2059,2 n.m.								

* “Waypoints-from Table 1” - near exact results obtained by linear interpolation; RL - Rhumb line.

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PRIMJER 2. / *Example 2*

From Lat. 30°00,0' N., Long. 070°00,0' W., to Lat 30°00,0' N., Long. 010°00,0' W., find the total distance on the great circle and 3 waypoints by using Latitude Equation of the Mid-longitude (i.e., find the waypoints on Long. 055°00,0' W., Long. 040°00,0' W. and Long. 025°00,0' W.).

Table 3. Calculation of waypoints – Example 2
Tablica 3. Izračun međutočaka – primjer 2

	Waypoints – from Table 1				Waypoints – from Equation (20) and (21)			
No	Lat	Long	Course RL	Dist (n.m.)	Lat	Long	Course RL	Dist (n.m.)
0	**30-00,0 N**	**070-00,0 W**	077,6	790,2	**30-00,0 N**	**070-00,0 W**	077,8	789,7
1*	**32-50,9 N** $\varphi 1=30°$ $\varphi 2=33{,}6901°$	**055-00,0 W** $\Delta Long=30°$	086,1	757,8	**32-46,8 N** $\varphi 1=30°$ $\varphi 2=33{,}6901°$	**055-00,0 W** $\Delta Long=30°$	085,9	758,3
2	**33-41,4 N** $\varphi 1=30°$ $\varphi 2=30°$	**040-00,0 W** $\Delta Long=60°$	093,9	757,8	**33-41,4 N** $\varphi 1=30°$ $\varphi 2=30°$	**040-00,0 W** $\Delta Long=60°$	094,1	758,3
3*	**32-50,9 N** $\varphi 1=33{,}6901°$ $\varphi 2=30°$	**025-00,0 W** $\Delta Long=30°$	102,4	790,2	**32-46,8 N** $\varphi 1=33{,}6901°$ $\varphi 2=30°$	**025-00,0 W** $\Delta Long=30°$	102,2	789,7
4	**30-00,0 N**	**010-00,0 W**			**30-00,0 N**	**010-00,0 W**		
			Total distance	**3096,0**			**Total distance**	**3096,0**
GC distance 3079,1 n.m. // RL distance 3117,7 n.m.								

* "Waypoints-from Table 1" - near exact results obtained by linear interpolation; RL - Rhumb line.

CONCLUSION / *Zaključak*

The issue of the great circle navigation is most frequently addressed by using special computer softwares or modern electronic aids to navigation (GPS, ECDIS, etc.). However, the same problem can be solved by using gnomonic charts or nautical tables. Most tables are relatively complicated and can be, for this reason, supplemented by a table of final results showing the latitude of the intermediate position which lies between two observed positions. The final results tables can help obtain approximate waypoints but they can also be used for an initial assessment of the great circle navigation, i.e. for checking the results obtained by other means. Moreover, the method of calculating the latitude of the mid-longitude is useful when using logarithmic tables (logarithms of trigonometric functions), which are featured within nautical tables [4][5], also when using a calculator in computing intermediate positions of the great circle.

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