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THINKING STRUCTURES AND MATHEMATICAL ACHIEVEMENTS IN CHILDREN WITH MILD INTELLECTUAL DISABILITIES

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ABSTRACT

The development of thinking structures in children has a significant influence on their mastering of mathematical concepts. The appearance of concrete logical thinking and its indicators (conservation concepts) influence positive school achievement regarding mathematical contents. By shaping the formal operations which represent the final and supreme phase of cognitive development, the most complex forms of conservations become available. Combinatorics is a feature which belongs to the last phase of cognitive development; it represents the generalization of the operations acquired in the concrete operational stage.

The purpose of this paper is to analyze the development of the operational thinking structures in relation to concrete logical operations (using the conservation of number, length, mass and volume) and formal operations (using combinatorics) as well as their influence on the mathematical achievement in students with mild intellectual disabilities at an later school age.

For the purpose of this study we have collected a sample of 120 students of both sexes with mild intellectual disabilities. The subjects were 12 to 15-year-old, corresponding to the school age of fifth to eighth graders.

Standard Piaget's conservation tasks (the assessment of number, length, mass and volume) and combinatorics have been used for the evaluation of the operational thinking. The mastery of mathematical contents was evaluated by means of a Criterium knowledge test constructed for the purposes of this study.

The results have shown statistically significant differences in the mathematical Criterium knowledge test achievement among the students in our population sample, regarding the attained level of operational thinking on all tasks that have been used.

Piaget's theory argues for the application of games that support reversibility, identity and conservation, which makes mathematical thinking more compliant, more active, broader, deeper, and more original.

***Key words**: children with mild intellectual disabilities, conservations, combinatorics, mathematical contents*

INTRODUCTION

The earliest forms of cognition in children consist of perceptive-practical activities. During the second year of life, thinking, which is the most developed and complex form of cognitive activities begins to develop. The processes of thinking are characterized by operating with empirical information, such as perception, imagination, word or idea. Thinking provides knowledge about various characteristics of an object; one compares by similarity, notices the differences, establishes relations among objects, groups the objects by analogies and notices the constant and the variable (Demetriou et al., 2010).

For the theoretical basis of this paper, we have utilized the theory laid out by a famous psychologist-epistemologist Jean Piaget (Jean Piaget - 1896-1980). The essence of his biologistic theory involves the assertion that learning is subordinate to the developmental process of a child, that it depends on the level of the development and that the influence of maturation has a decisive role in development (Piaget, Inhelder, 1996). According to Piaget, as emphasized by Donaldson (1997) and Živković (2007), thinking is a system of operations, i.e. a system of internal actions. Piaget identifies four phases in the development of cognitive structures: the sensorimotor stage (from birth to two years of age), the pre-operational stage (from two to seven years of age), the concrete operations stage (from seven to twelve years of age), and the formal operations stage (from twelve years of age on).

Jensen (2006) points out that the age at which the particular stages appear may vary to a certain extent, depending on the intelligence of a child and the environmental factors, but the order of appearance and the contents of the stages are invariable.

According to Piaget, the end of the sensorimotor period marks the beginning of the development of thinking. The sensorimotor intelligence operates with perception and practical activities, whereas concrete operations operate with ideas (perceptions and activities transferred to the internal plan). The actions the child performs are external in the beginning, but later on become internalized. At that time the operations become reversible (bi-directional - such operations are addition and subtraction, multiplication and division), there is integration in the composition of actions, (the creation of connected as opposed to the isolated thinking operations), and so the thought becomes mobile (Donaldson, 1997; Piaget, Inhelder, 1996).

Bruce and Muhammad (2009) analysed the operations being performed on the mental plan at this stage, and their results showed that the operation may appear and function only in relation with the concrete reality (thus the name concrete operations). There are various operations (e.g. mathematical, logical etc.), but they all develop in the interaction with the concrete reality. In this phase, children can only solve problems related to real objects or events, and not to abstract concepts or hypothetical tasks.

The thinking of a child is more logical, less egocentric and depends less on immediate perceptive experiences. The child is able to perform mental operations, such as calculation, to think about objects, about the classes of objects, as well as about the relations among these classes (Donaldson, 1997).

When an operation becomes the object of a higher degree operation, Piaget speaks about formal operations. At this level, the subject becomes aware of his own mental operations he was able to perform at the previous level, but not to analyse them. The abstract thinking is developed, and the child becomes capable of thinking in abstract categories, forming conclusions on the basis of abstract presumptions, formulating general laws and principles, and understanding metaphors. The new abilities increasingly direct the thinking of an adolescent to hypothetical and ideological issues. Formal operations are shaped during adolescence (Kuhn, 2008).

It is known that children with mild intellectual disabilities pass through the same developmental phases as the children who belong to the typical population, only much more slowly. Piaget and Inhelder (1996) think that children with intellectual disabilities cannot achieve a great progress towards the next stage in the development of intellectual and cognitive

abilities, because they are not able to build new forms and compositions of thinking, but remain tied to the infantile compositions deriving from the previous developmental stage, despite the fact that that stage does not correspond to the child's age any more. Inhelder (1968; according to Maćešić-Petrović, 1998), stresses that the characteristics of the mental development of a child with intellectual difficulties are reflected in longer duration of transitive forms between various developmental stages as opposed to the children belonging to the average population. In this way, the possibility for the opposition of two different compositions and levels of constructions in the development of an individual is created, which contributes to the destabilization of the person's development.

Globally, the manner of the organization of thinking structures is necessary for the proper direction of the educational process.

Mathematics initiates a child into the perception and understanding of the relations in the immediate environment, helps the development of a child's thinking and other psychological functions. Mathematical contents are abstract in their nature. In order for high abstractions to become clearer and for the development of these notions to follow the natural path of the child's development, Taub et al. (2008) stress that it is necessary to know the basic characteristics of that development.

In 2006, the Organisation for Economic Co-operation and Development (OECD) carried out an assessment of pupils' knowledge in mathematics which was performed at all educational levels. The unsatisfactory results obtained indicate the necessity to constantly re-evaluate the parameters which may influence the process of the mastering of mathematics. The success in mastering mathematical notions depends, among others, on the development of the thinking structures of a child. The appearance of concrete logical thinking is important for pupils' positive school achievement in terms of mathematical contents. Indicative of the appearance of concrete operations in thinking are conservation notions, i.e. the abilities to understand the immutability of the quantitative characteristics of an object, despite its visible transformations. The appearance of conservation indicates that the operations have become reversible. Mathematical thinking implies reversibility, which enables the child to gradually adopt the conservation of numbers, length, mass and volume, and at the same time understand that a certain value does not change unless something is added to it or subtracted from it. The operational correspondence relies on the idea of a number, which the child uses when he or she wishes to establish the equality of series. For the conservation operations to be mastered, the operation of identity is necessary, which is connected to the rational deduction and represents the

essence of the ability of conservation (Piaget, 1994; Santrock, 2007; Japundža-Milisavljević, 2008). Conservation tasks served Piaget in documenting his thesis on qualitative differences in children's thinking when compared with that of adults. The most complex forms of conservation become enabled by shaping formal operations, which is the last and the most developed phase in the development of thinking. The combinatorics represents the generalization of the operations acquired in the concrete operations stage and is a feature of the last phase of (Piaget, Inhelder, 1996; Donaldson, 1997; Krstic and Baucal, 2003).

Foreign researches studying children with slight intellectual difficulties confirm the importance of the development of the thinking functions in mastering the mathematics programme at young and later school ages, emphasizing that the immaturity of these structures in children with mild intellectual disabilities hinders their progress within this academic area (Taub et al., 2008). In their study conducted on the sample of 90 children with mild intellectual disabilities attending the third, fourth, and fifth grades, Maćešić-Petrović (1996) conclude that the processes of the creation of notions and of logical operation systems represent the framework for the mastering of the contents of teaching subjects, and of mathematics in particular.

We have found the basic idea for this work in the study which have dealt with the development of the thinking functions in children with mild intellectual disabilities and the relations between this development and the mastering of the mathematics programme (Maćešić-Petrović, 1996; Westwood, 2004; Đurić-Zdravković, 2007). In the literature we find an insufficient number of study dealing with the functioning of thinking, and its relation with the solving mathematical tasks in children with mild intellectual disabilities at later school ages (Kuhn, 2008; Taub et al., 2008) in relation to the study dealing with children with slight intellectual difficulties at younger school ages. We find it justified to find clear information on the level of the functioning of thinking in the pupils with mild intellectual disabilities of later school ages and the relation to their learning of mathematics, which could help teachers create methods for the adequate mastering of the mathematics programme, in accordance with the abilities of children with mild intellectual disabilities.

AIM AND HYPOTHESES

The aim of this paper is to assess the development of the thinking structures at the level of concrete logical operations (by achieving conservation of numbers, length, mass and volume) and at the level of formal operations (by achieving combinatorics) as well as to assess their relation with the mathematical achievements of the pupils with slight intellectual difficulties at the later school ages.

In this paper we set a general hypothesis related to the existence of an important correlation between the development of the thinking structures and the successful mastering of the mathematics programme by pupils with mild intellectual disabilities at later school ages (H1). The subhypotheses were based on the following presumptions:

H1.1 There is a statistically significant relation between the quality of the development of the thinking structures at the level of concrete operations in children with slight intellectual difficulties of later school ages and their achievements within the teaching subject of mathematics, whereby the children who have the structures of concrete operations more developed will have mastered the mathematics programme more efficiently.

H1.2 There is a statistically significant relation between the quality of the development of the thinking structures at the level of formal operations by children with mild intellectual disabilities, at later school ages and their achievements within the teaching subject of mathematics, whereby the children who have more developed structures of formal operations, will be more successful in solving mathematical tasks.

METHOD

Sample

The sample of this study was composed of 120 pupils of both genders. There were somewhat more male (56.7%) than female pupils (43.3%) in the sample. The criteria for the selection of the subjects implied the intelligence quotient of the pupils which varied from 50 to 69, and was assessed by the WISC assessment scale for the assessment of intellectual abilities. The children were 12 to 15-years-old corresponding to grades 5 to 8 of school age (30 pupils per class). The pupils did not have any neurological, psychiatric, highly emotional and combined disorders.

Measurement instruments

The study involved children of later school ages, which conditioned the need to assess the aspects of the operability of thinking processes which appear at that age. To assess the operability of thinking, we used the standard Piaget's Conservation assessment tasks (number, length, mass and volume), and the tasks used in the assessment of combinatorics (combinations of objects).

The responses of the subjects in solving the tasks of the conservation of number can be classified as follows:

1a) Pre-operational level - global spatial correspondence: the child brings more elements than it is necessary, creating a range of the same length, but with a much denser arrangement and with more elements

1b) Pre-operational level - visual correspondence: The child ensures the equality range by putting an element of one colour opposite to each element of another colour

2) Operational correspondence: The child counts the elements of one colour and brings as many elements of another colour and organizes them into the range.

Having interpreted the results of the tasks for the conservation of length, mass and volume, we have classified the responses of the subjects into three categories:

1) The absence of conservation: The child denies the equality when the original position is changed

2) Intermediary level: The child presumes that there is conservation, but is not sure about it and claims that it only exists by some transformations

3) Operational level: The child confirms that conservation is evident in all transformations.

The children who achieve the operational level explain the observed conservation in different ways, thereby using the following explanations typical of operational functioning:

3a) The first type of explanations refers to the empirical reversibility based on the understanding that the phenomenon of an object is kept constant because the earlier position can be reached by means of a reversible action

3b) The second type of explanations refers to the reversibility by the compensation of relations (e.g. the object is wider, but thinner)

3c) The third type of explanations refers to the sameness of the appearance (we subtracted nothing and we added nothing). A conspicuous

feature of this identity is that it gains the value of evidence for the conservation when the other kinds of evidence have already been discovered, whereas the children from the pre-operational level, who also know that it was "the same clay", draw no conclusion related to conservation

3d) The fourth type of explanations refers to the evident existence of conservation, but without an explanation given by the subject.

When solving the tasks of combinatorics using the combination of objects, there are three categories of success which can be specified:

1) The absence of combination (pre-operational level)

2) Intermediary level: the children with the ability to perform concrete operations execute only several combinations, using trial and error in solving the task

3) The success of combination (operational level): The child approaches the task systematically and determines all possibilities by exhausting the combinations of a given set.

To assess children's mastery of mathematical contents, we used Criteria knowledge tests, constructed for each grade separately and specifically designed for this study.

The results of the criteria assessment were divided into three categories:

The pupil has fully mastered the programme requirements of the grade he/she attends (+)

The pupil has partially mastered the programme requirements of the grade he/she attends (+-)

The pupil has not mastered the programme requirements of the grade he/she attends (-)

The other data necessary for the study were collected by means of the analysis of pupils' pedagogical documentation (gender, IQ, age, schoolyear, the absence of neurological, psychiatric, sensory, highly emotional, and combined disorders).

The study

The research was conducted in the elementary schools for children with mild intellectual disabilities in the city of Belgrade. The testing was conducted in continuity, without any breaks, and it was conducted with each pupil individually. The Criteria test was given to the pupils before the end of the school year and after the curriculum contents in mathematics had been fully realized.

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Data processing

The statistical processing of the data collected, which were processed by the programme SPSS v. 16.0, was carried out by means of descriptive statistical methods. The statistical procedures and measures used were frequencies, percentages, arithmetic means, standard deviation, Student's t-test, χ^2 test and the contingency coefficient (c).

RESULTS

The results will be presented with regard to the manner of data processing and the aims and hypotheses of the research.

The success of the pupils in solving the tasks of conservation and combinatorics

The data have been classified according to the responses obtained which are typical of the conservation of number (Table 1). In solving the tasks, the subjects from our sample have mainly relied on visual correspondence (60%), whereas the global spatial correspondence has been represented in 16.7% of the total sample examined.

The domination of the operational correspondence has been realized by 23.3% of the subjects. The results of the statistical analysis indicate that the relation between the level of education and the success of the pupils in solving the task of the conservation of number is statistically significant. The data obtained support the thesis that the children at a higher educational level achieve better results on the tasks of conservation.

Table 1. The assessment of subjects' success on the task of the conservation of number

	Categories of responses on the task of the conservation of number							
Class	Global		visual		operational		∑	
	%	n	%	n	%	n	%	n
V	6.7	8	15.0	18	3.33	4	25.0	30
VI	4.2	5	16.7	20	4.16	5	25.0	30
VII	5.0	6	13.3	16	6.66	8	25.0	30
VIII	0.8	1	15.0	18	9.15	11	25.0	30
∑	16.7	20	60.0	72	23.30	28	100.0	120
	χ^2= 8.20 df=3 c= +0.18 (p< 0.05)							

The research results presented in Table 2 indicate that the realization of the conservation length is present in 87.5% of subjects from our sample. The results of the statistical analysis indicate that the relation between the education level and the success of the pupils on the task of the conservation of length is statistically significant. This practically means that the children of the oldest educational level have achieved the best results in the assessment of the conservation of length, due to the fact that their level of processing is operational.

Table 2. The assessment of subjects' success on the task of the conservation of length

	Categories of responses on the task of the conservation of length							
class	absence of conservation		intermediary level		operational level		∑	
	%	n	%	n	%	n	%	n
V	1.7	2	5.8	7	17.5	21	25.0	30
VI	0.8	1	3.4	4	20.8	25	25.0	30
VII	0.0	0	0.8	1	24.2	29	25.0	30
VIII	0.0	0	0.0	0	25.0	30	25.0	30
∑	2.5	3	10.0	12	87.5	105	100.0	120
	χ^2 = 15.47 df=3 c= +0.34 (p< 0.01)							

The data presented in Table 3 show that the operational conservation of length is mostly mastered by means of identity, to a smaller extent by the compensation of operations, while the empirical reversibility is least represented.

Table 3. The assessment of subjects' success on the operational conservation of length

	categories of responses on the operational conservation of length							
class	3a)		3b)		3c)		3d)	
	%	n	%	n	%	n	%	n
V	3.3	4	5.8	7	4.2	5	4.2	5
VI	3.3	4	4.2	5	12.5	15	0.8	1
VII	2.5	3	6.7	8	15.0	18	0.0	0
VIII	1.7	2	10.0	12	13.3	16	0.0	0
∑	10.8	13	26.7	32	45.0	54	5.0	6

Legend: 3a - empirical reversibility
3b - reversibility by the compensation of operations
3c - identity
3d - no explanation

Table 4. The assessment of subjects' success on the task of the conservation of mass

	categories of responses on the task of the conservation of mass							
class	absence of conservation		intermediary level		operational level		∑	
	%	n	%	n	%	n	%	n
V	1.6	2	6.7	8	16.7	20	25.0	30
VI	1.6	2	3.4	4	19.9	24	25.0	30
VII	0.0	0	0.8	1	24.2	29	25.0	30
VIII	0.0	0	0.0	0	25.0	30	25.0	30
∑	3.2	4	10.9	13	85.8	103	100.0	120
	χ^2= 20.49 df=3 c= +0.38 (p< 0.01)							

The study results presented in Table 4 indicate that 85.8% of the children have mastered the conservation of mass. The results of the statistical analysis indicate that the relation between the level of education and pupils' success on the tasks of the conservation of mass is statistically significant. The children from our sample are most successful in solving the tasks which involve the conservation of mass if they are at the operational level.

The data presented in Table 5 show that the operational conservation of mass is scarcely mastered on the basis of identity and the compensation of relations.

Table 5. The assessment of subjects' success on the operational conservation of mass

	categories of responses on the operational conservation of mass							
class	3a)		3b)		3c)		3d)	
	%	n	%	n	%	n	%	n
V	10.0	12	6.7	8	0.0	0	0.0	0
VI	9.2	11	6.7	8	4.1	5	0.0	0
VII	9.2	11	8.3	10	6.7	8	0.0	0
VIII	6.6	8	11.6	14	6.7	8	0.0	0
∑	35.0	42	33.3	40	17.5	21	0.0	0

Legend: 3a - empirical reversibility
3b - reversibility by the compensation of operations
3c - identity
3d - no explanation

Table 6. The assessment of subjects' success on the task of the conservation of volume

	categories of responses on the task of the conservation of volume							
class	absence of conservation		intermediary level		Operational level		∑	
	%	n	%	n	%	n	%	n
V	7.5	9	11.7	14	5.8	7	25.0	30
VI	4.2	5	8.3	10	12.5	15	25.0	30
VII	0.8	1	7.5	9	16.6	20	25.0	30
VIII	0.0	0	4.2	5	20.9	25	25.0	30
∑	12.5	15	31.7	38	55.8	67	100.0	120
	χ^2= 17.32 df=6 c= +0.36 (p< 0.01)							

The data in Table 6 show that only 55.8% of the subjects achieve the conservation of volume, whereas as many as 31.7% of the subjects give passing solutions on this task. The results of the statistical analysis indicate that the relation between the level of education and the pupils' success on the tasks of the conservation of volume is statistically significant. The data obtained could practically indicate that the children of higher educational levels who have mastered the operational level solve the tasks which include the assessment of the conservation of volume.

Table 7. The assessment of subjects' success on the operational conservation of volume

	categories of responses on the operational conservation of volume							
class	3a)		3b)		3c)		3d)	
	%	n	%	n	%	n	%	n
V	2.5	3	0.8	1	0.0	0	2.5	3
VI	5.8	7	4.2	5	0.8	1	1.7	2
VII	12.6	15	3.4	4	0.8	1	0.0	0
VIII	9.9	12	4.9	6	5.9	7	0.0	0
∑	30.8	37	13.3	16	7.5	9	4.2	5

Legend: 3a - empirical reversibility
3b - reversibility by the compensation of operations
3c - identity
3d - no explanation

The results presented in Table 7 indicate that the operational conservation of volume is mastered mostly on the basis of the empirical reversibility, followed by the compensation of relations and finally by identity, while few subjects do not give any explanations for their claims.

The results of the tasks of the combination of objects are presented in Table 8. According to the results, 35.8% of the subjects show the absence of combination, i.e. they function at the pre-operational level, 51.7% were successful at the intermediary development stage, and in only 12.5% of the cases the subjects succeeded to combine the elements systematically, establishing relations among them, and finding all the possible combinations. The results of the statistical analysis indicate that the relation between the level of education and the pupils' success on the tasks of the combination of objects is not statistically significant. Higher educational level subjects are more successful in solving the tasks of the combination of objects in the operational sense in relation to the lower educational level subjects, but their number is not sufficient for this relation to be statistically significant.

Table 8. The assessment of subjects' success on the task of the combination of objects

	categories of responses on the task of the combination of objects							
class	absence of combination		Intermediary level		operational level		∑	
	%	n	%	n	%	N	%	n
V	10.0	12	15.0	18	0.0	0	25.0	30
VI	9.2	11	15.8	19	0.0	0	25.0	30
VII	8.3	10	10.9	13	5.8	7	25.0	30
VIII	8.3	10	10.0	12	6.7	8	25.0	30
∑	35.8	43	51.7	62	12.5	15	100.0	120
	χ^2 = 4.05 df=6 c= +0.18 (p> 0.05)							

The level of mastery in the curriculum contents in mathematics

The results presented in Table 9 show that our subjects do not achieve the 75% mastery of the curriculum contents at any educational level. This level of mastery is necessary for the curriculum to be considered adapted to the abilities of children with mild intellectual disabilities (Bojanin, 2002; according to Đurić-Zdravković, 2007). The distribution of the subjects according to their success in mastering the curriculum contents in mathematics in relation to their school age, i.e. the level of education, emphasizes the domination of subjects with a higher degree of mastery of the curriculum in mathematics in the 5th and 6th grades, whereas the subjects with a lower degree of mastery of the curriculum contents prevail in the 7th and 8th grades. A negative correlation between the examined

variables (c = – 0.45) has been found, which is statistically significant at the reliability level 0.01 (χ^2= 30.53 p < 0.01 for df= 6).

Table 9. The relation between the degree of mastering of the mathematics curriculum and the educational level of the pupils

	degree of mastering of the curriculum in mathematics							
class	did not master		partially mastered		mastered		∑	
	%	n	%	n	%	n	%	n
V	20.67	6	4.61	1	74.72	23	100	30
VI	33.64	10	15.26	5	51.10	15	100	30
VII	34.25	10	17.45	6	48.30	14	100	30
VIII	34.68	10	16.27	5	49.05	15	100	30
	χ^2 = 30.53 df=6 c= –0.45 (p< 0.01)							

The relation between the development of conservation and combinations qualities, and the level of mastering of the curriculum in mathematics

A statistically significant positive relation was found between the achieved degree of the operability of thinking on the tasks of the conservation of number, length, mass and volume, and combinatorics, and the level of mastering of the curriculum in mathematics (Table 10). Therefore, it was found that the subjects with a higher level of thinking functions solve mathematical tasks more successfully at the later school age.

A statistically significant relation was found between the achieved degree of the operability of thinking at the level of concrete operations and the success by the criteria test in mathematics (p<0.05 in all tasks), which confirms the first hypothesis of the study (h1.1). Moreover, a statistically significant relation was found between the degree of the functioning of thinking at the level of formal operations and the success measured by the Criteria test in mathematics (p<0.01), which confirms the second hypothesis as well (h1.2). Therefore, the general study hypothesis suggesting that there is a significant correlation between the development of the thinking structures and the successful mastering of the curriculum in mathematics in the pupils with mild intellectual disabilities of the later school age (h1) can be accepted.

Table 10. The relation between the tasks of conservation, combinatorics and the degree of mastering of the curriculum in mathematics

	degree of mastering of the curriculum in mathematics
conservation of number	χ^2= 5.80 df=2 c=+0.22 (p< 0.05)
conservation of length	χ^2= 9.37 df=2 c=+0.27 (p< 0.05)
conservation of mass	χ^2= 8.84 df=2 c=+0.26 (p< 0.05)
conservation of volume	χ^2= 8.36 df=4 c=+0.26 (p< 0.05)
combinatorics	χ^2=12.62 df=4 c=+0.31 (p< 0.01)

DISCUSSION

The data obtained indicate that the number of subjects who were unsuccessful in solving the tasks applied decreases with the increase in the educational level, and the number of subjects in the intermediary stage increases. The majority of the general population children master the conservation of number and length at the age of 7 and 8, the conservation of mass after the age of 8, and the conservation of volume after the age of 11 (Donaldson, 1997; Baucal and Stepanovic, 2006; Jensen, 2006). We therefore conclude that the children from our sample adopt logical operations with difficulties and delay in relation to their age and educational level. Our study points to the presence of a slower development of operations of conservation in children with slight intellectual difficulties at an later school age, when their adoption is expected. We conclude that the children who are successful on the tasks of conservation have concrete operational abilities, and the children who, for any reason (cognitive or not), make mistakes on the tasks of conservation have not developed concrete operations as the composition of internalized schemes even at an later school age.

Piaget believes that the logical-mathematical experience results in the cognition of the relations between objects (e.g. the child counts 5 little stones from left to right, then from right to left, then places them in a circle and counts them again, to create the notion of the number 5). Therefore, this experience exceeds the cognition of the physical properties of objects. The cognition created on the basis of that type of experience is the construction of thinking and it is conditioned by the complex intellectual activity. This research contributes to the thesis of the inaccessibility of the constructions of thinking in children with slight intellectual difficulties, as Barbel Inhelder suggests in her research. (Piaget, Inhelder, 1996), which we discussed in the introduction. The results of our study indicate that children

with mild intellectual disabilities are stopped in their development at the level of concrete operations. At this stage, partially built constructions of the operability of thinking are present, because some 15% of the children have not adopted the conservation of length and mass, almost half of them have not adopted the conservation of volume, whereas as many as 60% of the subjects rely on visual correspondence when solving the task of the conservation of number. Also, the operational conservation of length is mostly mastered on the basis of identity, whereas the operational conservation of mass is mastered in the smallest percentage by means of this category of responses. The conservation of mass is evenly adopted both by means of the empiric reversibility and the compensation of relations. The operational conservation of volume is mostly mastered by means of the empiric reversibility, followed by the means of the compensation of relations and finally by means of identity.

On the basis of such findings we conclude that the operational conservation of length is the first thinking structure the children with mild mental disabilities in our sample adopt, because the responses based on identity are represented in the largest percentage. We have already mentioned that a conspicuous feature of identity is the fact that it acquires the value of evidence for conservation only after the other types of evidence have been discovered (e.g. the empirical reversibility and the compensation of relations). Inhelder presents similar findings about the adoption of the conservation of length (Piaget, Inhelder, 1996). On the other hand, the adoptability of the conservation of mass in our sample includes prevalently more flexible reversibility, which still does not imply identity. The conservation of volume is mainly based on the explanation of simple reversibility. This data coincide with the findings of Inhelder, who mentions that the operational conservation of volume is present in the borderline cases and in higher levels of children with mild intellectual disabilities, but never in typical clinical features of intellectual disorders. The reasons for these findings are explained with the lack of insight into the notion of proportion, which is connected with the notion of volume. The notion of proportion depends on formal operations.

We concluded that, at the age at which the appearance of formal operations is typical, only 12.5% of the subjects have reached the operational level. Such results indicate that more complex forms of operational functioning are adopted with delay in relation to the mastering of these abilities in the general children population.

The authors of previous papers dealing with this area have emphasized the phenomenon of false balance, fixation and viscosity of the cognitive development in children with mild intellectual disabilities. The

fixation indicates slowing down and stopping in one of the developmental phases, and the viscosity is determined by a longer duration of the transitional status between two stages. Maćešić-Petrović (1996) conducted a study of 90 children with mild intellectual disabilities of older and younger school ages. They have found simultaneous presence of two compositions, which results in the presence of lower cognitive operations at the time when the appearance of new and more complex ones is expected.

It is considered that the inability of children to conserve derives from the inability to decentrate, both from the immediate perceptive situation and from the relation between a moment in time and the one that follows. Children focus on one feature of the immediate situation and neglect the other ones. They also focus on the present moment failing to mentally return to the previous situation. Therefore, they do not understand that one action is, in principle, reversible (Donaldson, 1997; Einfeld et al., 2007).

The results of our study indicate that 5th graders are the only ones who achieve about 75% of the mastering of mathematics. The other pupils achieve about 50% in total. The results obtained raise the question of the adaptation of curricular requirements to the developmental abilities of children with mild intellectual disabilities. We think that certain adaptations of the curriculum in mathematics, which would be formulated according to the cognitive abilities of the children with mild intellectual disabilities and completed by the permanent conducting of the re-educational treatment, would contribute to a more successful mastering of mathematics.

We concluded that children who have not mastered the tasks of conservation solved mathematical tasks by means of intuitive and perceptive tools, and that they adapted to the curricular requirements by creating passive and automatized models of processing mathematical information, the essence of which they do not understand. Finally, we found that the children who have successfully solved the tasks of conservation could understand the characteristics of natural ranges of numbers and the real meaning of arithmetical operations.

Bruce and Muhammad (2009) point to the significance of direct and systematic instructions involved in the stimulation of the operations for concrete thinking in children with mild intellectual disabilities, with the purpose of solving mathematical tasks more successfully. After a detailed assessment of the operability of thinking in this population, they recommend the training of visual and differentiated attention, the consideration of the characteristics of all objects used in a mathematical task, the detailed demonstration of the procedure for each task requirement, and the repeated naming of objects and actions.

CONCLUSION

The results of this study confirm the delay in the adoption of logical operations in children with mild intellectual disabilities in relation to their age and educational level, as well as the insufficient degree of mastering mathematics at the later school age. With the aim of an adequate stimulation of thinking functions and a more adequate mastering of the curriculum in mathematics, we propose that Piaget's theory be introduced into working with children with mild intellectual disabilities. In his theory Piaget argues for the application of the games stimulating reversibility, identity and conservation, and making mathematical thinking more flexible, active, broader, deeper and more original (Piaget, 1994). The games should be designed in accordance with the mental functioning of the child and be oriented towards the spontaneous learning and developmental processes. Such games should be focused on complex mental operations related to the addition, subtraction, amount, number, notions such as less than, greater than... (Japundža-Milisavljević, 2008). According to Piaget, children learn contents more efficiently through games than through teaching activities. The game represents an excellent trigger and guide in the learning process. Children adopt new knowledge through games, by means of their own experiences. Didactic games, games of fantasy and roles and games with rules are very important in the development of the abilities of thinking and concluding. The different forms of games are identified as significant, among others, in the development of conservations, operational correspondences, in creating mathematical notions, in solving problems (Demetriou et al., 2010).

Jukić (2007) gives an interesting suggestion involving the use of a set of domino dice as a means of mastering different mathematical notions and the development of thinking in children. One should emphasize that such an attitude is essential for the application of mathematical implications in children with mild intellectual disabilities, starting from the earliest school age, and transferring the same mode to the later school age. Methodological implications of this learning for the development of mathematical notions are reflected in the requirement that mathematical notions cannot be "transferred" to the children, but that the children build and construct them in their numerous contacts with objects and phenomena from their immediate environment, making connections and relations between them on the mental plan (Taub et al., 2008; Dejić, 2008; Klasnić, 2009). The immediate environment is therefore indispensable in the process of the development of the mathematical-logical structures in children with slight intellectual difficulties.

Contemporary authors emphasize the importance of estimation, taking into account the life experience of the child which represents one of the important methodological guidelines. This implies the need to consider the situations and events from the children's everyday lives when choosing the contents. In other words, mathematical contents should be connected with children's experiences and the real problems they face. This principle is consistent with the situational learning or the so-called empirical mathematics. Such mathematics is not only more interesting to children, but is also more logical, and children solve tasks with pleasure. It is therefore necessary that the educator be familiar with children's abilities in order to build the development of mathematical notions upon them, starting with the earliest school ages, and without failing to follow and assess children's abilities in their further development.

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MISAONE STRUKTURE I MATEMATIČKO POSTIGNUĆE DJECE S BLAŽIM INTELEKTUALNIM TEŠKOĆAMA

SAŽETAK

Razvoj misaonih struktura djeteta utječe na uspješnost u savladavanju matematičkih pojmova. Pojava konkretnog logičkog mišljenja i njegovi indikatori (pojmovi konzervacije) značajni su za pozitivno školsko postignuće učenika u okviru matematičkih sadržaja. Oblikovanjem formalnih operacija, kao posljednjom najsavršenijom fazom razvoja mišljenja, omogućuju se najsloženiji vidovi konzervacije. Kombinatorika predstavlja generalizaciju operacija stečenih u stadiju konkretnih operacija i obilježje posljednje faze razvoja mišljenja.

Cilj ovog rada usmjeren je na procjenu razvoja misaonih struktura konkretnih logičkih operacija (ostvarivanjem konzervacije broja, duljine, mase i volumena) i formalnih operacija (ostvarivanjem kombinatorike), kao i njihovu povezanost s matematičkim postignućem učenika s blažim intelektualnim teškoćama starije školske dobi.

U istraživanju je sudjelovalo 120 učenika oba spola (43,3% djevojčica i 56,7% dječaka), kronološke dobi od 12 do 15 godina. Sudionici su učenici od V do VIII razreda beogradskih osnovnih škola za djecu s blažim intelektualnim teškoćama.

Za procjenu operativnosti mišljenja korišteni su standardni Piagetovi zadaci za procjenu konzervacije (broja, duljine, mase i volumena) i kombinatorike, a za procjenu usvajanja matematičkih sadržaja korišten je kriterijski test znanja, posebno konstruiran za potrebe ovog istraživanja.

Rezultatima istraživanja je utvrđena povezanost između stupnja misaone razvijenosti na svim primijenjenim zadacima i razine savladavanja gradiva matematike.

Imajući u vidu loša postignuća ispitanika našeg istraživanja u svim segmentima rada, naglašavamo značaj prezentiranja Piagetovih i matematičkih zadataka kroz igru, kao što i *Piagetova teorija zastupa stav o primjeni igara kojima se potiču reverzibilnost, identitet i konzervacija te kojima se matematičko mišljenje čini gipkijim, aktivnijim, širim, dubljim i originalnijim.*

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UVOD

Najraniji oblici spoznaje kod djece sastoje se u opažajno-praktičnoj aktivnosti. Tijekom druge godine života počinje se razvijati mišljenje, kao najrazvijeniji i najsloženiji vid kognitivne djelatnosti. Procese mišljenja karakterizira operiranje iskustvenim informacijama, kao što su opažaj, predstava, riječ ili pojam. Mišljenjem se ostvaruje saznanje o različitim karakteristikama objekta, uspoređuje se po sličnosti, uočavaju se razlike, uspostavljaju se relacije među objektima, grupiraju se predmeti po analogijama, te se uočava ono što je konstantno i promjenljivo (Demetriou i sur., 2010).

Za teorijsku osnovu ovog rada iskoristili smo teoriju jednog od najpoznatijih psihologa-epistemologa Jeana Piageta (Jean Piaget 1896-1980). Suština njegove biologističke teorije sastoji se u tvrdnji da je učenje podređeno razvojnom procesu djeteta, da ono ovisi od razine tog razvoja i da je za razvoj presudan utjecaj maturacije (Piaget i Inhelder, 1996).

Po Piagetu, kako ističu Donaldson (1997) i Živković (2007), mišljenje je sustav operacija tj. sustav unutarnjih radnji. Piaget razlikuje četiri faze u razvoju kognitivnih struktura: Senzomotorički stadij (od rođenja do druge godine), Predoperacijski stadij (od druge do sedme godine), Stadij konkretnih operacija (od sedme do dvanaeste godine), Stadij formalnih operacija (počev od dvanaeste godine).

Jensen (2006) ukazuje na to da uzrast na kome se pojedini stadiji javljaju može varirati u izvjesnoj mjeri, ovisno od inteligencije djeteta i faktora okoline, ali su redoslijed javljanja stadija i njihov sadržaj nepromjenjivi.

Prema Piagetu, kraj senzomotoričkog razdoblja označava početak razvoja mišljenja. Senzomotorička inteligencija operira opažajima i praktičnim aktivnostima, dok konkretne operacije operiraju predodžbama (opažajima i aktivnostima prenijetim na unutrašnji plan). Akcije koje dijete izvodi u početku su vanjske, a kasnije postaju internalizirane. Tada operacije postaju reverzibilne (dvosmjerne, kao što su npr. zbrajanje i oduzimanje, množenje i dijeljenje), integrirane u sustav akcija (stvaranje povezanih, a ne izoliranih misaonih operacija) i time misao postaje pokretljiva (Donaldson, 1997; Piaget i Inhelder, 1996).

Bruce i Muhammad (2009) su analizirali operacije koje se odvijaju na mentalnom planu u ovom stadiju i rezultatima pokazali da one mogu nastati

i funkcionirati samo u odnosu sa konkretnom realnošću (otud naziv konkretne operacije). Postoje različite operacije (npr. matematičke, logičke, itd.), ali sve one se odvijaju prilikom interakcije sa konkretnom stvarnošću. Djeca u ovoj fazi mogu riješiti samo probleme koji se odnose na stvarne objekte ili događaje, a ne apstraktne koncepte ili hipotetičke zadatke.

Mišljenje djeteta je logičnije, manje egocentrično i manje ovisno od neposrednih perceptivnih iskustava. Dijete je u stanju vršiti mentalne operacije kao što je računanje, misliti o predmetima, o klasama predmeta, te o odnosima među tim klasama (Donaldson, 1997).

Kada operacija postane predmet operacije višeg reda, Piaget govori o formalnim operacijama. Na ovoj razini subjekt postaje svjestan vlastitih mentalnih operacija, koje je i na prethodnoj razini bio u stanju izvesti, ali ne ih i analizirati. Razvija se apstraktno mišljenje i dijete postaje sposobno misliti u apstraktnim kategorijama, donositi zaključke na osnovu apstraktnih pretpostavki, formulirati opće zakone i principe, te razumjeti metafore. Nove sposobnosti sve više usmjeravaju mišljenje adolescenta ka hipotetičkim i ideološkim pitanjima. Formalne operacije se formiraju tijekom adolescencije (Kuhn, 2008).

Poznato je da djeca s blažim intelektualnim teškoćama (u daljnjem tekstu BIT) prolaze kroz iste faze razvoja, kao i djeca tipične populacije, samo mnogo sporijom brzinom. Piaget i Inhelder (1996) smatraju da djeca s intelektualnim teškoćama ne mogu ostvariti progresivan napredak k sljedećem stadiju u razvoju intelektualnih i spoznajnih sposobnosti jer nisu u stanju da izgrade nove oblike i sustave mišljenja, već ostaju vezani za infantilne sustave koji potiču iz prethodnog stadija razvoja iako on više ne odgovara datom kalendarskom uzrastu djeteta.

Inhelder (1968; prema Maćešić-Petrović, 1998) ističe da se odlike mentalnog razvoja djeteta s intelektualnim teškoćama ogledaju u dužim trajanjima prelaznih formi između različitih stadija razvoja, nego što je to slučaj s djecom prosječne populacije. Na taj način stvara se mogućnost suprotstavljanja dva različita sustava i razine konstrukcija u razvoju individue, što doprinosi destabilizaciji tog razvoja.

Način organiziranosti misaonih struktura, u globalnom smislu, neophodan je za pravilno usmjeravanje edukativnog procesa.

Matematika uvodi dijete u percipiranje i shvaćanje odnosa u neposrednom okruženju, pomaže razvoju dječjeg mišljenja i drugih psihičkih funkcija. Matematički sadržaji su po prirodi apstraktni. Da bi se približile visoke apstrakcije i da bi razvoj ovih pojmova pratio prirodan put dječjeg razvoja, Taub i suradnici (2008) ističu da je potrebno poznavati osnovne karakteristike tog razvoja.

Nezadovoljavajući rezultati učenika ostvarenih tijekom provjeravanja znanja iz nastavnog predmeta Matematika na svim razinama obrazovanja, koje je provela OECD -Organisation for Economic Co-operation and Development (2006), ukazuju na neophodnost stalnog preispitivanja parametara koji mogu utjecati na proces njenog savladavanja.

Uspješnost u savladavanju matematičkih pojmova, između ostalih, zavisi od razvoja misaonih struktura djeteta. Za pozitivno školsko postignuće učenika u matematičkim sadržajima značajna je pojava konkretnog logičkog mišljenja. Indikator pojave konkretnih operacija u mišljenju su pojmovi konzervacije tj. sposobnosti shvaćanja nepromjenljivosti kvantitativnih svojstava objekta unatoč njegovim vidljivim transformacijama. Pojava konzervacija je znak da su operacije postale reverzibilne. Matematičko mišljenje podrazumijeva reverzibilnost, koja omogućuje djetetovo postupno ovladavanje konzervacijom broja, duljine, mase i volumena, shvaćajući da se određena veličina ne mijenja, ukoliko joj ništa nije dodano ni oduzeto. Operacijska korespondencija se oslanja na pojam broja kojom se dijete koristi kada želi utvrditi jednakost nizova. Za ovladavanje operacijama konzervacije neophodna je i operacija identiteta koja je vezana za racionalnu dedukciju i predstavlja suštinu sposobnosti konzervacije (Piaget, 1994; Santrock, 2007; Japundža-Milisavljević, 2008). Zadaci konzervacije su poslužili Piagetu da dokumentira svoju tezu o kvalitativnim razlikama u načinu mišljenja djece u odnosu na odrasle. Najsloženiji oblici konzervacije postaju omogućeni oblikovanjem formalnih operacija, kao posljednjom najrazvijenijom fazom razvoja mišljenja. Kombinatorika predstavlja generalizaciju operacija stečenih u stadiju konkretnih operacija i obilježje posljednje faze razvoja (Piaget i Inhelder, 1996; Donaldson, 1997; Krstic i Baucal, 2003).

Inozemna istraživanja kod djece s BIT potvrđuju važnost razine razvoja misaonog funkcioniranja u savladavanju programa matematike u mlađoj i starijoj školskoj dobi, te ističu da nerazvijenost ovih struktura kod djece s BIT otežava napredovanje u okviru ove akademske oblasti (Taub i sur., 2008). Maćešić-Petrović (1996) u istraživanju provedenom na uzorku od 90-ero djece s BIT III, IV i V razreda zaključuje da procesi formiranja pojmova i sistema logičkih operacija čine okosnicu savladavanja sadržaja nastavnih predmeta, pogotovo matematike.

Osnovnu ideju ovog rada nalazimo u rezultatima prethodnih istraživanja koja su se bavila razvojem misaonog funkcioniranja djece s BIT i njegovom povezanošću sa savladavanjem programa matematike (Maćešić-Petrović, 1996; Westwood, 2004; Đurić-Zdravković, 2007). U literaturi nalazimo nedovoljan broj istraživanja koji proučava razvoj misaonog funkcioniranja i povezanost s rješavanjem zadataka iz matematike kod djece

s BIT u starijoj školskoj dobi (Kuhn, 2008; Taub i sur., 2008) u odnosu na istrazivanja kod djece s BIT u mlađoj školskoj dobi.

Smatramo opravdanim utvrditi jasne informacije o razini misaonog funkcioniranja učenika s BIT starije školske dobi i povezanosti s učenjem matematike, koje bi mogle pomoći nastavnicima u kreiranju načina adekvatnog savladavanja programa matematike primjereno mogućnostima djeteta s BIT.

CILJ I HIPOTEZE

Cilj rada je procjena razvoja misaonih struktura u ravnini konkretnih logičkih operacija (ostvarivanjem konzervacije broja, duljine, mase i volumena) i formalnih operacija (ostvarivanjem kombinatorike), kao i njihove povezanosti s matematičkim postignućem učenika s BIT starije školske dobi.

U ovom radu, postavili smo opću hipotezu koja se odnosi na postojanje značajne korelacije između razvoja misaonih struktura i uspješnog savladavanja programa matematike kod učenika s BIT starije školske dobi (H1). Podhipoteze su se bazirale na sljedećim pretpostavkama:

H1.1: Postoji statistički značajna povezanost između kvalitete razvoja misaonih struktura u ravnini konkretnih operacija djece s BIT starije školske dobi i njihovog postignuća u okviru nastavnog predmeta Matematika, pri čemu će djeca koja imaju razvijenije strukture konkretnih operacija imati i bolju savladanost programa matematike.

H1.2: Postoji statistički značajna povezanost između kvalitete razvoja misaonih struktura u ravnini formalnih operacija djece s BIT starije školske dobi i njihovog postignuća u okviru nastavnog predmeta Matematika, pri čemu će djeca koja imaju razvijenije strukture formalnih operacija imati bolji uspjeh u rješavanju zadataka iz matematike.

METODA

Uzorak ispitanika

Uzorak ovog istraživanja čini 120 učenika, oba spola. U uzorku je bilo nešto više učenika muškog spola (56,7%) u odnosu na ispitanike ženskog spola (43,3%).

Kriteriji za izbor ispitanika podrazumijevali su količnik inteligencije učenika koji se kretao u okvirima od 50 do 69, procijenjen WISC skalom za procjenu intelektualnih sposobnosti, kalendarske dobi od 12 do 15 godina, školske dobi koja je podrazumijevala uključivanje učenika od V do VIII razreda (za svaki razred ispitano je po trideset učenika) te odsustvo neuroloških, psihijatrijskih, senzornih, izraženih emocionalnih i kombiniranih smetnji.

Mjerni instrumenti

Istraživanjem su obuhvaćena djeca starije školske dobi, što je uvjetovalo potrebu da se procjene oni aspekti operativnosti misaonih tokova čiju pojavu bilježimo u toj dobi. Za procjenu operativnosti mišljenja korišteni su standardni Piagetovi zadaci za procjenu konzervacije (broja, duljine, mase i volumena) i kombinatorike (kombiniranje objekata).

Odgovore ispitanika pri rješavanju zadatka konzervacije broja moguće je grupirati u sljedeće kategorije:

1a) Predoperacijska razina - globalna prostorna korespondencija: dijete donosi više elemenata nego što je potrebno ostvarujući niz iste dužine, ali sa znatno gušćim rasporedom i sa više elemenata

1b) Predoperacijska razina - vizualna korespondencija: dijete osigurava jednakost niza postavljanjem elementa jedne boje naspram svakog elementa druge boje

2) Operacijska korespondencija: dijete izbroji elemente jedne boje i donese isto toliko elemenata druge boje, te ih stavi u niz.

Odgovore ispitanika pri tumačenju rezultata na zadacima konzervacije duljine, mase i volumena, grupirali smo u tri kategorije:

1) Odsustvo konzervacije: dijete negira jednakost pri promjeni prvobitnog položaja

2) Intermedijalna razina: dijete pretpostavlja da postoji konzervacija, ali nije sigurno u to i tvrdi da postoji samo pri nekim transformacijama

3) Operacijska razina: dijete tvrdi očiglednost konzervacije pri svim transformacijama.

Djeca koja dostižu operacijsku razinu obrazlažu uočenu konzervaciju na različite načine, koristeći pri tom sljedeća objašnjenja koja su karakteristična za operacijsko funkcioniranje:

3a) prva vrsta obrazloženja poziva se na *empirijsku reverzibilnost* utemeljenu na shvaćanju da se pojavnost objekta održava stalnom, jer se povratnom akcijom može postići prvobitno stanje

3b) druga vrsta obrazloženja poziva se na *reverzibilnost kompenzacijom relacija* (npr. objekat je širi, ali tanji)

3c) treća vrsta obrazloženja se poziva na istovjetnost pojave (nismo ništa ni oduzeli ni dodali). Upadljivo svojstvo ovog *identiteta* je da on stječe vrijednost dokaza za konzervaciju onda kada su druge vrste dokaza već otkrivene, dok djeca sa predoperacijske razine, koja su također znala da je to "isti plastelin", nisu izvukla nikakav zaključak o konzervaciji

3d) četvrta vrsta obrazloženja poziva se na evidentno postojanje konzervacije, ali *bez datog objašnjenja* od strane ispitanika.

Pri rješavanju zadatka kombinatorike, kombiniranjem objekata, postoje tri kategorije uspješnosti:

1) Odsustvo kombiniranja (predoperacijska razina)

2) Intermedijalna razina: djeca sa sposobnošću izvođenja konkretnih operacija izvedu samo nekoliko kombinacija, radeći pokušajima i pogreškama prilikom izvođenja zadatka

3) Uspješnost kombiniranja (operacijska razina): ispitanik prilazi zadatku sustavno, te određuje sve mogućnosti iscrpljujući kombinacije danog skupa.

Za procjenu savladanosti matematičkih sadržaja korišteni su kriterijski testovi znanja, koji su, za potrebe ovog istraživanja, konstruirani za svaki razred posebno. Rezultati kriterijskog ocjenjivanja distribuirani su u tri kategorije:

- potpuno savladao programske zahtjeve razreda koji pohađa (+)
- djelomično savladao programske zahtjeve razreda koji pohađa (+ –)
- nije savladao programske zahtjeve razreda koji pohađa (–).

Ostali podaci koji su bili neophodni za potrebe istraživanja, preuzeti su analizom pedagoške dokumentacije učenika (spol, koeficijent inteligencije, kalendarska i školska dob, odsustvo neuroloških, psihijatrijskih, senzornih, izraženih emocionalnih i kombiniranih smetnji).

Način provođenja istraživanja

Istraživanje je provedeno na području grada Beograda, u osnovnim školama za djecu s BIT. Testiranje je provedeno u kontinuitetu, bez

vremenskih pauza, individualno, sa svakim učenikom posebno. Pred kraj školske godine zadan je kriterijski test znanja, kada su programski sadržaji iz matematike u potpunosti realizirani.

Obrada podataka

Statistička obrada prikupljenih podataka, smještenih u datoteku programa SPSS v. 16.0, vršila se metodama deskriptivne statistike. Od statističkih postupaka i mjera korišteni su: frekvencije, postotci, aritmetička sredina, standardna devijacija, Studentov t-test, χ^2 test i koeficijent kontingencije (c).

REZULTATI

Rezultati će biti prikazani s obzirom na način obrade podataka i dati sukladno cilju i hipotezama.

Uspješnost učenika u zadacima konzervacije i kombinatorike

Podaci su grupirani prema kategorijama odgovora karakterističnih za konzervaciju broja, te prikazani u Tablici 1. Rješavajući ovaj zadatak, ispitanici našeg uzorka se uglavnom oslanjaju na vizualnu korespondenciju (60%), dok je globalna prostorna korespondencija zastupljena sa 16,7% ukupnog ispitanog uzorka. Dominaciju operacijske korespondencije ostvaruje 23,3% ispitanika. Rezultati statističke analize ukazuju da je odnos između razine edukacije i uspješnosti učenika na zadatku konzervacije broja statistički značajan. Dobiveni podatak govori u prilog tezi da djeca koja su na višoj obrazovnoj razini ostvaruju bolje rezultate na zadacima konzervacije.

Tablica 1.

Rezultati istraživanja prikazani u Tablici 2 ukazuju da je ostvarivanje konzervacije duljine prisutno kod 87,5% ispitanika našeg uzorka. Rezultati statističke analize ukazuju da je odnos između razine edukacije i uspješnosti učenika na zadatku konzervacije duljine statistički značajan. Ovaj podatak znači da su djeca najstarije obrazovne razine pokazala najbolje rezultate pri procjeni konzervacije duljine s obzirom da se nalaze na operacijskoj razini.

Tablica 2.

Iz Tablice 3 uočavamo da se operacijska razina konzervacije duljine u najvećem postotku ovladava temeljem identiteta, nešto manje kompenzacijom relacija, a tek potom empirijskom reverzibilnošću.

Tablica 3.

Rezultati istraživanja prikazani u Tablici 4 ukazuju da 85,8% djece ovladava konzervacijom mase. Rezultati statističke analize ukazuju da je odnos između razine edukacije i uspješnosti učenika na zadatku konzervacije mase statistički značajan. Djeca u uzorku starije obrazovne razine su najuspješnija pri rješavanju zadataka koji obuhvaćaju konzervaciju mase ukoliko se nalaze na operacijskoj razini.

Tablica 4.

Iz Tablice 5 uočavamo da se operacijska razina konzervacije mase u najmanjem postotku ovladava temeljem identiteta, a podjednako empirijskom reverzibilnošću i kompenzacijom relacija.

Tablica 5.

Tablica 6 ukazuje da samo 55,8% ispitanika ostvaruje konzervaciju volumena, dok čak 31,7% ispitanika na ovom zadatku daje prijelazna rješenja. Rezultati statističke analize ukazuju da je odnos između razine edukacije i uspješnosti učenika na zadatku konzervacije volumena statistički značajan. Dobiveni podatak bi mogao ukazati da djeca starije obrazovne razine, a koja su ovladala operacijskom razinom, rješavaju zadatke koji obuhvaćaju procjenu konzervacije volumena.

Tablica 6.

Rezultati Tablice 7 ukazuju da se operacijska razina konzervacije volumena u najvećem postotku ovladava temeljem empirijske reverzibilnosti, potom kompenzacijom relacija, te identitetom, a najmanji broj ispitanika ne daje objašnjenje za svoju tvrdnju.

Tablica 7.

Na zadatku kombiniranja objekata, čiji su rezultati prikazani u Tablici 8, 35,8% ispitanika pokazuje odsustvo kombiniranja, tj. funkcionira na predoperacijskoj razini, 51,7% je bilo uspješno na intermedijalnom stadiju

razvoja, a samo u 12,5% slučaja ispitanici su uspjeli kombinirati elemente sistematično, uspostavljajući odnose među njima, te pronalazeći sve moguće kombinacije. Rezultati statističke analize ukazuju da odnos između razine edukacije i uspješnosti učenika na zadatku kombiniranja objekata nije statistički značajan. Ispitanici starije obrazovne razine su uspješniji pri rješavanju zadatka kombiniranja objekata u odnosu na ispitanike niže razine obrazovanja, ali njihov broj nije dovoljan da bi ovakav odnos bio statistički značajan.

Tablica 8.

Savladanost programskih sadržaja nastavnog predmeta Matematika

Iz prikazanih rezultata u Tablici 9 uočavamo da ispitanici našeg uzorka ni na jednom nivou edukacije ne postižu potrebnih 75% savladanosti programskih sadržaja, koliko je potrebno da bismo program smatrali primjerenim i prilagođenim sposobnostima djece s BIT (Bojanin, 2002; prema Đurić-Zdravković, 2007). Distribucija ispitanika prema uspješnosti u savladavanju programskih sadržaja iz matematike u odnosu na školsku dob, tj. razinu edukacije, upućuje na zaključak o dominaciji ispitanika s boljom savladanošću programa matematike u V i VI razredu, dok u VII i VIII razredu prevladavaju ispitanici s lošijom savladanošću programskih sadržaja iz matematike. U pitanju je negativna korelacija između ispitanih varijabli ($c = – 0,45$), koja je statistički značajna na razini pouzdanosti 0.01 ($\chi^2=30.53$ $p < 0,01$ za $df=6$).

Tablica 9.

Povezanost između kvalitete razvoja konzervacije i kombiniranja sa savladanošću programa matematike

Kod ispitanika našeg uzorka je utvrđena statistički značajna pozitivna povezanost između dostignutog stupnja operativnosti mišljenja na zadacima konzervacije broja, konzervacije duljine, konzervacije mase, konzervacije volumena, te kombinatorike i razine savladavanja programa matematike, koja je prikazana u Tablici 10. Dakle, utvrđeno je da ispitanici s višom razinom misaonog funkcioniranja uspješnije rješavaju matematičke zadatke u starijoj školskoj dobi.

Tablica 10.

Izloženi rezultati pokazuju da je povezanost između dostignutog stupnja operativnosti mišljenja u ravnini konkretnih operacija i uspjeha na Kriterijskom testu iz matematike statistički značajna ($p<0,05$ u svim zadacima), čime je potvrđena prva podhipoteza istraživanja (H1.1). Rezultati su, također, pokazali da postoji statistički značajna povezanost stupnja misaonog funkcioniranja u ravnini formalnih operacija i uspjeha na Kriterijskom testu iz matematike ($p<0,01$), čime je potvrđena i druga podhipoteza istraživanja (H1.2). Prema tome, opća hipoteza istraživanja o postojanju značajne korelacije između razvoja misaonih struktura i uspješnog savladavanja programa matematike kod učenika s BIT starije školske dobi (H1) može se prihvatiti.

RASPRAVA

Dobiveni podaci ukazuju da s porastom razine edukacije opada broj neuspješnih ispitanika na primijenjenim zadacima, a raste broj ispitanika koji se nalaze u intermedijalnom stadiju. Većina djece opće populacije ovladava konzervacijom broja i duljine na uzrastu od 7 do 8 godina, konzervacijom mase poslije osme godine starosti, a konzervacijom volumena poslije jedanaeste godine starosti (Donaldson, 1997; Baucal i Stepanovic, 2006; Jensen, 2006). Time zaključujemo da djeca našeg uzorka osvajaju logičke operacije s teškoćama i zakašnjenjem u odnosu na dob i razinu edukacije. Naše istraživanje ukazuje na prisustvo usporenog razvoja operacija konzervacija kod djece s BIT u starijoj školskoj dobi, kada bi se njihova osvojenost već trebala očekivati. Zaključujemo da djeca koja su uspješna u zadacima konzervacije imaju konkretne operacijske sposobnosti, a djeca koja iz bilo kojeg razloga (kognitivni ili ne) čine pogreške u zadacima konzervacije nisu razvila konkretne operacije kao sustav internaliziranih shema čak ni u starijoj školskoj dobi.

Piaget smatra da logičko-matematičko iskustvo rezultira spoznajom o odnosima između predmeta (npr. dijete broji pet kamenčića s lijeva na desno, pa s desna na lijevo, zatim ih postavlja u krug i opet broji, da bi konačno formiralo pojam broja 5). Ovo iskustvo, dakle, nadilazi spoznaju fizičkih svojstava predmeta. Spoznavanje nastalo na osnovi ove vrste iskustva je konstrukcija mišljenja i ono je uvjetovano složenom intelektualnom aktivnošću. Ovo istraživanje predstavlja prilog tezi o nedostižnosti konstrukcija mišljenja kod djece s BIT, kao što je napominjala Barbel Inhelder u svojim istraživanjima (Piaget i Inhelder, 1996), o čemu smo

već govorili u uvodnom dijelu rada. Rezultati našeg istraživanja ukazuju da su djeca s BIT zaustavljena u svom razvoju na razini konkretnih operacija. Čak i u ovom stadiju su prisutne nedograđene konstrukcije operativnosti mišljenja jer oko 15% djece nije osvojilo konzervaciju duljine i mase, gotovo polovica konzervaciju volumena, dok se čak 60% ispitanika oslanja na vizualnu korespondenciju rješavajući zadatak konzervacije broja. Također, operativna konzervacija duljine se u najvećem postotku ovladava temeljem identiteta, dok se operativna konzervacija mase u najmanjem postotku ovladava ovom kategorijom odgovora. Konzervacija mase se podjednako osvaja empirijskom reverzibilnošću i kompenzacijom relacija. Operativna konzervacija volumena se u najvećem postotku ovladava temeljem empirijske reverzibilnosti, potom kompenzacijom relacija, te identitetom.

Ovakvi podaci nas navode na zaključak, da je operacijska razina konzervacije duljine najprije osvojiva kod djece s BIT u našem uzorku jer su u najvećem postotku zastupljeni odgovori temeljeni na identitetu. Napomenuli smo već, da je upadljivo svojstvo identiteta da on stječe vrijednost dokaza za konzervaciju onda kada su druge vrste dokaza već otkrivene, kao što su empirijska reverzibilnost i kompenzacija relacija. Slične podatke navodi i Inhelder kod osvojenosti konzervacije duljine (Piaget i Inhelder, 1996). S druge strane, osvojenost konzervacije mase u našem uzorku obuhvaća pretežno istančanija reverzibilnost, koja još ne podrazumijeva identitet. Konzervacija volumena je uglavnom temeljena na objašnjenju proste reverzibilnosti. Ovi podaci se slažu s nalazima istraživanja Inhelderove, koja napominje da je operacijska razina konzervacije volumena prisutna kod graničnih slučajeva i viših razina djece s BIT, ali nikad kod tipičnih kliničnih slika intelektualnih teškoća. Razloge ovakvih nalaza objašnjavamo nemogućnošću uvida u pojam proporcije koji je povezan s pojmom volumena. Pojam proporcije ovisi o formalnim operacijama.

Zaključili smo da u dobi za koju je karakteristična pojava formalnih operacija svega 12,5% ispitanika doseže operacijsku razinu. Ovakvi rezultati ukazuju da se složenije forme operacijskog funkcioniranja osvajaju sa zakašnjenjem u odnosu na specifičnost ovladavanja ovih sposobnosti u općoj populaciji djece.

I u radovima prethodnih autora u ovoj oblasti, istaknuta je pojava lažne ravnoteže, fiksacije i viskoznosti kognitivnog razvoja djece s BIT. Fiksacija ukazuje na usporenost i zaustavljanje u jednoj od razvojnih faza, a viskoznost je određena duljim trajanjem statusa tranzicije između dva stadija. Maćešić-Petrović (1996) je provela istraživanje u kojem je sudjelovalo 90-ero djece s BIT mlađe i starije školske dobi, čiji rezultati upućuju na zaključak o istovremenom prisustvu dvaju sustava što rezultira

prisustvom nižih kognitivnih operacija u vrijeme kada se očekuje pojava novih, složenijih.

Smatra se da nesposobnost djece da konzerviraju proistječe iz nesposobnosti decentriranja, kako iz neposredne opažajne situacije, tako i iz odnosa između jednog trenutka u vremenu i onog koji mu slijedi. Dijete se centrira na jednu odliku neposredne situacije, a zanemaruje ostale. Također se centrira na sadašnji trenutak, propuštajući se u mislima vratiti na ranije stanje stvari, te ne sagledava da je jedan čin u načelu reverzibilan (Donaldson, 1997; Einfeld i sur., 2007).

Rezultati našeg istraživanja pokazuju da samo ispitanici V razreda postižu oko 75% savladanosti gradiva matematike. Ostali ispitanici postižu oko 50% ukupne savladanosti gradiva. Dobiveni rezultati navode nas na postavljanje pitanja o prilagođenosti programskih zahtjeva razvojnim sposobnostima djece s BIT. Smatramo da bi izvjesno prilagođavanje programa matematike, koje bi bilo formulirano prema kognitivnim sposobnostima djece s BIT i dopunjeno permanentnim provođenjem reedukativnog tretmana, doprinijelo uspješnijem savladavanju matematike.

Zaključili smo da su djeca, koja nisu ovladala zadacima konzervacije, rješavala matematičke zadatke intuitivnim i perceptivnim sredstvima te da su se prilagođavala programskim zahtjevima stvaranjem pasivnih i automatiziranih obrazaca obrade matematičkih informacija, čiju suštinu ne razumiju, a da su djeca koja su uspješno rješavala zadatke konzervacije mogla shvatiti svojstva prirodnog niza brojeva i pravo značenje aritmetičkih operacija.

Bruce i Muhammad (2009) ukazuju na značaj direktnih i sistematskih instrukcija vezanih za poticaje operacija konkretnog mišljenja kod djece s BIT, u svrhu uspješnijeg rješavanja matematičkih zadataka. Nakon detaljne procjene operativnosti mišljenja kod djece ove populacije, oni preporučuju vježbanje vizualne i diferencirane pažnje, razmatranje karakteristika svih objekata koji su primijenjeni u matematičkom zadatku, detaljno demonstriranje procedure svakog zahtjeva zadatka i ponovno imenovanje objekata i akcija.

ZAKLJUČAK

Rezultati ovog istraživanja potvrđuju kašnjenje u osvajaju logičkih operacija kod djece s BIT u odnosu na dob i razinu edukacije, kao i nedovoljnu savladanost gradiva matematike u starijoj školskoj dobi. U cilju adekvatnog poticanja misaonog funkcioniranja i adekvatnijeg savladavanja programa matematike predlažemo provođenje teorije Piageta, koja zastupa

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stav o primjeni igara kojima se potiču reverzibilnost, identitet i konzervacija te kojima se matematičko mišljenje čini gipkijim, aktivnijim, širim, dubljim i originalnijim (Piaget, 1994). Igre trebaju biti u skladu sa mentalnim funkcioniranjem djeteta i biti orjentirane ka spontanim procesima učenja i razvoja. Ovakve igre bi mogle biti fokusirane na složene mentalne operacije koje se odnose na dodavanje, oduzimanje, količinu, broj, pojmove manje od, veće od... (Japundža-Milisavljević, 2008). Po Piagetu, djeca kroz igru učinkovitije uče sadržaje nego kroz aktivnosti nastave. Igra predstavlja odličan pokretač i vodič u učenju. Dijete kroz igru dolazi do novih saznanja vlastitim iskustvom. Za razvijanje sposobnosti mišljenja i zaključivanja vrlo su važne didaktičke igre, igre mašte i uloga, igre sa gotovim pravilima. Različiti oblici igre navedeni su kao značajni, između ostalih, u razvoju konzervacija, operacijskih korespondencija, formiranju matematičkih pojmova, rješavanju problema (Demetriou i sur., 2010).

Jukić (2007) zanimljivo predlaže uporabu domino pločica, kao sredstva za savladavanje raznih matematičkih pojmova i razvoj mišljenja kod djece.

Treba istaći da je ovakav stav ključan za primjenu matematičkih implikacija kod djece s BIT, počevši od najranije školske dobi, te prenoseći isti modus i u starijoj školskoj dobi. Metodičke implikacije tog saznanja na razvoj matematičkih pojmova reflektiraju se u zahtjevu da se matematički pojmovi ne mogu „prenositi“ djeci, nego da ih dijete izgrađuje i konstruira u brojnim kontaktima s predmetima, objektima i pojavama iz neposredne okoline dovodeći ih u veze i odnose na mentalnom planu (Taub i sur., 2008; Dejić, 2008; Klasnić, 2009). Zbog toga je neposredna okolina nezamjenjiva u procesu razvoja logičko-matematičkih struktura kod djece s BIT.

Suvremeni autori ističu važnost uvažavanja životnog iskustva djeteta, što predstavlja jednu od važnih metodičkih smjernica. Pod tim podrazumijevaju potrebu da se pri izboru sadržaja rukovodi situacijama i događajima iz dječjeg svakodnevnog života, tj. da matematički sadržaji budu povezani s iskustvom i stvarnim problemima s kojima se dijete susreće. To je sukladno situacijskom učenju ili tzv. iskustvenoj matematici. Takva matematika nije samo interesantnija djeci, nego je i logičnija i čini da se matematički zadaci rješavaju sa zadovoljstvom. Zato je potrebno da edukator poznaje dječje sposobnosti i da na njima gradi razvoj matematičkih pojmova od najranije školske dobi, obavezno prateći i procjenjujući dječje mogućnosti tokom daljnjeg razvoja.