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MATHEMATICAL MODELLING WITH FUZZY SETS OF SUSTAINABLE TOURISM DEVELOPMENT

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ABSTRACT

In the first part of the study we introduce fuzzy sets that correspond to comparative indicators for measuring sustainable development of tourism. In the second part of the study it is shown, on the base of model created, how one can determine the value of sustainable tourism development in protected areas based on the following established groups of indicators: to assess the economic status, to assess the impact of tourism on the social component, to assess the impact of tourism on cultural identity, to assess the environmental conditions and indicators as well as to assess tourist satisfaction, all using fuzzy logic.

It is also shown how to test the confidence in the rules by which, according to experts, appropriate decisions can be created in order to protect biodiversity of protected areas.

KEY WORDS

mathematical modelling, sustainable tourism development, protected areas, fuzzy logic, comparative indicators

CLASSIFICATION

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INTRODUCTION

For managers of protected area the most important are changes that are taking place in the economic, political, socio-cultural, technological and ecological environment. Changes that occur in these environments managers must register but also understand and accept as a starting point for their decisions related to the management of the area. Furthermore, it is not only important to register these changes but to be able to evaluate their impact on biodiversity of the area. In order to evaluate the impact of tourism development on biodiversity we use the indicators that identify that impact. How to measure sustainable development indicators is still an open question. Many institutions dealing with sustainable tourism development have their own proposals for indicators which determine the sustainability of tourism development [1-9].

Important place occupy indicators suggested by the experts of the World Tourism Organization (WTO) and comparative indicators recommended by the experts of the European Union. For some of comparative indicators unique measurable parameters are not yet established and the indicators of the WTO are numerous and their determination is time-consuming process. This study deals with certain comparative indicators for measuring sustainable development of tourism in the protected area (PA) using fuzzy sets. The aim of this study is, using fuzzy sets, to reduce the possibility of wrong decisions that could be caused by imprecise measurement of indicator or by impossibility to determine the indicator itself and to reduce the impact of subjectivity that exists in evaluation of comparative indicators. We start with an idea to express comparative indicators in the form of fuzzy sets in order to avoid problems that occur because of the strict limits when we measure sustainability indicators and take appropriate actions related to the results applied to all destinations.

As every tourist destination is rich in its characteristics (especially expressed in PA as a tourist destination) it is expected to have large inaccuracy when the same indicator values are applied to different destinations. In order to evaluate the impact of tourism development on the PA's environment besides suggested indicators, it is, also, suggested an indicator of exploitation of forest ecosystems that can be used in the PA which has such ecosystems. For this indicator certain limits of impact, based on the way of PA's financing and the participation of PA's funds (generated by the exploitation of forest ecosystems) in the total budget necessary to maintain the PA's functioning, are suggested.

MAIN ATTITUDES AND SUSTAINABILITY EVALUATION OF TOURISM DEVELOPMENT BY COMPARATIVE INDICATORS

The suggested comparative indicators for evaluation of sustainability of development of tourist destinations have been made to integrate economic, ecological, social and cultural factors as well as measuring of tourists' satisfaction with the offered services. These factors were decisive in the grouping of indicators which measure sustainability (intensity of the sustainable development) of tourist destination. On this basis, comparable indicators for evaluation of sustainability of tourist destination are classified into the following groups:

1. **group of indicators for evaluation of the economic state:** shows the intensity of the economic impact of tourism business in the tourist place, destination or area,
2. **group of indicators for evaluation of social component:** reflects the social integrity of the local community in terms of subjective well-being and benefits that tourism brings to local population,
3. **group of indicators for evaluation of the impact on cultural identity**: express the level of preservation of cultural identity of local community under the influence of visitors who carry different cultural integrity,

4. **group of indicators for evaluation of environmental conditions:** identify environmental conditions under the influence of tourism activities in the monitored area,
5. **group of indicators for measuring satisfaction of tourists**: identify level of satisfaction of tourists that visit the destination as well as comments about the attractiveness of the destination [10, p. 36].

Based on the evaluation of the primary inputs control actions are determined in order to raise the management of destination to a higher level of sustainability. Management actions that managers take are conditioned by the indicators' value evaluation. These values are classified into three zones. These zones are:

- **red area**: conditions in the area were rated as critical and it is necessary to take appropriate actions in order to put under control further tourism development in the area and stop its destructive effect on environment. If necessary, on some parts of the area further tourist activities should be banned, either on a shorter period or permanently,
- **yellow area:** situation is evaluated as tolerable and further trend of progressive tourism development will create certain negative consequences for the environment and biodiversity and certain protective measures should be taken,
- **green area:** the condition of tourism development in observed tourist destination is evaluated as sustainable and destination management is good and liable.

Let us note, regarding the comparative indicators as suggested by the EU-experts, that for some of them precise limit values are determined (based on some earlier researches) while for other indicators there is no unique attitude regarding their signs and consequently no precise limit values, as lsited in Table 1.

Considering that imprecision follows this type of research, because of subjective or objective reasons, indicators have different forms with different researchers. Some of the inaccuracies that occur in evaluation of indicators are:

- every protected area has specific management goals related to the protection of biodiversity and opportunities of development of compatible tourism,
- values of indicators also contain subjective opinions and views of the person who measure and evaluate them so they have influence on decisions and measures taken on the basis of these results. As a result, these decisions are often unjust, rushed or wrong,
- some of the indicators are applicable only to measure the sustainability of tourism destinations where mass tourism is realized, and for protected area as a tourist destination there is no place for classical mass tourism,
- indicators of the environmental conditions do not contain information on biodiversity and as such are incomplete in the application for measuring the environmental conditions in the protected area. Considering that the most of the protected areas are placed in highland this indicator should contain information on the intensity of exploitation of forest ecosystems.

Uncertainty, imprecision and other ambiguities can be reduced by using the approximate method that will absorb them and which is based on the application of fuzzy sets theory. The idea is that each of the comparative indicators appear in the form of fuzzy number to recap the conclusion using fuzzy logic for each of the options that appears. During the modelling (by fuzzy theory) of a problem a number of possibilities (the rules) occur. Number of rules is related to the number of variables that appear in the analysis of the problem. If considered individually all the options that appear in this problem, then it would be a huge job. Besides

Table 1. Limit values for some of comparative indicators [10, p. 42].

Effects	Indicators	Limit values and meaning
Economic effects of tourism development	Income season character: Percentage of visits in full time season (3 months)	less than 40 % green area 40 % - 50 % yellow area more than 50 % red area
	Ratio of number of overnights and accommodation capacity	more than 150 green area 120 - 150 yellow area less than 120 red area
	Coefficient of local tourist gain	still not determined
Tourist satisfaction	Repeated visit - percentage of repeated visits in 5 years period	more than 50 % green area, 30 % - 50 % yellow area, less than 30 % red area
	Ratio of accommodation capacity and number of local people	less than 1,1:1 green area 1,1:1 - 1,5:1 yellow area, more than 1,6:1 red area
Cultural	Tourism intensity: ratio of number of overnights (in thousands) and local population (in hundreds)	less than 1,1:1 green area 1,1:1 - 1,5:1 yellow area more than 1,6:1 red area
	Participation of tourism in local netto social product	Should be compared with participation of tourism in the local employment
Social	Percentage of tourist that are not coming with tourist agencies	More than 70 % green area 50 % - 70 % yellow area less than 50 % red area
Environment conditions	Land – percentage of land where construction is allowed but not yet accomplished	less than 10 % green area 10 % - 20 % yellow area more than 20 % green area
	Utilization and occupation of land – percentage of changes in land occupation with buildings in 5 years time	not determined
	Traffic – percentage of tourists who are not coming with private vehicle	More than 20 % green area 10 % - 20 % yellow area less than 10 % red area

that, the subject of our interest is not to analyze these rules individually, but to demonstrate that using the theory of fuzzy logic we can avoid certain errors that occur when placing strict limits in evaluation of indicators and that it is possible to determine the intensity of sustainable tourism development using fuzzy logic. Testing of rules will be done using the software MATLABR12 *Fuzzy Logic Toolbox*.

BASIC TERMS AND OPERATIONS WITH FUZZY SETS

DEFINITION OF FUZZY SETS

In classical theory there are very precise boundaries that separate the elements that belong to a particular set from those elements that do not belong to it. In other words, for every observed element we easily determine if it belongs or not to a particular set.

Therefore, the classical set theory starts from the position that an element x of (universal) set X belongs or not to a particular subset M. Affiliation to set M is conditioned with characteristic of elements, in other words with conditions that element has to fulfil in order to belong to the M set. Function

$$\mu_M(x) = \begin{cases} 1, & x \in M, \\ 0, & x \notin M, \end{cases}$$

is called membership function of set M. So, $\mu_M(x) \in \{0, 1\}$is set of values of membership function. For example, for set $X = R$, the subset M is defined as a set whose elements are real numbers between 5 and 10, including 5 and 10,

$$M = \{x | x \in \mathrm{R},\ 5 \leq x \leq 10\}.$$

According to this definition, number 4,9 is not an element of set M, while number 5,1 is an element of M. Large number of sets that we use in everyday life has no precise limits that separate elements that belong to a set of those that do not.

Let us suppose that X represents character of evaluation of visit to the tourist destination in the one year time at the locality L_1 and that the visit is described with statements indicated as sets:
A_1: if the number of visitors is less than 17 000 visit is described as “weak”,
A_2: if the number of visitors ranges between 17 000 and 25 000 visit is described as “good”,
A_3: if the number of visitors is more than 25 000 visit is described as “excellent”.
We symbolically denote sets A_1, A_2 and A_3 as follows:

$$A_1 = \{x \mid x \in \mathbb{N} \wedge x < 17\,000\},\ A_2 = \{x \mid x \in \mathbb{N} \wedge 17\,000 \leq x \leq 25\,000\},\ A_3 = \{x \mid x \in \mathbb{N} \wedge x > 25\,000\}.$$

Membership functions of A_1, A_2 and A_3 basic sets are represented through values of characteristic functions, in case that we observed visits in the last several years (Fig. 1.):

x_1 = 16 800 - number of visitors in 2007, visit is described as “weak”,
x_2 = 17 100 - number of visitors in 2008, visit is described as “good”,
x_3 = 24 500 - number of visitors in 2009, visit is described as “good“,
x_4 = 25 100 - number of visitors in 2010, visit is described as “excellent“.

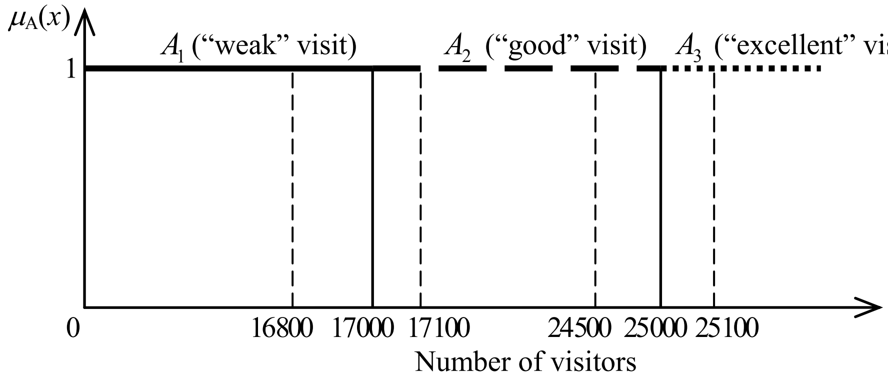

Figure 1. Membership function of A_1, A_2 and A_3 with x_1, x_2, x_3 and x_4 as arguments.

If we present membership function in the form of table, putting the value of 1 if $x_i \in A_i$ and 0 if $x_i \notin A_i$ (with i = 1, 2, 3 or 4 in both cases) then we obtain data as in Table 2.

If we replace {0, 1} with [0, 1], then affiliation of element to A_1, A_2 and A_3 becomes graduated. Denote, once again, with $\mu_A(x)$ membership function that defines in what amount some element of universal set X belongs to subset A. If there is “more truth” that element x belongs to subset A than element y then $\mu_A(x) > \mu_A(y)$. Correspondingly, element x in a greater degree fulfils the requirements of set A membership. For the membership function we have $0 \leq \mu_A(x) \leq 1$, $\forall x \in A$. We define fuzzy set A as set of ordered pairs:

$$A = \{(x, \mu_A(x)) \mid x \in X, \wedge \mu_A(x) \in 17\,000\},$$

Table 2. Values of membership function of A_i sets (sets with strict limits).

Number of visitors	Description of visit		
	Weak	Good	Excellent
16 800	1	0	0
17 100	0	1	0
24 500	0	1	0
25 100	0	0	1

where X represents universal set, and $\mu_A(x)$ is membership function of A for element x [11].

Every fuzzy set is uniquely defined with its membership function. If we represent sets A_1, A_2 and A_3 from our example, in the form of fuzzy sets, using, for example, trapezoidal, then membership function will have values in segment [0, 1], Figure 2.

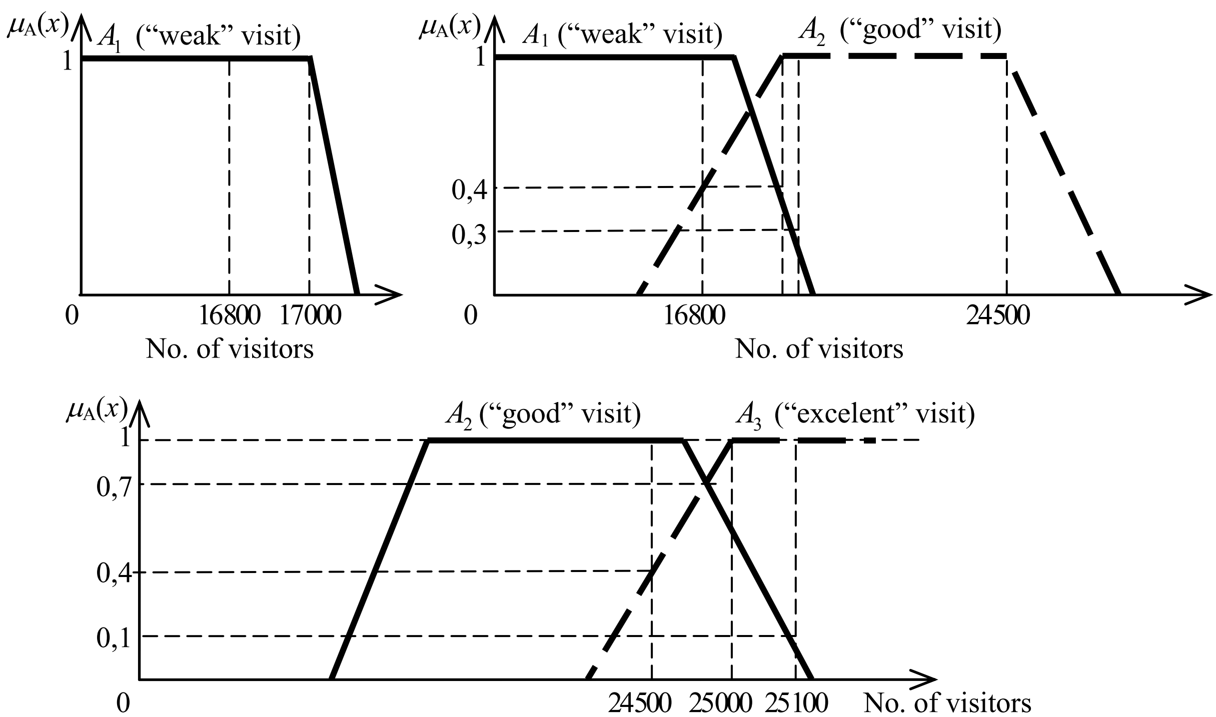

Figure 2. Sets A_1, A_2 and A_3 with x_1, x_2, x_3 and x_4 as arguments and theirs degrees of affiliation.

Hence the visit of 16 800 visitors is described as "weak" (observed with clasical membership function) because, according to definition of A_1, it belongs to A_1. If observed with modified membership function, the same visit of 16 800 visitors can be described as "weak" (value of membership function 1) but also as "good" (value of membership function 0,4). Similarly, number of visits in 2008, x_3 = 24 500, can be described as "good" (observed with classical membership function) or, if observed with modified membership function, as "excellent" (value of membership function 0,4).

Table 3. Values of membership function of fuzzy sets A_1, A_2 and A_3.

No of visitors	Weak	Good	Excellent
x_1 = 16 800	1	0,40	0
x_2 = 17 100	0,3	1	0
x_3 = 24 500	0	1	0,4
x_4 = 25 100	0	0,1	1

So if one needs to make certain management decisions on the basis of affiliation to sets with precise limits (first case in our example) such decisions will certainly produce some errors because non-objective inputs are used, while access to evaluation of affiliation to set with "soft" limits (fuzzy sets) will not have such anomalies. Therefore, the evaluation with fuzzy sets is more objective.

BASIC OPERATIONS WITH FUZZY SETS

Let us suppose that fuzzy sets *A* and *B* are given, subsets of the same universal set. Union of fuzzy sets *A* and *B* (denoted as $A \cup B$) is defined as the smallest fuzzy set that contains both the fuzzy set *A* and the fuzzy set *B*. Union corresponds to the operation "OR". Membership function is defined as follows: $\mu_{A\cup B}(x) = \max[\mu_A(x), \mu_B(x)]$.

Intersection of fuzzy sets *A* and *B* (denoted as $A \cap B$) is defined as the biggest fuzzy set which is contained in both fuzzy sets, *A* and *B*, at the same time. Intersection corresponds to operation "AND". Membership function is defined as follows: $\mu_{A\cap B}(x) = \max[\mu_A(x), \mu_B(x)]$.

Complement of fuzzy set *A* (denoted as $\bar{A}$ or A^C) represents fuzzy set which membership function is $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$ [11]. These operations are the basic operations, that define the min-max theory of fuzzy sets. Let us suppose that we are given fuzzy sets *A* and *B*, Figure 3. Then the membership functions of fuzzy sets $A \cup B$, $A \cap B$ and $\bar{A} = A^C$ are given in Figure 4 [12].

DEFINITION OF FUZZY NUMBER

Fuzzy number is normalized and convex fuzzy set, that is characterised by confidence interval $[a_1, a_2]$ and level of security α. Figure 5 represents fuzzy number *A* and appropriate confidence interval and level of security α for confidence interval [13, pp. 11-15].

SOME FORMS OF MEMBERSHIP FUNCTION

The most commonly used classes of fuzzy numbers are *triangular* and *trapezoidal* fuzzy numbers. Triangular fuzzy number (Fig. 6.) depends on the form of membership function. We present it in the form of ordered triple $A = (a_1, a_2, a_3)$ where a_1 is a bottom limit of fuzzy number, a_2 value of fuzzy number with highest level of affiliation and a_3 a top limit of fuzzy number.

Second class makes trapezoidal fuzzy numbers, presented in the form of ordered quadruple $A = (a_1, a_2, a_3, a_4)$, Figure 7. Value of variable *x*, for which $\mu_A(x) = 1$, is centre of fuzzy set *A*.

For practical application of fuzzy logic, for the purpose of system control, existence of a fuzzy set with one element, for which the membership function has value 1, is of a particular importance. This set is called a fuzzy set of singleton type.

FUZZY RELATIONSHIPS, RULES AND CONCLUSION

The complexity of human behavior cannot be described by mathematical theory, but it is simple to describe (by fuzzy numbers) the way how the man manages certain technical system (for example vehicle, camera, phone, number of vehicles present in the parking).

When the model of human thinking is formed, that is expressed in words and sentences of spoken language, its start position is based on fuzzy propositions. Fuzzy proposition basic form is "x is A", where *A* represents language value which is given with the fuzzy set, a set upon the definition area of the language variable *x*.

Fuzzy proposition determines the degree of affiliation of the variable *x* to the fuzzy set *A*. In general case variable *x* can be expressed by numerical value or as a fuzzy set. In linking in

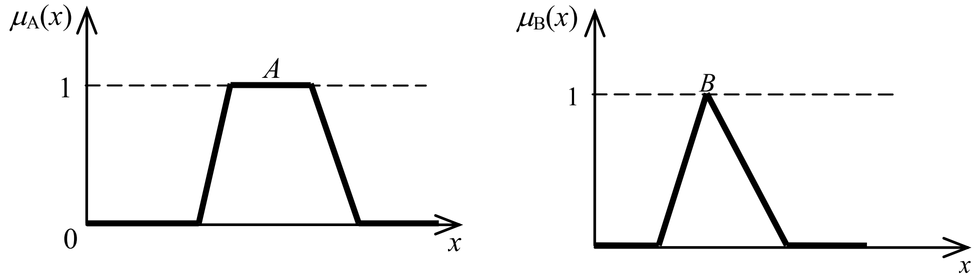

Figure 3. Graphical representations of sets A and B, $\mu(\cdot)$ – membership functions.

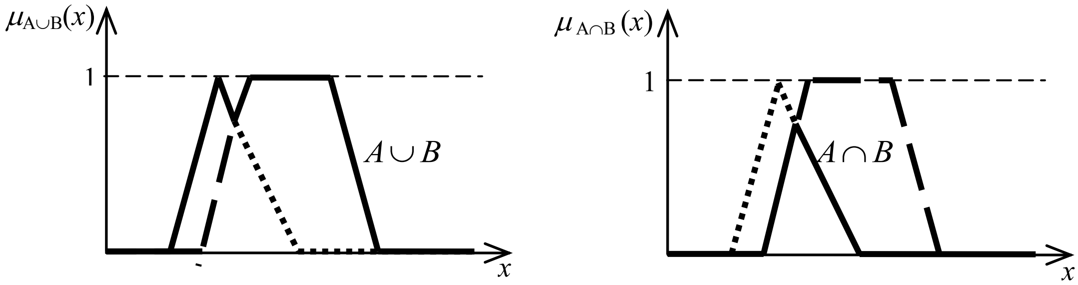

Figure 4. Membership functions of sets a) $A \cup B$ and b) $A \cap B$, for sets A and B shown in Fig. 3.

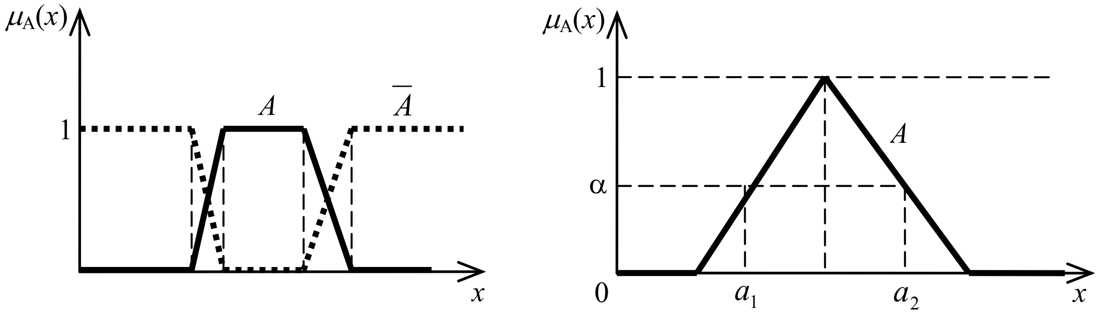

Figure 5. a) Membership function of fuzzy complement. b) Fuzzy number A with confidence interval $[a_1, a_2]$ and level of security α.

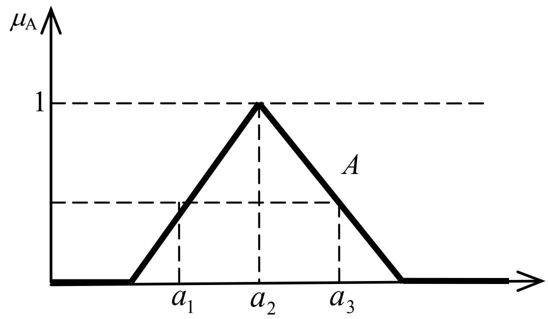

Figure 6. Triangular fuzzy number A.

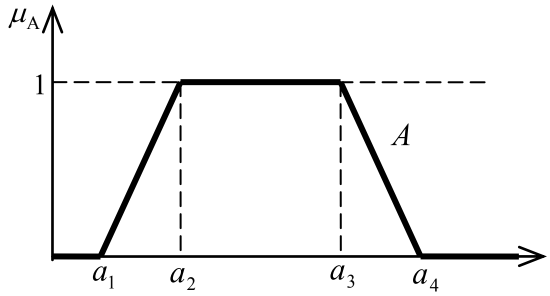

Figure 7. Trapezoidal fuzzy number A.

connecting of propositions certain conjunctions (operators) are used (and, or, if-then). In combination of propositions and conjunction we create a fuzzy rule, which mostly is stated as

IF x is A and y is B THEN z is C.

Statements "x is A" and "y is B" are premises (or condition), while "z is C" represents conclusion or consequence of a rule.

To describe the chosen system process or event we need large number of rules, therefore we talk about the set of fuzzy rules. For mathematical display of fuzzy rules we use fuzzy relations that define quantitative relation between variable conditions and variable conclusions. Certain possibility of partial truth with premise and conclusion is allowed in fuzzy logic. Applying the fuzzy set concept and associated procedure of fuzzy conclusions it is possible to quantify the idea of proximate conclusion which is the main characteristic of human thinking.

Managing model is reflected in a series of logical rules, and general conclusion form for the purpose of managing is:

IF (information about the system condition) THEN (managing information).

Information about condition and managing are language variables and related fuzzy sets are joined to them. Information about system condition is built by logical operations with fuzzy sets which are previously mentioned: (they are AND, OR, NO). Mainly used definitions are:

$$\text{(OR):} \quad C = A \cup B = \max\{A, B\}, \quad \mu_C(x) = \max\{\mu_A(x), \mu_B(x)\},$$

$$\text{(AND):} \quad C = A \cap B = \min\{A, B\}, \quad \mu_C(x) = \min\{\mu_A(x), \mu_B(x)\},$$

$$\text{(NO):} \quad C = \overline{A} \quad \mu_C(x) = 1 - \mu_A(x).$$

Models based on fuzzy logic often require more iterations. First we have to define set of rules and corresponding affiliation functions. After observing the result, we make (if necessary) corrections of certain rules and/or affiliation functions. Then, once more, using our modified rules and/or functions we test our model.

Let us use stated logical rules and economical comparative indicator given in Table to illustrate a correlation between fuzzy logical rules:

IF the visit seasonality is at the sustainable level and accommodation capacity utilization tolerable and an influence on the local tourist increase of tourism development has no importance

OR

IF the seasonality is tolerable and the accommodation capacity utilization unsustainable and local tourist increase has importance

OR

IF the visit seasonality is at the tolerable level and accommodation capacity utilization at an unsustainable level and the local tourist increase of tourism development has importance

OR

IF visit seasonality is alarming and utilization at the sustainable level and local tourist increase of tourism development has importance

OR

THEN economic influence of tourism development, in the protected area, is at the sustainable level.

A problem with fuzzy conclusions is associated with the cause and effect relation between two different statements. It is also associated with the defining of statement's truth value which occurs as a conclusion based upon the truth value of a statement which represents the premise. Relation between two statements describes fuzzy control rule.

Fuzzy control rule can be stated as:

$$R_i\text{: IF } d_j \text{ THEN } d_k \text{ (CF} = \mu_i\text{)}$$

where d_j and d_k are statements which may have fuzzy variables whose truth values ranges between 0 and 1. Factor μ_i is a certainty factor (CF). Its truth value ranges between 0 and 1, and it represents the degree of trust in rule's accuracy. The higher truth value, the greater trust in the rule. For example, with fuzzy control rule used here, we conclude about an action which has to be taken. A necessity for that action is based on the state evaluation of negative influence of tourism development on protection of biodiversity in a certain area of PA. That rule is expressed as follows:

R_1: IF state in the area is bad THEN it is necessary to reduce the number of tourists (CF = μ).

In this way we simulate the process of fuzzy conclusion where, based on the truth value of the "state is bad" statement, we make a conclusion about the truth value statement "necessary to reduce". If the fuzzy control rule includes "and" or "or" conjunctions, then it is called complex or mixed rule.

CONTROL OF FUZZY LOGIC MODEL

Fuzzy sets theory allow us to observe insufficiently precise phenomena which we are not able to model using the probability theory or interval mathematics. In other words, when intangibility derives from inaccuracies in communication among two people (e.g. tall people, low temperature, weak sale, great pollution, good visit), that intangibility is modeled by fuzzy sets theory.

Fuzzy description of a certain system, opposed to exact mathematical and static description, is not uniquely determined. Even though it seems somewhat arbitrary and irrational, it is still a very rational approach, especially when it comes to managing complex systems. When we want to describe a certain complex system with precise relations we come across more complex mathematical problems which require larger number of parameters and more complex numerical solving methods. With fuzzy approach, we rationally approximate the system description based on the model as seen by an expert for observation of technological process which solves the managing task.

Fuzzy control model can be split into three basic steps: (1) conversion of input information that come from the real world (system) into a conceptual model expressed by fuzzy logic, (2) application of conclusion procedures based on fuzzy logic, and (3) conversion of fuzzy conclusion results into real system managing variables.

First step where the input information are converted with into fuzzy sets is called fuzzyfication, while the reversed process of converting conclusions of fuzzy logic into managing values of the real system (managing actions) is called defuzzyfication.

Fuzzyfication is converting process of outer data into inner (fuzzy) form, so that it can be used by the determining system. We often use fuzzy singleton for converting numerical data into fuzzy set. Fuzzy singleton is a fuzzy set whose affiliation function has value 1 for $x = x_0$, though for other values $x \neq x_0$ it has value 0**.**

COMPARATIVE INDICATORS AND FUZZY NUMBERS

INDICATORS OF ECONOMIC STATE INFLUENCE

Visit seasonality

Economic benefit achieved by the protected area of the tourist trade that is related or is based on its attributes is largely dependent of the number of tourists who visit it. In other words, if fewer tourists visit the area we can, unquestionably, expect smaller economic benefit of the tourist activity that is implemented. Therefore, larger number of tourists brings bigger economic benefit. Besides the economic benefit, number of visitors is related with the negative effects that follow tourist trade in protected area.

In our approach of evaluation of the tourism development sustainability in protected area, as a ground base we will use results that are submitted by researchers who measured the indicators of tourism development sustainability. One may argue that the ideal situation of tourist distribution in the area in a year is the one in which during full time season (3 months) we make 30 % of the annual revenue (10 % a month) what is for most tourist areas elusive [10].

Because of that, UN suggested the following, more tolerable and in practice more realistic, attitude toward evaluating seasonal concentration of tourist trade in the high season (3 months) like:

1. if percentage of visits in full time season makes less than 40 % of annual tourist visits to the area than we consider, based on this economic indicator, that this visit distribution provides opportunities for sustainable tourism development in the area,
2. if percentage of visits in full time season ranges between 40 % and 50 % of annual tourist visit; in other words, if tourist trade in three months season makes 40 % - 50 % of annual tourist trade in protected area then it is necessary to take certain precautions because that seasonality trade is unsustainable, and further more it has negative impact on the protected area,
3. if percentage of visits in full time season makes more than 50 % of annual tourist visit in the protected area, then this situation is alarming and it has its economic and ecologic consequences.

Let us apply, regarding this criteria, fuzzy logic on seasonality indicators using following symbols and terminology: S is set of all fuzzy sets which we will use to describe an economic indicator related to the tourist trade implemented by the protected area in one year time, based on the realization percent of the trade in high season (trade percent that is made in the most profitable three months). Symbols are:

- S_0 is fuzzy number which indicates that “seasonality is sustainable”, in the case that tourist trade percent in season is lower than 0,4 (less than 40 %) of annual tourist trade,
- S_n is fuzzy number which indicates that “seasonality is unsustainable”, in the case that tourist trade value ranges between 0,4 and 0,5 (40 % - 50 %) of annual tourist trade,
- S_a is fuzzy number which indicates that “seasonality is alarming”, in the case that the tourist trade value is more than 0,5 (more than 50 %) of annual tourist trade.

Therefore, set S contains S_o, S_n, S_a, and $S = \{S_o, S_n, S_a\}$. Sets S_o, S_n and S_a indicate the state of visit seasonality which is made in high season (three months) in the protected area, which represents partial value of economic indicator of the tourist development activity.

Based on these indicators we must gain insight into economic effects which tourist activity has in the protected area. Let us show those sets using a membership function and trapezoidal form in displaying fuzzy numbers, Figure 8.

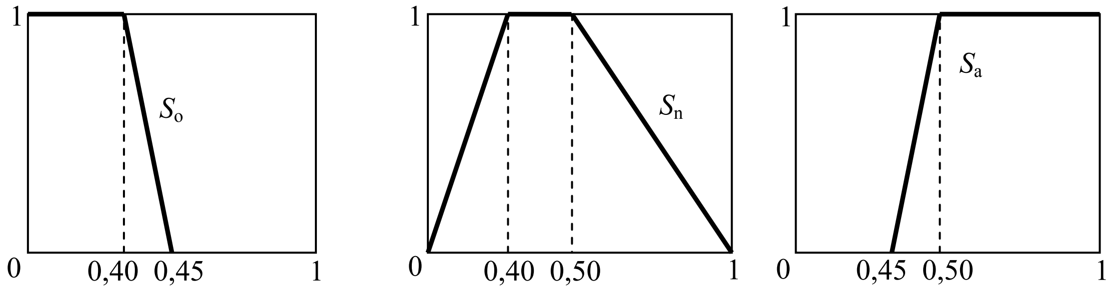

Figure 8. Membership functions of fuzzy sets S_o, S_n and S_a.

Relation between number of tourist's overnights and accommodation facilities

According to the spatial plan of protected areas, and according to the management area aims for the planned locations and its visitors accommodation facilities can be provided. Accommodation facilities can be various: (mountain huts, cottage resorts, rural households, hotels, motels…) depending of the implemented spatial planning activities planned for certain areas within or in the protected part of the protected area. Fulfilling the possibilities of visitors' accommodation in the protected area or on its locality (rural households) an opportunity is provided for the PA management to make certain financial effects of tourist activity.

Size of the accommodation facilities has to be preplanned by the spatial plan (of locality) and management plan (considering capacity of the area). Considering the extent of area accommodation facilities, maximum of economic effects that can be implemented by the PA are determined.

Besides that, relation between tourists' overnights and accommodation facilities that PA disposes with is used as an economic trade indicator which is realized in the protected area. As recommended, utilization of accommodation facilities in annual percent values is evaluated:

1. if the utilization of accommodation facilities in one year time is less than 33 %, economic benefit, that destination has, is unsustainable. In that case, protected area managers have to take certain measures in order to change momentary state,
2. if its utilization ranges between 33 % and 42 % we can say that its utilization is economically tolerable, but still not optimal, and we should seek for other options of improvement and tend to sustainable utilization,
3. in case that utilization is larger than 42 % then we can say that utilization is economically sustainable.

Furthermore, we can notice that this kind of evaluation might be imprecise in economic sustainability evaluation of tourist activity in the area. Indicator alone does not provide possibilities of precise evaluation of tourist trade which is related to the tourist activity in the protected area, because there is a large number of visitors who take part trading but they do not realise their overnights in the destination area (they stay with its friends, relatives, rural households...).

So let us say that K is set of all fuzzy sets with which the indicator of economic trade is described. Economic trade indicator is presented through relation between overnights' number and accommodation capacity with which the area disposes. In that case:

- K_n is fuzzy number that states for "low level of accommodation facility utilization" of the area. It is when the intensity of bed utilization during one year is smaller than 0,33,
- K_p is fuzzy number that states for "tolerable utilization of accommodation facilities" of the area. It is when the intensity ranges between 0,33 and 0,42, i.e. between 33 % and 42 %,
- K_o is fuzzy number that states for "sustainable level of accommodation facilities utilization" of the area. It is when the intensity of bed utilization during one year is larger than 0,43.

Therefore, set K includes elements K_n, K_p and K_o: $K = \{K_n, K_p, K_o\}$. Corresponding membership functions are given in Figure 9.

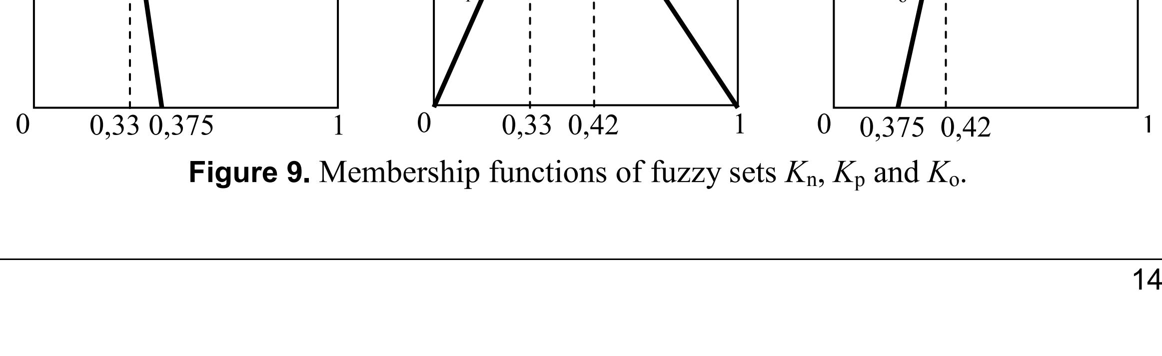

Figure 9. Membership functions of fuzzy sets K_n, K_p and K_o.

Coefficient of local tourist gain

Backbone of tourist activity influence in PA on economic benefit of local community (directly or through complementary trades) is made of segments that manifest this activity through:

- participation of products produced within territorial boundaries of PA of total tourist consumption,
- the share of overnights in municipalities that border with PA and in which PA is included in total number of overnights at country level,
- the share of consumption which is realized in the PA of total consumption of municipalities in which it is placed and which gravitate towards PA.

All these segments create components of economic benefit gain of local people. Therefore, in order to determine coefficient of local tourist gain we need to analyze all aforementioned factors which affect the change of economic situation of local community. The result of those indicators would represent the coefficient values of local tourist gain. It is obvious that determination process of that coefficient is difficult, imprecise and vague.

Considering difficulties with defining this indicator, European experts did not state its strict boundaries, which means that evaluation of this important indicator is given to local managers or tourist's destination experts. To reduce inaccuracy of the evaluation, we consider that PA's tourist activity has certain impact on life standard of local people and that intensity of that impact ranges between 0 % (minimum) and 100 % (maximum). Or, stated differently:

- *tourist gain has certain impact* on local tourist gain,
- in other words, PA's tourist trade *has no special importance* for local tourist gain.

Let us express this statements in fuzzy sets. Set L is a set of all fuzzy sets which describe tourist's activity influence on general gain of economic effects at local people.

- L_n is fuzzy number which states that PA's tourist activity "does not have any significant participation" in economic effect's gain of local people,
- L_z is fuzzy number which states that PA's tourist activity "has significant influence" on economic gain.

Set $L = \{L_n, L_z\}$ and corresponding membership functions are given in Figure 10. According to this representation of tourist's activity influence on local tourist gain, one may conclude that in both cases we can use information which we have, though in most cases it is incomplete.

Therefore, no matter what limits we set as criteria for determining boundaries of participation in local tourist gain, we cannot say that for some numbers the state is sustainable if the necessity for PA's sustainability and development of ecotourism in it does not have alternative, and if we are aware that without satisfied local people any PA can survive.

Furthermore, we can talk about smaller or bigger intensity of local community's participation in sharing of benefits that development of tourism in PA carries with it. More difficult alternative will occur if the role of tourist development in local community reached its maximum

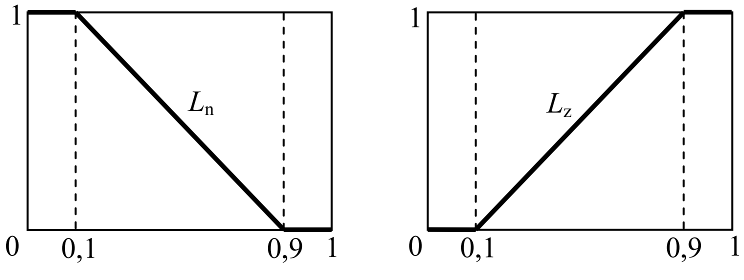

Figure 10. Membership function of fuzzy sets L_n and L_z.

and now went in opposite direction. In that case, consequences for the local community, which is accustomed to welfare, will be more severe than if it was development situation.

INDICATORS OF EVALUATION OF TOURISM INFLUENCE ON SOCIAL COMPONENT

Participation of tourism in local people's employment

Tourism trade in a local community can be pillar of local industry and, also, additional activity for the local people. And local people can be fully involved in tourism development.

One of the greatest benefits which controlled tourism development in PA provides, and which concern the local people, is enlargement of its general income and increase in number of employed people.

Local people's dependence on tourism trade reflects in determining the intensity of its economic dependence of the trade.

Partial indicator, that shows how PA's tourism trade development impacts on social component or how local people depend of tourism trade, does not have precisely defined limits. That is because each PA has its characteristics (as any tourist destination) displayed in the beauty of PA's nature and, also, in the necessity for employment of its local people (directly or indirectly).

Analyzing the percentage of vacancies created (directly or indirectly) by the tourism trade will show if the tourism trade has significant or insignificant role in general number of working population. Each evaluation will be subjective in certain part, because it depends of whose interests it was made for.

In other words, what does the client want to show?

1. If the client wants to show "greater importance of tourism for the area" then there will be vacancies initialized by tourism and those that are not so important for tourism trade, or
2. if the client wants "to point" that space used for tourism trade can be "more economically and rationally" utilized – and number of vacancies created by tourism trade will come to the point that "tourism activity has no significance".

Local population dependence on tourism trade in the PA can cause certain sociological problems.

Great reliance of local people on tourism trade can cause certain problems in cases of bad season or necessity for decreasing the tourism development intensity; in other words, reduce of working population. Anyway, some dependence of local people certainly exists, furthermore because of PA's presence.

Let us using fuzzy sets descriptively express local people's dependence on tourism trade.

U is set of all fuzzy sets with which we describe participation of tourism in employment of local people.

- U_m is fuzzy number stating that tourism trade is not significant for local population employment; participation of tourism in local population employment is small,
- U_v is fuzzy number stating that tourism trade is significant for local population employment; participation of tourism in local population employment is large.

Obviously, $U = \{U_m, U_v\}$. Since precise limits for the indicator are not determined, membership functions of fuzzy sets U_m and U_v are represented in Figure 11.

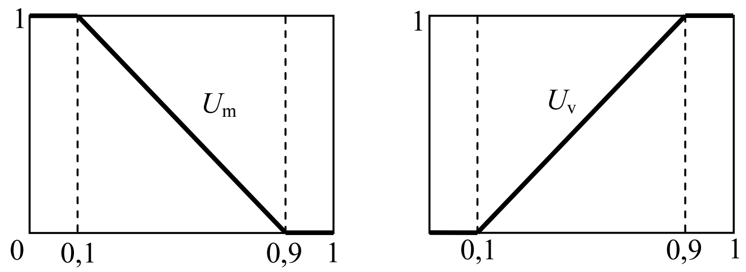

Figure 11. Membership functions of fuzzy sets U_m and U_v.

The percentage of tourists that came without mediation of tourist agency

Success of some tourism promotion is measured by the number of tourists that visit location, by their staying in, and by their effort in promoting the values of the protected area. With good presentation of protected area's qualities, we can expect that certain number of tourists visit the area without mediation of a tourist agency.

The goal of protected area management should be that tourists visit the area without the mediation of tourist agencies (especially international agencies). In that way, local people could feel the economic benefits of tourism in the area.

By the recommendation of EU experts, it is considered that tourist destination, in the sense of measuring this indicator of sociological effect of PA's tourism trade development, has previously set limits which should evaluate impact of this indicator on social component of tourism development. Before we specify these limits we should highlight the things that are symptomatic for these evaluation limits.

In fact, these limits can be priory used for the areas where mass-tourism is in progress. Since protected areas do not have that kind of opportunity for development of mass-tourism, it is obvious that, in attempt to apply these limits, certain mistakes will occur.

Set T represents set of all fuzzy sets that describe importance of number of tourists, that visit the protected area without mediation of tourist agencies, in general number of tourists in one year time. Let us say that:

- T_o represents fuzzy set that points that percentage of tourists, that visited the area without mediation of tourists agencies, is larger than 70 %, "visit is sustainable",
- T_p represents fuzzy set that points that percentage of tourists, that visited the area without mediation of tourist agencies, ranges between 50 % and 70 %, "visit is tolerable",
- T_n represents fuzzy set that points that percentage of tourists, that visited the area without mediation of tourist agencies, is smaller than 50 %, "visit is unsustainable".

Obviously, $T = \{T_n, T_p, T_o\}$. Membership functions of sets T_n, T_p and T_o are giv

en in Figure 12.

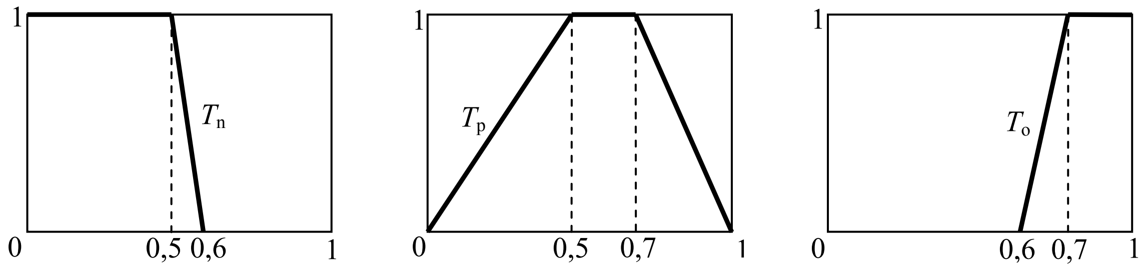

Figure 12. Membership function of fuzzy sets T_n, T_p and T_o.

INDICATORS OF EVALUATION OF INFLUENCE ON CULTURAL IDENTITY

Relationship between accommodation capacity and number of local population

Indicator that will show the influence of tourism development on cultural identity of the area, its architectural features, and necessity for changes caused by the construction of adequate infrastructure, is relationship between accommodation capacities and number of local population. This indicator will provide information about the pressure, which local community endure, caused by the intensity of tourism development in the protected area.

N represents set of all fuzzy sets that describe relationship between accommodation capacities and number of local population. Let us say that:

- N_n represents fuzzy set pointing that relationship between accommodation capacities and number of local population is smaller than 0,6, “relationship is unsustainable”,
- N_p represents fuzzy set pointing that relationship between accommodation capacities and number of local population ranges between 0,6 and 0,9, “relationship is tolerable”,
- N_o represents fuzzy set pointing that relationship between accommodation capacities and number of local population is larger than 0,9, “relationship is sustainable”.

Obviously, $N = \{N_n, N_p, N_o\}$. Fuzzy sets’ membership functions are given in Figure 13.

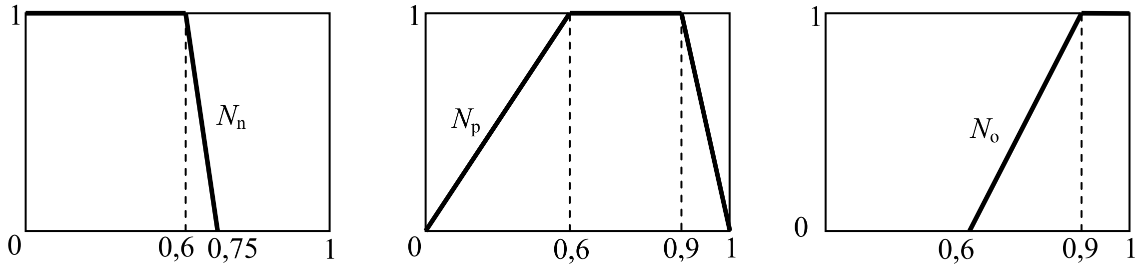

Figure 13. Membership functions of fuzzy sets N_n, N_p and N_o.

Intensity of tourism expressed in proportion of number of overnights and number of local population

Intensity of tourism development in protected area, by the recommendations of EU-experts, is measured in relationship between number of tourists’ overnights in one year time (expressed in thousands) and number of local population (expressed in hundreds). However, this method of evaluation of tourism influence on cultural identity of local people did not consider important segments of tourists: amateurs, picnickers, weekend visitors, and other visitors that visit the area for just a day without staying a night, but who influence, through total tourist trade, on cultural integrity of local people. With this indicator, as well, we encounter a problem with precise evaluation of tourism influence on cultural identity of local community. Probably, its evaluation, mainly, depends of an expert who evaluates it. Let us say that R represents set of all fuzzy sets that describe proportion of number of overnights in one year time (expressed in thousands) and number of local population (expressed in hundreds).

Let us suppose that:

- R_n represents fuzzy set that points that proportion of number of local population and number of overnights during one year is smaller than 0,6, fuzzy set “is unsustainable”,
- R_p represents fuzzy set that points that proportion of number of local population and number of overnights during one year ranges between 0,6 and 0,9; fuzzy set “is tolerable”,
- R_o represents fuzzy set that points that proportion of number of local population and number of overnights during one year is larger than 0,9, fuzzy set “is sustainable”.

Obviously, $R = \{R_n, R_p, R_o\}$. Corresponding membership functions are presented in Figure 14.

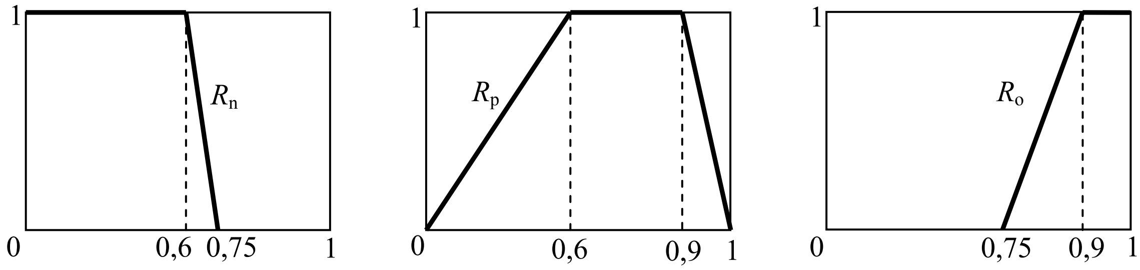

Figure 14. Membership function of fuzzy sets R_n, R_p and R_o.

Indicator that would consider larger number of visitors to the area is the one that evaluates relationship between number of visitors and number of available parking lots in the protected area. Let us say that proportion of 2,5 people per parking lot is a sustainable proportion and it would serve as standard in making spatial plans of national parks [10].

Once again, R represents set of all fuzzy sets that describe possible outcomes after evaluation of indicator of proportion of number of visitors per parking lot. Furthermore,

- R_o represents fuzzy set that points that proportion of number of visitors per parking lot is less or equal 2,5 and that proportion is convenient; fuzzy set of "proportion of number of visitors per parking lot is sustainable",
- R_n represents fuzzy set that points that proportion of number of visitors per parking lot is more than 2,5 and that state is unsustainable; fuzzy set of "proportion of number of visitors per parking lot is unsustainable".

Obviously $R = \{R_n, R_o\}$. Corresponding membership functions are given in Figure 15.

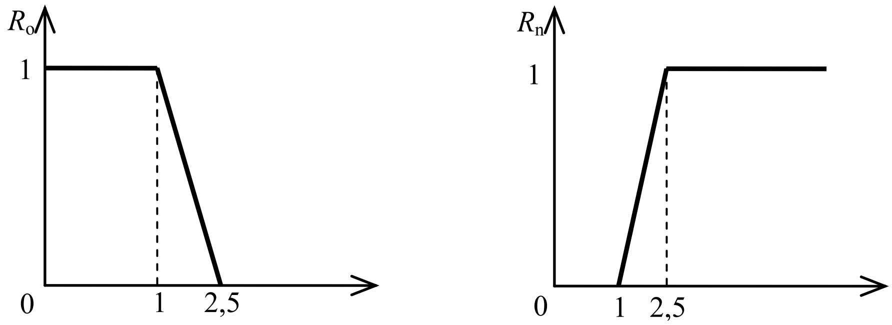

Figure 15. Membership functions of fuzzy sets R_o and R_n.

INDICATORS OF ENVIRONMENT'S CONDITION

Control of tourists' visits to protected area

One of indicators of influence of tourism development on environment and biodiversity of protected area is the pressure caused by use of traffic in order to visit the location. The percentage of tourists, that while visiting the location use their own transportation, is measured. It is considered that use of public transportation lessens the pressure on environment because more visitors use the same vehicle.

It is, furthermore, considered that if the percentage of visitors who visit the area by public transpiration:

- is larger than 20 %, the influence of traffic use onto environment and biodiversity is sustainable,
- ranges between 10 % and 20 %, then the influence is tolerable,

- is smaller than 10 %, the influence is negative and certain measures should be undertaken in order to prevent and stop further destruction of environment.

Let us say that Q represents set of all fuzzy sets that describe condition of traffic influence on environment and PA's biodiversity. And,

- Q_n represents fuzzy set that points that percentage of visitors, who use public transportation, is smaller than 10 % then "traffic influence on environment is unsustainable",
- Q_p represents fuzzy set that points that percentage of visitors, who use public transportation, ranges between 10 % and 20 % then "traffic influence on environment is tolerable",
- Q_o represents fuzzy set that points that percentage of visitors, who use public transportation, is larger than 20 % then "traffic influence on environment is sustainable".

Obviously, $Q = \{Q_n, Q_p, Q_o\}$. Membership functions of Q_n, Q_p and Q_o are given in Figure 16.

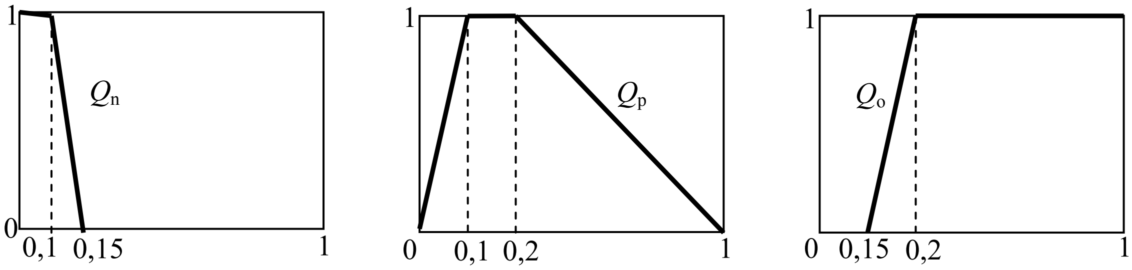

Figure 16. Membership functions of fuzzy sets Q_n, Q_p, and Q_o.

Intensity of realization of spatial plan for the needs of tourism development

What can be an indicator of negative influence of tourism development on environment is the intensity of the construction of infrastructure facilities for tourism needs and location operation; in other words, the percentage of the planned utilization of land for the construction of infrastructure facilities. Anyway, intensive construction causes high pressure on environment. Especially, if the construction was not smooth as it was planned by strategic documents (spatial plan), but caused by intensity of tourism development. Therefore, if in a short time starts the construction of infrastructure facilities, then significant negative impact on environment will occur. The intensity of realization of the construction represents the indicator of influence on environment.

If the percentage of land size in which construction is allowed but not realised:

- is larger than 20 %, it is considered that rapid construction in that area will get certain negative phenomena, and therefore "condition is unsustainable",
- ranges between 10 % and 20 %, it is considered that the intensity of construction for tourism needs is in the state of "condition is tolerable",
- is smaller than 10 %, the state is considered as "sustainable".

Let M denotes a set of all fuzzy sets that describe influence of intensity of implementation of area planned for the construction of infrastructure facilities for tourism needs. Then:

- M_o represents fuzzy set that points that percentage of undeveloped, but planned land for tourism needs less than 10 %; fuzzy set "percentage of unrealized construction is sustainable",
- M_p represents fuzzy set that points that percentage of undeveloped, but planned land for tourism needs ranges between 10 % and 20 %, fuzzy set "percentage of unrealized construction is tolerable",
- M_n fuzzy set that points that percentage of undeveloped, but planned land for tourism needs is more than 20 %; fuzzy set "percentage of unrealized construction is alarming".

Naturally, $M = \{M_n, M_p, M_o\}$ and membership functions of its fuzzy sets are given in Figure 17.

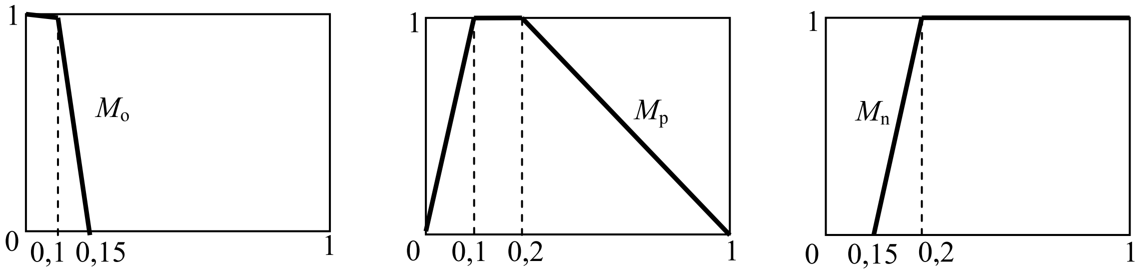

Figure 17. Membership function of fuzzy sets M_o, M_p and M_n.

Management of PA's forest ecosystem

One of very important indicators which should point the condition of environment is the way how we manage PA's forest ecosystems. Forest resources have multiple economic and social significance. The size of exploitation of those resources greatly influences on:

- providing necessary financial resources for the PA's functioning on the basis of exploitation of wood assortments,
- protection and soil quality,
- development of tourism, hunting and recreation,
- protection and development of environment etc.

Proper and controlled use of this natural resource is of particular importance for the very existence of the protected area.

Although funds got on the basis of forest management are significant for maintenance and functioning of the protected area, the base of PA's forest management should be sanitary felling that maintains hygiene and encourages natural rejuvenation.

PA's managers, in lack of financial resources, often make interventions in the way and size of exploitation. In other words, managers in lack of funds for usual financing of PA's basic functions resort to bigger interventions of forest exploitation. Violation of PA's forest ecosystem can be caused by illegal felling, that can imperil PA's forest ecosystem. We will consider relationship between imperil forest ecosystems and financial funds of PA. Based upon researches we can conclude that PA is financed by its own financial funds or from the national budget.

State's participation in PA's finance is various. It ranges from total absence of support and funds to full providing of all necessary means (what is rare). Anyway, we can say that State participation in PA's financing of necessary means ranges between 30 % and 40 %; for many protected areas this is what allows them to function and survive. Park managers should provide other means. These means should be provided through activities and functions that can be done in order with primary and secondary goals. There are protected areas where funds of forest exploitation make 80 % of financing of necessary means (e.g. parks in Bosnia and Herzegovina). This kind of approach, unquestionably, leads to degradation of protected area and destruction of ecosystem. The condition, based on utilization and management of forest ecosystems, can be described as:

- sustainable, if incomes of forest ecosystems exploitation are smaller than 30 % of necessary means for PA's functioning,
- tolerable, if the incomes range between 30 % and 40 %,
- unsustainable, if the incomes are larger than 40 %.

Let us say that set B represents set of all fuzzy sets that describe intensity of forest ecosystems utilization regarding the percentage of exploitation's participation in PA's financing. In that case,

- B_o is fuzzy set that points that incomes of forest's ecosystems exploitation are less than 30 % of means that are necessary for PA's functioning; fuzzy set "managing of forest ecosystems is sustainable",
- B_p is fuzzy set that points that incomes of forest's ecosystems exploitation range between 30 % and 40 % of means that are necessary for PA's functioning; fuzzy set "managing of forest ecosystems is tolerable",
- B_n is fuzzy set that points that incomes of forest's ecosystems exploitation are more than 40 % of means that are necessary for PA's functioning; fuzzy set "managing of forest ecosystems is unsustainable".

Obviously, $B = \{B_o, B_p, B_n\}$ and membership functions of its fuzzy sets are given in Figure 18.

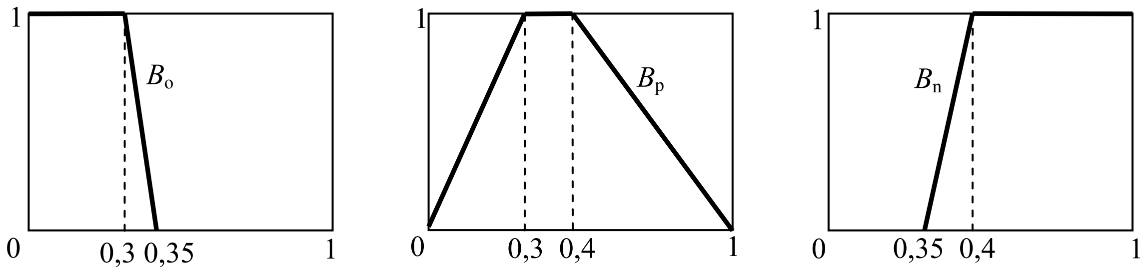

Figure 18. Membership function of fuzzy sets B_n, B_i and B_o.

REPEATED VISITS AS AN INDICATOR OF TOURISTS' SATISFACTION

There are various factors that influence on satisfaction of PA's visitors and which influence on their choice. The most important are:

- ambient values of area (natural and cultural-historical values, availability of walking trails, ...),
- tourism activity in the region,
- quality of manifestation tourism product that takes its place in the park area and its surroundings,
- how local population and PA's employees treat visitors.

Alhough there is neither unique nor simple approach in evaluation of satisfaction with service quality provided in tourism industry, EU-experts suggest that as an indicator for evaluation of satisfaction with service quality of we should consider percentage of tourists that repeated their visit to a certain tourist destination five years after first visit. This approach does not have consistent and uniform manner because these data can be gathered by surveying the visitors. Sometimes, survey is conducted in order to check the intentions of tourists' visits to a certain destination, and sometimes after one. Therefore, there is a lot of inaccuracy and subjectivity in evaluation of indicator values of tourists' satisfaction by their visit to some destination.

Depending of the percentage of visitors that repeated their visit, we consider that:

- if the percentage of visitors that repeated their visit ranges between 30 % and 40 % then we can say that this destination is interesting and that it gives great satisfaction to the tourists,
- if the percentage is larger than 50 % then the satisfaction is higher and tourist destination can influence on change in visitors' consciousness,
- if the percentage is smaller than 30 % then we can say that the destination lost its attraction.

Inaccuracy in evaluation of this indicator (among other) lies in the fact that survey is conducted on the people that visit the area for traditional, religious, cultural and historical reasons. Thus, there are many other reasons that cannot be factors in evaluating the destination's attraction but which cannot be avoided in measuring the percentage of tourists that repeated their visit.

Let us say that set Z represents set of all fuzzy sets that describe conditions of evaluation of destination's attraction, so:

- Z_n is fuzzy set that points that percentage of tourists that repeated their visit is smaller than 30 %; fuzzy set "destination is unattractive",
- Z_a is fuzzy set that points that percentage of tourists that repeated their visit ranges between 30 % and 40 %; fuzzy set "destination is still attractive",
- Z_v is fuzzy set that points that percentage of tourists that repeated their visit is larger than 50 %; fuzzy set "destination is very attractive".

Obviously, $Z = \{Z_n, Z_a, Z_v\}$, and fuzzy sets' membership functions are given in Figure 19.

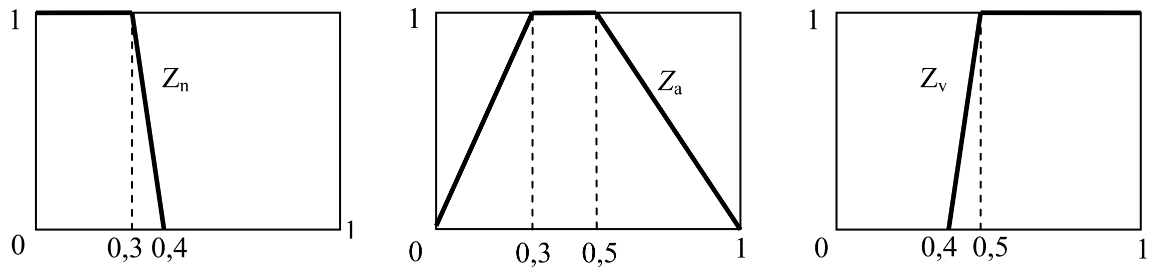

Figure 19. Membership function of fuzzy sets Z_n, Z_a and Z_v.

CONCLUSION RULES AND COMPARATIVE INDICATORS

Indicators of destination's economic development and fuzzy conclusion

In order to make certain conclusions regarding evaluation of economic benefits made from tourism development and sustainability of that development from the point of view of evaluation of these effects based on comparative indicators that are presented as fuzzy sets, we have to define rules by using fuzzy logic where by using fuzzy numbers as input components we determine values of output variables, evaluation of economic effects.

As we previously mentioned, the indicators of evaluation of economic effects gained by tourism development are:

1. evaluation of seasonality of tourism trade, set of all fuzzy numbers that describe the evaluated condition as $S = \{S_o, S_p, S_n\}$,
2. evaluation of intensity of destination's accommodation utilization is described by fuzzy sets $K = \{K_n, K_p, K_o\}$,
3. evaluation of influence of tourism trade on increase of economic income of local population; local tourism increase is evaluated by $L = \{L_n, L_z\}$ fuzzy set.

Insignia space where certain descriptions of possible evaluation of economic effects' conditions, which development of tourism has, is determined by fuzzy sets' results $S \times K \times L$. If E is set of all fuzzy sets that evaluate economic component of tourism development, then the space of possible conditions, upon which evaluation will be made, make arranged fuzzy sets' triples S, K, L:

$$\begin{aligned} E &= \{S_o, S_p, S_a\} \times \{K_n, K_p, K_o\} \times \{L_n, L_z\} = \\ &= \{S_oK_nL_n, S_oK_nL_z, S_oK_pL_n, S_oK_pL_z, S_oK_oL_n, S_oK_oL_z, S_pK_nL_n, S_pK_nL_z, S_pK_pL_n, \\ &\quad S_pK_pL_z, S_pK_oL_n, S_pK_oL_z, S_aK_nL_n, S_aK_nL_z, S_aK_pL_n, S_aK_pL_z, S_aK_oL_n, S_aK_oL_z\}, \end{aligned}$$

where each coordinate S, K, L represents language variable defined by set of attributes.

Characteristic function of attributes' influence

In order to establish certain criteria according which condition will be described by appropriate attribute, that represents fuzzy number, we use the fact that each attribute of evaluated indicator describes condition of influence of evaluated indicator, denoted by fuzzy number. Therefore, to each of these attributes we can join function that characterizes it. There is defined function for each attribute a_i:

$$\varphi : a_i \to \varphi(a_i) \in \begin{cases} 1, & \text{describes the positive condition of indicator influence,} \\ 0, & \text{describes improved condition of indicator influence,} \\ -1, & \text{describes negative condition of indicator influence,} \end{cases}$$

where $\varphi(a_i)$ represents characteristic function of attribute a_i condition and $\varphi(a_i) \in \{1, 0, -1\}$.

Attributes that describe the condition of intensity of economic influence of tourism development in PA are given in Table 4.

Table 4. Attributes of description of indicators' conditions that determine intensity of economic effect of tourism development.

Attribute / Evaluation indicator	Symbol	Possible condition of attribute $\varphi(a_i)$	Characteristic attribute function $\varphi(a_i)$
Seasonality of visit	S	S_o, S_p, S_a	$\varphi(S_o) = 1,\ \varphi(S_p) = 0,\ \varphi(S_a) = -1$
Intensity of accommodation utilization	K	K_n, K_p, K_o	$\varphi(K_n) = -1,\ \varphi(K_p) = 0,\ \varphi(K_o) = 1$
Impact of tourism development on local industry	L	L_n, L_z	$\varphi(L_n) = -1,\ \varphi(L_z) = 1$

Set containing evaluations of conditions of tourism impact onto economic component is $\mathsf{S}(E)$, with the following elements:

$$\mathsf{S}(E)=\{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_{11}, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}, s_{17}, s_{18}\},$$

where each element s_i ($i = 1, \dots 18$) represents one influence condition of evaluated indicators described by attributes (fuzzy numbers). So, $s = (a_1, a_2, a_3,)$ where a_1 belongs to S, a_2 to K and and a_3 to L.

Characteristic function of condition s is represented by

$$\varphi(s) = \varphi(a_1, a_2, a_3) = (\varphi(a_1), \varphi(a_2), \varphi(a_3)).$$

Let us define a function which will assign a value to each condition s from $\mathsf{S}(E)$, depending of individual value of attributes describing that very condition s. Let us refer to that function as a "severity influence function" and denote it by $\tau(s)$. It is equal to sum of individual attributes that make observed condition:

$$\tau(s) = \sum_{i=1}^{n} \varphi(a_i),$$

where $\varphi(a_i)$ is a function of influence of individual attributes' condition, s is condition of influence of evaluated indicators, $s = (a_1, a_2, \dots a_n)$ and a_i are individual attributes ($i = 1, 2, \dots, n$). Values of influence severity of individual conditions are as listed in Table 5.

Let us define in condition set $\mathsf{S}(E)$ the relation $\rho \equiv$ "to have the same influence severity". Equivalence axioms stand for ρ relation:

Table 5. Possible conditions of economic condition evaluation and severity functions' values of certain conditions.

L_n	$\tau(s)$	L_z	$\tau(s)$
$S_o K_n L_n$	-1	$S_o K_n L_z$	1
$S_o K_p L_n$	0	$S_o K_p L_z$	2
$S_o K_o L_n$	1	$S_o K_o L_z$	3
$S_p K_n L_n$	-2	$S_p K_n L_z$	0
$S_p K_p L_n$	-1	$S_p K_p L_z$	1
$S_p K_o L_n$	0	$S_p K_o L_z$	2
$S_a K_n L_n$	-3	$S_a K_n L_z$	-1
$S_a K_p L_n$	-2	$S_a K_p L_z$	0
$S_a K_o L_n$	-1	$S_a K_o L_z$	1

reflexivity: for each condition S from $\mathsf{S}(E)$, $s\rho s$ stands,
symmetry: $\forall\ s_1, s_2 \in \mathsf{S}(E)$, $s_1\rho s_2 \Rightarrow s_2\rho s_1$,
transitivity: $\forall\ s_1, s_2, s_3 \in \mathsf{S}(E)$ stands $(s_1\rho s_3\ \&\ s_2\rho s_3) \Rightarrow s_1\rho s_3$.

Therefore, in set $\mathsf{S}(E)$, the equivalence relation ρ is defined, which divides $\mathsf{S}(E)$ into equivalence classes. Let C be the set of all influence classes, so the equivalence classes are:

$$C = \{[-3], [-2], [-1], [0], [1], [2], [3]\}.$$

Elements of classes are conditions made by evaluation of measurement indicators (condition attributes):

$$C_{[-3]} = \{S_aK_nL_n\},\ C_{[-2]} = \{S_pK_nL_n,\ S_aK_pL_n\},\ C_{[-1]} = \{S_oK_nL_n,\ S_pK_pL_n,\ S_aK_oL_n,\ S_aK_nL_z\},$$
$$C_{[0]} = \{S_oK_pL_n,\ S_pK_oL_n,\ S_pK_nL_z,\ S_aK_pL_z\},\ \mathrm{C}_{[1]} = \{S_oK_oL_n,\ S_oK_nL_z,\ S_pK_pL_z,\ S_aK_oL_z\},$$
$$C_{[2]} = \{S_oK_pL_z,\ S_pK_oL_z\},\ \mathrm{C}_{[3]} = \{S_oK_oL_z\}.$$

Using previously mentioned attributes that describe state of economic indicator, we introduce fuzzy numbers that represent evaluation of condition of influence on economic component. Intensity of influence of tourism development on economic component represents function which depends of indicator values (S, K, L).

E represents set of all fuzzy sets that describe economic effects of tourism development, and:

- E_a represents economic opportunities used up to 25 %; fuzzy set points that economic effects, which local community makes of tourism development, represent "condition is alarming" (described by class conditions $C_{[-3]}$, $C_{[-2]}$),
- E_n represents economic opportunities used up to 50 %; fuzzy set points that economic effects, which local community makes of tourism development, represent "effects do not have significant influence" (described by class condition $C_{[-1]}$),
- E_p represents economic opportunities used from 50 % to 70 %; fuzzy set points that economic effects, which local community makes of tourism development, represent "condition is tolerable" (described by class conditions $C_{[0]}$, $C_{[1]}$),
- E_o represents economic opportunities used more than 75 %; fuzzy set points that economic effects, which local community makes of tourism development, represent "condition is sustainable" (described by class conditions $C_{[3]}$, $C_{[2]}$).

Then $E = \{E_a, E_n, E_p, E_o\}$. Corresponding membership functions are given in Figure 20.

Finally, let us analyse impact which some indicators have onto evaluation of economic effects of tourism development in particular destination.

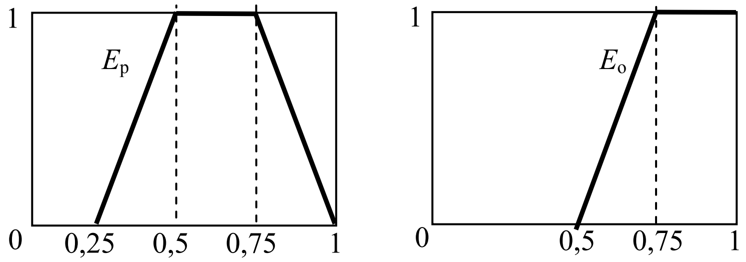

Figure 20. Membership functions of fuzzy sets E_p and E_o.

Using fuzzy implication "if P then R", which leads us to management function, we have an opportunity to create certain rules regarding which evaluation of economic indicator values will be defined. Rules upon which we evaluated local community's economic benefit of tourism development are:

$$
\begin{array}{lllll}
R_1\text{:} & \text{IF} & (x \text{ is } S_o \text{ AND } x \text{ is } K_o \text{ AND } x \text{ is } L_z) & \text{ELSE} & \\
 & & (x \text{ is } S_o \text{ AND } x \text{ is } K_p \text{ AND } x \text{ is } L_z) & \text{ELSE} & \\
 & & (x \text{ is } S_p \text{ AND } x \text{ is } K_o \text{ AND } x \text{ is } L_z) & \text{THEN} & (y \text{ is } E_o), \\
R_2\text{:} & \text{IF} & (x \text{ is } S_o \text{ AND } x \text{ is } K_p \text{ AND } x \text{ is } L_n) & \text{ELSE} & \\
 & & (x \text{ is } S_p \text{ AND } x \text{ is } K_n \text{ AND } x \text{ is } L_n) & \text{ELSE} & \\
 & & (x \text{ is } S_p \text{ AND } x \text{ is } K_n \text{ AND } x \text{ is } L_z) & \text{ELSE} & \\
 & & (x \text{ is } S_a \text{ AND } x \text{ is } K_p \text{ AND } x \text{ is } L_z) & \text{ELSE} & \\
 & & (x \text{ is } S_o \text{ AND } x \text{ is } K_o \text{ AND } x \text{ is } L_n) & \text{ELSE} & \\
 & & (x \text{ is } S_o \text{ AND } x \text{ is } K_n \text{ AND } x \text{ is } L_z) & \text{ELSE} & \\
 & & (x \text{ is } S_p \text{ AND } x \text{ is } K_p \text{ AND } x \text{ is } L_z) & \text{ELSE} & \\
 & & (x \text{ is } S_a \text{ AND } x \text{ is } K_o \text{ AND } x \text{ is } L_z) & \text{THEN} & (y \text{ is } E_p), \\
R_3\text{:} & \text{IF} & (x \text{ is } S_o \text{ AND } x \text{ is } K_n \text{ AND } x \text{ is } L_n) & \text{ELSE} & \\
 & & (x \text{ is } S_p \text{ AND } x \text{ is } K_p \text{ AND } x \text{ is } L_n) & \text{ELSE} & \\
 & & (x \text{ is } S_a \text{ AND } x \text{ is } K_o \text{ AND } x \text{ is } L_n) & \text{ELSE} & \\
 & & (x \text{ is } S_a \text{ AND } x \text{ is } K_n \text{ AND } x \text{ is } L_z) & \text{THEN} & (y \text{ is } E_n), \\
R_4\text{:} & \text{IF} & (x \text{ is } S_a \text{ AND } x \text{ is } K_n \text{ AND } x \text{ is } L_n) & \text{ELSE} & \\
 & & (x \text{ is } S_p \text{ AND } x \text{ is } K_n \text{ AND } x \text{ is } L_n) & \text{ELSE} & \\
 & & (x \text{ is } S_a \text{ AND } x \text{ is } K_p \text{ AND } x \text{ is } L_n) & \text{THEN} & (y \text{ is } E_a),
\end{array}
$$

For example, we read rule R_1:

IF

visit seasonality is *sustainable* and accommodation capacity utilization is *sustainable* AND tourism trade is *significant* for increase of local community's economic effects

OR

visit seasonality is *sustainable* and accommodation capacity utilization *is tolerable* AND tourism trade is *significant* for increase of local community's economic effects

OR

visit seasonality is *unsustainable* and accommodation capacity utilization is *sustainable* and tourism trade is *significant* for increase of local community's economic effects

THEN

economic effect's intensity caused by tourism development is *sustainable*,

and similarly other rules. Evaluation of economic effects dependance onto previously introduced indicators was checked using MATLAB *Fuzzy Logic Toolbox*. Previously defined fuzzy numbers were used in trapezodial form.

Let us consider analysis of the first rule of condition evaluation R_1, ... R_4 where condition of economic effects of tourism development expresses sustainable condition, Figs. 21 and 22. One can notice that each change in evaluated indicator's value causes change in certainty that condition evaluation is denoted as sustainable.

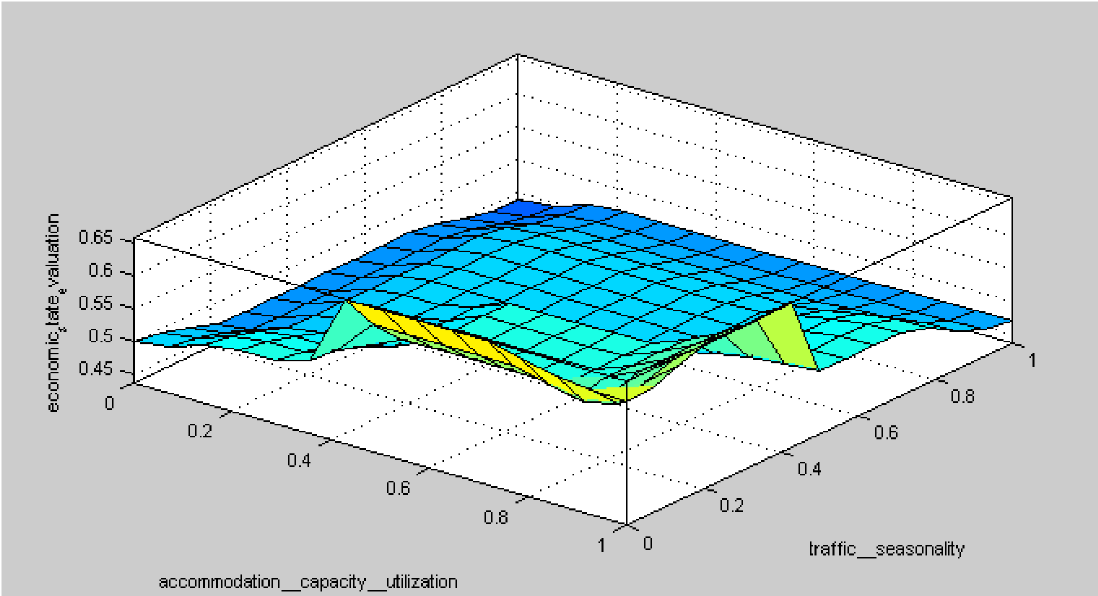

Figure 21. Rule R_4 of conclusion and defuzzification.

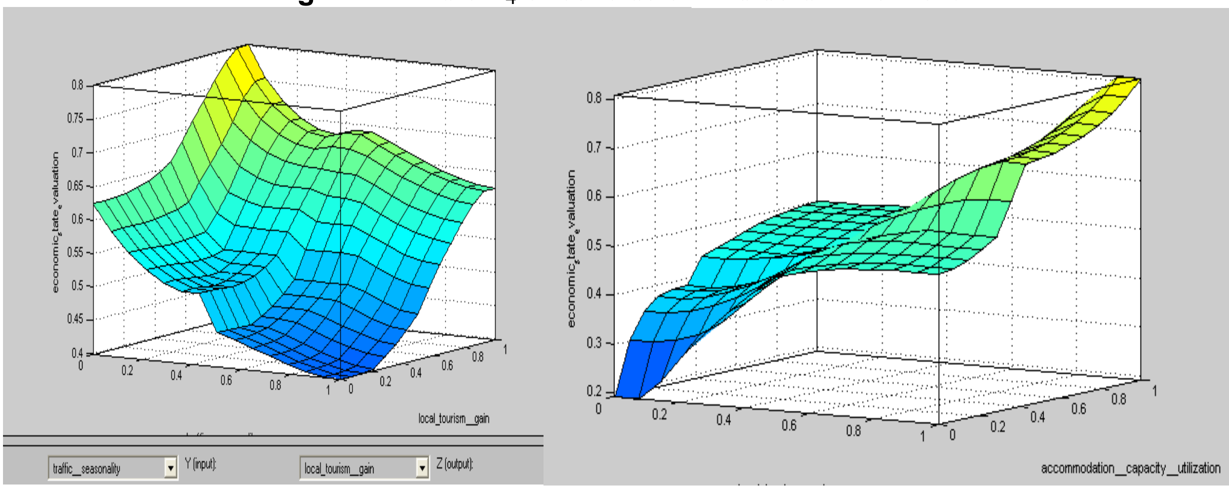

Figure 22. Representation of rules and membership function of economic effects' evaluation expressed by rules R_4 (left) and R_1 (right).

CONCLUSIONS

Measuring the sustainability development of tourism in comparative indicators, the classical method contains a lot of imprecision caused by the uncertainty in the limits of individual indicators, as well decisions are the result of the impact assessment of individual indicators in the traditional manner. The proposed model enables measurement of sustainable development tourism we are not able to dispose of precise values comparative indicators. Measuring sustainable development of tourism using fuzzy sets and fuzzy logic is just one area in which use fuzzy logic in tourism. Exposed methods for measuring the intensity of the sustainability of individual indicators can be applied to define the rules and reasoning to determine sustainable development destination with all the uncertainty that can occur in such measurements. Rules are defined for measuring the intensity of the sustainable development of tourism destinations using fuzzy logic, which will be further developed.

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MATEMATIČKO MODELIRANJE ODRŽIVOG RAZVOJA TURIZMA POMOĆU NEIZRAZITIH SKUPOVA

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SAŽETAK

U prvom dijelu rada razmatrani su neizraziti skupovi usporedbenih indikatora kojima se određuje održivost razvoja turizma. U drugom dijelu rada, na temelju postavljenog modela, pokazano je kako pomoću neizrazite logike učinkovito odrediti vrijednosti održivog razvoja turizma u zaštićenim područjima ako su postavljeni sljedeći skupovi indikatora: za ekonomski status, za utjecaj turizma na društvenu komponentu, za utjecaj turizma na kulturni identitet, za uvjete u okolini te za zadovoljstvo turista.

Također je pokazano kako utvrditi razinu pouzdanosti u pravila pomoću kojih se, prema stručnjacima, donose primjerene odluke za zaštitu biološke raznolikosti zaštićenih područja.

KLJUČNE RIJEČI

matematičko modeliranje, održivi razvoj turizma, zaštićena područja, neizrazita logika, usporedbeni indikatori