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LIMITING POSSIBILITIES OF RESOURCE EXCHANGE PROCESS IN COMPLEX OPEN MICROECONOMIC SYSTEM

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SUMMARY

A problem on extreme performance of microeconomic system with several firms is considered. Each firm aspires to increase the profit. Flows of the good between the firms determine the structure of the system. So, sequential structure corresponds to intermediaries (dealers) operating in the market, parallel structure corresponds to competition in the market. The system at issue is an open economic system because of presence of external flows from the sources described by a distribution of the value of the good. The problem is solved for the basic structures: maximal profit and corresponding prices are found for each firm.

KEY WORDS

open microeconomic systems, optimal prices, series and parallel structures

CLASSIFICATION

ACM: Categories and subject descriptors: J.4 [Computer Applications]; Social and behavioral sciences – Sociology,

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INTRODUCTION

Research of behaviour of the economic agents buying and selling the goods in the market is one of primary goals of microeconomics. Analogies between exchange processes in economic systems and heat- and mass exchange processes in thermodynamic systems are promising methods of such research [1]. Irreversible microeconomics [2 - 4], using analogies to finite-time thermodynamics [5], allows one to solve problems of optimal control of the prices in irreversible processes of resources exchange. Some open microeconomic systems either without a firm or containing the only firm are examined in [3, 6]. A firm means a subsystem managing the prices and taking money - a basic resource - from a system. The analysis of firm of complex structure is given in [7].

Further the open microeconomic systems consisting of several firms are considered. Such systems take place in the competitive markets, at presence of intermediaries (dealers), at economic independence of divisions of the enterprise and in other cases.

STATEMENT OF A PROBLEM

Let us consider the open microeconomic system consisting of n firms-intermediaries, buying and selling a scalar resource. If the system represents the firm buying and selling a resource to the environment then the system is a homogeneous system. In the opposite case, when the system consists of several firms which sell a resource each other and to the environment then the system is a complex system. Each of firms in complex system may operate the prices of sale of a resource and aspires to receive the maximal profit. Let us consider a stationary mode at which streams in system do not change in time. For this mode we shall determine intensity of resources flows, the prices of sale of the goods for each of firms and compare the total profit taken from the system, with the profit of a homogeneous firm.

The solution of this problem will be carried out for the linear functions of a supply and demand describing the resources exchange between the system and its environment (markets).

HOMOGENEOUS FIRM

First we describe a problem on maximal profit determination for a homogeneous firm exchanging a scalar resource with an environment, representing the distributed market with known density of distribution of value v (marginal rate of substitution between the goods and money) $f(v)$ (Fig. 1). As the firm in a stationary mode can not accumulate a stock of the goods, intensities of input and output flows of the goods should be equal. Let us denote this flow intensity as g. Then the profit of firm is

$$\varpi = (p_1 - p_2)\cdot g. \tag{1}$$

Control variable is the price of sale p_1. The price $p_2(p_1)$ is determined from equality of input and output flows of the goods. The flow g depends on the chosen price p_1 too.

FORMAL STATEMENT OF A PROBLEM

The problem at issue *to determine such price of sale of the goods that the profit of firm should be maximal*:

$$\varpi = [p_1 - p_2(p_1)]\cdot g(p_1) \to \max_{p_1}. \tag{2}$$

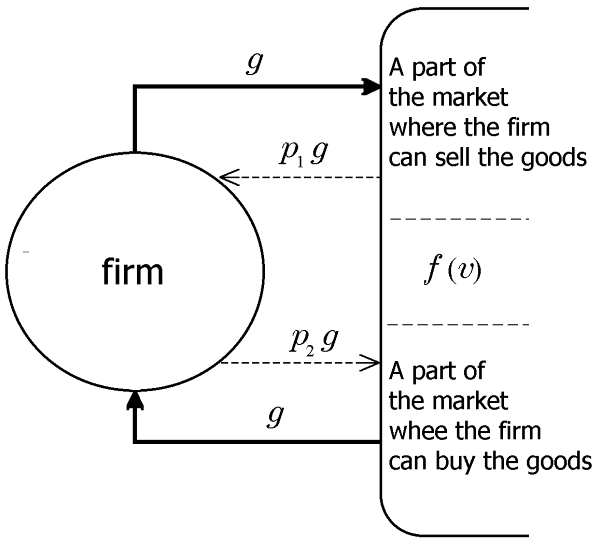

Figure 1. A homogeneous firm working in the distributed market.

ALGORITHM OF THE SOLUTION OF THE PROBLEM

1. To determine dependencies $p_2(p_1)$, $g(p_1)$ from the condition of equality of input and output flows intensities.
2. To substitute these dependences in (1) and to solve the obtained problem. As a result of the solution we find values p^*, $p_2^* = p_2(p_1)$, $g^* = g(p_1)$ and value of the maximal profit ϖ^*.

Let intensity of purchases of the goods be determined by firm as following

$$g = \beta \int_{-\infty}^{p_2} (p_2 - v) f(v) \mathrm{d}v = \beta \left[p_1 \int_{-\infty}^{p_2} f(v) \mathrm{d}v - \int_{-\infty}^{p_2} v f(v) \mathrm{d}v \right], \quad (3)$$

where β is a constant factor. Value $\int_{\infty}^{p_2} f(v)\mathrm{d}v$ represents a part of the market $x(p_2)$ carrying out sale of the goods, and

$$m_v(p_2) = \int_{-\infty}^{p_2} v f(v) \mathrm{d}v \bigg/ \int_{-\infty}^{p_2} f(v) \mathrm{d}v$$

is the expectation of the goods value at this part of the market. Therefore (3) can be rewritten as

$$g = \beta \breve{x}(p_2)[p_2 - \breve{m}_v(p_1)]. \quad (4)$$

Similarly,

$$g = \alpha \hat{x}(p_1)[\hat{m}_v(p_1) - p_1], \quad (5)$$

where α is a constant factor, $\hat{x}(p_1)$ is a part of the market carrying out purchase of the goods, $\hat{m}(p_1)$ is the expectation of the goods value at the part $\hat{x}$ of the market. Equating the right parts (4) and (5) and expressing p_2 we find dependence $p_2(p_1)$.

EXAMPLE 1

Let value v has the uniform distribution in segment $[0, p_0]$. Then

$$\hat{x}(p_2) = \frac{p_2}{p_0}, \qquad \hat{m}_v(p_2) = \frac{p_2}{2},$$
$$\hat{x}(p_1) = \frac{p_0 - p_1}{p_0}, \quad \hat{m}_v(p_1) = \frac{p_0 + p_1}{2}.$$

Intensity of the goods flow is equal to

$$g = \beta \frac{p_2}{p_0}\left(p_2 - \frac{p_2}{2}\right) = \beta \frac{p_2^2}{2p_0}. \tag{6}$$

or

$$g = \alpha \frac{p_0 - p_1}{p_0}\left(\frac{p_0 + p_1}{2} - p_1\right) = \alpha \frac{(p_0 - p_1)^2}{2p_0}. \tag{7}$$

whence we express p_2:

$$p_2(p_1) = \sqrt{\frac{\alpha}{\beta}} \cdot (p_0 - p_1). \tag{8}$$

With the account of (7) and (8) we rewrite the problem (2) as following

$$\varpi(p_1) = \left[p_1\left(1 + \sqrt{\frac{\alpha}{\beta}}\right) - p_0\sqrt{\frac{\alpha}{\beta}}\right]\alpha \frac{(p_0 - p_1)^2}{2p_0} \to \max_{p_1}. \tag{9}$$

The solution of this problem is the control price

$$p_1^* = \frac{p_0(3 + 3\sqrt{\alpha/\beta}) - p_0\xi(\alpha,\beta)}{3\left(1 + \sqrt{\alpha/\beta}\right)}. \tag{10}$$

and corresponding to this price values

$$p_2^* = \sqrt{\frac{\alpha}{\beta}}\frac{p_0[1 + \xi(\alpha,\beta)]}{3\left(1 + \sqrt{\alpha/\beta}\right)}, \quad g^* = \frac{\alpha}{18}\frac{p_0[1 + \xi(\alpha,\beta)]}{\left(1 + \sqrt{\alpha/\beta}\right)^2},$$
$$\varpi^* = \frac{\alpha\beta}{54}\frac{p_0^2}{\left(\sqrt{\alpha} + \sqrt{\beta}\right)^2}[2 - \xi(\alpha,\beta)]\cdot[1 + \xi(\alpha,\beta)]^2, \tag{11}$$

where

$$\xi(\alpha,\beta) = \sqrt{1 + 6\sqrt{\alpha/\beta} - 6(\alpha/\beta)\sqrt{\alpha/\beta}}.$$

So, at $\alpha = \beta[\xi(\alpha, \beta) = 1]$ optimal prices are $p_1^* = 2p_0/3$, $p_2^* = p_0/3$, the flow intensity corresponding to these prices is $g^* = \alpha p_0/18$, and maximal profit of the firm is $\varpi^* = \alpha p_0^2/54$.

EXAMPLE 2

Let the spectrum of distribution of the value v be set $\{\breve{v}, \hat{v}\}$, and supply and demand functions at $p_1, p_2 \in$ \in $\{\breve{v}, \hat{v}\}$ are given in a linear form

$$g = \beta(p_2 - \breve{v}), \tag{12}$$

$$g = \alpha(\hat{v} - p_1). \tag{13}$$

Equating (12) and (13), we receive dependence $p_2(p_1)$:

$$p_2 = \frac{\alpha}{\beta}(\hat{v} - p_1) + \breve{v}. \tag{14}$$

Accounting (13) and (14) we write down problem (2) as

$$\varpi(p_1) = \left[p_1 - \frac{\alpha}{\beta}(\hat{v} - p_1) + \breve{v} \right] \alpha(\hat{v} - p_1) \to \max_{p_1}$$

which solution looks like

$$p_1^* = \frac{(1 + 2\alpha/\beta)\hat{v} + \breve{v}}{2(1 + \alpha/\beta)}. \tag{15}$$

Values p_2^*, g^* and ϖ^* corresponding to the optimal solution are:

$$p_2^* = \frac{(1 + 2\beta/\alpha)\breve{v} + \hat{v}}{2(1 + \beta/\alpha)}, \quad g^* = \frac{\alpha(\hat{v} - \breve{v})}{2(1 + \alpha/\beta)}, \quad \varpi^* = \frac{\alpha\beta}{4(\alpha + \beta)}(\hat{v} - \breve{v})^2. \tag{16}$$

At $\alpha = \beta$

$$p_1^* = \frac{3}{4}\hat{v} + \frac{1}{4}\breve{v}, \quad p_2^* = \frac{1}{4}\hat{v} + \frac{3}{4}\breve{v}, \quad g^* = \frac{\alpha}{4}(\hat{v} - \breve{v}), \quad \varpi^* = \frac{\alpha}{8}(\hat{v} - \breve{v})^2,$$

that coincides with the solution received in [8].

If the spectrum of distribution of values of the goods can be divided into two subsets V_1, V_2, such that $p_1 < \min V_1$, $p_2 > \max V_2$, the solution of problem (2) for a system like (15), (16) at

$$\breve{v} = M[v \mid v \in V_2], \quad \hat{v} = M[v \mid v \in V_1].$$

SERIES STRUCTURE

Let us consider the problem on determination of limiting possibilities of resource exchange process in a complex (non-homogeneous) economic system. The solution of the problem depends on the structure of the system. We consider two simple structures, which are the basic structures for any complex system. These are series and parallel structures.

First, let us consider series structure. The elementary case of series structure is shown in Fig. 2.

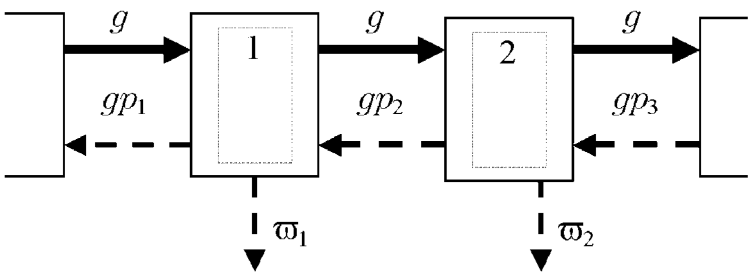

Figure 2. Consecutive structure of complex microeconomic system.

FORMAL STATEMENT OF THE PROBLEM

Each of firms solves a problem on determination of the sales price to maximize its profit:

$$\varpi_1 = g(p_3 - p_1) \to \max_{p_2}, \tag{17}$$

$$\varpi_2 = g(p_3 - p_2) \to \max_{p_3, g}. \tag{18}$$

Restrictions imposed on the solution set are supply and demand functions

$$g = f_1(p_3), \tag{19}$$

$$g = f_2(p_1). \tag{20}$$

ALGORITHM OF THE SOLUTION OF THE PROBLEM

1. To fix value p_2. Solving problem (18) and (19) we receive dependences $p_3(p_2)$, $g(p_2)$, $\varpi_2(p_2)$.
2. To solve problem (17) and (20) accounting the found dependence $g(p_2)$. As a result of the solution we find the optimum values p_2^*, g^*, ϖ_1^*.
3. To substitute values p_2^*, g^* in the dependences received on a first step of algorithm. Doing so we find p_3^* and ϖ_2^*.

EXAMPLE 3

As in previous example we set supply and demand functions in a linear form (12), (13):

$$g = \alpha(\hat{v} - p_3), \tag{21}$$

$$g = \alpha(p_1 - \breve{v}). \tag{22}$$

Then problem (18) has the form:

$$\varpi_2 = \alpha(\hat{v} - p_3)(p_3 - p_2) \to \max_{p_3}.$$

Its solution $p_3^* = (\hat{v} + p_2)/2$, and the goods flow corresponding for this solution is $g(p_2) = (\alpha/2)(\hat{v} - p_2)$. Expression of price p_1 through the received dependence $g(p_2)$ and substitution $g(p_2)$ and $p_1(p_2)$ leads to the following form of problem (17):

$$\varpi_1 = \frac{\alpha}{2}(\hat{v} - p_2)\left[p_2 - \frac{\alpha}{2\beta}(\hat{v} - p_2) - \breve{v}\right] \to \max_{p_2}.$$

Solving this problem we find p_2^*:

$$p_2^* = \frac{(1 + \alpha/\beta)\hat{v} + \breve{v}}{2 + \alpha/\beta}, \tag{23}$$

whence

$$g^* = \frac{\alpha\beta}{2(2\beta + \alpha)}(\hat{v} - \breve{v}), \quad p_1^* = \frac{\left(4 + \dfrac{\alpha}{\beta}\right)\hat{v} + \dfrac{\alpha}{\beta}\breve{v}}{2\left(2 + \dfrac{\alpha}{\beta}\right)}, \quad \varpi_1^* = \frac{\alpha\beta}{4(2\beta + \alpha)}(\hat{v} - \breve{v})^2. \tag{24}$$

Having substituted (24) in the received dependence $p_3^*(p_2)$, we find

$$p_3^* = \frac{\left(3 + 2\dfrac{\alpha}{\beta}\right)\hat{v} + \breve{v}}{2\left(2 + \dfrac{\alpha}{\beta}\right)}, \quad \varpi_2^* = \frac{\alpha\beta^2}{4(2\beta + \alpha)^2}(\hat{v} - \breve{v})^2,$$

and the total profit of the firms

$$\varpi = \varpi_1^* + \varpi_2^* = \frac{3\alpha\beta^2 + \alpha^2\beta}{4(2\beta + \alpha)^2}(\hat{v} - \breve{v})^2. \tag{25}$$

Value ϖ^* is less than profit of one firm working in the same markets calculated according to (16). Dependences of the profit extracted from the system with respect of number of the firms and ratio α/β are shown in Fig. 3.

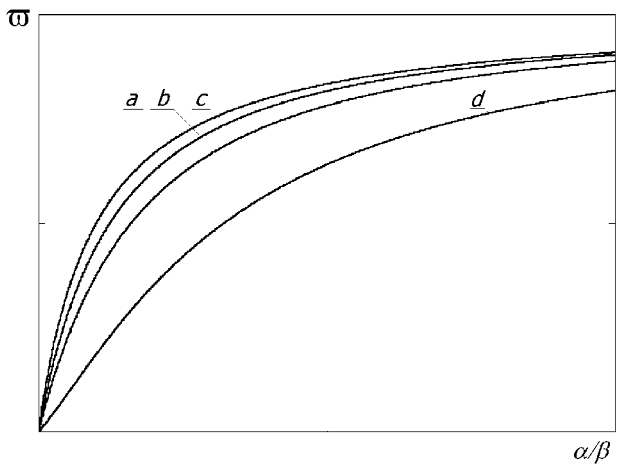

Figure 3. Dependence of the total profit of firms in system with series structure: a) homogeneous system, b) complex system with two, c) three, d) seven firms.

Note that there exists asymmetry of series structure: the first firm can choose the prices of sale of the resource, and receives more money than the second firm determining intensity of the resource exchange.

PARALLEL STRUCTURE

Let us consider the system of parallel structure consisting of two firms, exchanging a scalar resource with the markets (Fig. 4). Each of firms aspires to maximize the profit by controlling of the prices of output resources flows.

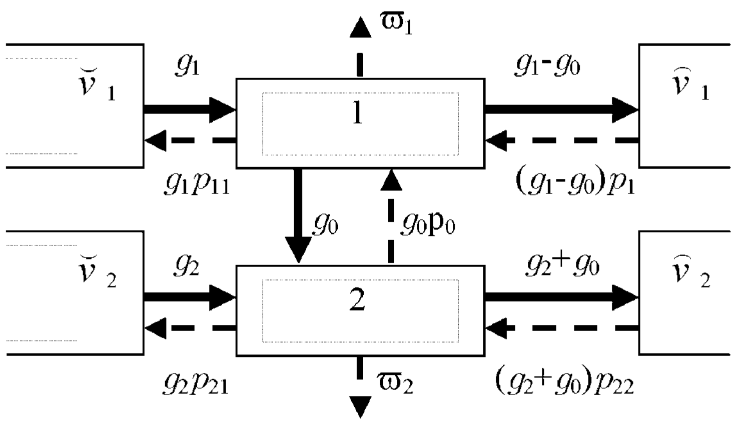

Figure 4. Complex microeconomic system with parallel structure.

FORMAL STATEMENT OF A PROBLEM

The problem at issue is *to determine the prices of the goods sale providing the maximal profit of each of the firms working in the market*:

$$\varpi_1 = (g_1 - g_0)p_{12} + g_0 p_0 - g_1 p_{11} \to \max_{p_0, p_{12}}, \tag{26}$$

$$\varpi_2 = (g_2 + g_0)p_{22} - g_0 p_0 - g_2 p_{21} \to \max_{g_0, p_{22}}, \tag{27}$$

subject to supply

$$g_1 = g_1(\hat{v}_1, p_{11}), \tag{28}$$

$$g_2 = g_2(\breve{v}_1, p_{21}), \tag{29}$$

and demand for resources

$$g_1 - g_0 = f_1(\hat{v}_1, p_{12}), \tag{30}$$

$$g_2 + g_0 = f_2(\hat{v}_2, p_{22}), \tag{31}$$

where $\breve{v}_i$, $\hat{v}_i$ ($i \in \{1, 2\}$) are values of resources in the markets.

ALGORITHM OF THE SOLUTION

1. Assuming the price p_0 to be fixed to solve problem (27), (29) and (31). As a result of the solution we receive dependences $p^*_{21}(p_0)$, $p^*_{22}(p_0)$, $g_0(p_0)$.
2. Using the received dependence $g_0(p_0)$ to solve problem (26), (28) and (30). We find optimum price strategy of the first firm p^*_{01}, p^*_{11}, p^*_{12}, and its maximal profit.
3. To substitute value p_0^* in dependences found on the first step of the algorithm. We find price strategy of the second firm and its maximal profit.

The received solution can be compared to limiting opportunities of one firm working in the distributed markets of raw material and finished commodity with sets of possible values $\{\breve{v}_1, \breve{v}_2\}$ and $\{\hat{v}_1, \hat{v}_2\}$ accordingly.

EXAMPLE 4

Let curves of a supply and demand be given in a linear form:

$$g_1 = \beta(p_{11} - \breve{v}_1); \quad g_1 - g_0 = \alpha(\hat{v}_1 - p_{12});$$
$$g_2 = \beta(p_{21} - \breve{v}_2); \quad g_2 + g_0 = \alpha(\hat{v}_2 - p_{22}).$$

At the fixed value p_0 we solve a problem

$$\varpi_2 = (g_2 + g_0)p_{22} - g_0 p_0 - g_2 p_{21} \to \max_{\substack{g_0, g_2, \\ p_{21}, p_{22}}} \left| \begin{array}{l} g_2 - \beta(p_{21} - \breve{v}_2) = 0; \\ g_2 + g_0 - \alpha(\hat{v}_2 - p_{22}) = 0. \end{array} \right. \tag{32}$$

We receive:

$$\begin{array}{ll} p_{21}(p_0) = \dfrac{\breve{v}_2 + p_0}{2}; & p_{22}(p_0) = \dfrac{\hat{v}_2 + p_0}{2}; \\ g_2 = \beta \dfrac{p_0 - \breve{v}_2}{2}; & g_0 = \dfrac{\beta\breve{v}_2 + \alpha\hat{v}_2}{2} - \dfrac{p_0(\alpha + \beta)}{2}. \end{array} \tag{33}$$

It is convenient to note the weighed average values of a resource in the markets as

$$v_{01} = \frac{\alpha\hat{v}_1 + b\breve{v}_1}{\alpha + \beta}, \quad v_{02} = \frac{\alpha\hat{v}_2 + b\breve{v}_2}{\alpha + \beta}.$$

In this notation the problem has the form

$$\varpi_1 = (g_1 - g_0)p_{12} + g_0 p_0 - g_1 p_{11} \to \max_{\substack{g_0, g_2, \\ p_0, p_{11}, p_{12}}} \left| \begin{array}{l} g_0 - \dfrac{\alpha + \beta}{2}(v_{02} - p_0) = 0; \\ g_1 - \beta(p_{11} - \breve{v}_1) = 0; \\ g_1 - g_0 - \alpha(\hat{v}_{12} - p_{12}) = 0, \end{array} \right. \tag{34}$$

and its solution is

$$p_0^* = \frac{1}{3} v_{01} + \frac{2}{3} v_{02}, \tag{35}$$

$$p_{11}^* = \frac{1}{3}v_{01} + \frac{1}{6}v_{02} + \frac{1}{2}\breve{v}_1; \quad p_{12}^* = \frac{1}{3}v_{01} + \frac{1}{6}v_{02} + \frac{1}{2}\hat{v}_1;$$
$$g_0^* = \frac{\alpha+\beta}{6}(v_{02} - v_{01}); \qquad g_1^* = \beta\left(\frac{v_{01}}{6} + \frac{v_{02}}{3} - \frac{\breve{v}_1}{2}\right).$$

Substituting the found value p_0^* (35) in (33) we find the optimal prices and flows intensities for the second firm:

$$p_{21}^* = \frac{v_{01}}{6} + \frac{v_{02}}{3} + \frac{\breve{v}_2}{2}, \quad p_{22}^* = \frac{v_{01}}{6} + \frac{v_{02}}{3} + \frac{\hat{v}_2}{2}, \qquad g_2^* = \beta\left(\frac{v_{01}}{6} + \frac{v_{02}}{3} - \frac{\breve{v}_1}{2}\right).$$

Finally, substituting these optimum values in expressions for the profit (32) and (34), we find total value of the profit extracted from the system:

$$\varpi^* = g_2^*(p_{22}^* - p_{21}^*) + g_0^*(p_{22}^* - p_{12}^*) + g_1^*(p_{12}^* - p_{11}^*) =$$
$$\beta\left(\frac{v_{01}}{6} + \frac{v_{02}}{3} - \frac{\breve{v}_1}{2}\right)\frac{\hat{v}_2 - \breve{v}_2}{2} + \beta\left(\frac{v_{01}}{3} + \frac{v_{02}}{6} - \frac{\breve{v}_1}{2}\right)\frac{\hat{v}_1 - \breve{v}_1}{2} +$$
$$+ \frac{\alpha+\beta}{6}(v_{02} - v_{01})\left(\frac{v_{02} - v_{01}}{6} + \frac{\hat{v}_2 - \hat{v}_1}{2}\right) \qquad (36)$$

Comparison of values of the profit calculated by (36) and the profit of the appropriate homogeneous system at $\alpha = \beta$ for various values of differences $\Delta v_1 = \hat{v}_1 - \breve{v}_1$ and $\Delta v_2 = \hat{v}_2 - \breve{v}_2$ is shown in Fig. 5.

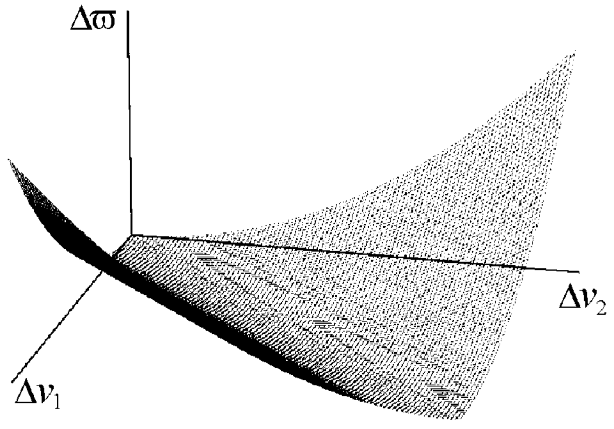

Figure 5. Increment of the profit $\Delta\varpi$ in system of parallel structure due to the prices diversification as a dependency on differences of the goods values in the markets.

CONCLUSIONS

The elementary structures of complex systems can be used for modelling the complex microeconomic systems consisting of series and parallel structures.

It is necessary to note, that series connections between firms result in reduction of the profit although parallel structure may result in increase of the total profit due to the prices diversification.

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KRAJNJE MOGUĆNOSTI IZMJENE RESURSA U SLOŽENOM, OTVORENOM, MIKROEKONOMSKOM SUSTAVU

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SAŽETAK

Razmatran je problem krajnjih mogućnosti mikroekonomskog sustava s nekoliko tvrtki. Svaka tvrtka teži povećanju dobiti. Tokovi dobara između tvrtki određuju strukturu sustava. Stoga sekvencijalna struktura odgovara posrednicima (engl. *dealers*) koji djeluju na tržištu, a paralelna struktura tržišnom natjecanju. Sustav u problemu je otvoreni, ekonomski sustav zbog eksternih tokova iz izvora opisanih raspodjelom vrijednosti dobra. Problem je riješen za osnovne strukture i za svaku tvrtku određene su najveća dobit i odgovarajuće cijene.

KLJUČNE RIJEČI

otvoreni mikroekonomski sustavi, optimalne cijene, serijske i paralelne strukture