CODEN STJSAO ISSN 0562-1887
ZX470/1502 UDK 681.516.3:681.532:621.38:531

Dynamic Model of Nanorobot Motion in Multipotential Field

***Branko NOVAKOVIĆ*[1]*, Josip KASAĆ*[1] and *Marijo KIROLA*[2]**

1) Fakultet strojarstva i brodogradnje Sveučilišta u Zagrebu, (Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb), Ivana Lučića 5, HR-10000 Zagreb, **Republic of Croatia**

2) Sveučilište u Dubrovniku (University of Dubrovnik), Ćira Carića 4, HR-20000 Dubrovnik, **Repubic of Croatia**

branko.novakovic@fsb.hr

Keywords
Canonical differential equations
Dynamic model of nanorobot motion
Multipotential field
Nanorobotics
Relativistic Hamiltonian

Ključne riječi
Dinamički model gibanja nanorobota
Kanonske diferencijalne jednadžbe
Multipotencijalno polje
Nanorobotika
Relativistički Hamiltonian

Received (primljeno): 2009-08-31
Accepted (prihvaćeno): 2011-01-25

Preliminary note

At the nanoscale the dynamics of the nanorobot motion is very complex and requires an interdisciplinary approach to designing it. Generally, a nanorobot is moving in a multipotential field. Therefore, in this paper a dynamic model of a nanorobot motion is described by the Hamiltonian canonical differential equations, as functions of the total potential energy of a nanorobot in a multipotential field. This model is derived for non-relativistic nanorobot motion, without quantum effects. The presented model is suitable for application to modern control algorithms such as an external linearization, optimal and adaptive control and an artificial intelligence control.

Dinamički model gibanja nanorobota u multipotencijalnom polju

Prethodno priopćenje

Dinamika gibanja nanorobota na nanoskali izuzetno je složena i zahtijeva interdisciplinarni pristup pri njezinu opisivanju. Općenito gledano, nanorobot se giba u multipotencijalnom polju. U tom smislu dinamika gibanja nanorobota opisana je ovdje Hamiltonovim kanonskim diferencijalnim jednadžbama, kao funkcijama totalne potencijalne energije nanorobota u multipotencijalnom polju. Ovaj je model izveden za nerelativističko gibanje nanorobota, bez kvantnih efekata. Prezentirani model pogodan je za primjenu modernih upravljačkih algoritama kao što su eksterna linearizacija, optimalno i adaptivno vođenje, te primjena algoritama umjetne inteligencije.

1. Introduction

Initially, the state of the art in nanorobotics region is briefly pointed out. As it is well known, nanorobotics is the multidisciplinary field that deals with the controlled manipulation with atomic and molecular-sized objects [1-2]. Generally, there are two main approaches for building useful devices from nanoscale components. The first one is based on self-assembly structures that can be realized in bionanorobotics [3, 8-9]. The second approach is based on direct application of mechanical forces, electromagnetic fields, and the other potential fields. The research in nanorobotics in the second approach has proceeded along two lines. The first one is devoted to the design of mechanical robots with nanoscale dimensions. This is a big technological problem.

The second line of nanorobotics research involves manipulation of nanoscale objects with macroscopic instruments and related potential fields. The techniques used in this approach are Scanning Probe Microscopy (SPM) [4] and Scanning Tunneling Microscope (STM) [5]. In the STM technique a quantum-mechanical effect, called tunneling process, and the piezoelectric actuators can be employed for the position control in a nanorobotics. Further technique is Atomic Force Microscope (AFM), which is sensitive directly to the forces between the tip and the sample (particle), rather than a tunneling current [6].

Recently, the Spin-Polarized Scanning-Tunneling Microscopy (SP-STM) and Magnetic Exchange Force Microscopy (MExFM) have been presented with application to metallic and electrically insulating

Symbols/Oznake

A_e	- vector potential of electromagnetic field, V - vektorski potencijal elektromagnetskog polja	M	- gravitational mass, kg - gravitacijska masa
A_g	- vector potential of gravitomagnetic field, m^2/s^2 - vektorski potencijal gravitomagnetskog polja	N	- matrix of velocities, m/s - matrica brzina
c	- speed of light in vacuum, m/s - brzina svjetlosti u vakuumu	p_x, p_y, p_z	- momentums in x, y and z directions, kg·m/s - momenti u x, y i z pravcima
ds	- line element, m - linijski element	r	- gravitational radius, m - gravitacijski radijus
E_e	- electric field, V/m - električno polje	U	- total potential energy, J - ukupna potencijalna energija
E_g	- gravitational field, m/s^2 - gravitacijsko polje	U_c	- control potential energy, J - upravljačka potencijalna energija
F_I	- interaction force, N - interakcijska sila	V_e	- scalar potential of electromagnetic field, V - skalarni potencijal elektromagnetskog polja
F_L	- Lorentz force, N - Lorentzova sila	V_g	- scalar potential of gravitational field, m^2/s^2 - skalarni potencijal gravitacijskog polja
F_p	- potential force, N - potencijalna sila	v_x, v_y, v_z	- velocities in x, y and z directions, m/s - brzine u x, y i z pravcima
F_t	- time-varying force, N - vremenski-varijabilna sila	$\mathcal{H}$	- Hamiltonian, J - Hamiltonijan
G	- gravitational constant, $N{\cdot}m^2/kg^2$ - gravitacijska konstanta	q	- elementary electric charge, C - elementarni električni naboj
H_e	- magnetic field, V/m - magnetsko polje	x, y, z	- positions in x, y and z directions, m - pozicije u x, y i z pravcima
H_g	- gravitational acceleration field, m/s^2 - gravitacijsko akceleracijsko polje	$\dot{x}, \dot{y}, \dot{z}$	- velocities in x, y and z directions, m/s - brzine u x, y i z pravcima
I	- interaction term, J - interakcijski član	$\ddot{x}, \ddot{y}, \ddot{z}$	- accelerations in x, y and z directions, m/s^2 - ubrzanja u x, y i z pravcima
m_0	- invariant (rest) mass of nanorobot, kg - invarijantna masa nanorobota		

magnetic nanostructures [7]. Potential applications of the nanorobots are expected in the tree important regions: nanomedicine [8, 10], nanotechnology [7] and space applications [11]. More information about applications of nanorobots in the mentioned regions can be found in the references [12]. Generally, an overview of the interaction between an artificial intelligence and robotics is presented in [13].

The main motivation in this paper is to help in designing the related dynamic models of a nanorobot motion in a multipotential field. Such models should be employable, among the others, to the description of nanorobot motion in the different potential fields like mechanical, electrical, electromagnetic, photonic, chemical and biomaterial, as well as in a gravitational field. At the nanoscale the dynamics of a nanorobot is very complex because there are very strong interaction between nanorobots and nanoenvironment. Thus, the first step in designing of the dynamic model of nanorobots motion is the development of the relativistic Hamiltonian for a multipotential field. This problem has been solved in the reference [12], where the concept of the variation principle and the generalized line element ds are employed. This is because ds^2 is a fundamental invariant of the four dimensional space-time continuum. The obtained relativistic Hamiltonian has been adapted for application to the non-relativistic quantum systems without spin by using the related Schrödinger equations [12]. The same relativistic Hamiltonian has also been transformed into the Dirac's like structure for application to the relativistic quantum system with spin [22].

The main goal in this paper is to derive non-relativistic and non-quantum canonical differential equations for nanorobot motion in a multipotential field. In that sense, the relativistic Hamiltonian derived in [12] is transformed into the non-relativistic one. It follows the application of the non-relativistic Hamiltonian to creation of the canonical differential equations for description of a nanorobot motion in a multipotential field. For validation of the general approach, the obtained dynamic model is applied to description of the nanorobot motion in the two-potential electromagnetic and gravitational field. This paper has been written by consideration of the related theories and fundamental laws of physics in the references [12, 14-24].

The paper is organized as follows. The second section presents formal system and problem statements. The third section shows a derivation process of the Hamiltonian canonic equations of the nanorobot motion in a multipotential field. In the fourth section a canonical model of nanorobot motion in a two-potential electromagnetic and gravitational field has been presented. An overview of the application of derived Hamiltonians to quantum systems has been considered in the fifth section. The conclusions of the paper with some comments are presented in the sixth section. Finally, the reference list is shown at the end of the paper.

2. Formal system and problem statements

Let the non-relativistic approximation of the Hamiltonian $\mathcal{H}$ for a nanorobot motion in a multipotential field is given by the relation derived in [12]:

$$\mathcal{H} \cong m_0 c^2 + \frac{1}{2m_0}\left[\left(p_x - \frac{v_x U}{c^2}\right)^2 + \left(p_y - \frac{v_y U}{c^2}\right)^2 + \left(p_z - \frac{v_z U}{c^2}\right)^2\right] + U. \tag{1}$$

Here m_0 is a rest mass of a nanorobot, c is a speed of the light in a vacuum, p_x, p_y and p_z, as well as v_x, v_y and v_z are momentums and velocities, respectively, in x, y, and z directions and U is a total potential energy of a nanorobot in a multipotential field. The momentums of the nanorobot motion can be calculated by the equations:

$$p_x = m_0 v_x, \qquad p_y = m_0 v_y, \qquad p_z = m_0 v_z. \tag{2}$$

At the nanoscale control of a nanorobot motion or/and manipulation we usually have the multi-potential field with n-potentials, plus an artificial control potential field of the nanorobot that influents to the nanorobot with a potential energy U_c. Thus, the related total potential energy of a nanorobot in a multipotential field is described by the following relation:

$$U = U_1 + U_2 + .. + U_n + U_c = \Sigma U_j + U_c\,, \quad j = 1,2,..,n. \tag{3}$$

In the relation (3) U_j is a potential energy of the nanorobot in the j-th potential field.

Thus, the problem is to derive the dynamic model of a nanorobot motion in a multipotential field, starting with the non-relativistic approximation of the Hamiltonian $\mathcal{H}$ from (1) and taking into account the relations (2) and (3). The derived model should be in the form of the Hamiltonian canonical differential equations that are suitable for applications of modern control algorithms.

3. Derivation of the Hamiltonian canonic equations of the nanorobot motion

In the case where there are no quantum mechanical effects one can employ classic Hamiltonian canonic forms for designing equations of the nanorobot motion [18]:

$$\dot{p}_x = -\frac{\partial\mathcal{H}}{\partial x}, \qquad \dot{p}_y = -\frac{\partial\mathcal{H}}{\partial y}, \qquad \dot{p}_z = -\frac{\partial\mathcal{H}}{\partial z},$$
$$\dot{x} = \frac{\partial\mathcal{H}}{\partial p_x}, \qquad \dot{y} = \frac{\partial\mathcal{H}}{\partial p_y}, \qquad \dot{z} = \frac{\partial\mathcal{H}}{\partial p_z}. \tag{4}$$

Applying the Hamiltonian (1) to the first line of (4) one obtains the canonical equations for the momentums in the following form:

$$\dot{p}_x = -\frac{\partial U}{\partial x} + \frac{1}{m_0 c}\begin{bmatrix} \left(p_x - \frac{v_x U}{c^2}\right)\frac{\partial}{\partial x}\left(\frac{v_x U}{c}\right) + \\ + \left(p_y - \frac{v_y U}{c^2}\right)\frac{\partial}{\partial x}\left(\frac{v_y U}{c}\right) + \\ + \left(p_z - \frac{v_z U}{c^2}\right)\frac{\partial}{\partial x}\left(\frac{v_z U}{c}\right) \end{bmatrix},$$

$$\dot{p}_y = -\frac{\partial U}{\partial y} + \frac{1}{m_0 c}\begin{bmatrix} \left(p_x - \frac{v_x U}{c^2}\right)\frac{\partial}{\partial y}\left(\frac{v_x U}{c}\right) + \\ + \left(p_y - \frac{v_y U}{c^2}\right)\frac{\partial}{\partial y}\left(\frac{v_y U}{c}\right) + \\ + \left(p_z - \frac{v_z U}{c^2}\right)\frac{\partial}{\partial y}\left(\frac{v_z U}{c}\right) \end{bmatrix}, \tag{5}$$

$$\dot{p}_z = -\frac{\partial U}{\partial z} + \frac{1}{m_0 c}\begin{bmatrix} \left(p_x - \frac{v_x U}{c^2}\right)\frac{\partial}{\partial z}\left(\frac{v_x U}{c}\right) + \\ + \left(p_y - \frac{v_y U}{c^2}\right)\frac{\partial}{\partial z}\left(\frac{v_y U}{c}\right) + \\ + \left(p_z - \frac{v_z U}{c^2}\right)\frac{\partial}{\partial z}\left(\frac{v_z U}{c}\right) \end{bmatrix}.$$

The application of the Hamiltonian (1) to the second line of (4) produces canonical equations for coordinates:

$$\dot{x}=\frac{1}{m_0}\left(p_x-\frac{v_xU}{c^2}\right),\quad \dot{y}=\frac{1}{m_0}\left(p_y-\frac{v_yU}{c^2}\right),\quad \dot{z}=\frac{1}{m_0}\left(p_z-\frac{v_zU}{c^2}\right). \tag{6}$$

Including (6), the relations (5) are transformed into the following canonical equations:

$$\begin{aligned}
\dot{p}_x&=-\frac{\partial U}{\partial x}+\frac{1}{c}\left[\dot{x}\frac{\partial}{\partial x}\left(\frac{v_xU}{c}\right)+\dot{y}\frac{\partial}{\partial x}\left(\frac{v_yU}{c}\right)+\dot{z}\frac{\partial}{\partial x}\left(\frac{v_zU}{c}\right)\right],\\
\dot{p}_y&=-\frac{\partial U}{\partial y}+\frac{1}{c}\left[\dot{x}\frac{\partial}{\partial y}\left(\frac{v_xU}{c}\right)+\dot{y}\frac{\partial}{\partial y}\left(\frac{v_yU}{c}\right)+\dot{z}\frac{\partial}{\partial y}\left(\frac{v_zU}{c}\right)\right],\\
\dot{p}_z&=-\frac{\partial U}{\partial z}+\frac{1}{c}\left[\dot{x}\frac{\partial}{\partial z}\left(\frac{v_xU}{c}\right)+\dot{y}\frac{\partial}{\partial z}\left(\frac{v_yU}{c}\right)+\dot{z}\frac{\partial}{\partial z}\left(\frac{v_zU}{c}\right)\right].
\end{aligned} \tag{7}$$

Now, one can define the so called interaction terms of a nanorobot motion in a multipotential field:

$$I_x=\frac{v_xU}{c},\quad I_y=\frac{v_yU}{c},\quad I_z=\frac{v_zU}{c}. \tag{11}$$

Applying (11) to the relations (10) one obtains the canonical dynamic equations of the nanorobot motion in a multipotential field as the functions of the interaction terms:

$$\begin{aligned}
m_0\ddot{x}&=-\frac{\partial U}{\partial x}-\frac{1}{c}\frac{\partial I_x}{\partial t}+\frac{1}{c}\left[\dot{y}\left(\frac{\partial I_y}{\partial x}-\frac{\partial I_x}{\partial y}\right)-\dot{z}\left(\frac{\partial I_x}{\partial z}-\frac{\partial I_z}{\partial x}\right)\right],\\
m_0\ddot{y}&=-\frac{\partial U}{\partial y}-\frac{1}{c}\frac{\partial I_y}{\partial t}+\frac{1}{c}\left[\dot{z}\left(\frac{\partial I_z}{\partial y}-\frac{\partial I_y}{\partial z}\right)-\dot{x}\left(\frac{\partial I_y}{\partial x}-\frac{\partial I_x}{\partial y}\right)\right],\\
m_0\ddot{z}&=-\frac{\partial U}{\partial z}-\frac{1}{c}\frac{\partial I_z}{\partial t}+\frac{1}{c}\left[\dot{x}\left(\frac{\partial I_x}{\partial z}-\frac{\partial I_z}{\partial x}\right)-\dot{y}\left(\frac{\partial I_z}{\partial y}-\frac{\partial I_y}{\partial z}\right)\right].
\end{aligned} \tag{12}$$

Now, one can employ the time derivatives of the equations (6):

$$\dot{p}_x=m_0\ddot{x}+\frac{1}{c}\frac{d}{dt}\left(\frac{v_xU}{c}\right),\quad \dot{p}_y=m_0\ddot{y}+\frac{1}{c}\frac{d}{dt}\left(\frac{v_yU}{c}\right),\quad \dot{p}_z=m_0\ddot{z}+\frac{1}{c}\frac{d}{dt}\left(\frac{v_zU}{c}\right). \tag{8}$$

The time derivation terms in (8) are presented by the following relations:

$$\begin{aligned}
\frac{d}{dt}\left(\frac{v_xU}{c}\right)&=\frac{\partial}{\partial x}\left(\frac{v_xU}{c}\right)\dot{x}+\frac{\partial}{\partial y}\left(\frac{v_xU}{c}\right)\dot{y}+\frac{\partial}{\partial z}\left(\frac{v_xU}{c}\right)\dot{z}+\frac{\partial}{\partial t}\left(\frac{v_xU}{c}\right),\\
\frac{d}{dt}\left(\frac{v_yU}{c}\right)&=\frac{\partial}{\partial x}\left(\frac{v_yU}{c}\right)\dot{x}+\frac{\partial}{\partial y}\left(\frac{v_yU}{c}\right)\dot{y}+\frac{\partial}{\partial z}\left(\frac{v_yU}{c}\right)\dot{z}+\frac{\partial}{\partial t}\left(\frac{v_yU}{c}\right),\\
\frac{d}{dt}\left(\frac{v_zU}{c}\right)&=\frac{\partial}{\partial x}\left(\frac{v_zU}{c}\right)\dot{x}+\frac{\partial}{\partial y}\left(\frac{v_zU}{c}\right)\dot{y}+\frac{\partial}{\partial z}\left(\frac{v_zU}{c}\right)\dot{z}+\frac{\partial}{\partial t}\left(\frac{v_zU}{c}\right).
\end{aligned} \tag{9}$$

The substitution of the equations (8) and (9) into the relations (7) gives the Hamiltonian canonical differential equations of the nanorobot motion in a multipotential field:

$$\begin{aligned}
m_0\ddot{x}&=-\frac{\partial U}{\partial x}-\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{v_xU}{c}\right)+\frac{1}{c}\left[\dot{y}\left(\frac{\partial}{\partial x}\left(\frac{v_yU}{c}\right)-\frac{\partial}{\partial y}\left(\frac{v_xU}{c}\right)\right)-\dot{z}\left(\frac{\partial}{\partial z}\left(\frac{v_xU}{c}\right)-\frac{\partial}{\partial x}\left(\frac{v_zU}{c}\right)\right)\right],\\
m_0\ddot{y}&=-\frac{\partial U}{\partial y}-\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{v_yU}{c}\right)+\frac{1}{c}\left[\dot{z}\left(\frac{\partial}{\partial y}\left(\frac{v_zU}{c}\right)-\frac{\partial}{\partial z}\left(\frac{v_yU}{c}\right)\right)-\dot{x}\left(\frac{\partial}{\partial x}\left(\frac{v_yU}{c}\right)-\frac{\partial}{\partial y}\left(\frac{v_xU}{c}\right)\right)\right],\\
m_0\ddot{z}&=-\frac{\partial U}{\partial z}-\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{v_zU}{c}\right)+\frac{1}{c}\left[\dot{x}\left(\frac{\partial}{\partial z}\left(\frac{v_xU}{c}\right)-\frac{\partial}{\partial x}\left(\frac{v_zU}{c}\right)\right)-\dot{y}\left(\frac{\partial}{\partial y}\left(\frac{v_zU}{c}\right)-\frac{\partial}{\partial z}\left(\frac{v_yU}{c}\right)\right)\right].
\end{aligned} \tag{10}$$

Further, one can define interaction forces as functions of the interaction terms:

$$F_{I_x} = \frac{\partial I_z}{\partial y} - \frac{\partial I_y}{\partial z},$$
$$F_{I_y} = \frac{\partial I_x}{\partial z} - \frac{\partial I_z}{\partial x}, \tag{13}$$
$$F_{I_z} = \frac{\partial I_y}{\partial x} - \frac{\partial I_x}{\partial y}.$$

The next definition is related to the time-varying forces as the functions of the interaction terms:

$$F_{t_x} = -\frac{1}{c}\frac{\partial I_x}{\partial t},$$
$$F_{t_y} = -\frac{1}{c}\frac{\partial I_y}{\partial t}, \tag{14}$$
$$F_{t_z} = -\frac{1}{c}\frac{\partial I_z}{\partial t}.$$

Finally, one can define the potential forces as the function of the total potential energy of a nanorobot in a multipotential field:

$$F_{p_x} = -\frac{\partial U}{\partial x},$$
$$F_{p_y} = -\frac{\partial U}{\partial y}, \tag{15}$$
$$F_{p_z} = -\frac{\partial U}{\partial z}.$$

Applying (13), (14) and (15) to the relations (12) one obtains the compact form of the canonical differential equations of the nanorobot motion in a multipotential field as the functions of the mentioned forces:

$$m_0\ddot{x} = F_{p_x} + F_{t_x} + \frac{1}{c}\left(\dot{y}F_{I_z} - \dot{z}F_{I_y}\right),$$
$$m_0\ddot{y} = F_{p_y} + F_{t_y} + \frac{1}{c}\left(\dot{z}F_{I_x} - \dot{x}F_{I_z}\right), \tag{16}$$
$$m_0\ddot{z} = F_{p_z} + F_{t_z} + \frac{1}{c}\left(\dot{x}F_{I_y} - \dot{y}F_{I_x}\right).$$

From the previous consideration one can introduces the following vectors:

$$\boldsymbol{X} = \left[x\ y\ z\right]^T, \dot{\boldsymbol{X}} = \left[\dot{x}\ \dot{y}\ \dot{z}\right]^T, \ddot{\boldsymbol{X}} = \left[\ddot{x}\ \ddot{y}\ \ddot{z}\right]^T,$$
$$\boldsymbol{F}_I = \left[F_{I_x}\ F_{I_y}\ F_{I_z}\right]^T, \boldsymbol{F}_t = \left[F_{t_x}\ F_{t_y}\ F_{t_z}\right]^T, \tag{17}$$
$$\boldsymbol{F}_p = \left[F_{p_x}\ F_{p_y}\ F_{p_z}\right]^T.$$

Including the vectors (17) into the relations (16) one generates the vector-matrix form of the canonical differential equations of the nanorobot motion in a multipotential field:

$$m_0\ddot{\boldsymbol{X}} = \boldsymbol{F}_p + \boldsymbol{F}_t + \frac{1}{c}\boldsymbol{N}\boldsymbol{F}_I, \boldsymbol{N} = \begin{bmatrix} 0 & -\dot{z} & \dot{y} \\ \dot{z} & 0 & -\dot{x} \\ -\dot{y} & \dot{x} & 0 \end{bmatrix}. \tag{18}$$

As one can see from the relations (18) the matrix $\boldsymbol{N}$ is an anti-symmetric matrix.

4. Canonical model of nanorobot motion in a two-potential electromagnetic and gravitational field

In order to validate of the general approach given in the section 3, the derived general model of a nanorobot motion in a multipotential field (16) is applied to two-potential electromagnetic and gravitational field. Let a nanorobot is an electric charged particle with charge q and rest mass m_0 that is moving with a non-relativistic velocity ($v << c$) in a combined electromagnetic and gravitational potential field. It is also assumed that a gravitational potential field belongs to a spherically symmetric non-charged body with a mass M. In that case the total potential energy of a nanorobot in that two-potential field is determined by the following equation:

$$U = qV_e + m_0V_g = qV_e + m_0\left(-\frac{GM}{r}\right). \tag{19}$$

In the relation (19) V_e and V_g are the related scalar potentials of an electromagnetic and a gravitational field. Parameter G is a gravitational constant, M is gravitational mass and r is a gravitational radius between a nanorobot and a center of a mass M. The interaction terms of a nanorobot in that two-potential field can be obtained by applying (19) to the relations (11):

$$I_x = \frac{v_x}{c}\left(qV_e + m_0V_g\right) = q\left(\frac{v_xV_e}{c}\right) + m_0\left(\frac{v_xV_g}{c}\right) = = qA_{e_x} + m_0A_{g_x},$$
$$I_y = \frac{v_y}{c}\left(qV_e + m_0V_g\right) = q\left(\frac{v_yV_e}{c}\right) + m_0\left(\frac{v_yV_g}{c}\right) = = qA_{e_y} + m_0A_{g_y}, \tag{20}$$
$$I_z = \frac{v_z}{c}\left(qV_e + m_0V_g\right) = q\left(\frac{v_zV_e}{c}\right) + m_0\left(\frac{v_zV_g}{c}\right) = = qA_{e_z} + m_0A_{g_z}.$$

Here (A_{ex}, A_{ey}, A_{ez}) is the vector potential of an electromagnetic field, while (A_{gx}, A_{gy}, A_{gz}) is an analog vector potential of a gravitational field. The related interaction forces as functions of the interaction terms can be calculated by using the relations (13) and (20):

$$F_{I_x} = \frac{\partial}{\partial y}\left(qA_{e_z} + m_0 A_{g_z}\right) - \frac{\partial}{\partial z}\left(qA_{e_y} + m_0 A_{g_y}\right),$$
$$F_{I_y} = \frac{\partial}{\partial z}\left(qA_{e_x} + m_0 A_{g_x}\right) - \frac{\partial}{\partial x}\left(qA_{e_z} + m_0 A_{g_z}\right), \quad (21)$$
$$F_{I_z} = \frac{\partial}{\partial x}\left(qA_{e_y} + m_0 A_{g_y}\right) - \frac{\partial}{\partial y}\left(qA_{e_x} + m_0 A_{g_x}\right).$$

The relations (21) can be transformed into the following equivalent form:

$$F_{I_x} = q\left[\frac{\partial A_{e_z}}{\partial y} - \frac{\partial A_{e_y}}{\partial z}\right] + m_0\left[\frac{\partial A_{g_z}}{\partial y} - \frac{\partial A_{g_y}}{\partial z}\right] =$$
$$= qH_{e_x} + m_0 H_{g_x},$$
$$F_{I_y} = q\left[\frac{\partial A_{e_x}}{\partial z} - \frac{\partial A_{e_z}}{\partial x}\right] + m_0\left[\frac{\partial A_{g_x}}{\partial z} - \frac{\partial A_{g_z}}{\partial x}\right] = \quad (22)$$
$$= qH_{e_y} + m_0 H_{g_y},$$
$$F_{I_z} = q\left[\frac{\partial A_{e_y}}{\partial x} - \frac{\partial A_{e_x}}{\partial y}\right] + m_0\left[\frac{\partial A_{g_y}}{\partial x} - \frac{\partial A_{g_x}}{\partial y}\right] =$$
$$= qH_{e_z} + m_0 H_{g_z}.$$

In the relations (22) the parameters H_{e_x}, H_{e_y} and H_{e_z} are the components of the magnetic field H_e, as the result of the interaction of nanorobot motion in an electrical field. On the other side, the parameters H_{g_x}, H_{g_y} and H_{g_z} are the components of the gravitational acceleration field H_g, as the result of the interaction of nanorobot motion in gravitational field. The time-varying forces of a nanorobot as the functions of the interaction terms can be obtained by employing (14) and (20):

$$F_{t_x} = -\frac{1}{c}\frac{\partial}{\partial t}\left(qA_{e_x} + m_0 A_{g_x}\right),$$
$$F_{t_y} = -\frac{1}{c}\frac{\partial}{\partial t}\left(qA_{e_y} + m_0 A_{g_y}\right), \quad (23)$$
$$F_{t_z} = -\frac{1}{c}\frac{\partial}{\partial t}\left(qA_{e_z} + m_0 A_{g_z}\right).$$

Finally, the potential forces of a nanorobot, as the function of the total potential energy of a nanorobot in a combined electromagnetic and gravitational potential field, can be obtained by using the relations (15) and (19):

$$F_{p_x} = -\frac{\partial}{\partial x}\left(qV_e + m_0 V_g\right),$$
$$F_{p_y} = -\frac{\partial}{\partial y}\left(qV_e + m_0 V_g\right), \quad (24)$$
$$F_{p_z} = -\frac{\partial}{\partial z}\left(qV_e + m_0 V_g\right).$$

Now, one can make the sum of the time-varying forces (23) and potential forces (24):

$$F_{p_x} + F_{t_x} = q\left(-\frac{\partial V_e}{\partial x} - \frac{1}{c}\frac{\partial A_{e_x}}{\partial t}\right) +$$
$$+ m_0\left(-\frac{\partial V_g}{\partial x} - \frac{1}{c}\frac{\partial A_{g_x}}{\partial t}\right) = qE_{e_x} + m_0 E_{g_x},$$
$$F_{p_y} + F_{t_y} = q\left(-\frac{\partial V_e}{\partial y} - \frac{1}{c}\frac{\partial A_{e_y}}{\partial t}\right) +$$
$$+ m_0\left(-\frac{\partial V_g}{\partial y} - \frac{1}{c}\frac{\partial A_{g_y}}{\partial t}\right) = qE_{e_y} + m_0 E_{g_y}, \quad (25)$$
$$F_{p_z} + F_{t_z} = q\left(-\frac{\partial V_e}{\partial z} - \frac{1}{c}\frac{\partial A_{e_z}}{\partial t}\right) +$$
$$+ m_0\left(-\frac{\partial V_g}{\partial z} - \frac{1}{c}\frac{\partial A_{g_z}}{\partial t}\right) = qE_{e_z} + m_0 E_{g_z}.$$

Here E_{e_x}, E_{e_y} and E_{e_z} are the components of the electric field $\boldsymbol{E}_e$, while E_{g_x}, E_{g_y} and E_{g_z} describe the analog influence of the gravitational field $\boldsymbol{E}_g$. Applying the interaction forces (22), and the sum of the time-varying forces and the potential forces (25) to the relations (16) one obtains the canonical differential equations of the nanorobot motion in a two-potential field, with the total potential energy given by (19):

$$m_0\ddot{x} = qE_{e_x} + \frac{q}{c}\left(\dot{y}H_{e_z} - \dot{z}H_{e_y}\right) + m_0 E_{g_x} +$$
$$+ \frac{m_0}{c}\left(\dot{y}H_{g_z} - \dot{z}H_{g_y}\right),$$
$$m_0\ddot{y} = qE_{e_y} + \frac{q}{c}\left(\dot{z}H_{e_x} - \dot{x}H_{e_z}\right) + m_0 E_{g_y} + \quad (26)$$
$$+ \frac{m_0}{c}\left(\dot{z}H_{g_x} - \dot{x}H_{g_z}\right),$$
$$m_0\ddot{z} = qE_{e_z} + \frac{q}{c}\left(\dot{x}H_{e_y} - \dot{y}H_{e_x}\right) + m_0 E_{g_z} +$$
$$+ \frac{m_0}{c}\left(\dot{x}H_{g_y} - \dot{y}H_{g_x}\right).$$

On that way, the validation of the general model (16) is successfully finished by (26). Generally, the first two terms on the right side of the relations (26) belongs to the well known equations of motion of a particle with charge q and mass m_0 in an electromagnetic field ($\boldsymbol{E}_e$, $\boldsymbol{H}_e$)[18]. In fact, the first two terms on the right side of the relations (26) represent the well known Lorentz force on a particle with charge q and mass m_0 in an electromagnetic field. The influence of the gravitational field to the motion of a particle with mass m_0 in that field is described by the third and fourth terms in the relations (26). Following the analogy to an electromagnetic field, this influence of a gravitational field often is called the equations of motion of a particle with mass m_0 in a gravitomagnetic field ($\boldsymbol{E}_g$, $\boldsymbol{H}_g$) [23-24]. In fact, the third and fourth terms on the right side of the relations (26) represent the analog to Lorentz force on a particle with mass m_0 in a gravitomagnetic field [23]. In that sense, the relation (26) can be transformed into the related vector equation as the explicit function of the mentioned Lorentz forces:

$$\begin{aligned} & m_0\ddot{\boldsymbol{X}} = q\left(\boldsymbol{E}_e + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{H}_e\right) + m_0\left(\boldsymbol{E}_g + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{H}_g\right) = \\ & = \boldsymbol{F}_{L_e} + \boldsymbol{F}_{L_g} \\ & \ddot{\boldsymbol{X}} = \left[\ddot{x}\ \ddot{y}\ \ddot{z}\right]^T, \quad \boldsymbol{E}_e = \left[E_{e_x}\ E_{e_y}\ E_{e_z}\right]^T, \\ & \boldsymbol{v} = \left[\dot{x}\ \dot{y}\ \dot{z}\right]^T, \quad \boldsymbol{E}_g = \left[E_{g_x}\ E_{g_y}\ E_{g_z}\right]^T. \end{aligned} \tag{27}$$

Here $\ddot{\boldsymbol{X}}$ is an acceleration vector, $\boldsymbol{v}$ is a velocity vector and $\boldsymbol{F}_{Le}$ and $\boldsymbol{F}_{Lg}$ are the related Lorentz forces in an electromagnetic and a gravitomagnetic field, respectively [23]. In this example a particle is a nanorobot with charge q and rest mass m_0. Therefore this nanorobot has the interactions with both an electromagnetic and a gravitational field. Thus, the relations (26), or (27) describe the dynamics of a nanorobot motion in a two-potential electromagnetic and gravitational field.

Further, from relations (19) and (20) one can calculate the components of the vector $\boldsymbol{A}_g$:

$$\begin{aligned} A_{g_x} &= \left(\frac{v_x V_g}{c}\right) = -\left(\frac{v_x GM}{rc}\right), \\ A_{g_y} &= \left(\frac{v_y V_g}{c}\right) = -\left(\frac{v_y GM}{rc}\right), \\ A_{g_z} &= \left(\frac{v_z V_g}{c}\right) = -\left(\frac{v_z GM}{rc}\right). \end{aligned} \tag{28}$$

Including (28) into the relations (22) one obtains the components of the vector $\boldsymbol{H}_g$:

$$\begin{aligned} H_{g_x} &= \left[\frac{\partial A_{g_z}}{\partial y} - \frac{\partial A_{g_y}}{\partial z}\right] = \frac{GM}{r^2}\left(\frac{v_z}{c}\frac{\partial r}{\partial y} - \frac{v_y}{c}\frac{\partial r}{\partial z}\right), \\ H_{g_y} &= \left[\frac{\partial A_{g_x}}{\partial z} - \frac{\partial A_{g_z}}{\partial x}\right] = \frac{GM}{r^2}\left(\frac{v_x}{c}\frac{\partial r}{\partial z} - \frac{v_z}{c}\frac{\partial r}{\partial x}\right), \\ H_{g_z} &= \left[\frac{\partial A_{g_y}}{\partial x} - \frac{\partial A_{g_x}}{\partial y}\right] = \frac{GM}{r^2}\left(\frac{v_y}{c}\frac{\partial r}{\partial x} - \frac{v_x}{c}\frac{\partial r}{\partial y}\right). \end{aligned} \tag{29}$$

Following (19), (25) and (28) one can calculate the components of the vector $\boldsymbol{E}_g$:

$$\begin{aligned} E_{g_x} &= \left(-\frac{\partial V_g}{\partial x} - \frac{1}{c}\frac{\partial A_{g_x}}{\partial t}\right) = \\ & -\left[\frac{GM}{r^2}\frac{\partial r}{\partial x} + \frac{1}{c^2}\left(-\dot{v}_x\frac{GM}{r} + v_x\frac{GM}{r^2}\dot{r}\right)\right], \\ E_{g_y} &= \left(-\frac{\partial V_g}{\partial y} - \frac{1}{c}\frac{\partial A_{g_y}}{\partial t}\right) = \\ & -\left[\frac{GM}{r^2}\frac{\partial r}{\partial y} + \frac{1}{c^2}\left(-\dot{v}_y\frac{GM}{r} + v_y\frac{GM}{r^2}\dot{r}\right)\right], \\ E_{g_z} &= \left(-\frac{\partial V_g}{\partial z} - \frac{1}{c}\frac{\partial A_{g_z}}{\partial t}\right) = \\ & -\left[\frac{GM}{r^2}\frac{\partial r}{\partial z} + \frac{1}{c^2}\left(-\dot{v}_z\frac{GM}{r} + v_z\frac{GM}{r^2}\dot{r}\right)\right]. \end{aligned} \tag{30}$$

In the non-relativistic case, the following condition $v << c$ should be satisfied. For that case and including a weak gravitational field, one obtains from (29) and (30) the new values for components of the vectors $\boldsymbol{H}_g$ and $\boldsymbol{E}_g$:

$$\begin{aligned} & H_{g_x} \simeq 0, \quad H_{g_y} \simeq 0, \quad H_{g_z} \simeq 0, \\ & E_{g_x} \simeq -\left(\frac{GM}{r^2}\frac{\partial r}{\partial x}\right), \quad E_{g_y} \simeq -\left(\frac{GM}{r^2}\frac{\partial r}{\partial y}\right), \\ & E_{g_z} \simeq -\left(\frac{GM}{r^2}\frac{\partial r}{\partial z}\right). \end{aligned} \tag{31}$$

Applying (31) to the relations (26) one obtains the dynamic model of the nanorobot motion in an electromagnetic field and in a weak gravitational field, valid for non-relativistic motion ($v << c$):

$$\begin{aligned} m_0\ddot{x} &= qE_{e_x} + \frac{q}{c}\left(\dot{y}H_{e_z} - \dot{z}H_{e_y}\right) - m_0\frac{GM}{r^2}\frac{\partial r}{\partial x}, \\ m_0\ddot{y} &= qE_{e_y} + \frac{q}{c}\left(\dot{z}H_{e_x} - \dot{x}H_{e_z}\right) - m_0\frac{GM}{r^2}\frac{\partial r}{\partial y}, \\ m_0\ddot{z} &= qE_{e_z} + \frac{q}{c}\left(\dot{x}H_{e_y} - \dot{y}H_{e_x}\right) - m_0\frac{GM}{r^2}\frac{\partial r}{\partial z}. \end{aligned} \tag{32}$$

Since the gravitational field of our planet Earth belongs to the weak potential fields, the relations (32) can be applied for calculation of a nanorobot motion in an electromagnetic field on the surface of our planet.

5. Application of derived Hamiltonians to quantum systems

In the case where quantum effects in nanorobotics are present there are two approaches to describe dynamics of a nanorobot motion. The first one is for non-relativistic quantum systems without spin, where condition $v << c$

is satisfied. In that case the non-relativistic Hamiltonian (1), derived for nanorobots motion in a multipotential field, should be transformed into the related quantum mechanical operator and applied to the well known Schrödinger equation [19]. This approach has been presented in the reference [12], where a model of nanorobot motion in a multipotential field has been employed with inclusion of related quantum effects. The second approach is referred to the relativistic quantum systems with spin. For inclusion of the spin effects one should employ the related Dirac's equations [15]. In derivation of the Dirac's equations for a nanorobot motion in a multipotential field the relativistic Hamiltonian derived in [12] has been employed. This approach has been considered in the reference [22]. Finally, dynamics of the quantum feedback systems and control concepts and applications are presented in the references [20] and [21], respectively.

6. Conclusion

At the nanoscale the dynamics of a nanorobot motion is very complex. This is because there are very strong interactions between nanorobots, manipulated objects (samples or particles) and nanoenvironment. Thus, the dynamic model of a nanorobot motion should be derived for application to a multipotential field. Starting with the non-relativistic approximation of Hamiltonian, derived in [12], in this paper the canonical form of differential equations for a nanorobot motion in a multipotential field has been derived. This model is suitable for application to modern control algorithms such as an external linearization, optimal and adaptive control and an artificial intelligence control. For validation of the presented general approach, the derived dynamic model of a nanorobot motion is applied to two-potential field combined of electromagnetic and gravitational fields.

At the small enough distances between particles (nanorobot tip - sample distances) a quantum mechanical effects can be appeared. Thus, for non-relativistic quantum systems without spin, the well known Schrödinger equation for a multipotential field should be applied for description of a nanorobot motion in that field. On the other side, for relativistic quantum systems with spin, the related Dirac's equation for description of a nanorobot motion in a multipotential field should be employed. The future work will be devoted to application of modern control algorithms for control of a nanorobot motion in a multipotential field.

Acknowledgement

This paper is the result of research on a scientific project: Artificial intelligence in control of complex nonlinear dynamic systems 120-1201842-3048. This project is supported by Ministry of Science, Education and Sport, Republic of Croatia.

REFERENCES

[1] REQUICHA, A. A. G.: *Nanorobotics*, Laboratory for Molecular Robotics and Computer Science Department, University of Southern California, Los Angeles, CA90089-0781 requicha@usc.edu, website http://www-lmr.usc.edu/~lmr, 2008.

[2] DREXLER, K. E.: *Nanosystems*. New York, NY, John Wiley & Sons, 1992.

[3] GÓMEZ – LÓPEZ, M.; PREECE, J. A.; STODDART, J. F.: *The Art and Science of Self-Assembling Molecular Machines,* Nanotechnology, 7(1996) 3, 183-192.

[4] WIESENDANGER, R.: *Scanning Probe Microscopy Methods and Applications*, Cambridge University Press, Cambridge, U.K. 1994.

[5] STROSCIO, J. A.; EIGLER, D. M.: *Atomic and Molecular Manipulation with the Scanning Tunneling Microscope*, Science, 254(1991) 5036, 1319-1326.

[6] BINNIG, G.; QUATE, C. F.; GERBER, Ch.: *Atomic Force Microscopy*, Phys. Rev. Lett. 56(1986), 930-933.

[7] WIESENDANGER, R.: *Spin Mapping at the Nanoscale and Atomic Scale*, Rev. Mod. Phys. 91(2009), 1495-1550.

[8] FREITAS, R. A., Jr.: *Nanomedicine, Volume I: Basic Capabilities*, Landes Bioscience, Georgetown, TX, 1999.

[9] NAMDI, M.; FERREIRA, A.; SHARMA, G.; MAVROIDIS, C.: *Prototyping Bio-Nanorobots using Molecular Dynamics Simulation and Virtual Reality*, Microelectronics Journal, 2007.

[10] DUBEY, A.; MAVROIDIS, C.; TOMASSONE, S.M.: *Molecular Dynamics Studies of Viral-Protein Based Nano-Actuators*, Journal of Comput. and Theoretical Nanoscience, 3(2006) 6, 885-897.

[11] MAVROIDIS, C.: *Bionano Machines for Space Applications*, Final Phase II Report to the NASA Institute of Advanced Concepts, July, 2006.

[12] NOVAKOVIĆ, B.: *Derivation of Hamilton Functions Including Artificial Control Fields in Nanorobotics* – Proceedings CIM 2009 Computer Integrated Manufacturing and High Speed Machining, Biograd, June 17-20, 2009, 139-146.

[13] NOVAKOVIĆ, B.: *Artificial Intelligence and Robotics,* Strojarstvo, 47(2005) 1-2, 3-4,14.

[14] EINSTEIN, A.: *Relativity: The Special and General Theory*, Crown, New York, 1961.

[15] DIRAC, P. A. M.: The Principles of Quantum Mechanics, Oxford, 1947.

[16] NOVAKOVIĆ, B.; NOVAKOVIĆ, D.; NOVAKOVIĆ, A.: *A New General Lorentz Transformation model*, CASYS'99, Ed. by D. M. Dubois, Publ. by The American Institute of Physics, AIP-CP517(2000), 437-450.

[17] NOVAKOVIĆ, B.; NOVAKOVIĆ, D.; NOVAKOVIĆ, A.: *A Metric Tensor of the New General Lorentz Transformation Model,* CASYS'2000, Ed. by D.M. Dubois, Publ. by CHAOS, Liège, Belgium, International Journal of Computing Anticipatory Systems, 10(2001), 199-217.

[18] SUPEK, I.: *Theoretical Physics and Structure of Matter*, Part 1 and Part 2, Školska knjiga, Zagreb, 1992 and 1990.

[19] SCHRÖDINGER, E.: *Schrödinger equation* – Wikipedia, the free encyclopedia, http://en.wikipedia.org/wiki/., 03.02.2009.

[20] YANAGISAWA, M.; KIMURA, H.: *Transfer Function Approach to Quantum Control – Part I : Dynamics of Quantum Feedback Systems*, IEEE Trans. on Automatic Control*,* 48(2003) 12, 2107-2120.

[21] YANAGISAWA, M.; KIMURA, H.: *Transfer Function Approach to Quantum Control – Part II: Control Concepts and Applications*, IEEE Trans. on Automatic Control*,* 48(2003) 12, 2121-2132.

[22] NOVAKOVIĆ, B.: *Generalized Relativistic Hamiltonian in an Alpha Field* – CASYS 2009 Ninth International Conference on Computing Anticipatory Systems, Abstract Book, D. Dubois (ed.), Liège, 2009.

[23] WILLIAMS, R. K.: *Gravitomagnetic Field and Penrose Scattering Processes*, Annals of the New York Academy of Sciences, 1045(2005), 232-245.

[24] RUGGIERO, M. L.: *Gravitomagnetic Gyroscope Precession in Palatini f(R) Gravity*, Phys. Rev. D 79(2009) 8, 4001-4005.