Interdisciplinary Description of Complex Systems 4(2), 136-143, 2006

SOCIAL FREE ENERGY OF A PARETO-LIKE RESOURCE DISTRIBUTION

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Category: Conference paper *Received:* 15 June, 2006. *Accepted:* 15 February, 2007.

SUMMARY

For an organisation with a Pareto-like distribution of the relevant resources we determine the social free energy and related social quantities using thermodynamical formalism. Macroscopic dynamics of the organisation is linked with the changes in the attributed thermodynamical quantities through changes in resource distribution function. It is argued that quantities of thermodynamical origin form the optimised set of organisation's state indicators, which is reliable expression of micro-dynamics.

KEY WORDS

resource distribution function, Pareto distribution, social free energy

CLASSIFICATION

JEL: C15

PACS: 05.70.-a, 89.65.Gh

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INTRODUCTION

Deepening the understanding of diverse human systems, their subsystems and smaller parts, is essential for a number of reasons, e.g. in order to enable reliable predictions about near-future states, improved allocation of scarce resources, improved planning of policies, increasing possibility for improvement of human living and working conditions, etc. The listed topics are important for different kinds of human systems, ranging from a family or a small group of persons to national and international systems. Within such systems there are institutions, processes and organisations the precise value of which is, in the socio-economic context, important both for them and for the environment. Here we particularly consider an organisation, not the whole system that it is a small part of. We assume that the organisation's resources are documented, e.g. book-kept. That may be due to internal procedures, relevant legislature, taking part in stock-markets, the transparency needed for external funding, etc. A straightforward consequence of documenting is that distribution of organisation's resources is known in different times. That enables their diverse statistical treatments, enlarging thereby the possibility to gain additional insight in resource structure and dynamics, i.e. to unveil possible, additional pieces of information.

Our aim is to develop a quantity which figures as an indicator of the state of the social system, and which may be intuitively and clearly determined from documented microscopic quantities.

A particular, interdisciplinary way of interpreting functioning of an organisation is possible through meta-theoretical description in which two system-oriented branches of theoretical physics, the thermodynamics and statistical physics, are exploited. That long-lasting approach has been recently revived by the idea that a part of socio-economic activity is expressible and measurable by way of social free energy. The social free energy is a measure of resources which can be transferred for a given purpose within a social system without changing its structure [1]. Formally, it was recently introduced meta-theoretically as the analogue of the physical free energy in a number of independent works: as the combination of innovation and conformity of a collective, profit, common benefit, availability, or free value of the canonical portfolio (see [1] and references therein). Substantially, it was recognised as a quantity with intrinsically social interpretation whose meta-theoretical origin enables its quantification and relation to other quantifiable functions [2]. In particular, that approach was introduced, developed in more detail, and accompanied by the modelling which in itself serves as a precursor to further application of the notion of social free energy on realistic cases [3].

In this article we start with the assumed form of a distribution of resources for a general organisation. In particular we take the distribution as one-parameter, Pareto-like distribution. Using it, we determine thermodynamical quantities. In that way we develop the notion of social free energy on the one hand, and contribute to searching for the existence of invariants in different organisations. While some results strictly depend on the form of the distribution chosen, others - including the most general ones - are independent of that choice. In particular, the power tail, the characteristic of Pareto distribution, was encountered in a large number of diverse systems [4] – ranging from original work on distribution of income, via distribution of outflows, via distribution of movies' popularity to distribution of flaws during quality control [5 – 7]. It is not clear whether in these examples there is some unifying underlying mechanism for generating Pareto distribution of observed resources, or not [6]. Nevertheless, this uncertainty about microscopic origin does not prevent the development of macroscopic approach in which underlying mechanisms are suppressed. Before proceeding, let us mention that the Pareto-like form chosen enables the development of compact expressions for relevant quantities.

In the second section of the article, we describe the distribution and argue about its adequacy. In the third section we determine thermodynamical quantities, that we analyse in the fourth section. Conclusions and projections of future work are given in the fifth section. The simplified case of the presented model is treated separately in the appendix.

MODEL

The model distribution function of the resources of a value u is the following function

$$p(u,a) = \frac{N(a)}{1+u^a}, \tag{1}$$

as shown in Fig. 1, with the norm $N(a)$ equal to

$$N(a) = \frac{\sin(\pi/a)}{\pi/a}. \tag{2}$$

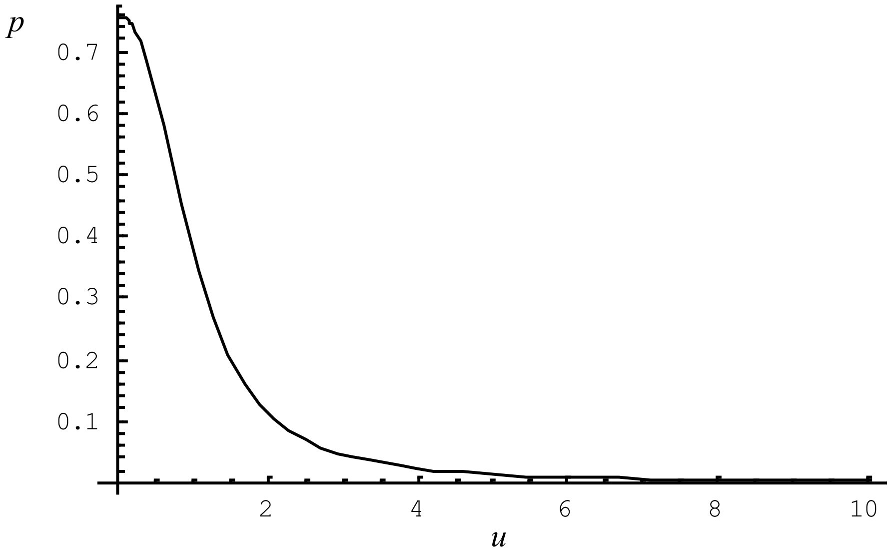

Fig. 1. Resource distribution function (1) for a = 2,5.

Resources u in (1) are considered to be non-dimensional quantities, which means that they are measured in units of some referent resource. We assume here that it is a constant during considered processes, and hence we leave it unspecified.

The distribution (1) requires $a > 2$. For large enough u, (1) becomes

$$N(a) \cdot u^{-a}, \tag{3}$$

which is Pareto distribution function with the exponent of the power law distribution equal to a (and Pareto distribution shape parameter k equal to $k = a - 1$) [4]. In realistic cases and in performed simulations for Pareto distribution, values of a range from 1,5 to 3. Generally, Pareto distribution is valid only in a certain, finite interval of resource values. In cases when it is used for income or outflows of individuals, or human groups, it includes the inegalitariness of the system it is attributed to [8]. The inegalitariness lessens for larger values of a. We consider as the measure of inegalitariness the portion of people whose resources are smaller than some constant, otherwise unspecified number. Independently of the value of that number, the portion is smaller for larger a. In addition, both originally and nowadays still prevalently Pareto distribution is related to income and outflow, i.e. transfer of resources distributions. Generally, one may argue that for owned, held resources it is not universally applicable. For the purpose of simplicity, we consider that Pareto distribution is relevant also in cases of representing distribution of book-kept resources, at least for certain classes of organisations. Nevertheless, the formalism described is straightforwardly applicable to other distributions, a topic that we will address in detail in future.

We describe the region $u \approx 0$ using a modified Pareto distribution (1), which denotes realistic, finite number of zero-value resources. Generally, distributions which in the large resource tail follow Pareto form, approach a plateau in the low resource part. The particular form of that plateau, introduced through the constant (taken to be equal to 1 in denominator in (1)) is, thus a representation of a realistic feature. It is expected that in realistic cases the region of low resource values will be rather insignificant for interpretations of the all-organisation characteristics, hence the precise form of the plateau is not crucial for the distribution.

Form (1) is a particular form of a whole family of possible forms representing organisations' distribution of resources. In that sense, the formalism to follow means more than just elaboration of facts for a particular choice of resource distribution, as it represents a general procedure for calculating the aggregated quantities for a general resource distribution function. For a detailed description of physically-interpreted microeconomic behaviour see [9].

In (1) a reference is an idealised situation of quasi-equilibrium organisation's operations, without turmoil in environment and other sources of significant fluctuations. Closely related to (1) is the resource distribution of an organisation which has just undergone a turmoil and which is, somewhat generalised, represented as a sudden and drastic change of a portion of resources in the following way:

$$p(u;\ \{u\},a) = \frac{N'(\{u\},a)}{1+u^a}\theta(u_0 - u)\left[\theta(u_< - u) + \theta(u - u_>)\right], \tag{4}$$

where {u} stands for $u_{<,\,>,\,0}$ (with $u_< < u_> < u_0$) while N' is a straightforwardly obtained norm. Expression (4) is nevertheless a somewhat cumbersome expression. In (4) the sudden lack of resources of a given interval of amounts is assumed. It is reasonable to consider a sudden enhancement of resources in a finite interval of values by substituting the bracket with $[1 + \theta(u - u_<) + \theta(u_> - u)]$.

THERMODYNAMICS

The total energy U of the system with resources described by (1) is given with

$$U(a) = \int_0^\infty up(u,a)du = \frac{1}{2\cos(\pi/a)} \quad . \tag{5}$$

The entropy $S(a)$ is determined using the statistical mechanics formula

$$S(a) = -\int_0^\infty p(u,a)\ln p(u,a)\cdot du\,, \tag{6}$$

$$S(a) = \ln\left|\frac{\pi/a}{\sin(\pi/a)}\right| - C - \Psi(1-1/a), \tag{7}$$

in which $\Psi(\cdot)$ is digamma function, and C = 0,577 is the Euler constant.

Temperature $T(a)$ is determined using

$$T\mathrm{d}S = \mathrm{d}U, \tag{8}$$

that gives

$$T(a) = \frac{\frac{\pi}{2}\frac{\sin(\pi/a)}{\cos^2(\pi/a)}}{\psi'(1-1/a) + a - \pi \mathrm{ctg}(\pi/a)}, \tag{9}$$

while free energy is obtained using

$$F(a) = U(a) - T(a)S(a). \tag{10}$$

The closed expression for (10) is omitted here because of its non-tractability. Dependence of entropy (7) and temperature (9) on a is shown in Figures 2 and 3, respectively, while the dependence of free energy on a is shown in Figure 4.

(10) may be interpreted from a formal and a substantial viewpoint. Formally, (10) is applicable in case of a quasi-equilibrium processes, because the expression (8) is appropriate in quasi-stationary dynamics of organisation's resources, with relatively small part of resources transferred in the relevant time unit. Because of that, flow of resources is relatively small, fluctuation in resource distribution as well, and consequently there is quasi-equilibrium situation for which (10) is relevant expression. Substantially, (10) means that individual processes in the organisation, which include resource transfer, are changed in a time interval which is relatively small compared to the time interval of a characteristic change in organisation's

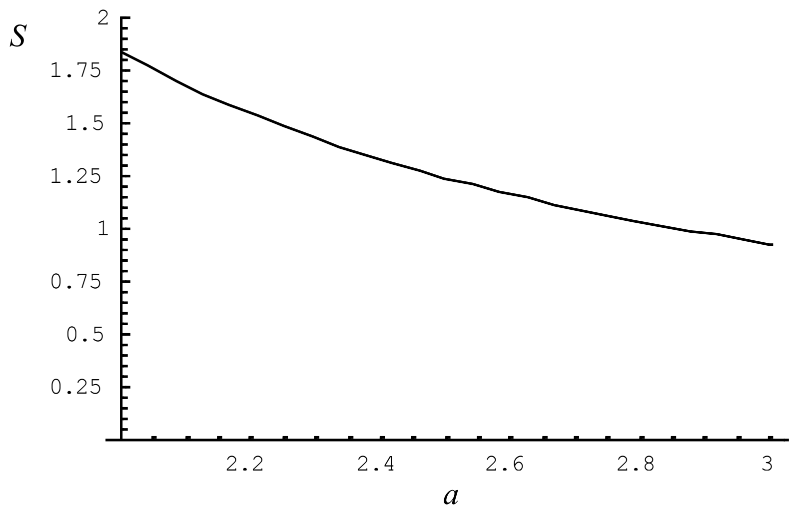

Fig. 2. Entropy as a function of a.

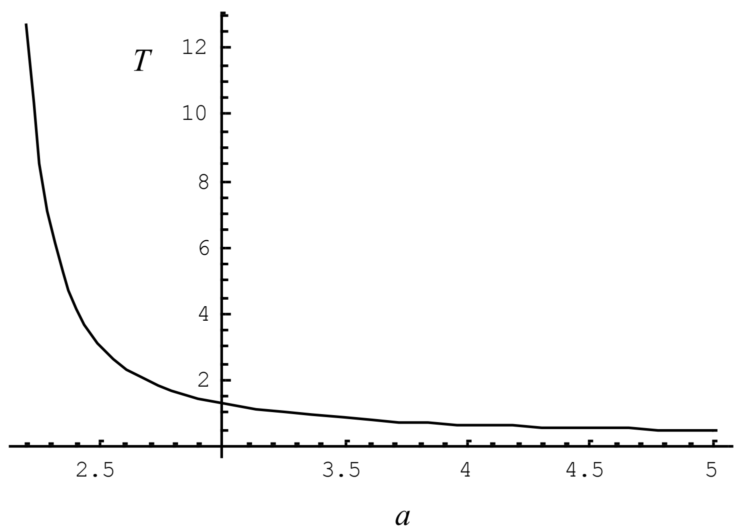

Fig. 3. Temperature as a function of a.

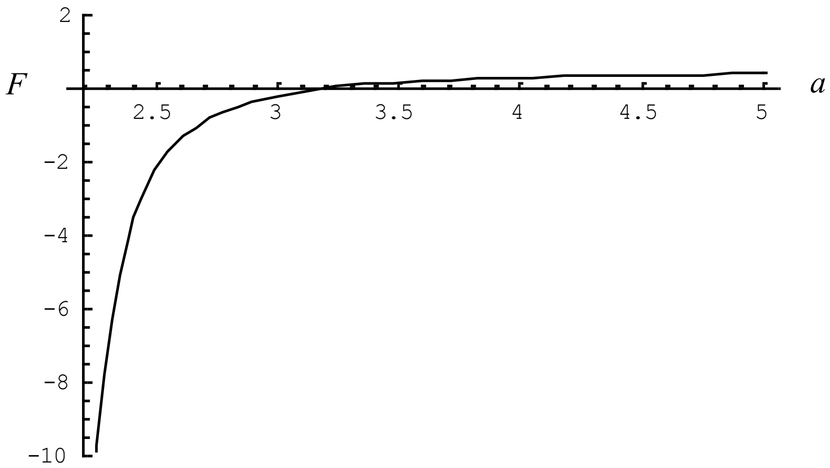

Fig. 4. Free energy given by (10).

environment. In that case, no matter what the environment state is, the organisation's resource distribution function has the assumed form (1). The change in the environment is thus projected onto the change of the parameter a of the resource distribution function.

Substantially, (12) enables the determination of the quantity of resources which are extractable from the organisation when its state changes from some initial i, to some final f:

$$\Delta F = F_\mathrm{f} - F_\mathrm{i.} \tag{11}$$

That quantity then gives the resources which are not bound to the organisation's dynamics but are exploitable for other purposes, one of which is regularly preventing or lowering the destructive influence of environment dynamics. Thus F measures the system level of adaptation. However, adaptation is not separated from other, intrinsically system quantities, like entropy. The higher level of adaptation brings about lower entropy, the interplay of which is discussed for general resource distribution elsewhere [1]. If $\Delta F > 0$ in (11), the system level of adaptation rises, and vice versa. Regarding changes in entropy, if $\Delta S > 0$, the system is considered to evolve spontaneously.

RESULTS AND DISCUSSION

Energy, entropy (Fig. 2) and temperature (Fig. 3) are lower for larger a, in accordance with the fact that for larger a, the portion of low-value resources is larger.

For $a < 3{,}187$ the social free energy is negative. We do not consider in detail the implications of F changing the sign, as we further in the section concentrate on the region $1{,}5 < a < 3$. With rising a the number of important resources of significant value decreases. In other words, system is more adapted with smaller number of valuable resources if we consider larger value of F as a sign of better adaptation.

Let us consider possible small fluctuations in distribution of resources, thus in values of a. Small changes in value of a are less important for the value of F in case of relatively large a. In that sense, larger a means that fluctuations are suppressed. This is because small fluctuations mean fluctuations in quantity of low-value resources, which are relatively large in number in case of a sufficiently large a. In order to preserve the functional form of the distribution function (1), the value of large-value resources should be somewhat changed, but without crucial changes in their quantity that would bring about a significant change in a. In cases of a relatively small value of a, its small fluctuations bring about relatively large change in F, because there is relatively large portion of large-value resources which can contribute to fluctuations.

Social free energy as a measure of adaptation means that changes in alignment with environment for relatively small values of *a* are reachable using intra-system redistribution of resources. To the contrary, changes in alignment (i.e. better adaptation) for relatively large values of *a* are brought about through changes in the number of small-valued resources. Thus, for large values of *a* there is structured response to environmental influences, and the corresponding systems can be considered as having some hierarchy because different resources function differently in response to environmental influences. Furthermore, this implies that large-valued resources contribute more to structure (or entropy) and intensity of dynamics than to free resources, i.e. the more-valued the resource is, the stronger it is bound to the system. This is valid in case of relatively small environmental influences, which is considered to be a regular characteristic of environment dynamics.

CONCLUSIONS

The process of determining macroscopic quantities from a given microscopic quantity – here a resource distribution function – is presented. In particular, for a realistic, Pareto-like resource distribution function, the social free energy is determined in characteristic situations. For the distribution chosen, the social free energy is constantly negative. Changing its value closer to zero is related to the model with better adaptation, the possibility of intensely localised-in-time dynamics and with rising proportion of low-value resources.

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SOCIJALNA SLOBODNA ENERGIJA PARETOVE RASPODJELE RESURSA

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SAŽETAK

Za organizaciju kojoj je raspodjela resursa opisana funkcijom sličnom Paretovom određujemo socijalnu slobodnu energiju i druge veličine primjenom termodinamičkog formalizma. Makroskopska dinamika organizacije povezana je s promjenama u pridruženim termodinamičkim veličinama putem promjena u funkciji raspodjele resursa. Diskutiramo kako veličine termodinamičkog porijekla tvore optimalni skup indikatora stanja organizacije, kao pouzdani iskaz mikrodinamike.

KLJUČNE RIJEČI

funkcija raspodjele resursa, Paretova raspodjela, socijalna slobodna energija