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ELASTIC LABOUR SUPPLY AND HOME PRODUCTION IN A MONETARY GROWTH MODEL

Wei-bin Zhang*

College of Asia Pacific Management, Ritsumeikan Asia Pacific University
Oita Prefecture, Japan

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ABSTRACT

This study constructs a monetary gender growth model with capital accumulation and endogenous labour supply. The real aspects of the model are based on the neoclassical growth theory and monetary aspects of the model are based on the cash-in-advance (CIA) approach. We show that the dynamics of the economy can be described by 2-dimensional differential equations. We simulate equilibrium and motion of the economy with specified monetary policies, household preference and technology. As the monetary economic system is unstable, the economy may either experience unlimited growth or economic crisis. We also study effects of changes in some parameters on the economic equilibrium. For instance, as the woman raises her propensity to stay at home, the capital intensity is not affected, which results in that the wage rates, the rate and output level per unit of labour input are not affected. Nevertheless, the man increases his work time and the woman reduces her work time, resulting in the fall of the labour supply. The money holing, durable goods used at home and consumption level per household are reduced; also the total wealth, total capital inputs, total durable goods and total output levels are reduced.

KEY WORDS

monetary growth, gender, capital accumulation, money, inflation policy, CIA approach

CLASSIFICATION

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*Corresponding author, η: wbz1@apu.ac.jp; +81 977 78 1020 (office), +81 977 73 9787 (home);
Ritsumeikan Asia Pacific University, 1-1 Jumonjibaru, Beppu-Shi, Oita-ken 874-8577, Japan

INTRODUCTION

This study examines effects of gender differences in preference and human capital and inflation policy on long-term economic growth. The supply side of our model is based on the neoclassical growth theory. We introduce gender and money into the neoclassical growth theory. Interactions between monetary policy and economic growth have been a significant topic in macroeconomics over the past four decades (see [1-6]. Specially, [7] provides a comprehensive review of the literature on monetary growth theory). Economists have examined various channels through which monetary policy can affect economic growth. Nevertheless, as observed, the literature remains silent on "providing a more complete characterization of local and global dynamics so as to help assess how money supply and banking policies affect macroeconomic performance in a short-run transition." [8; p. 1684]. It is important to provide a complete characterization of dynamic economic models and to follow the motion of the entire economic system over time. To deal with this important issue, we introduce money in a unified dynamic general equilibrium framework that is analytically tractable, applying an alternative approach to household behavior. In our approach, money is held because transactions need to be settled by payment via money. The first formal models with money as a medium of exchange that facilitate transactions were due to Baumol [9] and Tobin [10]. As cash balances are typically non-interest bearing, it is costly for individuals to hold money. Clower [11] builds a model with money as a medium of exchange through the so-called cash-in-advance constraint. The role that money plays in carrying out transactions is modeled by introducing transaction technology. The approach holds that goods cannot be exchanged for goods and only money can buy goods. Stockman [12] developed a growth model through CIA constraints. The model predicts that there is long-run superneutrality if only consumption expenditures are subject to a CIA constraint. If investment is also subject to a CIA constraint then steady state capital will fall when the growth rate of money rises.

Although there are many models of economic growth, there are only a few formal economic growth models which explicitly introduce gender into economic growth theory. As stated by Moe: "While men and women interact as economic agents, both within families and in the marketplace, historically, economists have modeled the behavior of *men* interacting in the *marketplace*. Women and families were virtually ignored in economic thought before the 1960s, and many argue that they continue to be marginalized in economic theory today." [13; p. 3]. Berik and Rodgers observed: "Gender-aware analysis since the early 1970s has produced a large literature showing men and women are affected differently by policies and processes associated with economic development." [14; p. 1]. Nevertheless, gender issues have been formally studied in macroeconomics only in the last decade. Earlier studies of gender economics were mostly microeconomic. As women have increased improved their formal education, improved work skills and experiences, and increased labour force attachment, women's effects on markets and macroeconomic performances cannot be ignored. As many empirical studies confirmed, gender differences in preferences and human capital lead to different economic phenomena within families as well as in economic development. In the neoclassical approach, it is generally believed that gender inequalities resulting from disparities in human capital will wither away in association with economic development [15-18]. According to Boserup, there will be a curvilinear relationship between economic growth [19]. Although productivity differences between men and women at low levels of economic development are not large, as economic conditions are improved, productivity differences tend to widen and a polarization and hierarchization of men's and women's work roles tend to ensure. Nevertheless, further economic growth will bring about a closing of the gap. The pace at which the gap is closed is dependent on many cultural, institutional, as well as economic factors. Over the years there have been a number of attempts to modify neoclassical consumer theory to deal with economic issues related to family and gender (see [20-26] for further literature on the subject). Nevertheless, one might argue that the contemporary economics has failed to develop analytical frameworks to properly introduce gender division of labour into economic growth theory with capital accumulation and inflation policies. It is necessary to design an analytical framework by which economic growth, family economy and gender differences be analyzed as an integrated whole. The paper is organized as follows. Section 2 defines the monetary growth model with gender and endogenous labour supply. Section 3 examines dynamic properties of the model, simulating the motion of the economic system and finding out the unique equilibrium point and its stability. Section 4 examines effects of changes in some parameters on the economic system. Section 5 concludes the study.

THE MONETARY GENDER GROWTH MODEL WITH HOME PRODUCTION

The production aspects of the economic system under consideration are similar to the one-sector neoclassical growth model proposed by Solow [27]. Production is generally described as combination of multiple production factors such as natural resources, labour, and capital. Time is represented continuously by a numerical variable which takes on all values from zero onwards. As our model exhibits constant returns to scale, the dynamics (in terms of per capita) will not be affected if we allow the population of all the groups to change in a constant growth rate over time. We use family as production units in economic analysis[1]. In addition to enjoyment, households also supply labour services, recreation, spiritual experiences, as well as conventional goods of the do-it-yourself variety. Household activities are conducted within houses as well as outside doors. These activities not only consume goods (like eat fruits and vegetables) but also utilize machines like housing, video, TV, cooking machines, cars, and the like. We further assume that each family consists of four members – father, mother, son and daughter. The total population[2] is $4N_0$. It is assumed that only the adults may work. The children get educated before they get married and joint the labour market. We assume that the husband and wife pass away at the same time. When the parents pass away, the son and the daughter respectively find their marriage partner within the same type of households and get married. The properties left by the parents are shared equally among the male and female children. The children are educated so that they have the same human capital as their parents. When a new family is formed, the young couple joins the labour market and has two children. As all the families are identical, the family structure is invariant over time under these assumptions. Let subscripts $j = 1$ and $j = 2$ stand for man and woman respectively. Let $T_j(t)$ stands for the work time of a representative household of group j and $N(t)$ for the flow of labour services used at time t for production. There is only a single production sector in the economy and labour is always fully employed. We measure $N(t)$ as follows:

$$N(t) = \sum_{j=1}^{2} h_j N_0 T_j(t), \tag{1}$$

where h_j are the level of human capital of group j. We assume h_j to be fixed. We have $N(t) = N_0\tilde{T}(t)$, where $\tilde{T}(t) = h_1 T_1(t) + h_2 T_2(t)$.

We use the conventional production function to describe a relationship between inputs and output. Total capital, $K(t)$ is distributed between the industrial sector and home use. We denote $K_i(t)$ and $K_h(t)$ as the capital stocks employed by the industrial sector and households, respectively. Then, we have

$$K(t) = K_i(t) + K_h(t).$$

Let $\bar{k}(t)$ stands for the value of capital owned per household. We have $\bar{k}(t) = K(t)/N_0$. We introduce two variables, $k_i(t)$ and $k_h(t)$

$$k_i(t) \frac{K_i(t)}{N_0}, \quad k_h(t) \equiv \frac{K_h(t)}{N_0}.$$

The production function is $F = A K_i^{\alpha} N^{\beta}$ where α and β are parameters. The marginal conditions are

$$r + \delta_k = \frac{\alpha f(k_i)}{k_i}, \ w_j(t) = h_j w(t), \ w = \beta f(k_i), \tag{2}$$

where $f = A k_i^{\alpha}$. The ratio of per unit time wage rates is equal to the ratio in human capital between man and woman. This implies that there is no gender discrimination in the labour market.

Money is introduced by assuming that a central bank distributes at no cost to the population a per capita amount of fiat money $M(t) > 0$. The scheme according to which the money stock evolves over time is deterministic and known to all agents. With μ being the constant net growth rate of the money stock, $M(t)$ evolves over time according to

$$\dot{M}(t) = \mu M(t), \ \mu > 0.$$

The government expenditure in real terms per capita, $\tau(t)$ is given by

$$\tau(t) = \frac{\dot{M}(t)}{P(t)} = \frac{\mu M(t)}{P(t)} = \mu m(t). \tag{3}$$

The representative household receives $\mu\, m(t)$ units of paper money from the government through a “helicopter drop”, also considered to be independent of his money holdings.

From $M = mP$, we have

$$\pi(t) = \frac{\dot{P}(t)}{P(t)} = \mu - \frac{\dot{m}(t)}{m(t)}. \tag{4}$$

The household makes decisions on leisure time, consumption levels of services and commodities as well as on how much to save. This study uses an alternative approach to consumers' behaviour[3]. We now describe behavior of consumers. A representative household's current income $y(t)$ is given by

$$y(t) = r(t)\bar{k}(t) + w_1(t)T_1(t) + w_2(t)T_2(t) - \pi(t)m(t) + \tau(t),$$

where $\pi(t)$ is inflation rate. The sum of money that consumers are using for consuming, saving, or transferring are not necessarily equal to the current income because consumers can sell wealth to pay, for instance, current consumption if the temporary income is not sufficient for purchasing goods and services. Retired people may live not only on the interest payment but also have to spend some of their wealth. The total value of wealth that a household can sell to purchase goods and to save is equal to $a(t)$ where $a(t) = \bar{k}(t) + m(t)$. Here, we do not allow borrowing for current consumption. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. This is evidently a strict consumption as it may take time to draw savings from bank or to sell one's properties. The disposable income of a household is defined as the sum of the current income and the wealth available for purchasing consumption goods and saving, $\hat{y}(t) = y(t) + a(t)$. That is

$$\hat{y}(t) = r(t)\bar{k}(t) + w_1(t)T_1(t) + w_2(t)T_2(t) - \pi(t)m(t) + \tau(t) + a(t).$$

Denote $\overline{T}_q(t)$ the leisure time at time t and the (fixed) available time for work and leisure by T_0. The time constraint is expressed by

$$T_q(t) + \overline{T}_q(t) = T_0, \quad q = 1, 2.$$

Substituting this time constraint into the disposable income yields

$$\hat{y}(t) = r(t)\bar{k}(t) + w_1(t)T_0 + w_2(t)T_0 - w_1(t)\bar{T}_1(t) - w_2(t)\bar{T}_2(t) - \pi(t)m(t) + \tau(t) + a(t). \quad (5)$$

Assume that cash has to be held in advance of purchasing goods[4]. The liquidity constraint of the household is formed as

$$m(t) = \chi\{c(t) + [r(t) + \delta_k]k_h(t)\}, \quad (6)$$

where χ is a positive parameter[5]. We require $0 < \chi \le 1$The budget constraint is given by

$$c(t) + [r(t) + \delta_k]k_h(t) + s(t) = \hat{y}(t).$$

Insert (5) and (6) in the above budget constraint

$$w_1(t)\bar{T}_1(t) + w_2(t)\bar{T}_2(t) + [1 + \chi\pi(t)]c(t) + [1 + \chi\pi(t)][r(t) + \delta_k]k_h(t) + s(t) = \hat{y}(t), \quad (7)$$

where

$$\bar{y}(t) = r(t)\bar{k}(t) + w_1(t)T_0 + w_2(t)T_0 + \tau(t) + a(t).$$

We assume that $U(t)$ is dependent on leisure times, consumption level, housing, and savings in the following way

$$U(t) = T_1^{\sigma_{01}}(t)T_2^{\sigma_{02}}(t)k_h^{\eta_0}(t)c^{\xi_0}(t)s^{\lambda_0}(t), \quad \sigma_0, \eta_0, \xi_0, \lambda_0 > 0,$$

where η_0 is called propensity to consume housing, σ_{01} and σ_{02} the husband and wife's propensities to stay at home, ξ_0 propensity to consume goods, and λ_0 propensity to own wealth. Becker [28] and Lancaster [29, 30] formally introduced the concept of household production function. Instead of receiving utility directly from goods purchased in the market, they assumed that consumers derive utility from the attributes possessed by these goods, and then only after some transformation is performed on these markets goods. A typical example is that the consumers purchase raw food in the market but they derive utility from the consumption of the completed meal, which has been produced by combining the raw food with labour and time in some environment. It can be seen that our utility function subject to the budget constrains include the idea of household production function. Here, the propensity to stay at home of a woman (or a man) is the aggregated measure of her propensity to enjoy leisure and the propensity to conduct home production. The parameter value of the propensity to stay at home is affected by many factors such as values, family game and economic conditions[6].

It should be noted that it is Reid who first explicitly treated the household as the locus of production as well as consumption, even though Becker is often considered as the father of the family economics[7] [31]. As pointed out by Ferber: "Becker … does not appear altogether serious about considering homemaking as real work, given his assertion that married women 'work' much less than single women." [32; p. 9] Stotsky identifies a number of phenomena related to gender differences and economic behavior [18]: (1) gender-based differences can influence macroeconomic variables, such as aggregate consumption, savings; (2) these differences may also affect the behavior of governments; (3) women tend to devote a larger share of household resources to the households' basic needs and the children's fostering; (4) women tend to have a higher propensity to save and to invest in productive activities and show greater caution in saving and investing; (5) women's lack of education and other economic and social opportunities, both absolutely and relative to men, inhibits economic growth. It can be seen that these gender differences can be taken account in our model by properly introducing endogenous human capital, discrimination in labour market, and household preference[8].

Maximizing U subject to (7) yields

$$w_1\bar{T}_1 = \sigma_1\bar{y}, \; w_2\bar{T}_2 = \sigma_2\bar{y}, \; (1 + \chi\pi)\cdot c = \xi\bar{y}, \; (1 + \chi\pi)\cdot(r + \delta_k)\cdot k_h = \eta\bar{y}, \; s = \lambda\bar{y}, \quad (8)$$

where

$$\sigma_1 \equiv \rho\sigma_{01},\ \sigma_2 \equiv \rho\,\sigma_{02},\ \xi \equiv \rho\xi_0,\ \eta \equiv \rho\eta_0,\ \lambda \equiv \rho\lambda_0,\ \rho \equiv (\sigma_{01}+\sigma_{02}+\xi_0+\eta_0+\lambda_0)^{-1}.$$

Previous equations mean that the housing consumption and consumption of the good are positively proportional to the net income and capital wealth, and the saving is positively proportional to the net income but negatively proportional to the wealth. The wealth accumulation is given by

$$\dot{a}(t) = s(t) - a(t). \tag{9}$$

The equation simply states that the change in wealth is equal to the savings minus the dissavings. It is straightforward to confirm that $K = K_i + K_h$ can expressed as

$$\bar{k}(t) = \tilde{T}(t)k_i(t) + k_h(t). \tag{10}$$

We have thus built the model. The rest of the paper examines properties of the dynamic system.

THE DYNAMICS AND STEADY STATE

We first show that the dynamics can be expressed as a two differential equations system. The following lemma is proved in the appendix.

LEMMA 1

The motion of the economic system is described by the following two differential equations with $k_i(t)$ and $m(t)$ as the variables

$$\dot{m} = \Lambda(m, k_i),$$
$$\dot{k}_i = \Omega(k_i, m),$$

where $\Lambda(m,\ k_i)$ and $\Omega(k_i,\ m)$ are functions m and k_i defined in the appendix. All the other variables are determined as functions of $k_i(t)$ and $m(t)$ by the following procedure: $\bar{k}$ by (A8) $\rightarrow k_h$ by (A1) $\rightarrow \bar{y}$ by (A3) $\rightarrow \pi$ by (A6) $\rightarrow a = \bar{k} + m$ $\rightarrow w_j$ and r by (2) $\rightarrow f = f(k_i) \rightarrow \bar{T}_j$ by (8) $\rightarrow T_j = T_0 - \bar{T}_j \rightarrow N \rightarrow F = f(k_i)N \rightarrow c$ and s by (8).

The lemma is important as it enables us to following the motion of the system with properly given initial conditions. We don't explicitly interpret the equations as the expressions are tedious. We now examine stead states. From $\pi = \mu - \dot{m}/m$ we have $\pi = \mu$ at a steady state. By (9), we have $\lambda\bar{y} = a$. From $\lambda\bar{y} = a$ and (A3), we have

$$\mu m = \Omega_0(k_i)\bar{k} + \beta h^* f,$$

where $1/\lambda - 1 - \mu$ and

$$\Omega_0(k_i) \equiv \delta + \frac{\alpha f}{k_i} - \frac{1}{\lambda}.$$

Substituting (A5) into the above equation yields

$$\Omega_0\left[h^* + \beta h^*\varphi_\pi + (1+\mu)\frac{m\,\varphi_\pi}{f}\right]\left[\frac{1}{k_i} - \left(\delta + \frac{\alpha f}{k_i}\right)\frac{\varphi_\pi}{f}\right]^{-1} + \beta h^* f - \bar{\mu} m = 0. \tag{11}$$

From (A5), $m = (\xi+\eta)\bar{y}/(\chi\tilde{\pi})$ and $\lambda\bar{y} = a$, we solve

$$m = \bar{\Omega}(k_i) \equiv (h^* + \beta h^*\varphi_\pi)\left\{\left[\frac{1}{k_i} - \left(\delta + \frac{\alpha f}{k_i}\right)\frac{\varphi_\pi}{f}\right]\left(\frac{\lambda\chi\tilde{\pi}}{\xi+\eta} - 1\right) - (1+\mu)\frac{\varphi_\pi}{f}\right\}^{-1} \tag{12}$$

Insert (12) in (11)

$$\Omega(k_i) \equiv \Omega_0\left[h^* + \beta h^*\varphi_\pi + (1+\mu)\frac{\bar{\Omega}\varphi_\pi}{f}\right]\left[\frac{1}{k_i} - \left(\delta + \frac{\alpha f}{k_i}\right)\frac{\varphi_\pi}{f}\right]^{-1} + \beta h^* f - \bar{\mu}\bar{\Omega} = 0. \tag{13}$$

LEMMA 2

An equilibrium value of k_i is determined by $\Omega(k_i) = 0$. For a given value of k_i, the values of the other variables are determined as the following procedure: $\pi = \mu \to m$ by (12) $\to \bar{k}$ by (A8) $\to k_h$ by (A1) $\to \bar{y}$ by (A3) $\to \pi$ by (A6) $\to a = \bar{k} + m \to w_j$ and r by (2) $\to f = f(k_i) \to \bar{T}_j$ by (8) $\to T_j = T_0 - \bar{T}_j \to N \to F = f(k_i)N \to c$ and s by (8).

Although it is straightforward to calculate the eigenvalues of the steady state, it is difficult to interpret the stability conditions because expressions are tedious. We omit to explicitly calculate the stability conditions. As the expressions are too tedious, we cannot easily interpret the analytical results. For illustration, we simulate the model, specifying the parameter values as follows

$$\alpha = 1/3, \quad A = 1{,}2, \quad N_0 = 10, \quad h_1 = 1{,}5, \quad h_2 = 1{,}3, \quad \chi = 0{,}8, \quad T_0 = 24, \quad \mu = 0{,}02,$$
$$\varepsilon_0 = 0{,}02, \quad \xi_0 = 0{,}09, \quad \lambda_0 = 0{,}84, \quad \eta_{01} = 0{,}08, \quad \sigma_{01} = 0{,}13, \quad \sigma_{02} = 0{,}15, \quad \delta_k = 0{,}05. \tag{14}$$

Man's propensity to stay at home is lower than woman's but man's human capital is higher than woman. The total productivity is specified at 1,2. Although the specified values are not based on empirical observations, the choice does not seem to be unrealistic. For instance, some empirical studies on the US economy demonstrate that the value of the parameter, α in the Cobb-Douglas production is approximately equal[9] to 0,3. With regard to the technological parameters, what are important in our study are their relative values. This is similarly true for the specified preference parameters. With the initial conditions, the changes of the variables over time are plotted in Figure 1. The real rate of interest rises and inflation rate falls over time. The output level per unit time and wage rates fall over time. Both man and woman reduce their work times. The consumption level rises and the total output and wealth are reduced[10].

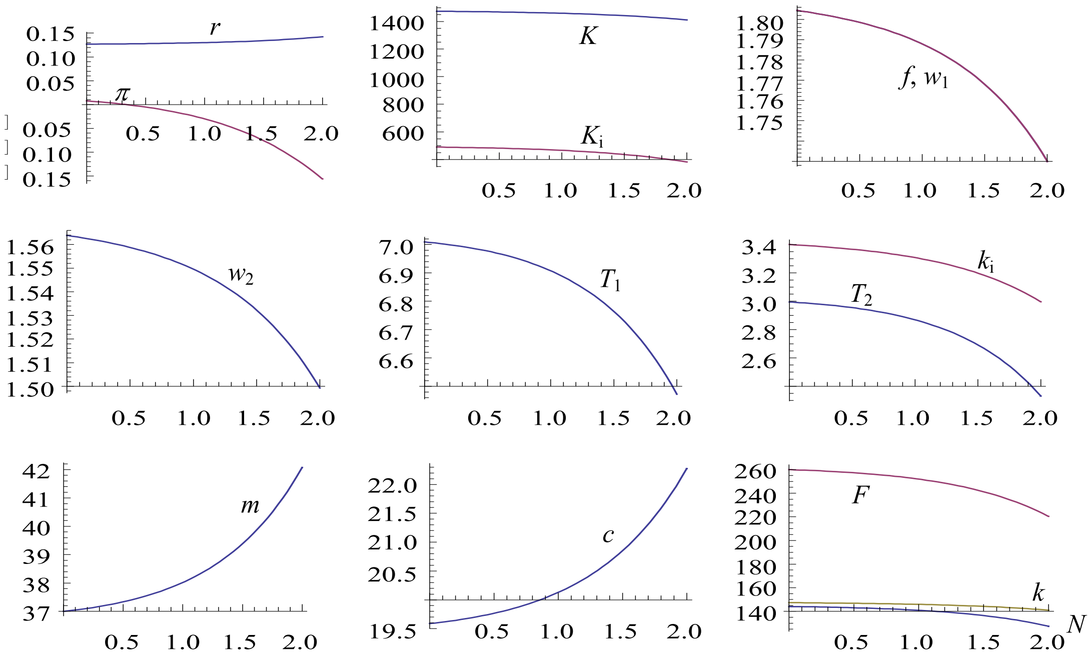

Figure 1. The motion of the system with money and division of labour.

With the parameter values specified as in (14), we find a unique meaningful solution of (13) as illustrated in Figure 2.

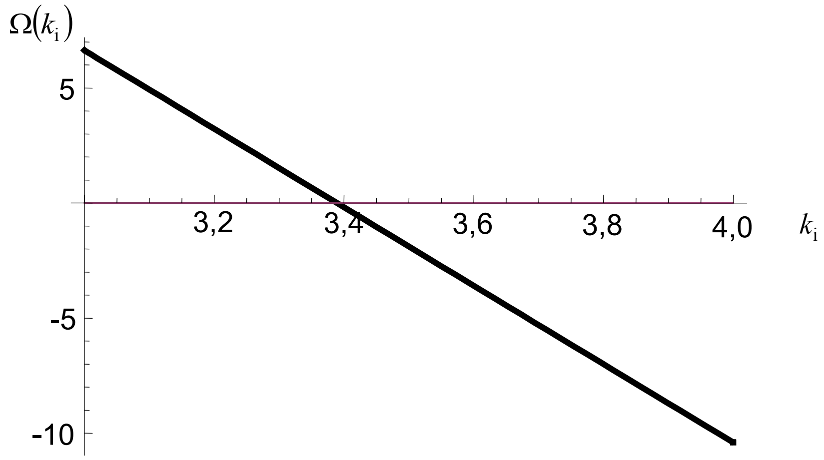

Figure 2. Graphical representation of the unique solution of equation (13).

The values of all the variables at the equilibrium point are given in

$$r = 0{,}13,\quad k_i = 3{,}39,\quad f = w_1 = 1{,}80,\quad w_2 = 1{,}56,\quad T_1 = 7{,}10,\quad T_2 = 3{,}10,$$
$$m = 36{,}42,\quad \bar{k} = 146{,}40,\quad k_h = 96{,}66,\quad c = 19{,}28,\quad N = 146{,}76,$$
$$K_i = 497{,}39,\quad K_h = 966{,}64,\quad K = 1464{,}03,\quad F = 264{,}54. \tag{15}$$

The two eigenvalues are respectively equal to 1,343 and –0,242. Hence, the unique equilibrium is unstable.

COMPARATIVE STATIC ANALYSIS

The previous section identifies the unique equilibrium of the global economy and demonstrates that the global economy is unstable. The monetary economic growth may suffer global crisis without government intervention[11]. This section examines impact of changes in some parameters on the long term equilibrium point[12]. First, we examine the case that all the parameters, except the woman's propensity to stay at home are the same as in (14). We increase σ_{02} from 0,15 to 0,16 The simulation results are listed in (16). Here, symbol $\bar{\Delta}$ stands for the change rate due to the parameter change. As the woman raises her propensity to stay at home, the capital intensity is not affected, which results in that the wage rates, the rate and output level per unit of labour input are not affected by the woman's preference change. Nevertheless, the man increases his work time and the woman reduces her work time. As the woman raises her propensity to stay at home, the woman tends to stay longer at home than before. As the work hours fall, the total labour supply is reduced. As the total labour supply is reduced, the total capital is also reduced. The net result is that the capital intensity is increased. As the difference in human capital between the woman and man is not very large but the woman reduces much long work time (in terms of percentage) than the man, the total labour input is reduced. As the total labour is reduced, we see that to maintain the system at the new equilibrium point, the money holing, durable goods used at home and consumption level per household are reduced; also the total wealth, total capital inputs, total durable goods and total output levels are reduced.

$$\bar{\Delta}r = \bar{\Delta}k_i = \bar{\Delta}f = \bar{\Delta}w_1 = \bar{\Delta}w_2 = 0,\quad \bar{\Delta}T_1 = 6{,}25,\quad \bar{\Delta}T_2 = -\,26{,}05,$$
$$\bar{\Delta}m = \bar{\Delta}\bar{k} = \bar{\Delta}k_h = \bar{\Delta}c = \bar{\Delta}N = \bar{\Delta}K_i = \bar{\Delta}K_h = \bar{\Delta}K = \bar{\Delta}F = -\,2{,}63. \tag{16}$$

We now increase the woman's human capital level, h_2, from 0,13 to 1,4. The results are listed in (17). The capital intensity is not affected, which results in that the man's wage rate, the rate and output level per unit of labour input are not affected by the woman's human capital. The woman's wage rate is increased due to her improvement in human capital. The money holing, durable goods used at home and consumption level per household are increased; also the total wealth, total capital inputs, total durable goods and total output levels are increased. We see that a rise in the tendency for woman to participate in labour market will benefit man as his wage rate is not affected, but his leisure time is increased and housing conditions are improved.

$$\bar{\Delta}r = \bar{\Delta}k_{\mathrm{i}} = \bar{\Delta}f = \bar{\Delta}w_1 = 0, \quad \bar{\Delta}w_2 = 7{,}69, \quad \bar{\Delta}T_1 = -8{,}51, \quad \bar{\Delta}T_2 = 25{,}78,$$
$$\bar{\Delta}m = \bar{\Delta}\bar{k} = \bar{\Delta}k_{\mathrm{h}} = \bar{\Delta}c = \bar{\Delta}N = \bar{\Delta}K_{\mathrm{i}} = \bar{\Delta}K_{\mathrm{h}} = \bar{\Delta}K = \bar{\Delta}F = 3{,}57. \tag{17}$$

The effects of a fall in the CIA parameter, χ: 0,8 $\Rightarrow$ 0,7, are summarized in (18). As less money is required for transactions, the rate of interest is increased. Both man and woman reduces their work time, the woman reduces a little more than the man. The fall in the capital intensity leads to the falls in the woman's and man's wage rates. As both the man and woman reduce work time, the total labour input is reduced. As the consumption level, the rate of interest and the durable goods are all increased, the net effect on money holding is positive. The total capital, the total output, the wealth and consumption levels per-capita are all reduced.

$$\bar{\Delta}r = 0{,}18, \quad \bar{\Delta}k_{\mathrm{i}} = -0{,}19, \quad \bar{\Delta}f = \bar{\Delta}w_1 = \bar{\Delta}w_2 = -0{,}07, \quad \bar{\Delta}T_1 = -0{,}08, \quad \bar{\Delta}T_2 = -0{,}22,$$
$$\bar{\Delta}m = 0{,}17, \quad \bar{\Delta}\bar{k} = -0{,}08, \quad \bar{\Delta}k_{\mathrm{h}} = 0{,}04, \quad \bar{\Delta}c = 0{,}17, \quad \bar{\Delta}N = -0{,}12,$$
$$\bar{\Delta}K_{\mathrm{i}} = -0{,}31, \quad \bar{\Delta}K_{\mathrm{h}} = 0{,}04, \quad \bar{\Delta}K = -0{,}08, \quad \bar{\Delta}F = -0{,}18. \tag{18}$$

We now increase the inflation policy, μ, from 0,02 to 0,03. The rate of interest is reduced and the capital intensity reduces which also leads to the rises in the wage rates. Both woman and man reduce their work time. Hence, a higher inflation rate makes people enjoy longer leisure hours and better housing conditions. Although the wealth per household is increased, the consumption level per household is reduced. The rest results are given in (19).

$$\bar{\Delta}r = -1{,}36, \quad \bar{\Delta}k_{\mathrm{i}} = 1{,}48, \quad \bar{\Delta}f = \bar{\Delta}w_1 = \bar{\Delta}w_2 = 0{,}49, \quad \bar{\Delta}T_1 = -0{,}41, \quad \bar{\Delta}T_2 = -1{,}16,$$
$$\bar{\Delta}m = -0{,}12, \quad \bar{\Delta}\bar{k} = 0{,}86, \quad \bar{\Delta}k_{\mathrm{h}} = 0{,}86, \quad \bar{\Delta}c = -0{,}12, \quad \bar{\Delta}N = -0{,}61,$$
$$\bar{\Delta}K_{\mathrm{i}} = 0{,}86, \quad \bar{\Delta}K_{\mathrm{h}} = 0{,}86, \quad \bar{\Delta}K = 0{,}86, \quad \bar{\Delta}F = -0{,}13. \tag{19}$$

CONCLUSIONS

This paper proposed a growth model with money, capital accumulation and endogenous labour supply. We showed how to calculate the time-dependent path of the dynamic system with given initial conditions and also simulated the motion of model with the Cobb-Douglas production function. As the monetary economic system is unstable, the perfectly competitive economy may either experience unlimited growth or economic crisis. As the economy without money will converge to its long-term equilibrium point, we see that the introduction of money makes the system unstable. We also carry out comparative static analysis with regard to some parameters. We may also generalize our model in different ways. It is well known that one-sector growth model has been generalized and extended in many directions. It is not difficult to generalize our model along these lines. It is straightforward to develop the model in discrete time. We may analyze behavior of the model with other forms of production or utility functions. There are multiple production sectors and households are not homogenous.

APPENDIX: PROVING LEMMA 1

First, from $r + \delta_{\mathrm{k}} = \alpha f/k_{\mathrm{i}}$ in equations (2) and $(1 + \chi\pi)(r + \delta_{\mathrm{k}})k_{\mathrm{h}} = \eta\bar{y}$, we obtain

$$k_{\mathrm{h}} = \frac{\eta\bar{y}k_{\mathrm{i}}}{\alpha\chi\tilde{\pi}f} \tag{A1}$$

where the time index is suppressed wherever no confusion and $\tilde{\pi} \equiv 1/\chi + \pi$. We note that r and w_{j} are uniquely determined as functions of k_{i} by (2). From $T_{\mathrm{q}} + \bar{T}_q = T_0$ (2) and (8), we calculate

$$w_2 = hw_1, \quad \overline{T}_2 = \sigma\overline{T}_1, \quad T_1 = T_0 - \overline{T}_1, \quad T_2 = T_0 - \sigma\overline{T}_1, \tag{A2}$$

where $h \equiv h_2/h_1$ and $\sigma \equiv \sigma_2/(h\sigma_1)$. From the definition of $\bar{y}$, we have

$$\bar{y} = \left(\delta + \frac{\alpha f}{k_i}\right)\bar{k} + \beta h^* f + (1+\mu)m, \tag{A3}$$

where $\delta \equiv 1 - \delta_k$ and $h^* \equiv (h_1 + h_2)T_0$, and we use (2), (3) and $a = \bar{k} + m$. From $w_1\overline{T}_1 = \sigma_1\bar{y}$ and (2), $\overline{T}_1 = (\sigma_1\bar{y})/(h_1\beta f)$ holds. From this equation, $\tilde{T} = h_1T_1 + h_2T_2$ and (A2), we have

$$\tilde{T} = h^* - \frac{(\sigma_1 + \sigma_2)\bar{y}}{\beta f}. \tag{A4}$$

From (10), (A1), (A3) and (A4), we have

$$\bar{k} = \left[h^* + \beta h^* \varphi_\pi + (1+\mu)\frac{m\varphi_\pi}{f}\right]\left[\frac{1}{k_i} - \left(\delta + \frac{\alpha f}{k_i}\right)\frac{\varphi_\pi}{f}\right]^{-1}, \tag{A5}$$

where

$$\varphi_\pi(\pi) \equiv \frac{\bar{\eta}}{\tilde{\pi}} - \tilde{\sigma}, \quad \bar{\eta} \equiv \frac{\eta}{\alpha\chi}, \quad \tilde{\sigma} \equiv \frac{(\sigma_1 + \sigma_2)}{\beta}.$$

Substituting (8) into (6) yields

$$m = \frac{(\xi + \eta)\bar{y}}{\chi\tilde{\pi}}.$$

Insert (A3) and (A5) in the above equation

$$\pi = \Lambda_0(m, k_i) \equiv \left[\frac{\beta h^* f + m + \mu m}{\delta k_i + \alpha f} + h^* + \frac{\chi\bar{\eta}m}{(\xi+\eta)f}\right]\frac{(\xi+\eta)f/(\chi m)}{\tilde{\sigma} + f(\delta k_i + \alpha f)^{-1}} - \frac{1}{\chi}. \tag{A6}$$

From this equation and $\pi = \mu - \dot{m}/m$, we obtain

$$\dot{m} = \Lambda(m, k_i) \equiv \mu m - m\Lambda_0(m, k_i). \tag{A7}$$

From (A5) and (A6), we have

$$\bar{k} = \overline{\Lambda}(m, k_i) \equiv \left[h^* + \beta h^* \varphi_\pi + (1+\mu)\frac{m\varphi_\pi}{f}\right]\left[\frac{1}{k_i} - \left(\delta + \frac{\alpha f}{k_i}\right)\frac{\varphi_\pi}{f}\right]^{-1}, \tag{A8}$$

Hence we can represent $\bar{k}$ as a function of m and k_i From (9), (A3), $s = \lambda\bar{y}$ and $a = \bar{k} + m$, we have

$$\dot{\bar{k}} = \lambda\left[\left(\delta + \frac{\alpha f}{k_i} - \frac{1}{\lambda}\right)\overline{\Lambda}(m, k_i) + \beta h^* f + \left(1 + \mu - \frac{1}{\lambda}\right)m\right] - \Lambda(m, k_i). \tag{A9}$$

Taking derivatives of (A8) with respect to t yields

$$\dot{\bar{k}} = \overline{\Lambda}_m\Lambda + \overline{\Lambda}_{k_i}\dot{k}_i,$$

where $\overline{\Lambda}_m$ and $\overline{\Lambda}_{k_i}$ are respectively partial derivatives of $\overline{\Lambda}$ with respect to m and k_i. Substituting (A7) and the above equation into (A9) yields

$$\dot{k}_i = \Omega(k_i, m) \equiv \frac{\lambda\left[(\delta + \alpha f/k_i - 1/\lambda)\overline{\Lambda} + \beta h^* f + (1+\mu-1/\lambda)m\right] - \Lambda - \overline{\Lambda}_m\Lambda}{\overline{\Lambda}_{k_i}}. \tag{A10}$$

REMARKS

[1]The model is a simplified version of the growth model with housing and location proposed by Zhang [33]. As pointed out, the stock of household capital actually exceeds market capital [34]. Benhabib *et al.* estimate that the output of the household sector may be as much as half that of the market sector and that labour hours in the home sector are almost as great as in the market

sector. Hence, proper introduction of household capital in growth theory is essential [35]. We refer monetary growth models with durable consumption goods to, for instance [36, 37].

[2]We neglect complicated issues related to endogenous fertility rates. It can be seen that on the basis of the literature of growth models with endogenous fertility, for instance [38-40], it is possible to generalize our model to take account of endogenous fertility.

[3]A recent explanation and its relations to traditional approaches to consumer behavior are provided by Zhang [7, 41].

[4]We only consider money for purchasing consumption goods and neglect possible money required for other purposes, such as investment. See [7] for how to take money for different purposes within the framework. By the way, Suen and Yip discuss how CIA constrains may affect the number of steady states and their stability properties in the Ramsey approach [26].

[5]According to Stockman, investment should also be taken into account. For simplifying the analysis, this study does not take account of the need of money for investment [12].

[6]As shown in [7], it is possible to make the propensities in the utility function as endogenous variables, dependent on conditions such as incomes, wages and social status.

[7]We also cite Ferber: "Actually, the honor of pioneering research on and analysis of the household as an economic unit properly belongs mainly to Margaret Reid [31], who in turn gave a great deal of credit to Hazel Kyrk, her teacher and mentor. … Interestingly both Kyrk and Reid were also on the faculty of the University of Chicago, which makes Becker's (and the profession's) failure to acknowledge their work all the more surprising" [32].

[8]See also [42, 43].

[9]See, for instance, [44, 45].

[10]As demonstrated late on, the model has a saddle point. Hence, the system may either grows infinite or collapses if not starting at a proper initial condition. Hence, we stimulate the model with very short period of time.

[11]As demonstrated in [7], if the role of money is neglected and the economy includes only "real variables", then the economic growth model is characterized of a unique stable equilibrium. The introduction of money causes instabilities in the model.

[12]It is important to examine impact of changes in parameters on dynamic processes of the system. As the system is unstable, we are concerned only with comparative static analysis.

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ELASTIČNA PONUDA RADA I DOMAĆA RADINOST U MODELU MONETARNOG RASTA

W.-B. Zhang

Azijsko pacifičko sveučilište Ritsumeikan
prefektura Oita, Japan

SAŽETAK

U radu se postavlja model spolno razlučivog monetarnog rasta s akumulacijom kapitala i endogenom ponudom radne snage. Model je utemeljen na neoklasičnoj teoriji rasta, dok su monetarni aspekti modela utemeljni na pristupu plaćanja unaprijed. Pokazuje se kako je dinamika pripadne ekonomije opisana dvodimenzionalnim diferencijalnim jednadžbama. Simulirana je ravnoteža i pomak ekonomije s postavljenim monetarnim politikama, prioritetima kućanstava i tehnologijom. Budući da je monetarna ekonomija nestabilna, ekonomija može ili pokazivati neograničeni rast, ili krizu. Također je proučeno kako promjene nekih parametara utječu na ekonomsku ravnotežu. Primjerice, kako žene povećavaju sklonost ostanku kod kuće, intenzitet kapitala se ne mijenja pa se slijedom toga ne mijenjaju niti iznosi nadnica, niti stopa rezultata rada po jedinici uloženog rada. Neovisno o tome, kako muškarci povećavaju svoje radno vrijeme a žene smanjuju, smanjuje se ponuda radne snage. Novac, trajnija dobra koja se upotrebljavaju u kućanstvima te razina potrošnje po kućanstvu se smanjuju. Također, ukupno bogatstvo, ukupni unos kapitala, ukupna količina trajnijih dobara i ukupni rezultati se smanjuju.

KLJUČNE RIJEČI

monetarni rast, rod, akumulacija kapitala, novac, inflacija, plaćanje unaprijed, CIA