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A TWO-SECTOR GROWTH MODEL WITH ENDOGENOUS HUMAN CAPITAL AND AMENITIES

Wei-Bin Zhang

College of Asia Pacific Management, Ritsumeikan Asia Pacific University
Oita Prefecture, Japan

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ABSTRACT

This paper examines issues related to urbanization with labour migration. The main departures from the traditional approaches to dynamics of economic structures are that the paper uses an alternative approach to consumer behaviour and introduces human capital accumulation via learning by doing. The model describes dynamic interactions among agricultural and industrial production, rural and urban amenities, distribution of production factors and preferences with endogenous capital and human capital accumulation. We show that the dynamic system may have either a single or multiple equilibrium points, depending upon returns to scale in the two sectors. We also examined effects of changes in some parameters.

KEY WORDS

two-sector model, agricultural sector, industrial sector, physical capital accumulation, human capital accumulation, rural and urban amenities

CLASSIFICATION

JEL: R11, R14

*Corresponding author, η: wbz1@apu.ac.jp; +81 977 73 9787, +81 977 78 1020;
Ritsumeikan Asia Pacific University, 1-1 Jumonjibaru, Beppu-Shi, Oita-ken 874-8577, Japan

INTRODUCTION

It is important to study urbanization with labour migration as in many less developed countries an important part of the population is still devoted to agriculture. In economies like India, China and some African countries, agricultural population shares a high percentage of the total population. On the other hand, industrialization and human capital accumulation are altering dramatically the labour distribution between agricultural and non-agricultural sectors in many developing economies. It is well known that the Harris-Todaro framework has played the role of a key model in analyzing industrialization with labour migration between the urban and rural areas[1].The Harris-Todaro model attempts to explain persistent rural-urban migration despite the high unemployment rates in cities, especially in developing economies. In this model, the formal-sector wage is fixed at a level far above the agricultural wage and the migration decision is based on expected earnings. To maintain the presumed equalization of expected earnings, some urban residents are unemployed as migration is costless. The change in the probability of formal employment is the principal mechanism that restores migration equilibrium in response to exogenous changes such as technical progress and job or wage growth in the city. As pointed out by Brueckner and Zenou [1], there are many other factors that limit urban growth. For instance, a rise in the city population raises the urban living cost, mostly likely through the land markets, which limits urban growth. As the urban population rises in response to positive shocks, land prices tend to rise, lowering the utility levels of all urban residents. The gap between rural utility and the expected utility of an urban resident is closed up by migration. This study examines migration equilibrium without unemployment. Although we still assume equalization of utility levels between the urban and rural areas, we analyze differences in urban and rural living conditions by assuming that urban and rural areas offer not only different wage rates but also different levels of amenity. This study shows how productivities, land and amenity interact to determine labour distribution between the urban and rural areas in the long term. By taking account of endogenous amenity and land and human capital accumulation, we try to offer an alternative approach to the economy described by the Harris-Todaro model.

Another important issue related to economic structural change is dynamics of human capital and technological change. It is well known that it is difficult to introduce both human capital and physical capital accumulation as endogenous variables into the Harris-Todaro framework because of analytical intractability. For instance, Matsuyama [2] examines how agricultural productivity influences economic growth and the process of industrialization. The model shows that the effect of agricultural productivity on growth is crucially dependent on openness to trade. Nevertheless, Matsuyama's analysis relies on the assumption that agriculture is backward and no technological progress will take place in the sector. The growth process is driven solely by learning by doing in manufacturing. Nevertheless, the assumption that human capital accumulation is negligible through learning by doing in agricultural sector is not realistic[2]. This study takes account of human capital accumulation both in industrial and agricultural sectors. Multiple equilibrium points exist when the two sectors exhibit increasing and decreasing returns to scale. Although our model is constructed in dynamics, because of the nature of the problem, it is difficult to carry out a complete dynamic analysis. This study is mainly concerned with issues of existence of equilibrium and comparative statics analysis.

The paper constructs a two-sector growth model with endogenous human capital and physical capital accumulation. The model tries to provide some insights into processes of industrialization and urbanization with labour migration. Although the paper studies issues similar to those addressed by the Harris-Todaro model and its various extensions, we deviate the traditional approach by proposing an alternative approach to household behaviour. The equilibrium mechanism of labour migration is expressed by equalizing utility levels in the urban and rural areas. Different from the Harris-Todaro approach, we use the concept of amenity to reflect living and work condition differences between the urban and rural areas. The wage rates differ between the industrial and agricultural sectors because the urban and rural areas offer different levels of amenity and land rent. It should be noted that this paper is an extension of a model proposed by Zhang [3; Ch. 6]. The main difference between this model and Zhang's model is that this study introduces differences in amenity between urban and rural areas, while Zhang's does not take account of possible differences in amenity in different professions and economic geography. The paper is organized as follows. Section 2 defines the two-sector growth model with physical and human capital accumulation. Section 3 provides the process to determine all the variables and demonstrate existence of equilibrium when the parameter values are specified. Section 4 examines effects of changes in the total productivity, the population, and propensity on the levels of physical and human capital and economic structure. Section 5 concludes the study. Appendix A.1 proves the process of finding equilibrium in Section 3. Appendix A.2 shows how to express the dynamics of the economic variables in a three differential equations system.

ECONOMIC GROWTH WITH PHYSICAL AND HUMAN CAPITAL ACCUMULATION

Similar to Harris and Todaro [4] and Irz and Roe [5], we consider an economic system consisting of agricultural and industrial sectors. The agricultural sector produces goods such as corn, rice and vegetables, which are only for consumption. The industrial sector produces commodities for investment and consumption. Industrial commodity is selected to serve as numeriare. It is assumed that labour force, land and capital are always fully employed[3]. The population is assumed to be homogenous in the sense that their preference and skill structures are identical. This implies that people can costlessly move from countryside to city, and vice versa. A person is free to choose his residential location. We assume that any person chooses the same area where he works and lives. Each area has fixed land. Land quality, climates, and environment are homogenous within each area, but they may vary between the areas. We neglect transportation cost of commodities[4]. As become evident later on, although it is conceptually not difficult to introduce transportation cost function and to provide balance conditions for demand and supply and for price equalization conditions with transportation cost, the problem will become analytically too complicated. The assumption of zero transportation cost of commodities implies price equality for the commodity over space. Nevertheless, as amenity and land are immobile, wage rates and land rent vary between the areas.

BEHAVIOUR OF PRODUCTION SECTORS

We denote $K(t)$, $r(t)$ and $p(t)$ the total capital, the rate of interest and price of agricultural commodity, respectively. We define the following indexes and variables

a, i – subscripts denoting agriculture and industry,
N – the total fixed labour force of the economy,
L_i and L – the fixed urban and rural areas,
$N_j(t)$ and $K_j(t)$ – the labour force and capital stocks employed by sector j ($j = a$, i) at time t,
$L_a(t)$ – the land employed by the agricultural sector,
$F_j(t)$ and $C_j(t)$ – sector j's output and consumption levels of product j, and
$w_j(t)$ – sector j's wage rate.

We assume that production processes can be described by some aggregate production functions. We assume that agricultural production is a process of combining land, labour force and capital. For simplicity, the production function of the agricultural sector is specified as follows

$$F_a(t) = A_a K_a^{\alpha_a} (H^{m_a/\beta_a} N_a)^{\beta_a} L_a^{\varsigma}, \ m_a \geq 0, \ \alpha_a + \beta_a + \varsigma = 1, \ \alpha_a, \beta_a, \varsigma > 0. \quad (1)$$

Here, the term $H^{m_a/\beta_a} \cdot N_a$ is the qualified labour input. The parameter m_a/β_a describes how effectively the agricultural sector utilizes human capital. The marginal conditions for the agricultural sector are given by

$$r + \delta_k = \frac{\alpha_a p F_a}{K_a}, \ w_a = \frac{\beta_a p F_a}{N_a}, \ R_a = \frac{\varsigma p F_a}{L_a}, \quad (2)$$

where δ_k is depreciation rate of physical capital.

The industrial production is a process of combining labour force and capital. The land use by the industrial sector is omitted[5]. The production function of the industrial sector is specified as follows

$$F_i(t) = A_i K_i^{\alpha_i} (H^{m_i/\beta_i} N_i)^{\beta_i}, \ m_i \geq 0, \ \alpha_i + \beta_i = 1, \ \alpha_i, \beta_i > 0. \quad (3)$$

The marginal conditions for the industrial sector are given by

$$r + \delta_k = \frac{\alpha_i F_i}{K_i}, \ w_i = \frac{\beta_i F_i}{N_i}. \quad (4)$$

We described behaviour of the production sectors.

CONSUMER BEHAVIOUR

Each worker may get income from land ownership, wealth ownership and wages. To simplify the model, we accept the assumption of "equally shared landownership" which means that the income of land rent is equally distributed among the population. The total land revenue is given by $R_i(t) \cdot L_i + R_a(t) L$, where $R_i(t)$ and $R_a(t)$ are the land rents in the city and the rural area, respectively. Each consumer obtains the following land revenue

$$\bar{r}(t) \equiv \frac{R_i(t) L_i + R_a(t) L}{N}. \quad (5)$$

This study uses the approach to consumers' behaviour proposed by Zhang in the early 1990s [3, 6]. This approach makes it possible to solve many national, international, urban, and interregional economic problems, such as growth problems with heterogeneous households, multi-sectors, and preference changes, which are analytically intractable by the traditional approaches in economics. Let $\bar{k}_j(t)$ stand for the per capita wealth (excluding land) owned by the typical household j Each household of area j obtains income

$$y_j(t) = r(t) \bar{k}_j(t) + w_j(t) + \bar{r}(t), \ \ j = i, a,$$

from the interest payment, $r\bar{k}_j$, and the wage payment, w_j and the land revenue, $\bar{r}$. We call y_i the current income in the sense that it comes from consumers' wages and current earnings from ownership of wealth. The sum of income that consumers are using for consuming, saving, or transferring are not necessarily equal to the current income because consumers can sell wealth to pay, for instance, the current consumption if the current income is not sufficient for buying food and touring the country. Retired people may live not only on the interest payment but also have to spend some of their wealth. The total value of the wealth that consumer j can sell to purchase goods and to save is equal to $p_i(t) \ \bar{k}_j(t)$ with $p_i(t) = 1$ at any t Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income is then equal to

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \tag{6}$$

The disposable income is used for saving and consumption. It should be noted that the value, $\bar{k}_j(t)$ (i.e., $p_i(t)\bar{k}_j(t)$), in the above equation is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we may consider $\bar{k}_j(t)$ as the amount of the income that the consumer obtains at time t by selling all of his wealth. Hence, at time t the consumer has the total amount of income equaling $\hat{y}_j(t)$ to distribute between consuming and saving. It should also be remarked that in the growth literature, for instance, in the Solow model, the saving is out of the current income, $y_i(t)$ while in this study the saving is out of the disposable income. This approach is discussed at length elsewhere [3, 6][6].

At each point of time, a consumer distributes the total available budget among housing, $l_j(t)$ saving, $s_j(t)$ consumption of agricultural goods, $c_{ja}(t)$ and consumption of industrial goods, $c_{ji}(t)$. The budget constraint is given by

$$R_j l_j + c_{ji} + p c_{ja} + s_j = \hat{y}_j. \tag{7}$$

Furthermore, at each point of time, consumers have four variables to decide. A consumer decides how much to consume housing, industrial and agricultural goods, and how much to save. Equation (7) means that consumption and savings exhaust the consumers' disposable personal income.

We assume that utility level, $U_j(t)$, that the consumer j obtains is dependent on $l_j(t)$, $c_j(t)$, $c_{ja}(t)$ and $s_j(t)$. The utility level of the typical consumer in region j is represented by

$$U_j(t) = \theta_j(t) l_j^{\eta_0}(t) c_j^{\xi_0}(t) c_{ja}^{\mu_0}(t) s_j^{\lambda_0}(t), \quad \eta_0, \xi_0, \mu_0, \lambda_0 > 0, \ \ j = a, i, \tag{8}$$

in which η_0, ξ_0, μ_0 and λ_0 are a typical person's elasticity of utility with regard to lot size, industrial goods, agricultural goods, and savings in area j, respectively. We call η_0, ξ_0, μ_0 and λ_0 the propensities to consume lot size, industrial goods, agricultural goods, and to hold wealth (save), respectively. In (8), $\theta_a(t)$ and $\theta_i(t)$ are respectively called the rural and urban amenity levels. Amenities are affected by, for instance, infrastructures, professional. In this study, we assume that amenity is affected by production and consumption activities. We specify θ_j as follows

$$\theta_j(t) = \bar{\theta}_j N_j^{d_j}(t), \ \ j = a, i, \tag{9}$$

where $\bar{\theta}_j$ (>0) and d_j are parameters. We do not specify sign of d_j as the population may have either positive or negative effects on the attractiveness of a location[7]. Maximizing $U_j(t)$ subject to the budget constraints yields

$$l_j(t) R_j(t) = \eta \hat{y}_j(t), \ \ c_j(t) = \xi \hat{y}_j(t), \ \ p(t) c_{ja}(t) = \mu \hat{y}_j(t), \ \ s_j(t) = \lambda \hat{y}_j(t). \tag{10}$$

where

$$\eta \equiv \rho \eta_0, \ \ \xi \equiv \rho \xi_0, \ \ \mu \equiv \rho \mu_0, \ \ \lambda \equiv \rho \lambda_0, \ \ \rho \equiv (\eta_0 + \xi_0 + \mu_0 + \lambda_0)^{-1}.$$

As shown in [3], the saving behaviour of the approach in this study is similar to these implied by the Keynesian consumption function and permanent income hypotheses, which are empirically more valid than the assumptions in the Solow model with a constant saving rate or the Ramsey model[8]. It should be remarked that the saving, $s(t)$ defined in this study is different from the saving in the Solow model. It can be shown that the approach to consumers' saving behaviour in this study can generate the same behaviour as in the Solow model or the Ramsey model when the propensity to save, λ, is assumed to be related to the wealth and income[9].

According to the definitions of $s_j(t)$ the wealth accumulation of the representative household in area j is given by

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{11}$$

As households are assumed to be freely mobile between the two areas, the utility level of people should be equal, irrespective of in which area they live, i.e.

$$U_i(t) = U_a(t). \tag{12}$$

We neglect possible costs for migration. In reality, even to change a house in a small town costs. Although it is not difficult to introduce migration costs into the model, it will become far more difficult to explicitly get analytical results. In this study, instead of wage equalization (which is often used as the equilibrium mechanism of population distribution), we assume that consumers obtain the same level of utility in different professions as the equilibrium mechanism of population distribution between the professions. Although the condition of utility equalization is often used in the literature of urban economics, the assumption of utility equalization is not often used in the literature of economic dynamics as the temporary equilibrium condition of population distribution. It is argued that this assumption is more reasonable than the assumption of wage equalization.

The total capital stock employed by the production sectors is equal to the total wealth owned by all the regions. That is

$$K(t) = K_a(t) + K_i(t) = \bar{k}_a(t)N_a(t) + \bar{k}_i(t)N_i(t). \tag{13}$$

The national demand for and supply of agricultural goods is equal. That is

$$c_{aa}(t)N_a(t) + c_{ia}(t)N_i(t) = F_a(t) \tag{14}$$

The national production of industrial goods is equal to the national consumption and national net saving. That is

$$C(t)+S(t) - K(t)+\delta_k K(t)=F_i(t), \tag{15}$$

where

$$C(t) \equiv c_a(t)N_a(t) + c_i(t)N_i(t),\ S(t) \equiv s_a(t)N_a(t) + s_i(t)N_i(t).$$

The assumption that labour force and land are fully employed is represented by

$$N_a(t) + N_i(t) = N,\ l_i(t)N_i(t) = L_i,\ L_a(t) + l_a(t)N_a(t) = L. \tag{16}$$

HUMAN CAPITAL ACCUMULATION

We assume that there are two sources of improving human capital, through learning by doing[10]. Arrow [7] first introduced learning by doing into growth theory. We specify the following dynamics[11]

$$\dot{H} = \frac{\tau_a N_a F_a}{NH^{\delta_a}} + \frac{\tau_i N_i F_i}{NH^{\delta_i}} - \delta_h H, \tag{17}$$

where τ_j, ε_j and δ_h are parameters. The term $\delta_h \cdot H$ describes depreciation of human capital, where δ_h is the depreciation rate of human capital. We interpret $\tau_j \cdot N_j \cdot F_j / NH^{\varepsilon_j}$ as effects of learning by doing of each worker in sector j upon accumulation of human capital. The contribution of the production sector to human capital improvement is positively related to its production scale, F_j, and is dependent on the level of human capital. The term H^{ε_j} takes account of returns to scale effects in human capital accumulation. The case of ε_j > (<) 0 implies that as human capital is increased it is more difficult (easier) to further improve the level of human capital. The term, N_j/N measures sector j's relative contribution to the improvement of human capital.

We have thus established the economic dynamics with endogenous economic structure, physical capital and human capital. We now examine dynamic properties of the system.

ECONOMIC EQUILIBRIUM

This section shows that the dynamic system may have either a unique or none or multiple equilibrium points. Since a complete dynamic analysis system is too complicated, we are only concerned with existence of equilibrium[12]. Before stating the main analytical results, we introduce two parameters

$$x_a \equiv \frac{\alpha_a m_i}{\beta_i} + m_a - \varepsilon_a - 1, \;\; x_i \equiv \frac{m_i}{\beta_i} - \varepsilon_i - 1.$$

The following proposition is proved in Appendix A1.

PROPOSITION

The equilibrium values of r and N_i are uniquely given by equations (A11) and (A17). For $0 < N_i < N$ and $r > 0$ if $x_a < 0$ and $x_i < 0$ (or $x_a > 0$ and $x_i > 0$), the system has a unique equilibrium; and if $x_a < 0$ and $x_i > 0$ ($x_a > 0$ and $x_i < 0$), the system may have none, one, or two equilibrium points. For a positive value of H determined by (A20), the equilibrium values of all the other variables are uniquely determined by the following procedure: $N_a = N - N_i \rightarrow \bar{k}_j$ (j = a, i) by (A19) $\rightarrow K_i$ by (A18) $\rightarrow R_j$ by (A14) $\rightarrow w_i$ by (A11) $\rightarrow w_a$ by (A12) $\rightarrow \bar{r}$ by (A10) $\rightarrow \hat{y}_j = \bar{k}_j/\lambda \rightarrow K$ by (A9) $\rightarrow K_a$ by (A8) $\rightarrow p$ by (A2) $\rightarrow l_i = L_i/N_i \rightarrow l_a$ by (10) $\rightarrow c_a$, c_{aa} and s_a by (10) $\rightarrow c_i$, c_{ia} and s_i by (10) $\rightarrow F_i$ by (3) $\rightarrow F_a$ by (1).

By the definitions of x_a and x_i, we interpret x_a and x_i as measurements of returns to scale of the agricultural and industrial sectors in the dynamic system, respectively. When $x_j <$ (>) 0 we say that sector j displays decreasing (increasing) returns to scale in the dynamic economy. The above proposition tells us that if the sectors both display decreasing (increasing) returns, the dynamic system has a unique equilibrium; if one sector displays decreasing (increasing) returns and the other sector exhibits increasing (decreasing), the system may have none, one, or two equilibrium points. As shown in Appendix A2, it is difficult to explicitly judge stability properties of the dynamic system. Nevertheless, if the urban and rural areas have the same level of constant amenity, then the dynamic analysis becomes much easier[13]. The following corollary is proved in [3].

COROLLARY

Assume that the urban and rural areas have the same level of constant amenity, that is, $\theta_i = \theta_a$. Then, if $x_a < 0$ and $x_i < 0$ (or $x_a > 0$ and $x_i > 0$), the system has a unique stable (unstable) equilibrium point; and if $x_a < 0$ and $x_i > 0$ ($x_a > 0$ and $x_i < 0$), the system may have none, one, or two equilibrium points. When the system has two equilibrium points, the one with higher value of H is stable and the other one is unstable.

The assumption of $\theta_i = \theta_a$ and the same level of land rents for different types of land use imply that the same level and consumption pattern of households, irrespective of their professions and location. Under this strict requirement, we can explicitly determine the dynamic properties of the model. Nevertheless, like in the Harris-Todaro model, this study is concerned with effects of rural and urban production and living condition differences upon labour migration. As shown in Appendix A2, it is difficult to analyze dynamic properties of the model. For illustration, we specify values of the parameters and simulate the model to examine the behaviour of the economic system. We specify the parameters as follows

$$\alpha_i = 0{,}45,\ \alpha_a = 0{,}25,\ \beta_a = 0{,}35,\ A_i = 1{,}1,\ A_a = 0{,}9,\ d_a = 0,\ d_i = -0{,}05,\ \bar{\theta}_i = 4,\ \bar{\theta}_a = 5,$$
$$\eta_0 = 0{,}07,\ \xi_0 = 0{,}1,\ \mu_0 = 0{,}05,\ \lambda_0 = 0{,}7,\ \tau_a = 0{,}01,\ \tau_i = 0{,}05,\ \varepsilon_a = 0{,}5,\ \varepsilon_i = 0{,}05,$$
$$m_a = 0{,}3,\ m_i = 0{,}7,\ N = 10,\ L_i = 1,\ L = 10,\ \delta_k = 0{,}03,\ \delta_h = 0{,}1. \tag{18}$$

The capital shares in the industrial and agricultural sectors, α_i and α_i are equal to 0,45 and 0,25, respectively. This implies that the industrial sector is relatively capital-intensive compared with agriculture[14]. The total productivity levels of the industrial and agricultural sectors, A_i and A_a are 1,1 and 0,9, respectively. The level of the industrial sector is higher than the agricultural sector. The amenity coefficients of the urban and rural areas, $\bar{\theta}_i$ and $\bar{\theta}_a$ are fixed at 4 and 5, respectively. As shown in the proposition, what matters is the ratio $\bar{\theta}_i / \bar{\theta}_a$, rather than their absolute values. The lower the ratio is, the more attractive the rural area becomes, with all other conditions equal. For simplicity, we assume that the rural amenity is constant and the urban amenity falls as the city's population rises[15]. The propensity to save, λ_0 is 0,7. The specified values of η_0, ξ_0 and μ_0 imply that the ratio between the expenditures on the housing and agricultural goods is 1,4 and the ratio between the expenditures on the industrial goods and agricultural goods is 2. The total population is 10 and the rural territory size is 10 times of the urban territory size. The conditions $\varepsilon_a = 0{,}5$ and $\varepsilon_i = 0{,}05$ mean respectively that the learning by producing exhibits decreasing effects in human capital; the agricultural sector's decreasing effect is much stronger than the industrial sector's.

Under (18) we have $x_a = -0{,}822$ and $x_i = -0{,}223$ This implies that the agricultural sector's learning by doing exhibits decreasing returns and the industrial sector increasing returns. As shown in the proposition, the system may have two equilibrium points. From the proposition we know that the variables, r, N_i, N_a, L_a, l_a and l_i are determined, independent of the two variables H and K_i. This implies that when the system has two equilibrium points, the rate of interest, the labour distribution and land-use distribution are equal at the two points[16]. The variable N_i is determined by equation (A17), $\Phi_N(N_i) = 0$. Figure 1 shows that the equation has a unique solution.

We uniquely determine r, N_i, N_a, L_a, l_a and l_i as in Table 1. We see that most of the labour force is located in the city. The farmer's lot size is much larger than the urban worker's lot size The variable H is determined by equation (A20), $\Phi_H(H) = 0$. Figure 2 shows that the equation

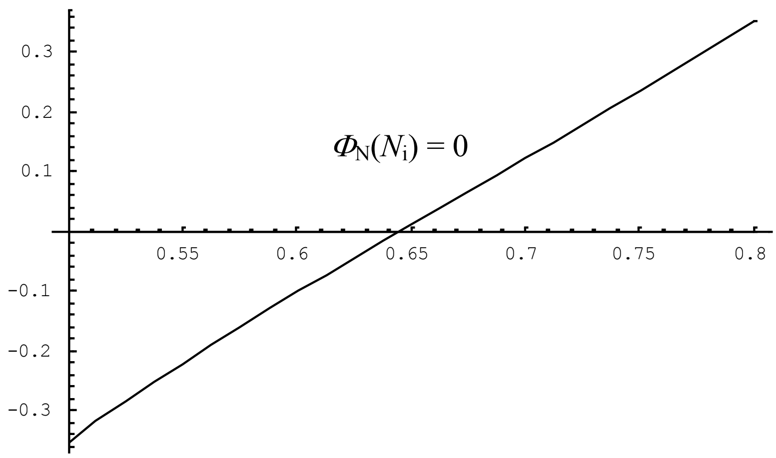

Figure 1. The unique labour distribution.

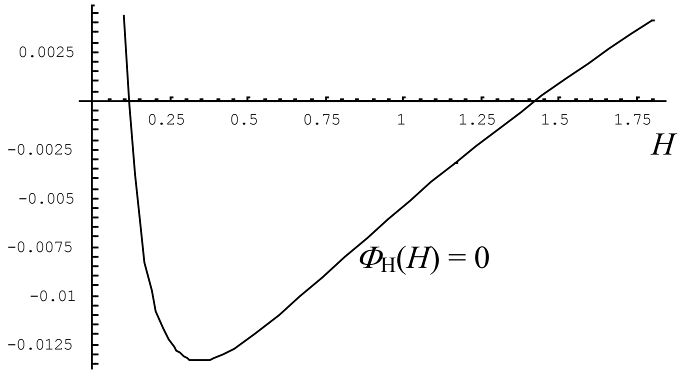

Figure 2. The existence of two equilibrium points of human capital.

has two solutions: H_1 = 0,115 and H_2 = 1,427. We denote the two equilibrium points using subscripts 1 and 2. We call the two equilibrium points as advanced equilibrium (AE) and underdeveloped equilibrium (UE). Following the proposition, we determine the equilibrium values of the other variables, which are summarized as in Table 1.

Table 1. The variables' values at the two equilibrium points, H_1 and H_2.

r	N_i	N_a	L_a	l_a	l_i
0,065	0,645	0,355	8,347	4,655	1,551

Variable	Equilibrium point		Variable	Equilibrium point		Variable	Equilibrium point	
	H_1	H_2		H_1	H_2		H_1	H_2
F	0,211	8,504	F_a	0,504	2,388	$\bar{k}_a$	0,161	3,946
H	0,115	1,427	F_i	0,174	4,280	$\bar{k}_i$	0,223	5,473
K	1,001	24,761	w_a	0,071	1,743	c_{aa}	0,016	0,159
p	0,072	1,769	w_i	0,149	3,651	c_a	0,023	0,564
$\bar{r}$	0,130	3,184	R_a	0,003	0,085	c_{ia}	0,021	0,221
$\bar{K}_a$	0,188	4,623	R_i	0,014	0,353	c_i	0,016	0,391
$\bar{K}_i$	0,820	20,138						

We see that the difference in the levels of human capital at the two equilibrium points is very large. The level of the human capital at the AE is more than 12 times higher than that at the UE. The ratio between the national output levels, F $(= F_i + p{\cdot}F_a)$, is 42 times. The price of the agricultural goods, land rent, and the wage rate at the AE are all much higher than the corresponding variables at the UE. The output levels of the two sectors, the per capita wealth levels of the rural and urban residents, and the per capita consumption levels of the two products by the rural and urban residents at the AE are all much higher than the corresponding variables at the UE.

In the literature of economic development, it is well known that there may be multiple equilibrium points for the same type of economy when market imperfections or endogenous human capital are introduced into economic dynamics. This implies, for instance, that two seemingly identical regions may follow radically different development paths, one leading to prosperity, the other to stagnation. Taiwan and Mainland China may provide a proper case for this result. Although they had similar backgrounds in terms of cultural heritage, values, and initial human capital, Taiwan and Mainland China had experienced totally different paths of industrialization during the period 1950-1980 – the former rapidly moved to the high equilibrium point, while the latter cycled around the low equilibrium point. It should be remarked that Canning [8] proposes a two-sector model with increasing returns to scale in the industrial sector and diminishing returns in agriculture. The model demonstrates that increasing demand for food coupled with diminishing returns in agriculture may not be a barrier to economic growth[17]. Canning's model shows that the growth of the economy may be unlimited, despite ever increasing demand for agricultural procedure and in the absence of technical progress, if the increasing demands in the capital goods industry are sufficient to outweigh the diminishing returns to capital in agriculture. The equilibrium point with the higher level of human capital in our model explains what the Canning model predicts. It should be remarked that the concerns of classical economists, such as Ricardo, about capital accumulation with agriculture and industry can be explained by the case of the decreasing returns to scale in the two sectors.

CHANGES IN THE PRODUCTIVITY LEVEL, THE POPULATION, AND THE PROPENSITY TO SAVE

We now examine how the parameters affect the economic structure and labour distribution. First, we examine the case that all the parameters, except the productivity of the industrial sector A_i, are the same as in (18). We increase the productivity level A_i from 1,1 to 1,15. We introduce a symbol $\bar{\Delta}$ to stand for the change rate of the equilibrium value of a variable in percentage due to the change in a parameter value from. For instance, with regard to a variable x_j, assuming the change of a parameter A_i from its current value A_{i0} (which equals 1,1 in this case) to the new value A_{i1} (equal to 1,15), we have

$$\bar{\Delta} x_j \equiv \frac{x_j(A_{i1}) - x_j(A_{i0})}{x_j(A_{i0})} \times 100.$$

As A_i rises from 1,1 to 1,15 the variables, r, N_i, N_a, L_a, l_a and l_i are not affected. Figure 3 shows how the two solutions of H are affected. The equilibrium values of the other variables are listed in Table 2. We can see that an increase in the industrial sector's total productivity has the opposite effects upon the variables at the AE and the UE. The equilibrium values at the UE are increased and the equilibrium values at the AE are reduced. This implies that for the economy under consideration, as the total productivity is improved, in order to maintain the system at equilibrium the lower equilibrium point is improved and the higher equilibrium point is lowered. Intuitively, it is easy to interpret the effects upon the UE as this point is characterized of decreasing returns to scale. An increase in the productivity will improve the economic performance of the economy. To interpret the effects upon the AE, first we note that this point is characterized of increasing returns to scale. Although we fail to prove its stability properties, this point is seemingly unstable. This implies that if the system is located near the AE, it has possibility of unlimited growth as the system will rarely remind at unstable equilibrium in the long term. If the equilibrium point is lowered, it is easier for the economy to sustain economic growth in the long term[18].

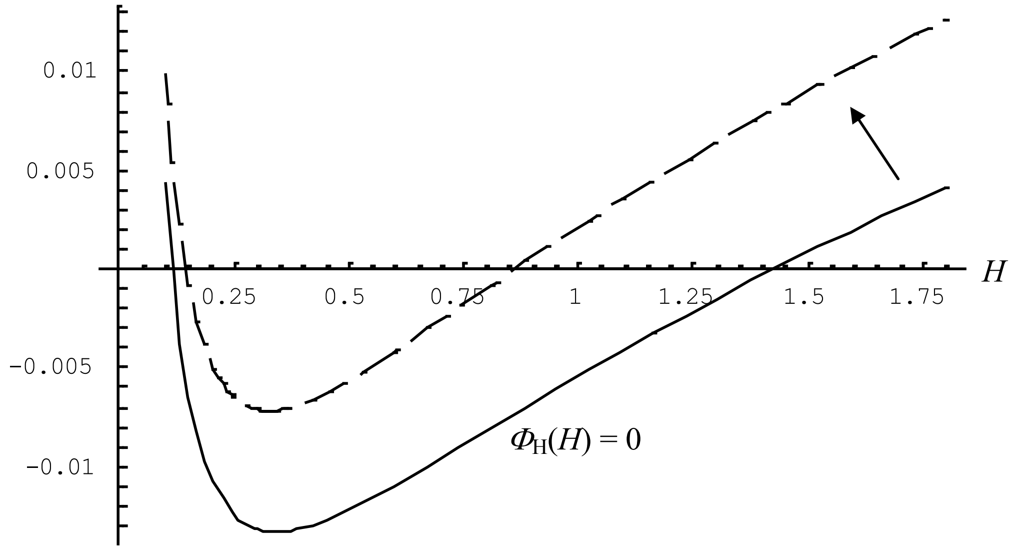

Figure 3. Shifts of the two equilibrium points as A_i rises.

Table 2. The effects as the industrial sector's total productivity rises. Values of variables are given at two equilibrium points, H_1 and H_2.

Variable	Equilibrium point		Variable	Equilibrium point		Variable	Equilibrium point	
	H_1	H_2		H_1	H_2		H_1	H_2
$\bar{\Delta} F$	49,971	–50,099	$\bar{\Delta} F_a$	17,724	–25,932	$\bar{\Delta} \bar{k}_a$	45,524	–42,932
$\bar{\Delta} H$	26,020	–39,936	$\bar{\Delta} F_i$	45,524	–42,932	$\bar{\Delta} \bar{k}_i$	45,524	–42,932
$\bar{\Delta} K$	45,524	–42,932	$\bar{\Delta} w_a$	45,524	–42,932	$\bar{\Delta} c_{aa}$	0	0
$\bar{\Delta} p$	45,524	–42,932	$\bar{\Delta} w_i$	45,524	–42,932	$\bar{\Delta} c_a$	45,524	–42,932
$\bar{\Delta} \bar{r}$	45,524	–42,932	$\bar{\Delta} R_a$	45,524	–42,932	$\bar{\Delta} c_{ia}$	0	0
$\bar{\Delta} \bar{K}_a$	45,524	–42,932	$\bar{\Delta} R_i$	45,524	–42,932	$\bar{\Delta} c_i$	45,524	–42,932
$\bar{\Delta} \bar{K}_i$	45,524	–42,932						

We also analysed the case when A_a: 0,9 $\to$ 1. It can be shown that the variables r, N_i, N_a, L_a, l_a and l_i are not affected and the two solutions of H are changed similarly to Figure 3. The change directions in the variables are the same as in Table 2, except that the per capita consumption levels of the farmers are affected but the consumption levels of the urban workers are not affected. As we increase τ_i: 0,05 $\to$ 0,055 the variables, r, N_i, N_a, L_a, l_a and l_i are not affected and the two solutions of H are changed similarly to Figure 3. The change directions in the variables are the same as in Table 2. We now allow the population to rise as follows: N: 1,0 $\to$ 1,1. It is demonstrated that the two solutions of H are changed similarly to Figure 3. The level of human capital at UE is increased and the level at the AE is reduced. The changes in the equilibrium values of the variables are given in Table 3. As the total population rises, the rate of interest is not affected. The urban population rises by 9,6 % and the rural population rises by 10,8 %. The residential lot sizes in the rural and urban area fall respectively by 9,4 % and 8,7 %. The level of human capital at the UE rises by 41 % and the level of human capital at the AE falls by 44 %. We see that an increase in the population has the opposite effects upon the variables at the AE and the UE, except for the per capital levels of consumption of the agricultural goods which are reduced at the both equilibrium points. The equilibrium values at the UE are increased and the equilibrium values at the AE are reduced. A larger population benefits the long-term economic growth. As the population is increased, the UE is improved and the AE is lowered.

Table 3. The effects as the population rises.

$\overline{\Delta} r$	$\overline{\Delta} N_i$	$\overline{\Delta} N_a$	$\overline{\Delta} L_a$	$\overline{\Delta} l_a$	$\overline{\Delta} l_i$
0,065	0,645	0,355	8,347	4,655	1,551

Variable	Equilibrium point		Variable	Equilibrium point		Variable	Equilibrium point	
	H_1	H_2		H_1	H_2		H_1	H_2
$\overline{\Delta} F$	78,649	–54,263	$\overline{\Delta} F_a$	31,073	–25,852	$\overline{\Delta} \bar{k}_a$	53,750	–52,419
$\overline{\Delta} H$	40,945	–43,918	$\overline{\Delta} F_i$	69,563	–47,526	$\overline{\Delta} \bar{k}_i$	54,483	–52,192
$\overline{\Delta} K$	69,583	–47,526	$\overline{\Delta} w_a$	53,023	–52,644	$\overline{\Delta} c_{aa}$	–9,326	–9,326
$\overline{\Delta} p$	69,583	–47,526	$\overline{\Delta} w_i$	54,775	–52,102	$\overline{\Delta} c_a$	53,750	–52,419
$\overline{\Delta} \bar{r}$	54,148	–52,296	$\overline{\Delta} R_a$	69,563	–47,484	$\overline{\Delta} c_{ia}$	–8,893	–8,894
$\overline{\Delta} \bar{K}_a$	69,563	–47,526	$\overline{\Delta} R_i$	69,563	–47,626	$\overline{\Delta} c_i$	54,483	–52,419
$\overline{\Delta} \bar{K}_i$	69,563	–47,526						

An important issue in growth theory is related to interdependence between the propensity to save and national wealth. The study of individual thrift and national wealth has long been important in economics because national saving is the source of the supply of capital, a main factor of production affecting the productivity of labour. Thrift had traditionally been regarded as a virtuous, socially beneficial act. Admit Smith argued that capital is increased by parsimony and diminished by prodigality. He believed that parsimony, and not industry, is the immediate cause of the increase in capital. Smith said that prodigals are public enemies. This belief was strongly challenged by Keynes in the General Theory. He suggested that saving is potentially disruptive to the economy and harmful to social welfare. High propensity to save may reduce consumption, without systematically and automatically giving rise to an offsetting expansion in investment. This might thus cause demand to fall lower than proper level and hence output and employment lower than the capacity of the economy. We show that in economies with returns to scale the impact of the propensity to hold wealth are situation-dependent. An increase in the propensity to save may either increase or reduce the national wealth, depending on the current situations of the system. This implies that both Smith and Keynes are right under some situations and wrong under others.

We increase the propensity to save as follows, λ_0: 0,7 $\rightarrow$ 0,73. The two solutions of H are changed similarly to Figure 3. The changes in the equilibrium values of the variables are listed in Table 4. As the propensity to save rises, the rate of interest falls. The urban population rises and the rural population falls. The land for agricultural use is increased. The lot sizes in the urban and rural areas are reduced. The levels of human capital and national output are reduced at the AE and increased at the UE.

Similar to the impact of an increase in the industrial sector's total productivity, an increase in the propensity to save has the opposite effects upon the variables at the AE and the UE, except the variables, p and $\bar{r}$ which are increased, and c_{aa} and c_{ia} which are reduced at the both equilibrium points. To see how c_{aa} and c_{ia} are reduced at the both equilibrium points, we note that as the propensity to save rises, the propensities to consume lot size, industrial goods, and agricultural goods fall relatively. The falls in the propensities tend to reduce the lot size, the consumption levels of the industrial agricultural goods and affect the prices of these goods. On the other hand, the changes in the incomes also affect the consumption levels of these variables and their prices. The net effects upon the consumption levels of the agricultural goods are negative at the both equilibrium points[19].

Table 4. The effects as the propensity to save rises.

$\overline{\Delta} r$	$\overline{\Delta} N_i$	$\overline{\Delta} N_a$	$\overline{\Delta} L_a$	$\overline{\Delta} l_a$	$\overline{\Delta} l_i$
–5,153	1,254	–2,276	0,799	–1,797	–1,238

Variable	Equilibrium point		Variable	Equilibrium point		Variable	Equilibrium point	
	H_1	H_2		H_1	H_2		H_1	H_2
$\overline{\Delta} F$	20,050	–36,361	$\overline{\Delta} F_a$	7,821	–16,760	$\overline{\Delta} \bar{k}_a$	19,622	–29,763
$\overline{\Delta} H$	10,633	–27,176	$\overline{\Delta} F_i$	18,613	–30,355	$\overline{\Delta} \bar{k}_i$	18,325	–30,524
$\overline{\Delta} K$	22,783	–27,906	$\overline{\Delta} w_a$	20,480	–29,259	$\overline{\Delta} c_{aa}$	–2,575	–2,575
$\overline{\Delta} p$	17,737	30,869	$\overline{\Delta} w_i$	17,145	–31,217	$\overline{\Delta} c_a$	14,706	–32,649
$\overline{\Delta} \bar{r}$	17,737	30,869	$\overline{\Delta} R_a$	16,805	–31,417	$\overline{\Delta} c_{ia}$	–3,631	–3,631
$\overline{\Delta} \bar{K}_a$	22,045	–28,340	$\overline{\Delta} R_i$	14,885	–32,544	$\overline{\Delta} c_i$	13,462	–33,379
$\overline{\Delta} \bar{K}_i$	22,953	–27,807						

CONCLUDING REMARKS

This paper examines issues related to urbanization with labour migration. The model is influenced by the Harris-Todaro model. Main departures from the traditional approach are that this paper uses an alternative approach to consumer and introduces human capital accumulation via learning by doing. The Harris-Todaro assumes unemployment of labour force in the urban area, while our study assumes full employment in the city as well. We explain differences in living conditions and wages between the urban and rural areas by introducing endogenous amenities. As amenities and technology vary between the city and rural area, the wage rates, housing rents and consumption levels are different between the city and rural area are different. The economic system consists of one production sector and one education sector. The model describes dynamic interactions among agricultural and industrial production, rural and urban amenities, distribution of production factors and preferences with endogenous capital and human capital accumulation. We show that the dynamic system may have either a single or multiple equilibrium points, depending on returns to scale parameters. We also examined effects of changes in some parameters. We get some insights into important issues related to relationships between living conditions and population growth, and the effects of propensity to save. For instance, we showed that in economies with returns to scale the impact of the propensity to hold wealth are situation-dependent. An increase in the propensity to save may either increase or reduce the national wealth, depending on the current situations of the system. This implies that both Smith and Keynes are right under some situations and wrong under others. Finally, it should be remarked that our comparative statics analysis is based on the specified parameter values. It is not difficult to see that effects of changes in any parameters are situation-dependent in the economy. We may extend the model in some directions. For instance, we may introduce some kind of government intervention in education into the model. It is also desirable to treat leisure time as an endogenous variable.

APPENDICES

A1: PROVING THE PROPOSITION

By equations (11) and (17) at equilibrium we have $s_j = \bar{k}_j$, $j = a, i$.

$$\frac{\tau_a N_a F_a}{NH^{\delta_a}} + \frac{\tau_i N_i F_i}{NH^{\delta_i}} = \delta_h H . \tag{A1}$$

From equations (2) and (4) we have

$$p = \frac{\alpha_i K_a F_i}{\alpha_a K_i F_a} \tag{A2}$$

Substitute $L_a = \varsigma \cdot p \cdot F_a / R_a$ in (2) and lj$\cdot R_j = \eta \cdot \hat{y}_j$ in (10) into the land constraints (16)

$$\eta \hat{y}_i N_i = R_i L_i , \ \varsigma p F_a + \eta \hat{y}_a N_a = R_a L \tag{A3}$$

Adding the two equations in (A3), we obtain

$$\bar{r} = (\varsigma p F_a + \eta \hat{y}_i N_i + \eta \hat{y}_a N_a)/N. \tag{A4}$$

From $s_j = \bar{k}_j$ in (A1) and $s_j = \lambda \cdot \hat{y}_j$ we have $\hat{y}_j = \bar{k}_j / \lambda$. Substitute that and equation (A2) into equation (A4) we obtain

$$\bar{r} = (\alpha_0 K_a F_i / K_i + \eta K / \lambda_a)/N. \tag{A5}$$

where we used equation (13) and $\alpha_0 \equiv \alpha_i \varsigma / \alpha_a$. Substitute $p \cdot c_{ja} = \mu \cdot \hat{y}_j$ in (10) into equation (14)

$$\mu \cdot K = \lambda \cdot p \cdot F_a. \tag{A6}$$

Substituting equations (10) into equation (15) yields

$$\delta_0 \cdot K = F_i \tag{A7}$$

where $\delta_0 \equiv \xi / \lambda + \delta_k$. Substituting equations (A2) and (A7) into equation (A6) yields

$$K_a = \delta_\lambda \cdot K_i, \tag{A8}$$

where $\delta_\lambda \equiv \mu \cdot \alpha_a / \alpha_i (\xi + \delta_k \cdot \lambda)$. From equations (A8) and (13), we have

$$K = (1 + \delta_\lambda) K_i \tag{A9}$$

Equations (A8) and (A9) determine K_a and K as unique functions of K_i. Substitute equations (A7) into (A9) into (A5)

$$\bar{r} = \bar{r}_0 K_i , \tag{A10}$$

where

$$\bar{r}_0 = (1 + \delta_\lambda) \cdot (\alpha_0 \delta_\lambda \delta_0 + \eta / \lambda)/N.$$

From equations (4), (A7) and (A9), we have

$$r = \alpha_i \delta_0 (1 + \delta_\lambda) - \delta_k , \ w_i = \frac{\beta_i \delta_0 (1 + \delta_\lambda) K_i}{N_i}. \tag{A11}$$

From equations (2) and (A8), we obtain

$$pF_a = \left(\frac{r + \delta_k}{\alpha_a} \right) \delta_\lambda K_i , \ w_a = \beta_a \delta_\lambda \left(\frac{r + \delta_k}{\alpha_a} \right) \frac{K_i}{N_a}. \tag{A12}$$

Inserting equations (10) into utility functions (8) and then applying equation (12), we obtain

$$\left(\frac{R_i}{R_a} \right)^\eta = \frac{\bar{\theta}_i^\rho \bar{k}_i N_i^{\rho d_i}}{\bar{\theta}_a^\rho \bar{k}_a N_a^{\rho d_a}}, \tag{A13}$$

in which we use $\hat{y}_j = \bar{k}_j/\lambda$ and equations (9). Substitute $\hat{y}_j = \bar{k}_j/\lambda$ and $p \cdot F_a$ in (A12) into equations (A3)

$$R_i = m_1 \bar{k}_i N_i, \quad R_a = m_2 K_i + m_3 \bar{k}_a N_a, \tag{A14}$$

where

$$m_1 \equiv \frac{\eta}{\lambda L_i}, \quad m_2 \equiv \left(\frac{r + \delta_k}{\alpha_a}\right)\frac{\varsigma \delta_\lambda}{L}, \quad m_3 \equiv \frac{\eta}{\lambda L}.$$

Insert equations (A14) into equation (A13)

$$\left(\frac{m_1 \bar{k}_i N_i}{m_2 K_i + m_3 \bar{k}_a N_a}\right)^{\eta} = \frac{\bar{\theta}_i^{\rho} \bar{k}_i N_i^{\rho d_i}}{\bar{\theta}_a^{\rho} \bar{k}_a N_a^{\rho d_a}}. \tag{A15}$$

From $\hat{y}_j = \bar{k}_j/\lambda$ and equations (A10) and (6), we have $w_j + \bar{r}_0 K_i = r_1 \bar{k}_j$ where $r_1 \equiv 1/\lambda - r$. Substituting equations (A8), (A11) and (A12) into the above equations, we have

$$\bar{k}_i \equiv \left(m_5 + \bar{r}_0 N_i\right)\frac{K_i}{r_1 N_i}, \quad \bar{k}_a = \left(m_6 - \bar{r}_0 N_i\right)\frac{K_i}{r_1 \left(N - N_i\right)}, \tag{A16}$$

where

$$m_5 \equiv \beta_i \delta_0 \left(1 + \delta_\lambda\right), \quad m_6 \equiv \left(r + \delta_k\right)\frac{\beta_a \delta_\lambda}{\alpha_a} + \bar{r}_0 N.$$

Substitute equations (A16) yields into equation (A15)

$$\Phi_N\left(N_i\right) \equiv \left(\frac{m_1\left(m_5 + \bar{r}_0 N_i\right)}{r_1 m_2 + m_3\left(m_6 - \bar{r}_0 N_i\right)}\right)^{\eta} - \frac{\bar{\theta}_i^{\rho} N_i^{\rho d_i - 1}}{\bar{\theta}_a^{\rho}\left(N - N_i\right)^{\rho d_a - 1}}\left(\frac{m_5 + \bar{r}_0 N_i}{m_6 - \bar{r}_0 N_i}\right) = 0, \tag{A17}$$

where we use $N_a + N_i = N$ and equations (13) and (A9). This equation contains a single variable. The labour distribution is determined by a positive N_i such that

$$\Phi_N(N_i) = 0, \; 0 < N_i < N.$$

We require $\rho d_j - 1 < 0$, $j = a, i$. As $\Phi_N(N) > 0$ we see that the problem has at least one meaningful solution. As it is difficult to discuss conditions whether the problem has a unique solution, we will confirm whether the labour distribution is unique when simulating the model. From equation (A17) and $N_a = N - N_i$ we determine the labour distribution as a function of the population and other parameters.

For any given N_i from equations (3) and (4), we solve K_i as a function of H

$$K_i = m_0 H^{m_i/\beta_i}, \tag{A18}$$

in which

$$m_0 \equiv \left(\frac{\alpha_i A_i}{r + \delta_k}\right)^{1/\beta_i} N_i.$$

From equations (A17) and (A11) we may consider m_0 as a parameter. From equations (A16) we solve $\bar{k}_i$ and $\bar{k}_a$ as functions of H

$$\bar{k}_j = q_j H^{m_i/\beta_i}, \quad j = a, i, \tag{A19}$$

where

$$q_i \equiv \left(m_5 + \bar{r}_0 N_i\right)\frac{m_0}{r_1 N_i}, \quad q_a \equiv \left(m_6 - \bar{r}_0 N_i\right)\frac{m_0}{r_1\left(N - N_i\right)}.$$

From equations (A14) and (A16), we get $R_a = m_4 K_i$, where $m_4 \equiv m_2 + (m_6 - \bar{r}_0 N_i) m_3 / r_1$. From $R_a = m_4 K_i$ and equations (2) and (A12), we obtain

$$L_a = \frac{\varsigma \delta_\lambda}{m_4} \frac{r + \delta_k}{\alpha_a}.$$

We now determine H. Substituting equations (1) and (3) into the last equation in (A1), we obtain the following equation

$$\Phi_h(H) \equiv \Phi_a(H) + \Phi_i(H) - \delta_h = 0, \tag{A20}$$

where we use equations (A8) and (A18) and

$$\Phi_a(H) \equiv \frac{\tau_a A_a N_a^{1+\beta_a} L_a^{\varsigma} \delta_\lambda^{\alpha_a} m_0^{\alpha_a}}{N} H^{x_a}, \quad \Phi_i(H) \equiv \frac{\tau_i A_i N_i^{1+\beta_i} m_0^{\alpha_i}}{N} H^{x_i},$$

$$x_a \equiv \frac{\alpha_a m_i}{\beta_i} + m_a - \varepsilon_a - 1, \quad x_i \equiv \frac{m_i}{\beta_i} - \varepsilon_i - 1.$$

We omit the case of $x_a = x_i = 0$. Equilibrium of the system is given by a positive H such that $\Phi(H) = 0$. When $x_a > 0$ and $x_a > 0$ equation $\Phi(H) = 0$ has a unique positive solution as $\Phi' = 0$ for any positive H, $\Phi(H) < 0$ and $\Phi(\infty) > 0$. Similarly, if $x_a < 0$ and $x_i < 0$ the equation $\Phi(H) = 0$ has a unique positive solution. It is easy to check that if either $x_a = 0$, or $x_i = 0$ then the system has a unique positive solution under certain conditions. We now prove that if $x_a > 0$and $x_i < 0$ (or $x_a < 0$ and $x_i > 0$), then the system has either two solutions or no solution. It is sufficient for us to examine one case, for instance that with $x_a > 0$ and $x_i < 0$. Since $\Phi(H) > 0$, $\Phi(\infty) > 0$ we see that $\Phi(H) = 0$ cannot have a unique solution. That is, the equation $\Phi(H) = 0$ has either multiple solutions, or no solution. On the other hand, as $\Phi'(H) = 0$ has a unique positive solution, we conclude that $\Phi(H) = 0$ has two solutions if $\Phi(H)$ has solutions. The necessary and sufficient condition for the existence of two solutions is that there exists a positive value H_1 of H such that $\Phi(H_1) < 0$ and $\Phi'(H_1) = 0$. We have thus proved the proposition.

A2: DESCRIBING THE MOTION WITH THREE DIFFERENTIAL EQUATIONS

We now show a procedure to determine dynamic properties of the system. We omit time index in expressions in Appendix A2. Similar to equation (A4), we have

$$\bar{r} = \frac{\alpha_0 K_a F_i / K_i + \eta \hat{y}_i N_i + \eta \hat{y}_a N_a}{N}, \tag{A21}$$

where we use equation (A2) and $\alpha_0 \equiv \alpha_i \varsigma / \alpha_a$. Substitute $p \cdot c_{ja} = \mu \cdot \hat{y}_j$ in (10) into equation (14)

$$\mu \left(\hat{y}_a N_a + \hat{y}_i N_i \right) = p F_a. \tag{A22}$$

Substituting equations (10) into equation (15) yields

$$\left(\xi + \lambda \right) \left(\hat{y}_a N_a + \hat{y}_i N_i \right) = F_i + \delta K. \tag{A23}$$

From equations (A22) and (A23), we have

$$\frac{F_i + \delta K}{p F_a} = \frac{\xi + \lambda}{\mu}. \tag{A24}$$

Substituting equation (A2) into equation (A24) yields

$$K_a = \frac{\alpha_a \mu}{\alpha_i (\xi + \lambda)} \frac{(F_i + \delta K) K_i}{F_i}. \tag{A25}$$

From equation (A25) and $K = K_a + K_i$ we solve

$$K = \frac{(1 + \alpha) K_i F_i}{F_i - \alpha \delta K_i}, \tag{A26}$$

in which $\alpha \equiv \alpha_a \mu / \alpha_i (\xi + \lambda)$. Equation (A26) determines $K(t)$ as a unique function of $K_i(t)$ and $H(t)$. We see that $K_a(t)$ is also determined as a function of $K_i(t)$ and $H(t)$. Substitute equation (A21) into the definitions of $\hat{y}_j$ in (6), we obtain

$$\hat{y}_j = (1 + r)\bar{k}_j + w_j + \frac{\alpha_0 K_a F_i / K_i + \eta \hat{y}_i N_i + \eta \hat{y}_a N_a}{N}, \quad j = a, i. \tag{A27}$$

Solving previous equations, we obtain

$$\hat{y}_i = \frac{(1 + r)N\bar{k}_i + N w_i + \alpha_0 K_a F_i / K_i + \eta N_a \bar{k}}{(1 - \eta)N}, \quad \hat{y}_a = \bar{k} + \hat{y}_i, \tag{A28}$$

in which $\bar{k} = (1 + r)(\bar{k}_a - \bar{k}_i) + w_a - w_i$. From equations (4), we see that r and w_i can be considered as functions of K_i, H and N_i. From equations (A3), we have

$$w_a = \frac{\beta_a (r + \delta_k) K_a}{\alpha_a N_a}. \tag{A29}$$

By equations (A2) and (A29), p and w_a are also functions of K_i, H and N_i. Using $N_a = N - N_i$ and equations (A2) and (A29), we can express $\hat{y}_i$ as a unique function of $\bar{k}_a$, $\bar{k}_i$, K_i, H and N_i:

$$\bar{k} = \Psi_k(\bar{k}_i, \bar{k}_a, K_i, H, N_i),$$
$$\hat{y}_i = \Psi(\bar{k}_i, \bar{k}_a, K_i, H, N_i). \tag{A30}$$

Inserting equations (10) into utility functions (8) and then applying equation (12), we obtain

$$\left(\frac{R_i}{R_a}\right)^{\eta} = \frac{\bar{\theta}_i \hat{y}_i N_i^{d_i}}{\bar{\theta}_a \hat{y}_a N_a^{d_a}}, \tag{A31}$$

in which we also use (9). Substitute equations (A3) into equation (A31)

$$\left(\frac{\alpha_0 K_a F_i}{K_i} + \eta \hat{y}_a N_a\right)^{-\eta} = \bar{\theta}\, \frac{\hat{y}_i^{1-\eta} N_i^{d_i - \eta}}{\hat{y}_a N_a^{d_a}}, \tag{A32}$$

where we use (A2) and

$$\bar{\theta} \equiv \frac{\bar{\theta}_i}{\bar{\theta}_a}\left(\frac{L_i}{\eta L_a}\right)^{\eta}.$$

Substitute equations (A28) into equation (A30)

$$\bar{\Psi}(\bar{k}_i, \bar{k}_a, K_i, H, N_i) \equiv \left(\frac{\alpha_0 K_a F_i}{K_i} + \eta (N - N_i)(\Psi_k + \Psi)\right)^{-\eta} - \frac{\bar{\theta} \Psi^{1-\eta} N_i^{d_i - \eta}}{(\Psi_k + \Psi)(N - N_i)^{d_a}} = 0. \tag{A33}$$

Assume that from equation (A33) we determine N_i as a function of $\bar{k}_a$, $\bar{k}_i$, K_i and H:

$$N_i \equiv \bar{\Psi}_N(\bar{k}_i, \bar{k}_a, K_i, H). \tag{A34}$$

From equation (A34) and $N_a = N - N_i$, we determine the labour distribution as functions of $\bar{k}_a$, $\bar{k}_i$, K_i and H. From equation (13) and $N_a = N - N_i$ we have

$$K_a + K_i = \bar{k}_a N + (\bar{k}_i - \bar{k}_a)\bar{\Psi}_N(\bar{k}_i, \bar{k}_a, K_i, H). \tag{A35}$$

Equation (A35) contains four variables: $\bar{k}_a$, $\bar{k}_i$, K_i and H. Assume that we solve $\bar{k}_a$ as a function of $\bar{k}_i$, K_i and H as follows

$$\bar{k}_a = \bar{\Psi}_0(\bar{k}_i, K_i, H). \tag{A36}$$

By the following procedure, we can determine all the variables as functions of $\bar{k}_i(t)$, $K_i(t)$ and $H(t)$ at any point of time: $\bar{k}_a$ by equation (A36) → N_i by equation (A34) → $N_a = N - N_i$ → $\bar{k}$ and $\hat{y}_i$ by equations (A30) → r and w_i by equations (4) → K by (A26) → p by equation (A2) → $\bar{r}$

by equation (A27) → w_a by (A29) → $\hat{y}_a$ by (A0) → $l_i = L_i/N_i$ → R_a by (A3) → l_a and R_i by equations (10) → c_a, c_{aa} and s_a by equations (10) → c_i, c_{ia} and s_i by equations (10) → F_i by equation (3) → F_a by equation (1). From equations (11), (17) and (A36), we have

$$\dot{\bar{k}}_a(t) = \Lambda_a\left(\bar{k}_i(t),\, K_i(t),\, H(t)\right) \equiv s_a(t) - \overline{\Psi}_0, \tag{A37}$$

$$\dot{\bar{k}}_i(t) = \Lambda_i\left(\bar{k}_i(t),\, K_i(t),\, H(t)\right) \equiv s_i(t) - \bar{k}_i(t),$$

$$\dot{H}(t) = \Lambda_H\left(\bar{k}_i(t),\, K_i(t),\, H(t)\right) \equiv \frac{\tau_a N_a(t) F_a(t)}{N H^{\varepsilon_a}(t)} + \frac{\tau_i N_i(t) F_i(t)}{N H^{\varepsilon_i}(t)} - \delta_h H(t). \tag{A38}$$

Taking derivatives of equation (13) with respect to t we obtain

$$\dot{\bar{k}}_a = \overline{\Psi}_{0k}\dot{\bar{k}}_i + \overline{\Psi}_{0K}\dot{K}_i + \overline{\Psi}_{0H}\dot{H}, \tag{A39}$$

where $\overline{\Psi}_{0k}$, $\overline{\Psi}_{0K}$ and $\overline{\Psi}_{0H}$ are partial derivatives of $\overline{\Psi}_0$ with respect to $\bar{k}_i(t)$ $K_i(t)$ and $H(t)$, respectively. From equations (A37) and (A39), we delete $\dot{\bar{k}}_a$ and obtain

$$\dot{K}_i(t) = \Lambda_K\left(\bar{k}_i(t),\, K_i(t),\, H(t)\right) \equiv \frac{\Lambda_a - \overline{\Psi}_{0H}\Lambda_H - \overline{\Psi}_{0k}\Lambda_i}{\overline{\Psi}_{0K}}. \tag{A40}$$

Equations (A38) and (A40) contain three variables: $\bar{k}_i(t)$, $K_i(t)$ and $H(t)$. The three differential equations determine the motion of $\bar{k}_i(t)$, $K_i(t)$ and $H(t)$ over time. All other variables are determined as functions of the three variables at any point of time. As the expressions are tedious, t is difficult to interpret analytical results. We are concerned only with equilibrium issues.

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REMARKS

[1]The model was first presented in [9] and [10]. The original static model has been extended in different ways (see for instance, [10 – 12]). As mentioned by Fields [13], the model has been extended to allow for an urban informal sector, on-the-job search from agriculture, duality within the rural sector, educational differences among workers, job fixity, mobile capital, endogenous urban wage setting, risk-aversion, a system of demand of goods and many other factors. The list of extensions can be much longer.

[2]The assumption by Matsuyama is not supported by the empirical evidence presented in [14] and [15]. It is demonstrated that growth in total factor productivity in agriculture is not only strictly positive but, in most cases, larger than total factor productivity growth in industry. It should also be remarked that the two-sector model presented in [8] fixes the saving rate and does not consider endogenous change in human capital.

[3]The assumption of full utilization of factor resources is strict. However, as shown in [3] for a two-sector economy with constant human capital, it is conceptually not difficult to relax the assumption of full employment of labour force. Nevertheless, the model with unemployment and human capital will become difficult to analyze.

[4]Although this assumption is often accepted in the literature of urbanization with agriculture (see [17, 18]), some studies try to examine impact of transportation costs upon urban-rural labour distribution (e.g., [19, 20]).

[5]As the urban land used for industrial sector is not large, the omission of industrial land use is acceptable.

[6]Zhang has also examined the relations between his approach and the Solow growth theory, the Ramsey growth theory, the permanent income hypothesis, and the Keynesian consumption function in details.

[7]The concept of amenity is often used in the literature of urban and regional economics (see, for instance, [6, 21 – 24]). The concept has recently been introduced into the Ramsey growth model in [25].

[8]The Keynesian consumption function and permanent income hypotheses (which are not the same) are similar to our approach in the sense that the propensity to save is affected by wealth. It should be noted that Zhang's approach is very general in the sense that by introducing endogenous taste change, Zhang's approach generates the same consumer behaviour as described by the traditional approaches (see [3]).

[9]Another important issue is about taste change. In any basic course in microeconomics, concepts of normal, inferior, and luxury goods are introduced. For illustration, we now point out possible ways to take account of a household's preference change due to changes in income. Let there be n kinds of goods and services. The household's utility function is given, for instance, by

$$U(t) = s^{\lambda(t)}(t)\prod_{j=1}^{n} c_j^{\xi_j(t)}(t),$$

where $c_j(t)$ is the consumption level of goods j, $s(t)$ is the saving, and the preference parameters are defined similarly as in (8). The budget constraint is given by

$$\sum_{j=1}^{n} p_j(t)c_j(t) + s(t) = \hat{y}(t)$$

where $\hat{y}(t)$ is the disposable income. The optimal solution is

$$s(t) = \lambda(t)\hat{y}(t),\ c_j(t) = \xi_j(t)\,\hat{y}(t)/p_j(t),\quad j = 1, \ldots, n.$$

Here, we consider that the propensities are influenced by the household's disposable income (and/or wage and wealth), his age, and other factors like relative social status in the following way:

$$\lambda(t) = \lambda[\hat{y}(t), t],\ \xi_j(t) = \xi_j[\hat{y}(t), t],\quad j = 1, \ldots, n.$$

For instance, if good 1 is an inferior good, and the others are normal, we may specify the preference change as follows: $\xi_1(t) = \xi_{10} - \xi_{11}\cdot\hat{y}(t)$, $\xi_1(t) > 0$, where ξ_{10} and ξ_{11} are constants and the rest of the parameters are kept constant. The preference change may be nonlinear. We will not examine taste change in this study as the analysis is already very complicated.

[10]In the contemporary literature of growth theory, different sources of human capital, such as education, are introduced to explain economic growth and development (see, e.g. [26 – 29]). This study is limited the case of learning by doing. It should be noted that Zhang [30] takes account of three sources of learning, learning by doing, learning by leisure, and learning by education.

[11]For simplicity, we assume a linear relation between the outputs and growth rate of human capital. It is important to examine what will happen to the system if the growth rate is related to the outputs with some reasonable nonlinear relations.

[12]Although we failed to explicitly give stability conditions, Appendix A2 shows the procedure of finding out the dynamic equations of the economic system.

[13]As mentioned before, the main extension of this study is to introduce amenity differences between the rural and urban areas (which are the key factors for explaining wage, consumption and land rent differences). In [3] the total land is not fixed and the transformation form one type of land use to another is costless and instantaneous.

[14]This assumption is accepted, for instance in [5].

[15]The specification is strict. For instance, as the urban area is expanded, the city may become more attractive.

[16]These properties are mainly due to the specified forms of the utility and production functions.

[17]The problem of increasing demand for food coupled with diminishing returns in agriculture was central to the classical growth theories of Malthus and Ricardo. In [10], Panagariya and Succar introduce economies of scale to the Harris-Todaro framework with fixed capital within a static framework.

[18]See [3] for more detailed discussions on multiple equilibrium points with different levels of human capital.

[19]We also demonstrate that the urban amenity parameter is improved, some people will migrate from the rural area to the urban area. The urban lot size falls and the rural lot size and agricultural land use are increased. The effects of the urban amenity improvement are similar to those caused by the productivity improvement.

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MODEL RASTA DVA SEKTORA S ENDOGENIM LJUDSKIM RESURSIMA I MOGUĆNOSTIMA

W.-B. Zhang

Azijsko pacifičko sveučilište Ritsumeikan
prefektura Oita, Japan

SAŽETAK

Rad razmatra pitanja vezana uz urbanizaciju s migracijom radne snage. Glavna odstupanja od tradicionalnih pristupa dinamici ekonomskih struktura su što se u radu koristi alternativni pristup ponašanju potrošača te što se uvodi akumulacija ljudskog kapitala putem učenja stečenog djelovanjem. Model opisuje dinamičko međudjelovanje između poljoprivredne i industrijske proizvodnje, ruralne i urbane mogućnosti, distribuciju faktora proizvodnje i preferencija kao i akumulaciju endogenog kapitala i ljudskih resursa. Pokazujemo kako dinamički sustav može imati ili jedno, ili više ravnotežnih stanja, ovisno o povratku na skalu u dva sektora. Također smo ispitali učinke promjena pojedinih parametara modela.

KLJUČNE RIJEČI

Model dva sektora, poljoprivredni sektor, industrijski sektor, fizička akumulacija kapitala, akumulacija ljudskih resursa, ruralne i urbane mogućnosti